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MHD Double-Diffusive and Viscous Dissipative Boundary Layer Flow over a Vertical Plate with Heat Source, Reacting Species, and Thermal and Mass Transfer Gradients

Okuyade Ighoroje Wilson Ata[1, *], Mebine Promise[2]

[1]Department of Mathematics/Statistics, Federal Polytechnic of Oil and Gas, Bonny Island, Nigeria

[2]Department of Mathematics/Computer Science, Niger Delta University, Wilberforce Island, Nigeria

Email address:

wiaokuyade@gmail.com (Okuyade Ighoroje Wilson Ata), p.mebine@yahoo.com (Mebine Promise)

[*]Corresponding author

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Abstract: Fluid flow problems with convective boundary conditions have applications in the science and engineering worlds. Specifically, they are relevant in the heating and cooling processes observed in glass fiber production, and aerodynamic extrusion. This paper investigates the problem of steady MHD double-diffusive, viscous dissipative boundary layer flow over a vertical plate with heat source, reacting species, and thermal and mass transfer gradients effects. Usually, the problem of flow through porous media is examined using the Boussinesq's approximations. The governing nonlinear partial differential equations are coupled and complex. Making them tractable, they are linearized into a set of ordinary differential equations using the similarity transform. The evolving set of ordinary differential equations is solved numerically using the fifth-order Runge-Kutta Fehlberg Method and Maple 21 mathematical computational software. The results obtained for the concentration, temperature, and velocity are presented graphically. The analysis of results shows, amongst others, that an increase in the magnetic field parameter increases the temperature and concentration, but decreases the velocity of the fluid; an increase in the Biot number increases the temperature, concentration, and velocity of the fluid; an increases in the concentration difference parameter increases the temperature, but decreases the concentration and velocity of the fluid; an increase in the Eckert number increases the concentration, but decreases the temperature and velocity of the fluid.

Keywords: Double-Diffusion, Heat Source, MHD, Reacting Species, Thermal/Mass Transfer Gradients, Viscous Dissipation

1. Introduction

Flow with Convective boundary conditions has industrial applications in glass fiber production, condensation in plate cooling baths, glass, and aerodynamic extrusion.

In heat transfer problems, the convective boundary condition (Newtonian boundary condition) corresponds to the convective heating (or cooling) at the surface (of an object/plate) and is obtained from the surface energy balance. For its applications, convective flow problems have attracted the attention of many research workers. Specifically, convective flow over moving vertical plates with convective boundary conditions has been considered from different perspectives. For example, Aziz [1] studied laminar thermal boundary layer flow using the method of similarity transformation; Ajadi et al [2] examined the boundary layer flow in the presence of slip velocity; Makinde [3] studied numerically the mixed convective flow using similarity solution. Makinde [4] looked at the heat and mass effects on the MHD fluid flow; Makinde and Olarewaju [5] considered the thermal boundary layer flow; Makinde [6] studied the effects of internal heat generation and convective boundary condition on the flow using similarity transformation method of solution; Makinde [7] considered the MHD mixed convective nth-order chemically reactive flow; Gangadhar et al [8] gave the analysis of the flow using similarity transformation. Rout et al [9] investigated the steady MHD chemically reacting flow with heat source using similarity transformation and fourth-order Runge-Kulta numerical approaches; Abbasi et al [10] examined the hydrodynamic and thermal boundary layers flow using an analytic approach; Emmanuel et al [11] worked on the effects of thermal radiation and viscous dissipation on the flow. Etwire ans Seini [12] investigated the flow using a numerical approach, and noticed among others that an increase in Eckert number increases the skin friction, plate surface temperature, thermal and velocity boundary layer but decreases the local Nusselt number; an increase in the Biot number increases the plate temperature and local Nusselt number; increase in the Hartmann number increases the wall shear stress, plate surface temperature, thermal and velocity boundary layer thicknesses but decreases the local Nusselt number; Imoro et al [13] looked at the flow in the presence of viscous dissipation and nth-order chemical reaction; Shateyi [14] analyzed the effects of viscous dissipation on the heat and mass transfer flow.

Furthermore, convective flow over moving vertical plates with constant boundary conditions has been considered from different perspectives. Some research reports are found in Hossain and Takhar [15], Rahman and Sattar [16], Ibrahim et al [17], Rajeswari et al [18], Makinde and Ogulu [19], Mohammed and Abo-Dahab [20], Makinde and Ogulu [21], Pal and Talukdar [22], Narayana et al [23], Parida et al. [24], Rajput and Kumar [25], Yazdt et al [26], Khan et al. [27], Devi et al. [28], Umamaheswar et al [29], Okuyade et al [30], Othman and Mahdy [31], Kharabela et al [32], Okuyade and Okor [33], and Okuyade and Okor [34].

Upon the above research reports, this paper aims at studying the steady MHD double-diffusive and viscous dissipative boundary layer flow over a porous vertical plate with heat source, reacting species, thermal and mass transfer gradients numerically using similarity transformation and the fifth-order Runge-Kutta Fehlberg numerical method.

This paper is presented in the following format: section 2 holds the materials and method; section 3 gives the results; section 4 the discussion, and section 5 gives the conclusion.

2. Materials and Methods

2.1. Physics of the Problem and Mathematical Formulation

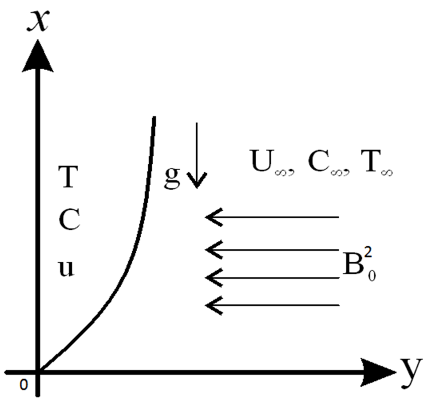

Figure 1. *The model of a vertically accelerating plate in a fluid.*

We investigate a steady 2-D flow of an electrically conducting, double-diffusive, and viscous dissipative boundary layer flow over a porous vertical plate with heat source, reacting species, and thermal and mass transfer gradients effects. The model is formulated on the following assumptions: the fluid is electrically conducting and chemically reactive, and heated at the bottom such that the hot fluid layer at the bottom heats the plate from the bottom to the upper surface to generate convection; the physical properties of the fluid such as the specific heat at constant pressure, thermal and mass conductivities, and density remain constant throughout the fluid; the fluid is mixed with a chemical species at a higher concentration to initiate a chemical reaction; the plate is porous and Darcian; a magnetic field of uniform strength and negligible induction effect is applied transverse to the plate; the plate is not heated to a high-temperature regime, and so thermal radiation is absent. In the model, the x-axis is taken to be in the vertical direction of the plate and the y-axis is normal to it. Now, if (u,v) are the velocity components in the (x,y) coordinates; T_∞ and C_∞ are the fluid temperature and concentration at equilibrium; C and T are the fluid and temperature; then, by the Boussinesq's approximations, the equations of mass balance, momentum, energy, and diffusion governing the flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\rho g\beta_t\left(T-T_\infty\right)$$

