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Dynamics of Planar Five-Link Hinged RRRRT Type Mechanism with Two Degrees of Freedom

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Abstract: The paper dwells on dynamic study of a 2-DOF planar five-link mechanism of RRRRT type. There is described the kinematic analysis of the mechanisms, by means of the results of which the formulas of kinetic energy, the reduced moment and the reduced force have been derived, and the non-linear differential equations of motion of second kind determining the motions, velocities and accelerations of the input links of a mechanisms have been obtained. The paper describes the example of solving the dynamic problem. The results obtained for the ideal and real mechanisms are shown in graphs.

Keywords: Planar Five-Link Mechanism, Dynamics, Two Degrees of Freedom, Kinetic Energy, Differential Equations of Motion

1. Introduction

The five-link hinged mechanisms with two degrees of freedom stand out from the numerous hinged mechanisms being used in practice. Such mechanisms allow for simulating and transforming the various and complex motions.

The particular feature of mechanisms with two degrees of freedom is the possibility of change in a trajectory of movement of a coupling point curve from simple to complex, during the velocity change of the input links taking into account the motion direction and their relative initial position, at constant lengths of the links.

These mechanisms, unlike mechanisms with one degree of freedom, can implement more complex preset laws of motion, while ensuring high machine productivity and having a major impact on optimization of technological process. It is those circumstances, which are directly involved in choosing the lengths of the output links of mechanisms, and are pointing to the necessary preconditions for the creation of compact, lightweight mechanisms, especially in the space and robotic spheres of industry, while reducing the material and energy costs.

It is known [1] that in a planar five-link kinematic chain with two rotating input links, depending on the number and combination of sliding and rotating kinematic pairs, it is possible to obtain more than twenty different types of mechanisms, including the RRRRT-type mechanism with one rotating and one translational input links.

Such mechanisms are started to be used in the special-purpose devices, especially in the opened space reflector-type parasites. By using such mechanism, fixed on the frame in the central part of the reflector antenna, high accuracy of the required geometry of the antenna screen surface is reached.

That provision reflects that the understudied for today planar five-link RRRRT-type mechanism needs further study, while regarding the dynamic studies of the planar five-link RRRRT type mechanisms there are numerous works performed by V. V. Dobrovolskiy [2], R. Beier [3], B. M. Abramov [4], S. N. Kozhevnikov [5], V. A. Zinovyev and A. P. Bessonov [6].

When considering the dynamics of a planar five-link mechanism of RRRRT type in the work of A. P. Bessonov and V. A. Ponomaryov [7], the motion of this mechanism is described by two differential equations describing the motions of two rotating input links.

When studying the dynamics of a two degrees of freedom five-link mechanism of RRRRT type, L. G. Ovakimov [8] describes its motions in the form of the equation of mass point dynamics with a mass similar to effective mass of the mechanism.

An analysis of works on dynamic studies of the planar five-link of RRRRT type mechanisms with two degrees of freedom has shown that the questions of kinematics of the mentioned mechanisms can be reputed partially studied, but their dynamic problems require the solution of some actual problems.

The author [1] work dwells on a dynamic analysis of the planar five-link mechanisms with two degrees of freedom, when the input links are the rotating ones (RRRRT). The dynamics of the mentioned mechanisms is studied with account for friction and clearances in the kinematic pairs. There has been studied the process of strike in the kinematic pairs with a clearance, and the analysis of the mechanisms with account for the elasticity of links has been carried out.

The recent works have appeared [9, 10, 11], which describe the problems of kinematic synthesis of the spatial [9], spherical [10] and planar [11] five-link hinged mechanisms of RRRRT type with two degrees of freedom.

This paper dwells on the dynamics of a planar five-link hinged mechanism of RRRRT type with two degrees of freedom (with one rotating and one translational input links).

Thus, we can state that there have been studied mostly the planar five-link mechanisms with two rotating pairs of RRRRT type, the available works on which need further study, since they are started to be used in the special-purpose devices. To increase the accuracy of reproduction of the required geometry of the antenna screen surface, the first stage of the study must involve the kinematic and dynamic analysis of a planar five-link mechanism of RRRRT type, as well as the comparative analysis of the obtained results of the ideal and real mechanisms, with establishing the possibility of further determination of kinematic (technological) and dynamic accuracy of mechanism.

2. Basic Part

2.1. Some General Data of the Analysis of the Planar Five-Link Hinged RRRRT Type Mechanisms of with Two Degrees of Freedom

Consider a kinematic scheme of a planar five-link RRRRT type mechanism with two degrees of freedom ABCD (Fig. 1). Designate the sizes of the links 2, 3 and 4 by ℓ_2, ℓ_3, ℓ_4, accordingly; the initial positions of the input links AB and D (sliding block) – by α_0 and S_{D_0}. The laws of motion of the input links can be expressed in the following form

$$\varphi_2 = \varphi_2(t) \text{ and } S_D = S_D(t), \tag{1}$$

or

$$\varphi_2 = \alpha_0 + \alpha(t) \text{ and } S_D = S_{D_0} + S(t), \tag{2}$$

where t –time; α and S–variable angle and displacement characterizing the motions of the input links, i.e. their current values.

