

Novel Energy Level Structure of Dirac Oscillator in Magnetic Field

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Abstract: We have presented an elegant high energy quantum problem, namely, the full Dirac oscillator under axial magnetic field with its full solution. We have found the energy spectrum which is rich and at the same time has a novel structure. The quantized energy levels show coupling of the oscillator frequency with the Larmor frequency in the 2D surface where the electrons under consideration follow a 2D oscillator. The axis in which magnetic field is pointed, the electrons follow a 1D oscillator. There is also coupling between spin and orbital motion and also a coupling between a resultant effect of orbital and spin motion with Larmor precession.

Keywords: (3+1) Dimensional Dirac Oscillator, Magnetic Field, Novel Energy Level Structure

1. Introduction

Moshinsky and Szczepaniak [1] have introduced an interesting interaction in the Dirac equation such that the Dirac Hamiltonian is linear in both momentum and the coordinates. The resulting Dirac equation is known as Dirac oscillator because it turns into a harmonic oscillator with strong spin-orbit coupling in the non-relativistic limit. The oscillator is achieved by introducing the coupling in the Dirac equation as [1]

$$i\hbar \frac{\partial \Psi}{\partial t} = [c\vec{\alpha} \cdot (\vec{p} - im\beta\omega\vec{r}) + \beta mc^2] \Psi,$$

where Ψ is the four component bispinor corresponding to spin-half particle, like electron, c is the speed of light, $\vec{\alpha}$ and β are standard representation of Dirac matrices, ω is the classical frequency of the oscillator. When $\omega = 0$, we recover the ordinary Dirac equation [2].

The Dirac oscillator plays a vital role in the depiction of relativistic many body problems [3-5] and super symmetric relativistic quantum mechanics [6-8]. Dirac oscillator framework is also proposed in quantum chromodynamics,

mainly in connection with quark confinement models in baryons and mesons [9]. Most thrilling aspects of this oscillator are its association to quantum optics [10]. It also maps into Anti-Jaynes-Cummings model for describing the interaction of a atomic transition in two level systems [11-13]. Moreover, a lot of attention and many physical applications in various branches of physics of this oscillator have been found, for example, in semiconductor physics [14], graphene [15] etc. An experimental demonstration of this oscillator has been initiated [16]. Equivalence has been found between the Dirac oscillator and a free particle in a rotating frame of reference [17]. Using previous results on the harmonic oscillator in a Snyder algebra, the Dirac oscillator with a minimal length is solved [18]. Recently, a tabletop implementation of a Dirac oscillator system based on a spin-orbit coupled ultracold atomic sample that allows for a direct observation of the corresponding analog of virtual pair creation on quantum measurement backaction has been proposed [19]. Last year, the propagator of a two dimensional Dirac oscillator in the presence of a uniform electric field is derived by a path integral technic [20].

In this context, we consider (3+1) dimensional Dirac oscillator in an external homogeneous magnetic field. We investigated mathematical formulation of the oscillator

problem and solved it. The quantized energy spectrum for up and down spin is derived. In the non-relativistic limit, this energy spectrum reduces to the spectrum of three dimensional harmonic oscillator with a spin-orbit contribution, when the magnetic field is absent. We present this problem and give a solution in section 2 and in section 3, we give a brief conclusion of the work.

2. Dirac Oscillator in Magnetic Field

In presence of an external magnetic field, the (3+1) dimensional Dirac oscillator is given by

$$\left[c\vec{\alpha} \cdot \left(\vec{p} + \frac{e}{c} \vec{A} - im\beta\omega\vec{r} \right) + \beta mc^2 \right] \psi = E\psi, \quad (1)$$

where \vec{A} is vector potential and $-e(e > 0)$ is the charge of the oscillator which is considered to be an electron here. We consider the uniform magnetic field to be along the z - direction and vector potential to be in the symmetric gauge as $\vec{A} = \left(-\frac{B}{2}y, \frac{B}{2}x, 0 \right)$, where B is the magnetic field strength.