$$+\rho g\beta_c\left(C-C_\infty\right)-\left(\frac{\sigma_e {B_o}^2}{\mu\mu_m}-\frac{\mu}{\kappa}\right)u \tag{2}$$

$$\rho C_p\left(u\frac{dT}{dx}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_r\left(C-C_\infty\right) \tag{4}$$

where μ is the dynamic viscosity; ρ is the density; g is the acceleration due to gravity; β_t is the coefficient of volumetric expansion due to temperature; β_C is the coefficient of volumetric expansion due to concentration; k is the thermal conductivity of the fluid; μ_m is the magnetic field permittivity; Q is the heat source; σ_e is the electrical conductivity of the fluid; B_o^2 is the magnetic field flux; C_p is the specific heat capacity at constant pressure; D is the coefficient of mass transfer/diffusivity, κ is the permittivity of the porous plate; k_r is the chemical reaction term for the species concentration.

2.2. Boundary Conditions

As the fluid bearing the vertical plate is heated at the bottom, the hot layer at the bottom with the temperature T_f heats up the plate from the bottom to the top and generates convection that gives a heat transfer coefficient such that $T_f > T_\infty$ for heating, and $T_f < T_\infty$ for cooling (Aziz, 2009). Significantly, convection enters the flow system in two ways: the heated fluid moving from bottom to top, and the heated plate influence on the fluid. On one hand, the heated fluid particles at the bottom, being buoyant, rise to the top, and are replaced by the cold and denser ones from the top such that convection current is generated in the fluid. On the other hand, being heated at the bottom, the ions on the surface of the plate absorb heat through conduction, and the plate is heated to the top. The heated plate in contact with the moving fluid also generates convective current in the fluid. Now, to generate the boundary conditions at the plate/wall, the convection through the heated plate is emphasized. By this, there is a convective temperature gradient between the bottom and upper surface of the plate with a heat source at the bottom and sink at the top. This heating phenomenon applies to mass diffusion; hence a heat and mass transfer relation exists for a particular geometry wherein the result obtained for convective heat transfer can be interchangeably used for convective mass transfer. Similarly, we have both heat and mass transfer Biot numbers (Bi) that determine the thermal or mass conductivity of a surface [35]. Additionally, since the plate is accelerating in the moving fluid, its velocity is the same as that of the fluid. Therefore, the velocity situation of the fluid is applicable to that of the plate. Upon these, the boundary conditions are

$$u(x,0) = \gamma U_o(x), v(x,0) = 0,$$

$$-k\frac{\partial T}{\partial y}(x,0) = h_f(T_f - T(x,0)),$$

$$-D\frac{\partial C}{\partial y}(x,0) = h_m\left(C_f - C(x,0)\right) \text{ at } y = 0 \tag{5}$$

$$u(x,\infty) = 0, v(x,\infty) = 0, T(x,\infty) \to T_\infty,\ C(x,\infty) \to C_\infty$$

$$at\ \ y \to \infty \tag{6}$$

where is $U_o(x)$ the characteristic velocity of the fluid/plate; $\gamma = 0$ for static plate; $\gamma = 1$ for accelerating plate (Aziz, 2009).

We introduce the following non-dimensionalized quantities

$$\Theta = \frac{T - T_\infty}{T_w - T_\infty},\ \Phi = \frac{C - C_\infty}{C_w - C_\infty},\ Gr_x = \frac{g\beta_t(T_w - T_\infty)x}{U_o^2},$$

$$Gc_x = \frac{g\beta_c(C_w - C_\infty)x}{U_o^2},\ M_x = \frac{\sigma_e B_o^2 \upsilon x}{\mu_m U_o^2},\ \chi_x = \frac{\upsilon}{\kappa},$$

$$\Pr_x = \frac{\upsilon}{k},\ Sc_x = \frac{\upsilon}{D},\ \delta_x = \frac{k_r x}{D}, H_x = \frac{Qx}{\rho k C_p U_o},$$

$$Ec_x = \frac{U_o^2}{kC_p(T_f - T_\infty)},\ Bi_{tx} = \frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_o}},\ Bi_{cx} = \frac{h_m}{D}\sqrt{\frac{\upsilon x}{U_o}},$$

$$N_c = \frac{C_\infty}{C_w - C_\infty} \tag{7}$$

where Θ and Φ are the non-dimensionalized temperature and concentration, respectively; H_x is the heat generation/absorption parameter; M_x is the magnetic field parameter; Ec_x is the Eckert number; Gr_x is the Grashof number due to temperature difference; Gc_x is the Grashof number due to concentration difference; χ_x is the porosity parameter; $\Pr_x$ is the Prandtl number; Sc_x is the Schmidt number; δ_x is the chemical reaction rate, N_c is the concentration difference parameter; Bi_{tx} is the convective heat transfer coefficient; Bi_{cx} is the convective mass transfer coefficient; h_f and h_m are the heat and mass transfer coefficient, respectively, which depend on the physical properties of the fluid, and physical situations.

Furthermore, studies have shown that the local parameters are functions of x. Therefore, there is the need to generate local similarity transforms to make them independent of x. We assumed

$$h_f = \frac{c}{\sqrt{x}},\ \sigma_e = \frac{d}{x},\ \beta_t = \frac{e}{x},\ \beta_c = \frac{h}{x},\ Q = \frac{m}{x},\ \delta = \frac{n}{x},$$

$$\chi = \frac{p}{x}. \tag{8}$$

where c, d, e, h, m, n and p are constant with appropriate dimensions.