Accordingly, we can write,

$$S_D = S_D(\varphi_2).$$

Kinematical connection between the input links 2 and 5 can be expressed in the form of the following ratios, which are the transmission ratios:

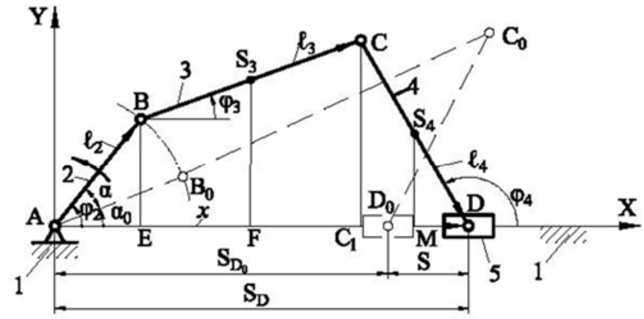

Figure 1. *To kinematic analysis of a planar five-link mechanism of RRRRT type.*

$$u_{52} = \frac{S_D}{\varphi_2}, \left(\mathrm{m}/\mathrm{rad}\right); \tag{3}$$

$$u_{25} = \frac{\varphi_2}{S_D}, \left(\mathrm{rad}/\mathrm{m}\right). \tag{4}$$

In accordance with the expression $S_D = S_D(\varphi_2)$ for the velocity of a sliding block 5, we obtain

$$V_D = \frac{dS_D}{dt} = \frac{dS_D}{d\varphi_2}\frac{d\varphi_2}{dt} = u_{52}\dot{\varphi}_2, \left(m/sec\right), \tag{5}$$

where u_{52} is a transmission ratio (analogue to the linear velocity of a sliding block) of the link 5 to the link 2 with the dimension of a meter.

Generally, the input links 2 and 5 can move at a variable velocity, thus the transmission ratio u_{52} may be a variable, but the motion in this case is aperiodical.

In case, when u_{52}=const, the motion is periodical.

In a mechanism under consideration, the positions of the links BC and CD and their points, as well as point C and their velocities and accelerations must be determined. We have

$$\begin{aligned} \varphi_3 = \varphi_3(t) \quad &\text{and} \quad \varphi_4 = \varphi_4(t); \\ x_i = x_i(t) \quad &\text{and} \quad y_i = y_i(t);\ i = S_3, S_4, C. \end{aligned} \tag{6}$$

or

$$\begin{aligned} \varphi_3 = \varphi_3(\varphi_2, S_D) \quad &\text{and} \quad \varphi_4 = \varphi_4(\varphi_2, S_D); \\ x_i = x_i(\varphi_2, S_D) \quad &\text{and} \quad y_i = y_i(\varphi_2, S_D). \end{aligned} \tag{7}$$

In order to determine the positions of the links 3 and 4, we shall consider a closed vector contour ABCDA (Fig. 1); we shall write

$$\overrightarrow{\ell_2} + \overrightarrow{\ell_3} + \overrightarrow{\ell_4} = \overrightarrow{S_D}. \tag{8}$$

By projecting this vectorial equation on the axis of the XAY coordinate system, we obtain:

$$\begin{aligned} \ell_2 \cos\varphi_2 + \ell_3 \cos\varphi_3 - \ell_4 \cos\varphi_4 = S_D; \\ \ell_2 \sin\varphi_2 + \ell_3 \sin\varphi_3 + \ell_4 \sin\varphi_4 = 0. \end{aligned} \tag{9}$$

By solving the system (9), we obtain the quadratic equation with respect to φ_3:

$$\cos^2\varphi_3(1+B^2)+2A\cos\varphi_3+(A^2-B^2)=0, \qquad (10)$$

where

$$A=\frac{(\ell_2\cos\varphi_2-S_D)^2+\ell_2^2\sin^2\varphi_2+\ell_3^2-\ell_4^2}{2\ell_3(\ell_2 cos\varphi_2-S_D)};$$

$$B=\frac{\ell_2\sin\varphi_2}{\ell_2\cos\varphi_2-S_D}.$$

From the first equation of a system (9), we have the φ_4 angle value:

$$\cos\varphi_4=(\ell_4)^{-1}(\ell_2\cos\varphi_2+\ell_3\cos\varphi_3-S_D). \qquad (11)$$

The position of the points S_3, C and S_3 is determined by the expressions:

$$\ell_{S_3}=(x_{S_3}^2+y_{S_3}^2)^{1/2}; \qquad (12)$$

$$\ell_c=(x_c^2+y_c^2)^{1/2}; \qquad (13)$$

$$\ell_{S_4}=(x_{S_4}^2+y_{S_4}^2)^{1/2}, \qquad (14)$$

where

$$x_{S_3}=\ell_2\cos\varphi_2+\frac{\ell_3}{2}\cos\varphi_3;\quad y_{S_3}=\ell_2\sin\varphi_2+\frac{\ell_3}{2}\sin\varphi_3; \qquad (15)$$

$$x_C=\ell_2\cos\varphi_2+\ell_3\cos\varphi_3;\quad y_C=\ell_2\sin\varphi_2+\ell_3\sin\varphi_3; \qquad (16)$$

$$x_{S_4}=S_D+\frac{\ell_4}{2}\cos\varphi_4;\quad y_{S_4}=\frac{\ell_4}{2}\sin\varphi_4; \qquad (17)$$

In order to determine the velocity and acceleration of the links 3 and 4 and their points of a mechanism, first we differentiate the first equality (7). Let's write

$$\omega_3=\frac{d\varphi_3}{dt}=\frac{d\varphi_3}{d\varphi_2}\frac{d\varphi_2}{dt}+\frac{d\varphi_3}{dS_D}\frac{dS_D}{dt}=$$
$$=u_{32}\omega_2+u_{35}V_D, \qquad (18)$$

where

ω_2- the angular velocity of the input link 2; V_D– the linear velocity of the input link 5 (sliding block); u_{32}- a dimensionless quantity – the gearing ratio from the link 3 to the link 2; u_{35}– the gearing ratio from the link 3 to the link5 (sliding block) with the dimension of $1/m$.