Using the following standard form of $\vec{\alpha}$ and β :

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

and

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (2)$$

and two-component form for ψ as

$$\psi = \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix}, \quad (3)$$

we obtain the following two simultaneous coupled equations:

$$(c\vec{\sigma} \cdot \vec{p} + e\vec{\sigma} \cdot \vec{A}) \psi_b + imc\omega\vec{\sigma} \cdot \vec{r} \psi_b = (E - mc^2) \psi_a \quad (4)$$

and

$$(c\vec{\sigma} \cdot \vec{p} + e\vec{\sigma} \cdot \vec{A}) \psi_a - imc\omega\vec{\sigma} \cdot \vec{r} \psi_a = (E + mc^2) \psi_b. \quad (5)$$

The system gives the following two equations

$$\begin{aligned} [c^2(\vec{\sigma} \cdot \vec{p})^2 + ce(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{p}) + ce(\vec{\sigma} \cdot \vec{p})(\vec{\sigma} \cdot \vec{A}) + e^2(\vec{\sigma} \cdot \vec{A})^2 + imc^2\omega[(\vec{\sigma} \cdot \vec{r}), (\vec{\sigma} \cdot \vec{p})] \\ + imce\omega[(\vec{\sigma} \cdot \vec{r}), (\vec{\sigma} \cdot \vec{A})] + m^2c^2\omega^2(\vec{\sigma} \cdot \vec{r})^2] \psi_a = (E^2 - m^2c^4) \psi_a \end{aligned} \quad (6)$$

and

$$\begin{aligned} [c^2(\vec{\sigma} \cdot \vec{p})^2 + ce(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{p}) + ce(\vec{\sigma} \cdot \vec{p})(\vec{\sigma} \cdot \vec{A}) + e^2(\vec{\sigma} \cdot \vec{A})^2 - imc^2\omega[(\vec{\sigma} \cdot \vec{r}), (\vec{\sigma} \cdot \vec{p})] \\ - imce\omega[(\vec{\sigma} \cdot \vec{r}), (\vec{\sigma} \cdot \vec{A})] + m^2c^2\omega^2(\vec{\sigma} \cdot \vec{r})^2] \psi_b = (E^2 - m^2c^4) \psi_b. \end{aligned} \quad (7)$$

One can easily show that

$$(\vec{\sigma} \cdot \vec{p})^2 = p^2, \quad (8)$$

$$(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{p}) + (\vec{\sigma} \cdot \vec{p})(\vec{\sigma} \cdot \vec{A}) = BL_z + B\hbar\sigma_z, \quad (9)$$

$$(\vec{\sigma} \cdot \vec{A})^2 = A^2, \quad (10)$$

$$[(\vec{\sigma} \cdot \vec{r}), (\vec{\sigma} \cdot \vec{p})] = \frac{4i}{\hbar} \vec{L} \cdot \vec{S} + 3i\hbar, \quad (11)$$

where electron spin $\vec{S} = \frac{\hbar}{2} \vec{\sigma}$,

$$[(\vec{\sigma} \cdot \vec{r}), (\vec{\sigma} \cdot \vec{A})] = ir^2[\vec{\sigma} \cdot \vec{B} - (\vec{\sigma} \cdot \vec{r})(\vec{B} \cdot \vec{r})] \quad (12)$$

and

$$(\vec{\sigma} \cdot \vec{r})^2 = r^2. \quad (13)$$

Using these, Eqs. (6) and (7) can be cast into

$$[c^2 p^2 + m^2 c^2 \omega^2 r^2 + ceB(L_z + \hbar \sigma_z) + e^2 A^2 - \frac{4mc^2 \omega}{\hbar} \vec{L} \cdot \vec{S} - mce\omega r^2 [\sigma_z B - (\vec{\sigma} \cdot \vec{r}) (\vec{B} \cdot \vec{r})]] \psi_a = (E^2 - m^2 c^4 + 3mc^2 \hbar \omega) \psi_a \quad (14)$$

and

$$[c^2 p^2 + m^2 c^2 \omega^2 r^2 + ceB(L_z + \hbar \sigma_z) + e^2 A^2 + \frac{4mc^2 \omega}{\hbar} \vec{L} \cdot \vec{S} + mce\omega r^2 [\sigma_z B - (\vec{\sigma} \cdot \vec{r}) (\vec{B} \cdot \vec{r})]] \psi_b = (E^2 - m^2 c^4 - 3mc^2 \hbar \omega) \psi_b. \quad (15)$$