Substituting equations (7) and (8) consecutively into equations (1)-(6), we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M_1 u + Gr\Theta + Gc\Phi \tag{10}$$

$$u\frac{\partial \Theta}{\partial x} + v\frac{\partial \Theta}{\partial y} = \frac{\partial^2 \Theta}{\partial y^2} + H\Pr\Theta + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{11}$$

$$u\frac{\partial \Phi}{\partial x} + v\frac{\partial \Phi}{\partial y} = \frac{\partial^2 \Phi}{\partial y^2} + Sc\delta\left(\Phi + N_c\right) \tag{12}$$

with the boundary conditions

$$u = 1, v = 0,\ \Theta' = Bi_t\left(\Theta(0) - 1\right),\ \Phi' = Bi_c\left(\Phi(0) - 1\right)$$

$$\text{at } y = 0 \tag{13}$$

$$u = 0, v = 0,\ \Theta = 0,\ \Phi = 0 \text{ at } y = \infty \tag{14}$$

where $M_1 = M + \chi$

Equations (10) - (12) are coupled. We linearize them using the similarity transformation

$$\Psi(x, y) = \sqrt{U_o \upsilon x} f(\eta),\ \eta(x, y) = y\sqrt{U_o / \upsilon x} \tag{15}$$

$$u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \Psi}{\partial x}$$

such that

$$u = \frac{\partial \Psi}{\partial y} = f'(\eta), v = \frac{1}{2}\left(\frac{U_o \upsilon}{x}\right)^{1/2} [\eta f'(\eta) - f(\eta)]$$

where Ψ stream function, η similarity independent variable, f velocity in the similarity transformed form.

Now, by equation (15), equations (10)-(14) become

$$f''' + \frac{1}{2} ff'' - Mf' = -Gr\Theta - Gc\Phi \tag{16}$$

$$\Theta'' + \frac{1}{2} \Pr f\Theta' + H \Pr \Theta = Ec\left(f''\right)^2 \tag{17}$$

$$\Phi'' + \frac{1}{2} Scf\Phi' - Sc\delta\Phi = Sc\delta N_c \tag{18}$$

with the boundary conditions

$$f = 0, f' = 1, \Theta' = Bi\left(\Theta(0) - 1\right),\ \Phi' = Bi\left(\Phi(0) - 1\right)$$

$$\text{at } \eta = 0 \tag{19}$$

$$f' = 0, \Theta = 0, \Phi = 0 \text{ at } \eta = \infty \tag{20}$$

Equation (9) is satisfied by the stream function.

Other factors influencing the flow are the Nusselt number (or thermal conductivity of the fluid), Sherwood number (or species conductivity of the fluid), and wall shear stress (or the force the fluid exerts on the wall), and these are prescribed non-dimensionally as

$$Nu = -\Theta'(\eta)\big|_{\eta=0} \tag{21}$$

$$Sh = -\Phi'(\eta)\big|_{\eta=0} \tag{22}$$

$$Cf = f''(\eta)\big|_{\eta=0} \tag{23}$$

2.3. Method of Solution

Equations (16) - (20) are solved numerically using the fifth-order Runge-Kutta Fehlberg method and Maple 21 computational software.

3. Results

The solutions of the concentration, temperature, and velocity obtained are computed and presented graphically. The effects of the magnetic field, chemical reaction rate, concentration difference ratio, heat source/sink parameters, and Schmidt, Grashof, and Biot numbers are investigated. For physically realistic constant values of $\chi = 0.1$, $Gc = 1.0$, $\Pr = 0.71$ and varied values of $M, \delta, Nc, H, Sc, Gr, Bi$, we obtained the results shown in Figure 2 – Figure 25.