Similarly, for the link 4, we obtain

$$\omega_4=\frac{d\varphi_4}{dt}=\frac{d\varphi_4}{d\varphi_2}\frac{d\varphi_2}{dt}+\frac{d\varphi_4}{dS_D}\frac{dS_D}{dt}=$$
$$=u_{42}\omega_2+u_{45}V_D, \qquad (19)$$

where

u_{42}- the gearing ratio from the link 4 to the link 2; u_{45} – the gearing ratio from the link 4 to the link5; ($1/m$.)

For the x_i and y_i coordinates: according to the second expressions (7), we obtain

$$V_{x_i}=\frac{dx_i}{dt}=\frac{dx_i}{d\varphi_2}\frac{d\varphi_2}{dt}+\frac{dx_i}{dS_D}\frac{dS_D}{dt}=u_{x_i}\omega_2+u_{x_id}V_D;$$
$$V_{y_i}=\frac{dy_i}{dt}=\frac{dy_i}{d\varphi_2}\frac{d\varphi_2}{dt}+\frac{dy_i}{dS_D}\frac{dS_D}{dt}=u_{y_i}\omega_2+u_{y_id}V_D, \qquad (20)$$

where

u_{x_i} and u_{y_i}- the gearing ratios with the dimension of (m);u_{x_id} and u_{y_id}- the dimensionless gearing ratios.

We shall notice that by differentiating the equations (10) and (11) with respect to the generalized coordinates φ_2 and S_D, we shall obtain the expressions determining u_{32}, u_{35}, u_{42}, u_{45}, the values of which are inserted into the formulas (18) and (19), and the real values of the angular velocities of the links 3 and 4 will be found.

For u_{32}, u_{35}, u_{42}, u_{45} from the expressions (10) and (11), we obtain:

$$u_{32}=\frac{D}{C};\ u_{35}=\frac{E}{C};\ u_{42}=\frac{F}{L};\ u_{45}=\frac{M}{L},$$

where

$$D=\frac{dA}{d\varphi_2}(\cos\varphi_3+A)-B\frac{dB}{d\varphi_2}\sin^2\varphi_3;$$

$$C=A\sin\varphi_3+\frac{1}{2}\sin 2\varphi_3(1+B^2);$$

$$E=\frac{dA}{dS_D}(\cos\varphi_3+A)-B\frac{dB}{dS_D}\sin^2\varphi_3;$$

$$F=\ell_2 sin\varphi_2+\ell_3 u_{32}\,sin\varphi_3-u_{52};$$

$$L=\ell_4 sin\varphi_4;$$

$$M=\ell_2 u_{25}\,sin\varphi_2-\ell_3 u_{35}\,sin\varphi_3-1.$$

Inversely,

$$\frac{dA}{d\varphi_2}=[2\ell_3(\ell_2 cos\varphi_2-S_D)]^{-1}\cdot$$
$$\cdot[2(\ell_2 cos\varphi_2-S_D)(-\ell_2 sin\varphi_2-u_{52})+$$
$$+\ell_2^2\sin 2\varphi_2+2\ell_3 A(\ell_2 sin\varphi_2+U_{52})];$$

$$\frac{dB}{d\varphi_2}=(\ell_2 cos\varphi_2-S_D)^{-1}[\ell_2 cos\varphi_2+$$
$$+B(\ell_2 sin\varphi_2-u_{52})];$$

$$\frac{dA}{dS_D}=[2\ell_3(\ell_2 cos\varphi_2-S_D)]^{-1}[2(\ell_2 cos\varphi_2-S_D)\cdot$$
$$\cdot(-\ell_2 sin\varphi_2-u_{52})+\ell_2^2 u_{25}\sin 2\varphi_2+$$
$$+A2\ell_3(\ell_2 u_{25} sin\varphi_2-1)];$$

$$\frac{dB}{dS_D}=(\ell_2 cos\varphi_2-S_D)^{-1}[\ell_2 u_{52} cos\varphi_2+$$

$$+B(u_{52}\ell_2 sin\varphi_2 - 1)].$$

The squared linear velocities of the points S_3, C, S_4 are determined in the same way:

$$V_{S_3}^2 = \dot{x}_{S_3}^2 + \dot{y}_{S_3}^2;\ V_c^2 = \dot{x_c^2} + \dot{y_c^2};\ V_{S_4}^2 = \dot{x}_{S_4}^2 + \dot{y}_{S_4}^2, \qquad (21)$$

where the values of $\dot{x}_{S_3}$, $\dot{y}_{S_3}$, $\dot{x}_c$, $\dot{y}_c$, $\dot{x}_{S_4}$ and $\dot{y}_{S_4}$ - are determined by differentiating the expressions (15), (16) and (17) with respect to time *t*. We obtain:

$$\dot{x}_{S_3} = -\dot{\varphi}_2\ell_2\sin\varphi_2 - \dot{\varphi}_3\frac{\ell_3}{2}\sin\varphi_3 =$$
$$= -\dot{\varphi}_2\ell_2\sin\varphi_2 - \frac{\ell_3}{2}\sin\varphi_3\left(u_{32}\dot{\varphi}_2 + u_{35}\dot{S}_D\right) =$$
$$= -\dot{\varphi}_2\left(\ell_2\sin\varphi_2 - \frac{\ell_3}{2}u_{32}\sin\varphi_3\right) -$$
$$-\dot{S}_D\frac{\ell_3}{2}u_{35}\sin\varphi_3\,;$$

$$\dot{y}_{S_3} = \dot{\varphi}_2\ell_2\cos\varphi_2 + \dot{\varphi}_3\frac{\ell_3}{2}\cos\varphi_3 =$$
$$= \dot{\varphi}_2\left(\ell_2\cos\varphi_2 + \frac{\ell_3}{2}u_{32}\cos\varphi_3\right) +$$
$$+\dot{S}_D\frac{\ell_3}{2}u_{35}\sin\varphi_3\,;$$