Since \vec{B} is along the z-direction, and in magnetic field spin of an electron precesses around \vec{B} , the effective component of $\vec{\sigma}$ is σ_z . Hence, the second term within the square bracket in the 6th term of the both equations should reduce as follows:

$$(\vec{\sigma} \cdot \vec{r}) (\vec{B} \cdot \vec{r}) = \sigma_z B \cos^2 \theta,$$

where θ is the azimuthal angle.

Then we have

$$[c^2 p^2 + m^2 c^2 \omega^2 r^2 + ceB(L_z + \hbar \sigma_z) + \frac{e^2 B^2}{4} r^2 \sin^2 \theta - \frac{4mc^2 \omega}{\hbar} \vec{L} \cdot \vec{S} - mce\omega \sigma_z B r^2 \sin^2 \theta] \psi_a = (E^2 - m^2 c^4 + 3mc^2 \hbar \omega) \psi_a \quad (16)$$

and

$$[c^2 p^2 + m^2 c^2 \omega^2 r^2 + ceB(L_z + \hbar \sigma_z) + \frac{e^2 B^2}{4} r^2 \sin^2 \theta + \frac{4mc^2 \omega}{\hbar} \vec{L} \cdot \vec{S} + mce\omega \sigma_z B r^2 \sin^2 \theta] \psi_b = (E^2 - m^2 c^4 - 3mc^2 \hbar \omega) \psi_b. \quad (17)$$

The first two terms of the left hand side of Eqs. (16) and (17) constitute Hamiltonian of 3 dimensional harmonic oscillator. Third term reveals an interaction between z component of total magnetic moment and the magnetic field and the fourth term reveals interaction between electric charge and the magnetic field. The fifth term appears due to spin-orbit interaction and sixth term due to an interaction between electron spin and magnetic field.

We can find closed form solution of Eqs. (16) and (17) using either $|Y_l^{m_l}(\theta, \phi)\chi_{m_s}\rangle$ basis or $|e^{im_l \phi}\chi_{m_s}\rangle$ basis, where the quantum number m_l is associated with the

eigenfunctions $e^{im_l \phi}$ of L_z and $S_z \chi_{m_s} = m_s \hbar \chi_{m_s}$, $m_s = \pm \frac{1}{2}$. In both of those bases, the eigen value of $\vec{L} \cdot \vec{S}$ is $\hbar^2 m_l m_s$. We prefer now to work with the $|e^{im_l \phi}\chi_{m_s}\rangle$ basis.

Since the system possesses cylindrical symmetry, in cylindrical coordinates ρ , ϕ and z , where $\rho^2 = x^2 + y^2$, Eqs. (16) and (17) can be written respectively as

$$\left[-c^2 \hbar^2 \left\{ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right\} + m^2 c^2 \omega^2 \rho^2 + m^2 c^2 \omega^2 z^2 + ceB(L_z + \hbar \sigma_z) + \frac{e^2 B^2}{4} \rho^2 - \frac{4mc^2 \omega}{\hbar} \vec{L} \cdot \vec{S} - mce\omega \sigma_z B \rho^2 \right] \psi_a = (E^2 - m^2 c^4 + 3mc^2 \hbar \omega) \psi_a \quad (18)$$

and

$$\left[-c^2 \hbar^2 \left\{ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right\} + m^2 c^2 \omega^2 \rho^2 + m^2 c^2 \omega^2 z^2 + ceB(L_z + \hbar \sigma_z) + \frac{e^2 B^2}{4} \rho^2 + \frac{4mc^2 \omega}{\hbar} \vec{L} \cdot \vec{S} + mce\omega \sigma_z B \rho^2 \right] \psi_b = (E^2 - m^2 c^4 - 3mc^2 \hbar \omega) \psi_b. \quad (19)$$