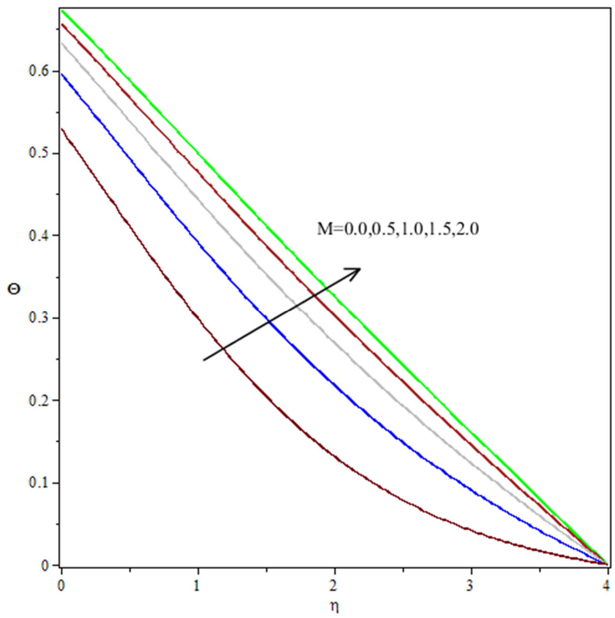

Figure 2. *Temperature-Magnetic Field (M) Parameter Profiles.*

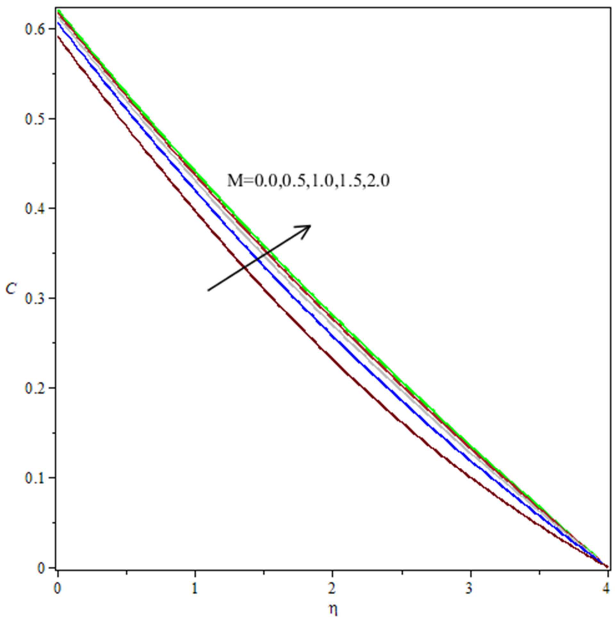

Figure 3. *Concentration-Magnetic Field (M) Parameter Profiles.*

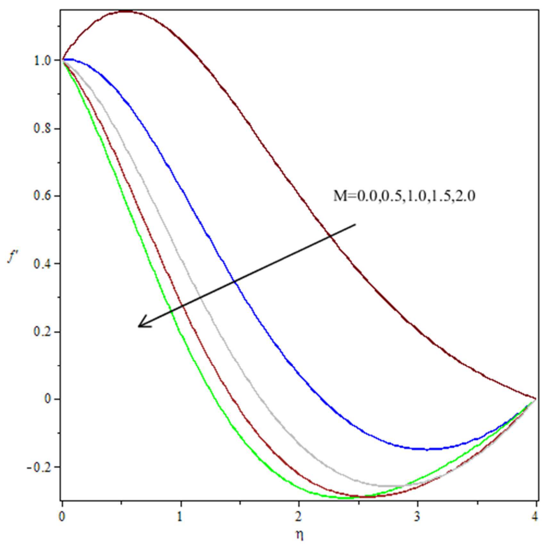

Figure 4. *Velocity-Magnetic Field (M) Parameter Profiles.*

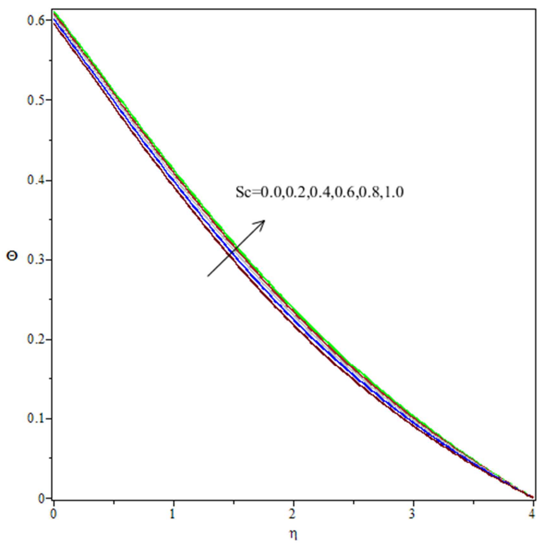

Figure 5. *Temperature-Schmidt Number (Sc) Profiles.*

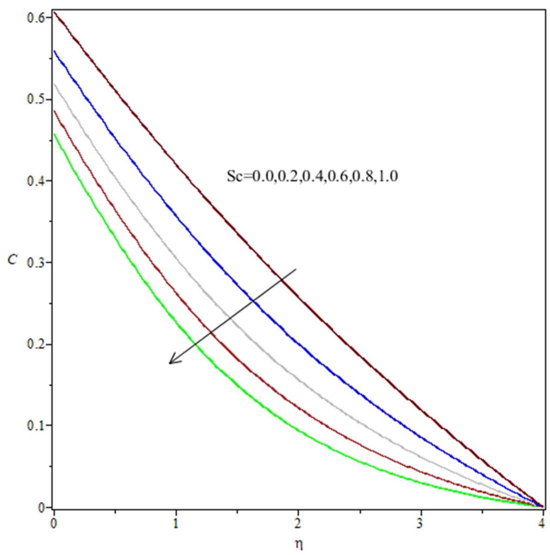

Figure 6. *Concentration- Schmidt Number (Sc) Profiles.*

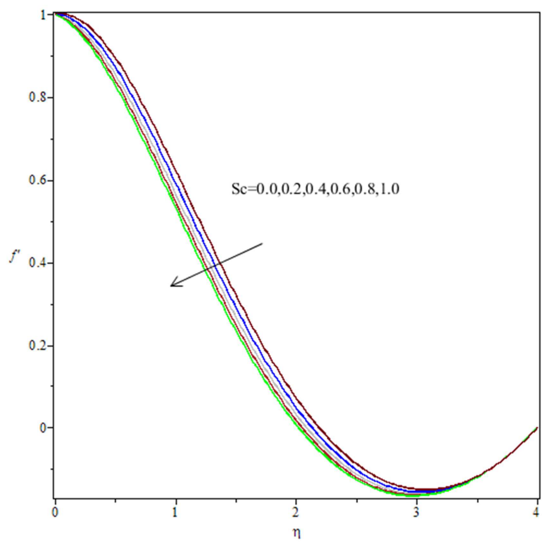

Figure 7. *Velocity- Schmidt Number (Sc) Profiles.*

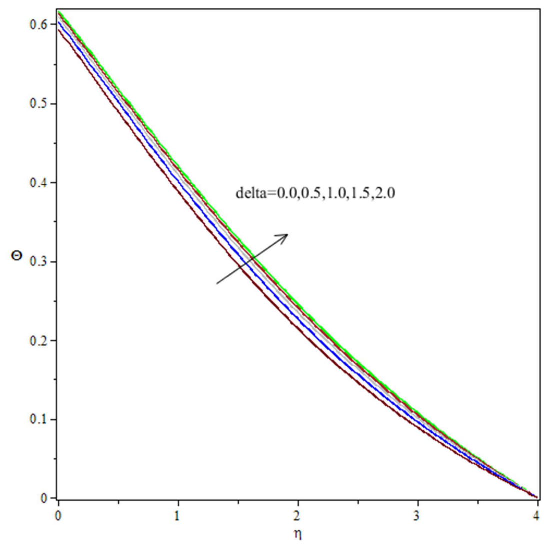