$$\dot{x}_C = -\dot{\varphi}_2(\ell_2\sin\varphi_2 - \ell_3u_{32}\sin\varphi_3) -$$
$$-\dot{S}_D\ell_3u_{35}\sin\varphi_3;$$

$$\dot{y}_C = \dot{\varphi}_2(\ell_2\cos\varphi_2 + \ell_3u_{32}\cos\varphi_3) +$$
$$+\dot{S}_D\ell_3u_{35}\cos\varphi_3;$$

$$\dot{x}_{S_4} = \dot{S}_D - \dot{\varphi}_4\frac{\ell_4}{2}\sin\varphi_4 = \dot{S}_D -$$
$$-\dot{\varphi}_2u_{45}\frac{\ell_4}{2}\sin\varphi_4 - \dot{S}_Du_{45}\frac{\ell_4}{2}\sin\varphi_4 =$$
$$= \dot{S}_D\left(1 - u_{45}\frac{\ell_4}{2}\sin\varphi_4\right) - \dot{\varphi}_2u_{42}\frac{\ell_4}{2}\sin\varphi_4\,;$$

$$\dot{y}_{S_4} = \dot{\varphi}_4\frac{\ell_4}{2}\cos\varphi_4 =$$
$$= \frac{\ell_4}{2}\cos\varphi_4\left(u_{42}\dot{\varphi}_2 + u_{45}\dot{S}_D\right) =$$
$$= \dot{\varphi}_2\frac{\ell_4}{2}u_{42}\cos\varphi_2 + \dot{S}_D\frac{\ell_4}{2}u_{45}\cos\varphi_4.$$

According to the expression (21), the squared velocities of the points S_3, C and S_4 take the following form:

$$V_{S_3}^2 = \left[-\dot{\varphi}_2\left(\ell_2 sin\varphi_2 + \frac{\ell_3}{2}u_{32}\sin\varphi_3\right) -\right.$$
$$\left. -\dot{S}_D\frac{\ell_3}{2}u_{35}\sin\varphi_3\right]^2 +$$
$$+\left[\dot{\varphi}_2\left(\ell_2 cos\varphi_2 + \frac{\ell_3}{2}u_{32}\cos\varphi_3\right) +\right.$$
$$\left. +\dot{S}_D\frac{\ell_3}{2}u_{35}\cos\varphi_3\right]^2;$$

$$V_C^2 = [-\dot{\varphi}_2(\ell_2 sin\varphi_2 + \ell_3u_{32}\sin\varphi_3) +$$
$$-\dot{S}_D\ell_3u_{35}\sin\varphi_3]^2 +$$
$$+[\dot{\varphi}_2(\ell_2 cos\varphi_2 + \ell_3u_{32}\cos\varphi_3) +$$
$$+\dot{S}_D\ell_3u_{35}\cos\varphi_3]^2;$$

$$V_{S_4}^2 = \left[\dot{S}_D\left(1 - u_{45}\frac{\ell_4}{2}\sin\varphi_4\right) -\right.$$
$$\left. -\dot{\varphi}_2u_{42}\frac{\ell_4}{2}sin\varphi_4\right]^2 +$$
$$+\left[\dot{\varphi}_2\frac{\ell_4}{2}u_{42}\cos\varphi_4 + \dot{S}_D\frac{\ell_4}{2}u_{45}\cos\varphi_4\right]^2.$$

2.2. Determining Kinetic Energy of a Planar Five-Link Mechanism of RRRRT Type

Consider a planar five-link RRRRT type mechanism with two degrees of freedom, whose dynamic model is shown in Fig. 2. The mechanism is influenced by different forces and moments, the input link 2 is influenced by the drive moment $M_D = M_2$, but the input link 5 - by the drive force F_{dr}, resistance force F_R and the gravity G_5. In the hinge C there are acted the outside force F_{ext} and the outside moment M_{ext}. The gravity forces G_3 and G_4, and moments of couple inertia forces M_3 and M_4, are exerted on the gravity points S_3 and S_4 of the links BC and CD, accordingly.

To derive the equation of motion of a planar five-link mechanism with two degrees of freedom, we shall use the Lagrange's equations of the second kind

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}_2}\right) - \frac{\partial T}{\partial\varphi_2} = M_{red}; \qquad (22)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{S}_D}\right) - \frac{\partial T}{\partial S_D} = F_{red}, \qquad (23)$$

where

T- kinetic energy of a mechanism; M_{red} and F_{red} − the reduced (generalized) moment and force on the φ_2 and S_D coordinates, accordingly.

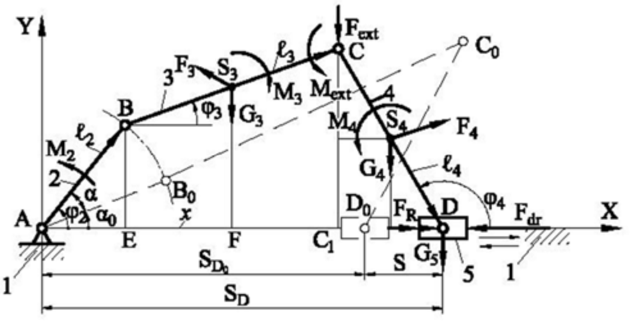

Figure 2. A dynamical model of a planar five-link mechanism of RRRRT type.

Kinetic energy of a 2-DOF planar hinged mechanism with two input links (crank and sliding block) and two connecting-rods, has the following form:

$$T = \frac{1}{2}\left(I_2\dot{\varphi}_2^2 + m_3V_{S_3}^2 + I_{S_3}\dot{\varphi}_3^2 + m_4V_{S_4}^2 +\right.$$

$$+I_{S_4}\dot{\varphi}_4^2 + m_5 V_D^2\big), \text{ (Nm)}, \tag{24}$$

where

I_2 – a given moment of energy of the link 2 relatively to a center of mass, which coincides with a center A of rotation of crank; m_3, m_4 and m_5 – the masses of the links 3, 4 and 5; $V_{S_3}^2$, $V_{S_4}^2$ and V_D^2 – squared linear velocities S_3, S_4 and D; I_{S_3} and I_{S_4} - the given moments of inertia of the links 3 and 4; $\dot{\varphi}_3^2$ and $\dot{\varphi}_4^2$ - squared angular velocities of the links 3 and 4.