As the equations involve partial derivatives of ρ and z and operators of z -component of angular momentum and spin, we take the solutions in the form:

$$\psi_a = F_a(\rho)G_a(z)e^{im_l\phi}\chi_{m_s=\frac{1}{2}} \quad (20)$$

and

$$\psi_b = F_b(\rho)G_b(z)e^{im_l\phi}\chi_{m_s=\frac{1}{2}}. \quad (21)$$

Then we get from Eq. (18)

$$\begin{aligned} & -\frac{1}{F_a(\rho)}c^2\hbar^2\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial F_a(\rho)}{\partial\rho}\right)+c^2\hbar^2\frac{m_l^2}{\rho^2}+\left(\frac{B^2e^2}{4}+mce\omega B+m^2c^2\omega^2\right)\rho^2 \\ & -c^2\hbar^2\frac{1}{G_a(z)}\frac{\partial^2 G_a(z)}{\partial z^2}+m^2c^2\omega^2z^2=E^2-m^2c^4+3mc^2\hbar\omega-ceB(m_l-1)\hbar-2mc^2\omega m_l\hbar. \end{aligned} \quad (22)$$

Let

$$E^2-m^2c^4+3mc^2\hbar\omega-ceB(m_l-1)\hbar-2mc^2\omega m_l\hbar=\lambda, \quad (23)$$

where λ is a constant. Hence, the Eq. (22) demands

$$-c^2\hbar^2\frac{1}{G_a(z)}\frac{d^2 G_a(z)}{dz^2}+m^2c^2\omega^2z^2=\epsilon \quad (24)$$

where ϵ is another constant. Then we have

$$-c^2\hbar^2\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dF_a(\rho)}{d\rho}\right)+c^2\hbar^2\frac{m_l^2}{\rho^2}F_a(\rho)+\left(\frac{B^2e^2}{4}+mce\omega B+m^2c^2\omega^2\right)\rho^2F_a(\rho)+(\epsilon-\lambda)F_a(\rho)=0. \quad (25)$$

Multiplying Eq. (24) by $\frac{1}{2mc^2}$, we have

$$-\frac{\hbar^2}{2m}\frac{d^2 G_a(z)}{dz^2}+\frac{1}{2}m\omega^2z^2G_a(z)=\frac{\epsilon}{2mc^2}G_a(z). \quad (26)$$

This is the simple harmonic oscillator Schrodinger equation and hence,

$$\epsilon=2\left(n+\frac{1}{2}\right)\hbar\omega mc^2, \quad n=0,1,2\dots \quad (27)$$

and $G_a(z)$ is the wave function of simple harmonic oscillator.

Eq. (25) can be written as

$$\frac{d^2 F_a(\rho)}{d\rho^2}+\frac{1}{\rho}\frac{dF_a(\rho)}{d\rho}-\frac{m_l^2}{\rho^2}F_a(\rho)-\left(\frac{B^2e^2}{4c^2\hbar^2}+\frac{me\omega B}{\hbar^2c}+\frac{m^2\omega^2}{\hbar^2}\right)\rho^2F_a(\rho)+\frac{1}{c^2\hbar^2}(\lambda-\epsilon)F_a(\rho)=0 \quad (28)$$

Now, performing a change of variable

$$\xi=\sqrt[4]{\frac{B^2e^2}{4c^2\hbar^2}+\frac{me\omega B}{\hbar^2c}+\frac{m^2\omega^2}{\hbar^2}}\rho, \quad (29)$$

we obtain from Eq. (28)

$$\frac{d^2 F_a(\xi)}{d\xi^2} + \frac{1}{\xi} \frac{dF_a(\xi)}{d\xi} - \frac{m_l^2}{\xi^2} F_a(\xi) - \xi^2 F_a(\xi) + DF_a(\xi) = 0, \tag{30}$$

where

$$D = \frac{\lambda - \epsilon}{c^2 \hbar^2 \sqrt{\frac{B^2 e^2}{4c^2 \hbar^2} + \frac{m e \omega B}{\hbar^2 c} + \frac{m^2 \omega^2}{\hbar^2}}}. \tag{31}$$