Figure 8. *Temperature- Chemical Reaction Rate (delta) Parameter Profiles.*

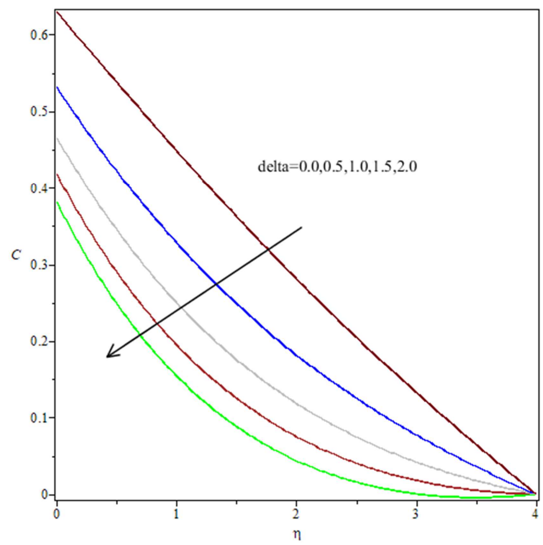

Figure 9. *Concentration- Chemical Reaction Rate (delta) Parameter Profiles.*

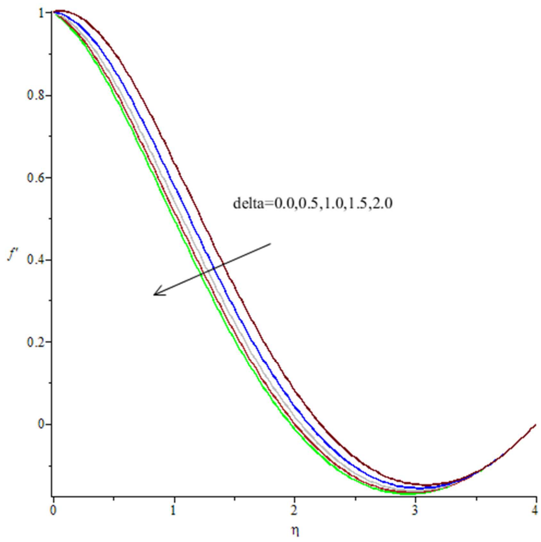

Figure 10. Velocity-Chemical Reaction Rate (delta) Parameter Profiles.

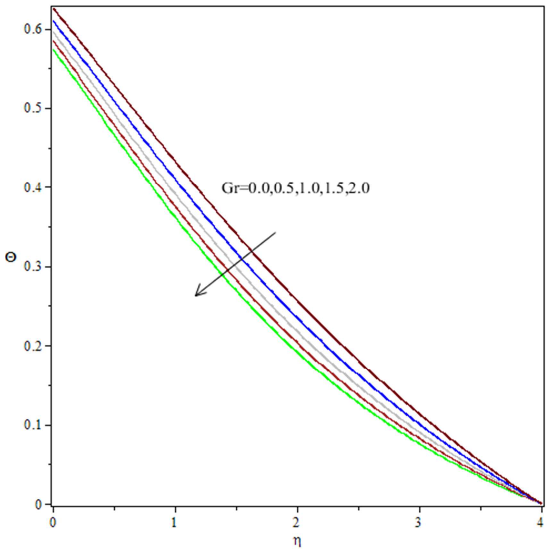

Figure 11. Temperature- Grashof Number (Gr) Profiles.

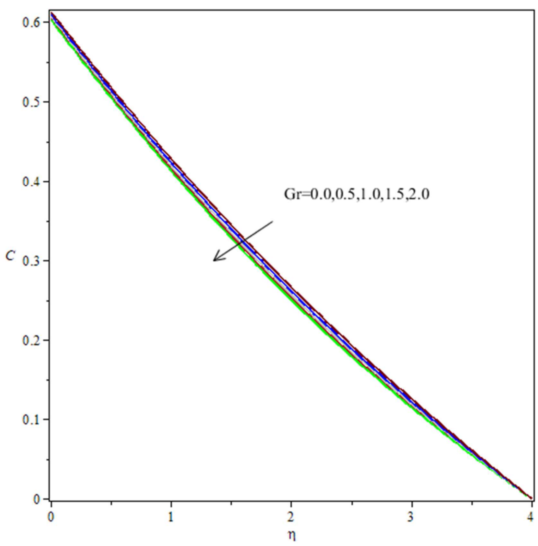

Figure 12. Concentration- Grashof Number (Gr) Profiles.

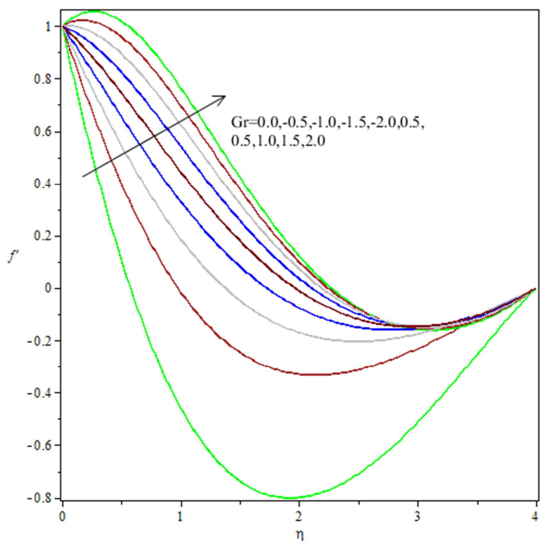

Figure 13. Velocity-Grashof Number (Gr) Profiles.

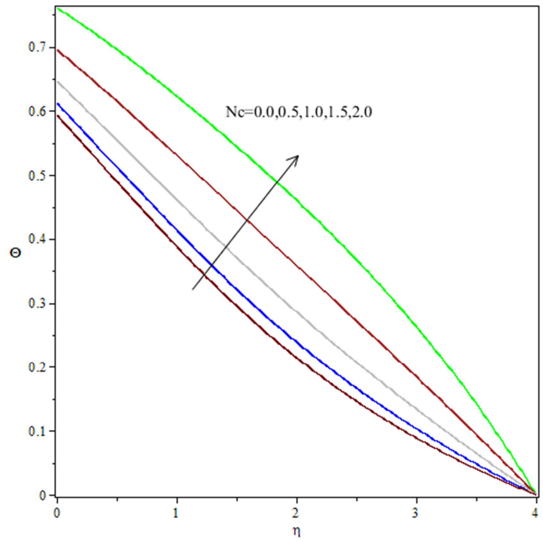

Figure 14. Temperature- Concentration Difference (Nc) Parameter Profiles.