The values $V_{S_3}^2$, $V_{S_4}^2$, V_D^2, $\dot{\varphi}_3^2$ and $\dot{\varphi}_4^2$ we insert into the formula (24). We obtain:

$$T = \frac{1}{2}\Big\{I_2\dot{\varphi}_2^2 + m_3\Big[\Big(-\dot{\varphi}_2^2(\ell_2 sin\varphi_2 + \\ +\frac{\ell_3}{2}u_{32}\sin\varphi_3\Big) - \dot{S}_D\frac{\ell_3}{2}u_{35}\sin\varphi_3\Big)^2 + \\ +\Big(\dot{\varphi}_2\Big(\ell_2\cos\varphi_2 + \frac{\ell_3}{2}u_{32}\cos\varphi_3\Big) + \\ +\Big(+\dot{S}_D\frac{\ell_3}{2}u_{35}\cos\varphi_3\Big)^2\Big] + I_{S_3}\big(u_{32}\dot{\varphi}_2 + u_{35}\dot{S}_D\big)^2 + \\ +m_4\Big[\Big(\dot{S}_D\Big(1 - u_{45}\frac{\ell_4}{2}\sin\varphi_4\Big) - \dot{\varphi}_2 u_{42}\frac{\ell_4}{2}\sin\varphi_4\Big)^2 + \\ +\frac{\ell_4^2}{4}\cos^2\varphi_4\big(u_{42}\dot{\varphi}_2 + u_{45}\dot{S}_D\big)^2\Big] + \\ +I_{S_4}\big(u_{42}\dot{\varphi}_2 + u_{45}\dot{S}_D\big)^2 + m_5 u_{52}^2\dot{\varphi}_2^2\Big\}. \tag{25}$$

By transforming the formula (25), kinetic energy of a mechanism will finally take the following form:

$$T = \tfrac{1}{2}\dot{\varphi}_2^2 I_{22} + \dot{\varphi}_2\dot{S}_D I_{25} + \tfrac{1}{2}\dot{S}_D^2 I_{55}, \tag{26}$$

where

$$I_{22} = I_2 + m_3\Big[\ell_2^2 + \frac{\ell_3^2}{4}u_{32}^2 + \\ +\Big[2\ell_2\frac{\ell_3}{2}\cos(\varphi_2 - \varphi_3)\Big] + I_{S_3}u_{32}^2 + \\ +m_4 u_{42}^2\frac{\ell_4^2}{4} + I_{S_4}u_{42}^2 + m_5 u_{52}^2;$$

$$I_{25} = m_3\Big(\ell_2\frac{\ell_3}{2}u_{35}\cos(\varphi_2 - \varphi_3) + \\ +\frac{\ell_3^2}{4}u_{32}u_{35}\Big) + I_{S_3}u_{32}u_{35} - \\ -m_4\Big(\frac{\ell_4}{2}u_{42}\sin\varphi_4 - \frac{\ell_4^2}{4}u_{42}u_{45}\cdot \\ \cdot(\sin\varphi_3\sin\varphi_4 + \cos^2\varphi_4)\Big) + I_{S_4}u_{42}u_{45};$$

$$I_{55} = m_3\frac{\ell_3^2}{4}u_{35}^2 + I_{S_3}u_{35}^2 + \\ +m_4\Big(\Big(1 - u_{45}\frac{\ell_4}{2}\sin\varphi_4\Big)^2 + \\ +\frac{\ell_4^2}{4}u_{45}^2\cos^2\varphi_4\Big) + I_{S_4}u_{45}^2.$$

$$\cdot\Big(\Big(1 - u_{45}\frac{\ell_4}{2}\sin\varphi_4\Big)^2 + \frac{\ell_4^2}{4}u_{45}^2\cos^2\varphi_4\Big) + \\ +I_{S_4}u_{45}^2.$$

In the formula of kinetic energy (26) I_{22}, I_{25} and I_{55} are the reduced moments of energy, which depend only on the geometry of masses and generalized φ_2 and S_D coordinates. These moments are de facto the reduced moments to the link 2(I_{22}), to the link 5 (I_{55}) and to two links 2 and 5 (I_{25}), simultaneously.

2.3. Determining the Reduced Moment and Reduced Force of a Planar Five-Link Mechanism of RRRRT Type

According to the Lagrange's equations (22), (23), the reduced moment and the reduced force can be derived from the condition of equality between elementary works of the reduced moment and force and elementary works of all outside forces, with appropriate variations of the generalized coordinates.

For a planar five-link mechanism under consideration, the equality of the reduced power and amount of the reduced powers, i.e. the power developing by the substituting force should be equivalent to the amount of powers developing by the forces exerted on the links of a mechanism under study.

Based on the above stated, we can write the equation of power developing by all forces exerted on a mechanism

$$P = M_{D_2}\dot{\varphi}_2 + G_3 V_{S_3} + M_{ext}\dot{\varphi}_3 + M_{ext}\dot{\varphi}_4 + \\ +_{ext}V_C + G_4 V_{S_4} + (G_5 + F_D)V_D = \\ = M_{red_2}\dot{\varphi}_2 + F_{red}\dot{S}_D, \tag{27}$$

where V_{S_3}, V_C and V_{S_4} - the linear velocities of the points S_3, C and S_4, accordingly; $\dot{\varphi}_3=\omega_3$, $\dot{\varphi}_4=\omega_4$ - the angular velocities of the links 3 and 4 obtained by the formulas (18) and (19); $M_{D_2} = M_2$ - the drive moment; G_3, G_4 and G_5 – the weight of the links 3, 4 and 5; M_{ext} – the external moment; F_{ext} – the external force; F_D – the drive force exerted on a sliding block.