According to [21], the corresponding solutions are

$$D = 2(N + 1), \quad N = 0, 1, 2, \dots \tag{32}$$

and

$$F_a(\xi) = A e^{\frac{1}{2}\xi^2} f_{m_l+k}^{m_l}(\xi), \tag{33}$$

where A is a normalization constant and $f_{m_l+k}^{m_l}(\xi)$ satisfy the equation

$$\xi \frac{d^2 f_{m_l+k}^{m_l}(\xi)}{d\xi^2} + (m_l + 1 - \xi) \frac{df_{m_l+k}^{m_l}(\xi)}{d\xi} + k f_{m_l+k}^{m_l}(\xi) = 0, \tag{34}$$

$$k = \frac{1}{2}(N - m_l) = \begin{cases} 0, 1, 2, \dots, \frac{N}{2}, & \text{for } N \text{ is an even integer} \\ 0, 1, 2, \dots, \frac{N-1}{2}, & \text{for } N \text{ is an odd integer} \end{cases} \tag{35}$$

and m_l can take $(N/2) + 1$ values for an even integer N and $N + 1$ values for an odd integer N .

Equation (32) gives the energy spectrum of the (3+1) dimensional Dirac oscillator in presence of magnetic field:

$$E^2 - m^2 c^4 = 2mc^2 \left[(N + 1)\hbar(\omega + \omega_L) + \left(n + \frac{1}{2}\right)\hbar\omega + \left(m_l - \frac{3}{2}\right)\hbar\omega + (m_l - 1)\hbar\omega_L \right] \quad \text{for } m_s = -\frac{1}{2}, \tag{36}$$

where the Larmor frequency ω_L is

$$\omega_L = \frac{eB}{2mc}. \tag{37}$$

The Larmor frequency occurs in the energy spectrum due to interaction between electric charge and the magnetic field. The third term in the square bracket of Eq. (36) appears for spin-orbit coupling and the fourth term for an interaction between total magnetic moment and the magnetic field.

Now, the other component of ψ is

$$\psi_b = F_b(\rho) G_b(z) e^{im_l \phi} \chi_{m_s=\frac{1}{2}} = F_b(\xi) G_b(z) e^{im_l \phi} \chi_{m_s=\frac{1}{2}} = A' e^{\frac{1}{2}\xi^2} f_{m_l+k}^{m_l}(\xi) G_b(z) e^{im_l \phi} \chi_{m_s=\frac{1}{2}} \tag{38}$$

and the corresponding energy spectrum is

$$E^2 - m^2 c^4 = 2mc^2 \left[(N + 1)\hbar(\omega + \omega_L) + \left(n + \frac{1}{2}\right)\hbar\omega + \left(m_l + \frac{3}{2}\right)\hbar\omega + (m_l + 1)\hbar\omega_L \right] \tag{39}$$

for $m_s = \frac{1}{2}$.

For $B = 0$

$$\begin{aligned}
E^2 - m^2 c^4 &= 2mc^2 \left[\left(N + n + \frac{3}{2} \right) \hbar \omega + \left(m_l - \frac{3}{2} \right) \hbar \omega \right] \\
&= 2mc^2 \left[\left(n' + \frac{3}{2} \right) \hbar \omega + \left(m_l - \frac{3}{2} \right) \hbar \omega \right] \quad \text{for } m_s = -\frac{1}{2}
\end{aligned} \tag{40}$$

and

$$\begin{aligned}
E^2 - m^2 c^4 &= 2mc^2 \left[\left(N + n + \frac{3}{2} \right) \hbar \omega + \left(m_l + \frac{3}{2} \right) \hbar \omega \right] \\
&= 2mc^2 \left[\left(n' + \frac{3}{2} \right) \hbar \omega + \left(m_l + \frac{3}{2} \right) \hbar \omega \right] \quad \text{for } m_s = \frac{1}{2},
\end{aligned} \tag{41}$$

where

$$n' = N + n = 0, 1, 2, \dots \tag{42}$$

These (Eqs. (40) and (41)) are of the same nature as in spherically symmetric Dirac oscillator [22]; the difference occurred only because of the cylindrical symmetry we have assumed from the very beginning of the solution procedure.