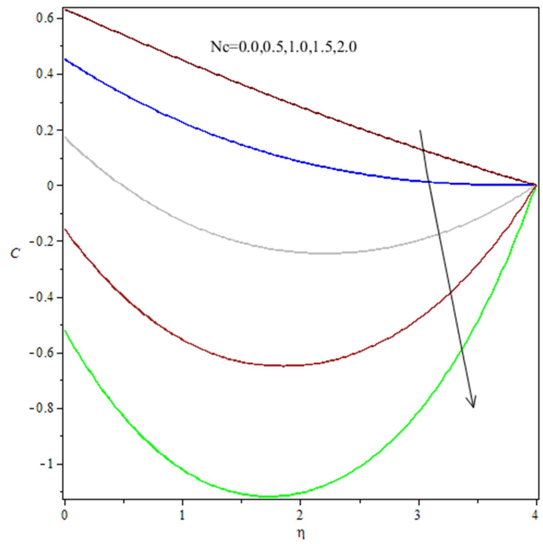

Figure 15. Concentration- Concentration Difference (Nc) Parameter Profiles.

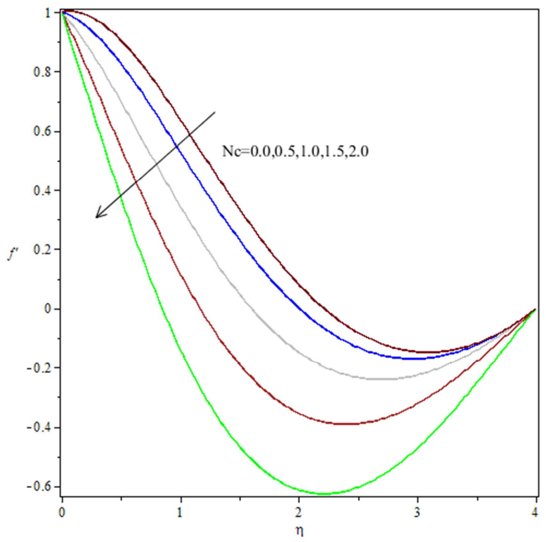

Figure 16. *Velocity-Concentration Difference (Nc) Parameter Profiles.*

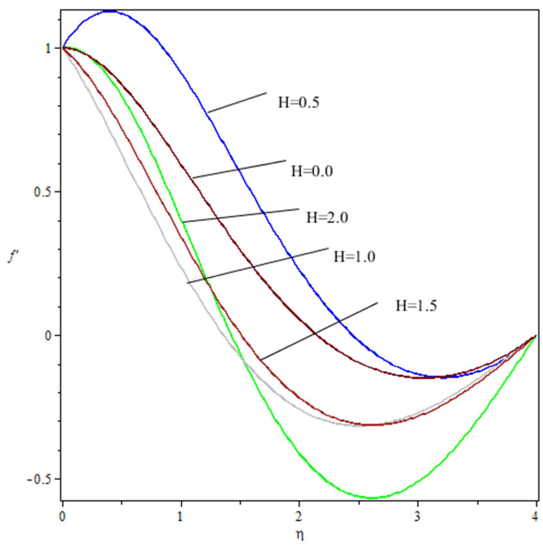

Figure 19. *Velocity- Heat Source/Sink (H) Parameter Profiles.*

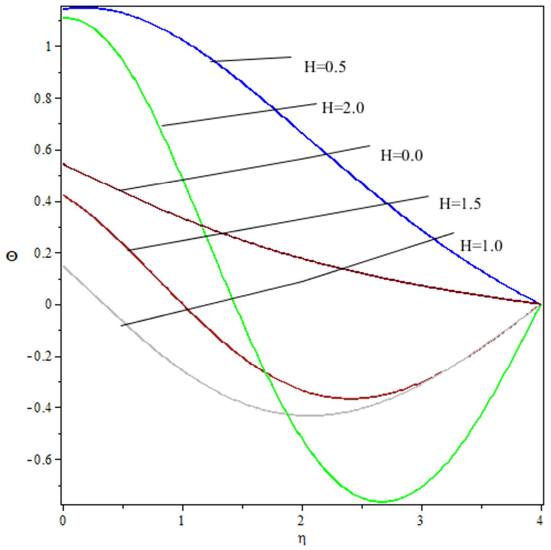

Figure 17. *Temperature- Heat Source/Sink (H) Parameter Profiles.*

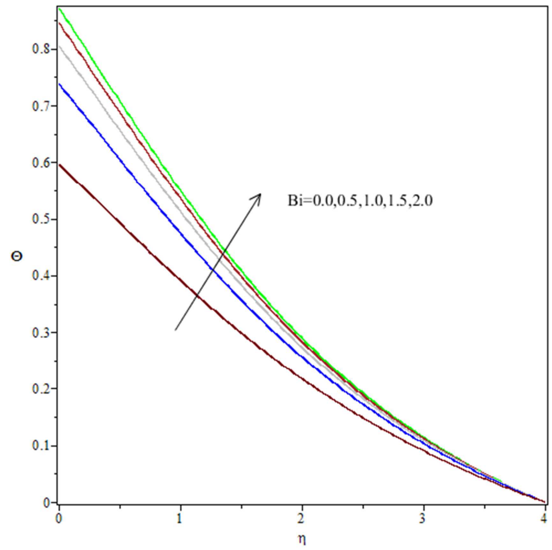

Figure 20. *Temperature- Biot Number (Bi) Profiles.*

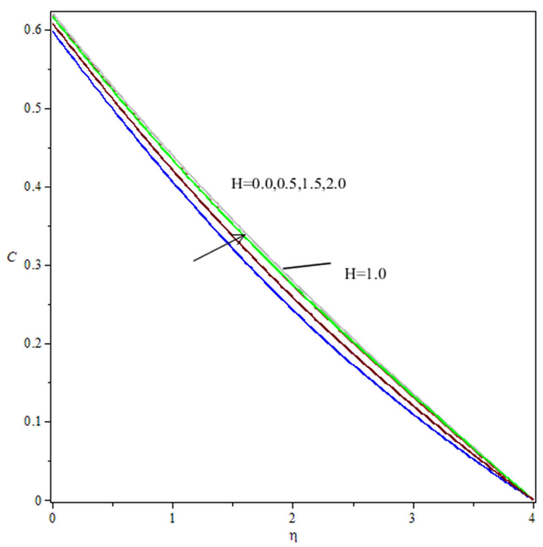

Figure 18. *Concentration-Heat Source/Sink (H) Parameter Profiles.*

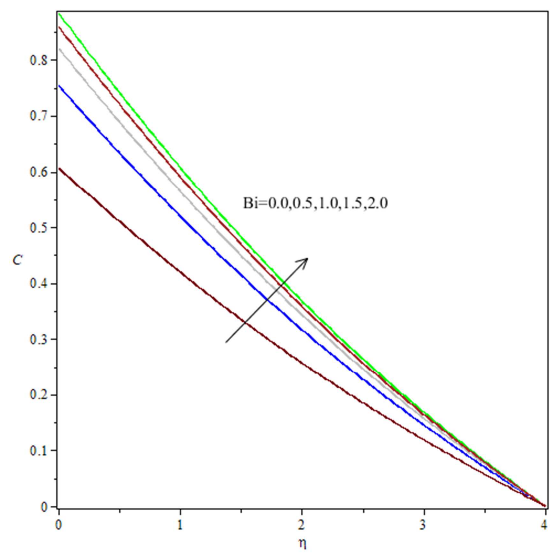

Figure 21. *Concentration- Biot Number (Bi) Profiles.*

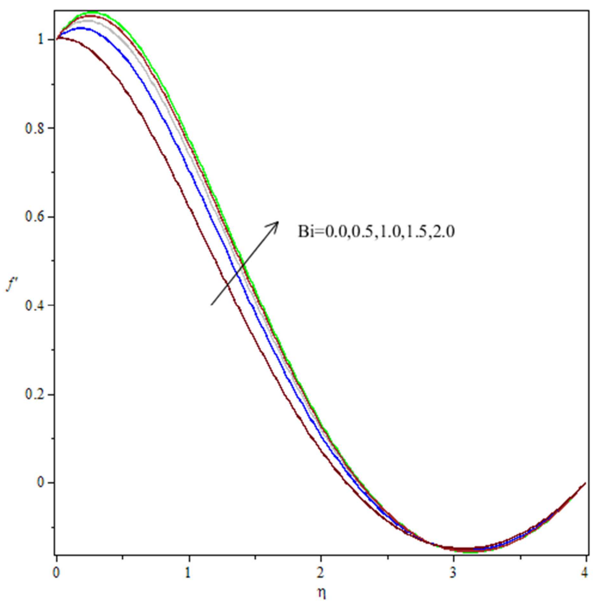

Figure 22. *Velocity-Biot Number (Bi) Profiles.*

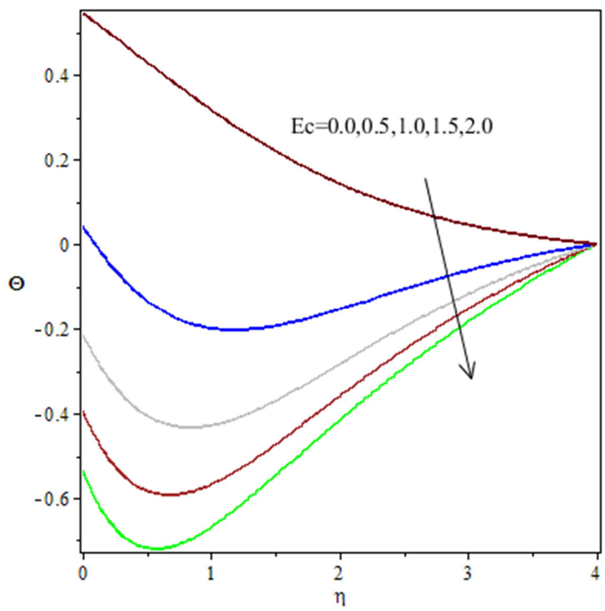

Figure 23. *Temperature-Eckert Number (Ec) Profiles.*

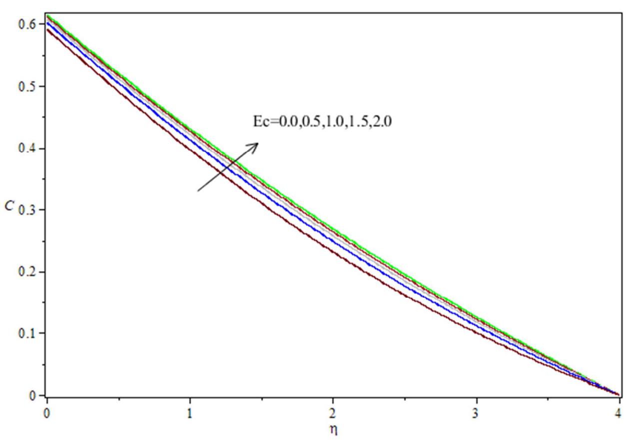

Figure 24. *Concentration-Eckert Number (Ec) Profiles.*

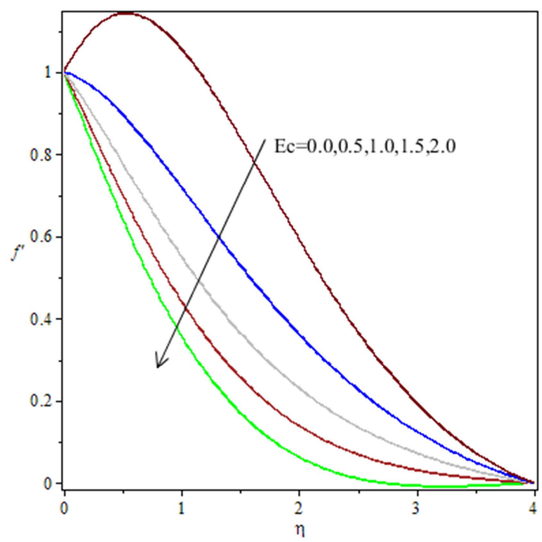

Figure 25. *Velocity-Eckert Number (Ec) Profiles.*

4. Discussion

The effects of the magnetic field parameter on the flow are shown in Figure 2 - Figure 4. They depict that an increase in the magnetic field parameter increases the temperature and concentration but decreases the flow velocity. As the fluid is chemically reactive, its particles exist as electric charges or ions. In the presence of an applied magnetic field, they generate electric current. And, the interaction of the electric current with the magnetic field produces the Lorentz force $F = j \wedge B_O$ (a mechanical force whose resistive effect has the potency of freezing up the flow velocity), where j the electric current density is induced by Faraday law, and B_O is the magnetic induction. This accounts for what is seen in Figure 4. Similarly, since temperature and concentration are velocity-dependent, Figure 2 and Figure 3 are supposed to decrease as a follow-through. Therefore, the increase in the temperature and concentration might have been influenced by some other factors in the presence of the magnetic field parameter.