The linear velocities V_{S_3}, V_C and V_{S_4} of the points S_3, C and S_4 will be found by differentiating the expressions (15) –(17) with respect to the generalized φ_2 and S_D coordinates.

We shall notice that generally, for all points S_3, C and S_4, we have:

$$\ell_{S_3} = \ell_{S_3}(\varphi_2, S_D); \quad \ell_C = \ell_C(\varphi_2, S_D); \\ \ell_{S_4} = \ell_{S_4}(\varphi_2, S_D). \tag{28}$$

Accordingly, the linear velocities of the points S_3, C and S_4 take the following form:

$$V_{S_3} = \frac{d\ell_{S_3}}{dt} = \frac{d\ell_{S_3}}{d\varphi_2}\frac{d\varphi_2}{dt} + \frac{d\ell_{S_3}}{dS_D}\frac{dS_D}{dt} = \\ = V_{S_3(\varphi_2)}\dot{\varphi}_2 + U_{S_3}\dot{S}_D; \tag{29}$$

$$V_C = \frac{d\ell_C}{dt} = \frac{d\ell_C}{d\varphi_2}\frac{d\varphi_2}{dt} + \frac{d\ell_C}{dS_D}\frac{dS_D}{dt} = \\ = V_{C(\varphi_2)}\dot{\varphi}_2 + U_C\dot{S}_D; \tag{30}$$

$$V_{S_4} = \frac{d\ell_{S_4}}{dt} = \frac{d\ell_{S_4}}{d\varphi_2}\frac{d\varphi_2}{dt} + \frac{d\ell_{S_4}}{dS_D}\frac{dS_D}{dt} =$$

$$= V_{S_4(\varphi_2)}\dot{\varphi}_2 + U_{S_4}\dot{S}_D, \quad (31)$$

where

$V_{S_3(\varphi_2)}, V_{C(\varphi_2)}$ and $V_{S_4(\varphi_2)}$ - the analogues of the linear velocities of the points S_3, C and S_4, accordingly, with the dimension of m; U_{S_3}, U_c and U_{S_4} - the analogues of the linear velocities of the same points with no dimension.

The values of the analogues of the linear velocities are determined by differentiating the expressions (15)-(17) with respect to the generalized φ_2 and S_D coordinates.

We insert the expressions (18), (19) and (29)-(31) into the formula (27). After some transformations, we shall obtain:

$$M_{red_2}\dot{\varphi}_2 + F_{red}\dot{S}_D = M_{D_2}\dot{\varphi}_2 + G_3V_{S_3(\varphi_2)}\dot{\varphi}_2 +$$
$$+M_{ext}\dot{\varphi}_2(u_{32}+u_{42}) + F_{ext}V_{C(\varphi_2)}\dot{\varphi}_2 +$$
$$+G_4V_{S_4(\varphi_2)}\dot{\varphi}_2 + G_3U_{S_3}\dot{S}_D + M_{ext}\dot{S}_D(u_{35}+u_{45}) +$$
$$+F_{ext}U_C\dot{S}_D + G_4U_{S_4}\dot{S}_D + (G_5+F_D)u_{52}\dot{\varphi}_2 +$$
$$+(G_5+F_D)\dot{S}_D, \quad (32)$$

from which, the reduced moment M_{red_2} and the reduced force F_{red} – take the following form:

$$M_{red_2} = M_{D_2} + G_3V_{S_3(\varphi_2)} + M_{ext}(u_{32}+u_{42}) +$$
$$+F_{ext}V_{C(\varphi_2)} + G_4V_{S_4(\varphi_2)} + (G_5+F_D)u_{52}; \quad (33)$$

$$F_{red} = (G_5+F_D) + G_3U_{S_3} + M_{ext}(u_{35}+u_{45}) +$$
$$+F_{ext}U_C + G_4U_{S_4}. \quad (34)$$

Since the analogues of the angular and,linear velocities are the functions of two variables φ_2 and S_D, the reduced moment M_{red_2} and the reduced force F_{red} will also be the functions of these variables – generalized coordinates.

2.4. The Differential Equations of Motion of a Planar Five-Link Mechanism of RRRRT Type

To derive the differential equations of motion of a planar five-link RRRRT type mechanism with two degrees of freedom, in accordance with the equations (22) and (23), we find partial and total derivatives of kinetic energy (26). We shall obtain

$$\left.\begin{aligned}
&\frac{\partial T}{\partial\varphi_2} = \frac{1}{2}\frac{\partial I_{22}}{\partial\varphi_2}\dot{\varphi}_2^2 + \frac{\partial I_{25}}{\partial\varphi_2}\dot{\varphi}_2\dot{S}_D + \frac{1}{2}\frac{\partial I_{55}}{\partial\varphi_2}\dot{S}_D^2;\\
&\frac{\partial T}{\partial\dot{\varphi}_2} = I_{22}\dot{\varphi}_2^2 + I_{25}\dot{S}_D;\\
&\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}_2}\right) = I_{22}\ddot{\varphi}_2 + \frac{\partial I_{22}}{\partial\varphi_2}\dot{\varphi}_2^2 + \frac{\partial I_{22}}{\partial S_D}\dot{\varphi}_2\dot{S}_D +\\
&+I_{25}\ddot{S}_D + \frac{\partial I_{25}}{\partial\varphi_2}\dot{\varphi}_2\dot{S}_D + \frac{\partial I_{25}}{\partial S_D}\dot{S}_D^2;
\end{aligned}\right\} \quad (35)$$