The non relativistic limit of the energy spectrum is obtained by setting $E = mc^2 + K$ with the consideration $K \ll mc^2$ and we get

$$K = (N + 1)\hbar(\omega + \omega_L) + \left(n + \frac{1}{2} \right) \hbar \omega + \left(m_l - \frac{3}{2} \right) \hbar \omega + (m_l - 1)\hbar \omega_L \quad \text{for } m_s = -\frac{1}{2} \tag{43}$$

and

$$K = (N + 1)\hbar(\omega + \omega_L) + \left(n + \frac{1}{2} \right) \hbar \omega + \left(m_l + \frac{3}{2} \right) \hbar \omega + (m_l + 1)\hbar \omega_L \quad \text{for } m_s = \frac{1}{2}. \tag{44}$$

For $B = 0$

$$K = \left(n' + \frac{3}{2} \right) \hbar \omega + \left(m_l - \frac{3}{2} \right) \hbar \omega \quad \text{for } m_s = -\frac{1}{2} \tag{45}$$

and

$$K = \left(n' + \frac{3}{2} \right) \hbar \omega + \left(m_l + \frac{3}{2} \right) \hbar \omega \quad \text{for } m_s = \frac{1}{2}. \tag{46}$$

Except spin-orbit contribution terms $\left(m_l - \frac{3}{2} \right) \hbar \omega$ in Eq. (45) and $\left(m_l + \frac{3}{2} \right) \hbar \omega$ in Eq. (46), these are exactly the energy spectrum of 3D non-relativistic harmonic oscillator.

If we take the solutions in the form:

$$\psi_a = F_a(\rho) G_a(z) e^{im_l \phi} \chi_{m_s = \frac{1}{2}} \tag{47}$$

and

$$\psi_b = F_b(\rho) G_b(z) e^{im_l \phi} \chi_{m_s = -\frac{1}{2}}, \tag{48}$$

the corresponding energy spectrum of ψ_a and ψ_b are given by respectively as

$$E^2 - m^2 c^4 = 2mc^2 \left[(N+1)\hbar(\omega - \omega_L) + \left(n + \frac{1}{2}\right)\hbar\omega - \left(m_l + \frac{3}{2}\right)\hbar\omega + (m_l + 1)\hbar\omega_L \right]$$

for $m_s = \frac{1}{2}$ (49)

and

$$E^2 - m^2 c^4 = 2mc^2 \left[(N+1)\hbar(\omega - \omega_L) + \left(n + \frac{1}{2}\right)\hbar\omega - \left(m_l - \frac{3}{2}\right)\hbar\omega + (m_l - 1)\hbar\omega_L \right]$$

for $m_s = -\frac{1}{2}$. (50)

3. Conclusion

We have solved the (3+1) Dirac oscillator with magnetic field and found the energy spectrum. The system is cylindrically symmetric: there are basically two oscillators, one is a 2D oscillator in the x - y plane and the other is a 1D oscillator in the z - direction. The energy spectrum shows this splitting exactly. Moreover, there appears an oscillation which is basically a Larmor precession, coupled (i) with the 2D oscillator in x - y plane and (ii) coupled with a net effect of orbital and spin angular momentum. The 2nd coupling is shown in the last term in Eqs. (36) and (39) or (49) and (50). There is also spin-orbit coupling coupled with the Dirac oscillator in the third term in Eqs. (36) and (39) or (49) and (50). The non-relativistic limit of the energy spectrum is rightly that of 3D harmonic oscillators plus spin-orbit coupling. Everything appeared smoothly in our calculation because of the initial decomposition of the problem by a cylindrical symmetry. In the absence of magnetic field, the problem shows characteristics of a 3D Dirac oscillator as can be verified with what is available in literature. The energy spectrum is novel and rich. This oscillator might find application in quantum optics. The system can easily be manipulated using photon beams or electromagnetic signal and made to absorb and emit radiation with energy $\hbar\omega$ or $\hbar\omega_L$ or $\hbar(\omega + \omega_L)$ or $\hbar(\omega - \omega_L)$. This is clear from the expression of the energy levels.

In conclusion, a very attractive quantum system has been elucidated which is a 3D Dirac oscillator immersed in axial magnetic field.

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