More so, the effects of the Schmidt number on the flow are shown in Figure 5 - Figure 7. They show that an increase in the Schmidt number decreases the temperature, concentration, and flow velocity. The Schmidt number shows the ratio of the interaction of the momentum diffusivity with the mass diffusivity of the fluid, and this varies from one fluid to another. Importantly, the mass diffusivity/diffusion coefficient, which is a function of the fluid concentration, size of the molecules (large molecules diffuse slowly), temperature, pore structures, and the rate of mixing with water, has inverse effects. The inverse effect produces a decreasing effect on the concentration. Schmidt number is similar to the Prandtl number in heat diffusion. These account for what is seen in Figure 5 and Figure 6. Similarly the velocity, as a function of temperature and concentration decreases in the same suit; thus accounting for the result in Figure 7.

Also, the effects of the chemical reaction parameter on the flow are shown in Figure 8 - Figure 10. They show that an increase in the rate of chemical reaction parameter increases the temperature but decreases the concentration and velocity. Normally, in a chemically reacting system, species are created or consumed; heat is generated in exothermic reactions and consumed in endothermic reactions. The creation or consumption of the species may strengthen or weaken the concentration of the fluid system. More so, the rate of the chemical reaction is dependent on some factors: concentration strength, nature of the reactants, presence/absence of heat, etc. The chemical reaction rate may be slow, abrupt and destructive, or normal. Therefore, the rate of a chemical reaction determines the level of the fluid particles interaction. Ordinarily, with an increased rate of chemical reaction, heat may be generated, showing an exothermic reaction, as in Figure 8. Also, in a chemical reaction, the original species/reactants are consumed to produce new ones, and this may reduce the concentration of the mixing fluids; thus structures like Figure 9 may be seen. Similarly, velocity as a function of temperature and concentration depends on both; thus the decrease in the velocity, as seen in Figure 10 may be due to the decrease in the concentration.

Furthermore, the effects of Grashof number on the flow are seen in Figure 11 - Figure 13. They show that an increase in the Grashof number decreases the temperature and concentration but increases the velocity. Being a heat generating problem, it is assumed that the temperature at the plate is higher than that of the fluid at equilibrium. Therefore, heat is absorbed into the fluid from the external/wall. The absorbed external heat ought to increase the fluid temperature, while that at the wall decreases. So, the decrease in temperature may be due to other factors. Specifically, the presence of temperature or concentration differential and volumetric expansion in a fluid flow system reduces the fluid viscosity, thus enhancing the interaction of the fluid particles. This accounts for the velocity flow structure in Figure 13.

The effects of the concentration difference parameter on the flow are seen in Figure 14 - Figure 16. They show that an increase in the concentration difference parameter increases the temperature but decreases the concentration and velocity.

Similarly, the effects of the heat source parameter are shown in Figure 17 - Figure 19. They show that an increase in the heat source parameter introduces fluctuation in the flow structures. For example, in Figure 17, as the heat source increases the temperature is highest for $H = 0.5$ and lowest for $H = 1.0$; for Figure 18 the concentration structure is highest for $H = 1.0$, then increases within $H = 0.0, 0.5, 1.5, 2.0$, thus jumping $H = 1.0$. A heat source in a flow system could mean that heat is applied from an external source or generated by the system due to an exothermic reaction occurring therein, and this increases the temperature of the system. More so, for a sink wherein heat is removed or the system is endothermic the temperature of the system is reduced. The heat/ applied/generated or absorbed heat affects the fluid particles interaction with one another and their motion. The increase or decrease in the level of the particles interaction increases or decreases the fluid temperature and velocity. Therefore, the observed fluctuation in flow in the presence of heat source may be due to some other factors.

The effects of the Biot number on the flow are shown in Figure 20 - Figure 22. They show that an increase in the Biot number increases the temperature, concentration, and velocity of the flow system. The Biot number is the ratio of internal conductive resistance within the body to the external convective resistance at the surface of the body. It shows how conduction and convection heat transfer phenomena are related. Small values of the Biot number show that conduction dominates the heat transfer method, while high values indicate that convection dominates the heat transfer mechanism. The same phenomenon applies to mass diffusion. Both heat/mass conduction and convection in a fluid system energize the fluid particles, thus the temperature, concentration, and velocity are enhanced; as in Figure 20 - Figure 22.

Additionally, the effects of the Eckert number are shown in Figure 23 - Figure 25. They show that an increase in the Eckert number increases the concentration but decreases the temperature and velocity of the flow structures. As a measure of the kinetic energy of the flow to the enthalpy difference across the thermal boundary layer, the Eckert number characterizes the heat dissipation in high-speed flow. Specifically, the Eckert number determines the level of conversion of heat energy into mechanical energy, which may account for wasted heat. Though energy is conserved in principle, heat is reduced. This may account for the trends in Figure 23. The decrease in the temperature would cause a decrease in the velocity, as seen in Figure 25.

5. Conclusion

Steady MHD double-diffusive and viscous dissipative boundary layer flow over a vertical plate with heat source/sink, reacting species, and thermal and mass transfer gradients is considered, and wherein the effects of the magnetic field, chemical reaction rate, concentration difference, and heat source parameters; Grashof, Schmidt, Biot and Eckert numbers on the flow are investigated. The analyses of results show that an increase in the:

1. Magnetic field parameter increases the temperature and concentration, but decreases the velocity;
2. Schmidt number decreases the temperature, concentration, and velocity;
3. Chemical reaction parameter increases the temperature, but decreases the concentration and velocity;
4. Grashof number decreases the temperature and concentration, but increases the velocity;
5. Concentration difference parameter increases the temperature, but decreases the concentration and velocity;
6. Heat source parameter produces fluctuation in the temperature, concentration, and velocity structures;
7. Biot number increases the temperature, concentration, and velocity of the flow system;
8. Eckert number increases the concentration, but

decreases the temperature, and velocity structures.

Author Contributions

Author 1 formulated the problem, developed the mathematical model and discussed the results; Author 2 performed the computer programming. Both authors read through the manuscript and approved the final version.

Authors' Declaration of Interest

The authors have declared that there is no conflict of interest.

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