$$\left.\begin{aligned}
&\frac{\partial T}{\partial S_D} = \frac{1}{2}\frac{\partial I_{22}}{\partial S_D}\dot{\varphi}_2^2 + \frac{\partial I_{25}}{\partial S_D}\dot{\varphi}_2\dot{S}_D + \frac{1}{2}\frac{\partial I_{55}}{\partial S_D}\dot{S}_D^2;\\
&\frac{\partial T}{\partial\dot{S}_D} = I_{25}\dot{\varphi}_2^2 + I_{55}\dot{S}_D;\\
&\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{S}_D}\right) = I_{25}\ddot{\varphi}_2 + \frac{\partial I_{25}}{\partial S_D}\dot{\varphi}_2\dot{S}_D + \frac{\partial I_{25}}{\partial S_D}\dot{\varphi}_2^2 +\\
&+I_{55}\ddot{S}_D + \frac{\partial I_{55}}{\partial S_D}\dot{S}_D^2 + \frac{\partial I_{55}}{\partial\varphi_2}\dot{\varphi}_2\dot{S}_D,
\end{aligned}\right\} \quad (36)$$

where

$$\frac{\partial I_{22}}{\partial\varphi_2} = I_{22(\varphi_2)}, \frac{\partial I_{25}}{\partial\varphi_2} = I_{25(\varphi_2)}, \frac{\partial I_{55}}{\partial\varphi_2} = I_{55(\varphi_2)},$$

$$\frac{\partial I_{22}}{\partial S_D} = I_{22(S_D)}, \frac{\partial I_{25}}{\partial S_D} = I_{25(S_D)} \text{ and } \frac{\partial I_{55}}{\partial S_D} = I_{55(S_D)} -$$

are determined by differentiating the expressions I_{22}, I_{25} and I_{55} with respect to the generalized φ_2 and S_D coordinates. We have:

$$I_{22(\varphi_2)} = 2I_{S_3}u_{32}u'_{32} + 2\frac{\ell_3^2}{4}u_{32}u'_{32}m_3 -$$
$$-2m_3\ell_2\frac{\ell_3}{2}(1-u_{32})\sin(\varphi_2-\varphi_3) +$$
$$+2m_4u_{42}u'_{42}\frac{\ell_4^2}{2} + 2I_{S_4}u_{42}u'_{42} +$$
$$+2m_5u_{52}u'_{52};$$

$$I_{25(\varphi_2)} = m_3\left[\frac{\ell_3^2}{4}(u'_{32}u_{35} + u'_{35}u_{32}) +\right.$$
$$+\ell_2\frac{\ell_3}{2}(u'_{35}\cos(\varphi_2-\varphi_3) -$$
$$\left.-u_{35}(1-u_{32})\sin(\varphi_3-\varphi_3))\right] +$$
$$+I_{S_3}(u'_{32}u_{35} + u'_{35}u_{32}) -$$
$$-m_4\left[\frac{\ell_4^2}{2}(u_{42}sin\varphi_4 + u'_{42}cos\varphi_4) -\right.$$
$$+\frac{\ell_4^2}{2}(u'_{42}u_{45} + u'_{45}u_{42})(\sin\varphi_3 sin\varphi_4 +$$
$$+cos^2\varphi_4) + u_{42}u_{45}(u_{32}cos\varphi_3\sin\varphi_4 +$$
$$\left.+u_{42}\cos\varphi_4\sin\varphi_3 - u_{42}sin2\varphi_4)\right] +$$
$$+I_{S_4}(u'_{42}u_{45} + u'_{45}u_{42}) + m_5u'^2_{52};$$

$$I_{55(\varphi_2)} = 2m_3\frac{\ell_3^2}{4}u_{35}u'_{35} +$$
$$+2I_{S_3}u_{35}u'_{35} + m_4\left[2\left(1-u_{45}\frac{\ell_4}{2}\sin\varphi_3\right)\cdot\right.$$
$$\cdot\left(-\frac{\ell_4}{2}u'_{45}\sin\varphi_4 - \frac{\ell_4}{2}u_{32}u_{45}\cos\varphi_3\right)\cdot$$
$$+\frac{\ell_4^2}{4}(2u_{45}u'_{45}\cos^2\varphi_4 - u_{42}u_{45}^2\sin2\varphi_4) +$$

$$+2I_{S_4}u_{45}u'_{45};$$

$$I_{22(S_D)} = 2I_{S_3}u_{32}u'_{32(S_D)} + m_3\left[2\frac{\ell_3^2}{4}u_{32}u'_{32(S_D)} - \right.$$

$$\left. - 2\ell_2\frac{\ell_3}{2}(u_{25} - u_{35})\sin(\varphi_2 - \varphi_3)\right] +$$

$$+2I_{S_3}u_{32}u'_{32(S_D)} + 2\frac{\ell_4^2}{4}m_4u_{42}u'_{42(S_D)} +$$

$$+2I_{S_4}u_{42}u'_{42(S_D)} + 2m_5u_{52}u'_{52(S_D)};$$

$$I_{25(S_D)} = m_3\left[\ell_2\frac{\ell_3}{2}\left(u'_{35(S_D)}\cos(\varphi_2 - \varphi_3) - \right.\right.$$

$$-u_{35}(u_{25} - u_{35})\sin(\varphi_3 - \varphi_3)) +$$

$$\left.+\frac{\ell_3^2}{4}\left(u'_{32(S_D)}u_{35} + u'_{35(S_D)}u_{32}\right)\right] +$$

$$+I_{S_3}\left(u'_{32(S_D)}u_{35} + u'_{35(S_D)}u_{32}\right) -$$

$$-m_4\left[\frac{\ell_4}{2}\left(u'_{42(S_D)}sin\varphi_4 + \right.\right.$$

$$\left.+u_{42}u_{45(S_D)}\cos\varphi_4\right) - \frac{\ell_4^2}{4}\left(u'_{42(S_D)}u_{45} + \right.$$

$$+u_{45(S_D)}u'_{42}\big)(\sin\varphi_3\sin\varphi_4 + \cos^2\varphi_4) +$$

$$+u_{42}u_{45}\big(u_{35(S_D)}cos\varphi_3\sin\varphi_4 +$$

$$+u_{45(S_D)}\cos\varphi_4\sin\varphi_3 +$$

$$+u_{45(S_D)}sin2\varphi_4\big)\big] +$$

$$+I_{S_4}\left(u'_{42(S_D)}u_{45} + u'_{45(S_D)}u_{42}\right);$$

$$I_{55(S_D)} = 2m_3\frac{\ell_3^2}{4}u_{35}u'_{35(S_D)} +$$

$$+2I_{S_3}u_{35}u'_{35(S_D)} +$$

$$+m_4\left[2\left(1 - u_{45}\frac{\ell_4}{2}\sin\varphi_4\right)\cdot\right.$$

$$\cdot\left(u'_{45(S_D)}\frac{\ell_4}{2}\sin\varphi_4 - u_{45}^2\frac{\ell_4}{2}\cos\varphi_4\right) +$$

$$+2\frac{\ell_4^2}{4}u_{45}u'_{45(S_D)}\cos^2\varphi_4 -$$

$$\left.-\frac{\ell_4^2}{4}u_{45}^3sin2\varphi_4\right) + 2I_{S_4}u_{45}u'_{45(S_D)}.$$

Inversely,

$u'_{32}, u'_{42}, u'_{35}, u'_{45}$- the analogues of the angular velocities of the links 3 and 4;

$u'_{32(S_D)}$, $u'_{42(S_D)}$, $u'_{35(S_D)}, u'_{45(S_D)}$- the mixed analogues of the angular accelerations.

By inserting the values of (35) and (36) into the equations (22) and (23), after some transformations, we obtain:

$$I_{22(\varphi_2)}\ddot{\varphi}_2 + I_{25(S_D)}\ddot{S}_D + \frac{1}{2}I_{22(\varphi_2)}\dot{\varphi}_2^2 +$$

$$+I_{22(S_D)}\dot{\varphi}_2\dot{S}_D + \big(I_{25(S_D)} -$$

$$-\tfrac{1}{2}I_{55(\varphi_2)}\Big)\dot{S}_D^2 = M_{np}; \tag{37}$$

$$I_{55(\varphi_2)}\ddot{S}_D\ell + I_{25(S_D)}\ddot{\varphi}_2\ell +$$

$$+\left(I_{25(\varphi_2)} - \frac{1}{2}I_{22(S_D)}\right)\dot{\varphi}_2^2\ell + \frac{1}{2}I_{55(S_D)}\dot{S}_D^2\ell +$$

$$+I_{55(\varphi_2)}\dot{\varphi}_2\dot{S}_D\ell = F_{np}\ell, \tag{38}$$

where in the equation (38), the multiplication of all members by factor of ℓ (unit length) is caused by the necessity of presenting all members of the differential equations (37) and (38) with a same dimension (N·m).

The non-linear differential equations of second kind (37) and (38) can be solved with respect to φ_2 and S_D, and the laws of motion of the input links 2 and 5 can be determined by the Runge-Kutta method, or by using numerical methods developed for solving such systems.

By way of example, we shall consider the case, when the sizes of the links of a mechanism are as follows: ℓ_2=0,060 m, ℓ_3=0,200 m; ℓ_4=0,140 m, the motion of a sliding block H=0,060m; S_{D_O}=ℓ_{AD_0}=0,160 m; the number of rotations of crank is n_2=60 min^{-1}; accordingly $\dot{\varphi}_2$=6,28 sec^{-1} and u_{52}=0,019 m; the law of motion of the second input link (sliding block) $S_D = S_D(t)$ is shown graphically (Fig.3); the masses of the links: $m_2 = 0.5\,kg$; $m_3 = 1{,}8\,kg$; $m_4 = 1{,}3\,kg$; $m_5 = 3\,kg$; accordingly, $G_2 = 4{,}9\,N$; $G_3 = 17{,}6\,N$; $G_4 = 12{,}6\,N$; $G_5 = 29{,}4\,N$; the external force $F_{ext} = 150\,N$; the external moment $M_{ext} = 200\,Nm$; the drive moment $M_{D2} = M_2 = 420\,Nm$; the drive force $F_D = 300\,N$; $I_2 = 0.026\,kg\cdot m^2$, $I_3 = 0{,}082\,kg\cdot m^2, I_4 = 0{,}063\,kg\cdot m^2$.

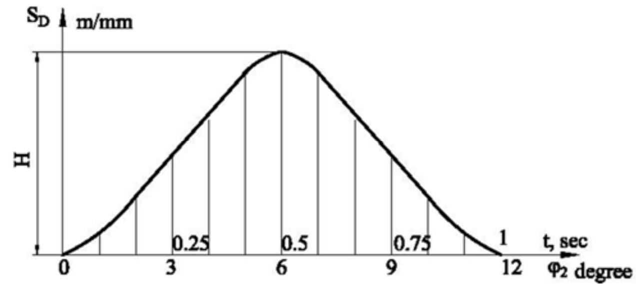

Figure 3. *The law of motion of the link 5 (a sliding block).*

By solving the differential equations (37) and (38), we obtain the laws of motion of the input links 2 and 5, as well as the angular and linear velocities and accelerations of these links.

Figures 4-7 illustrate the graphs of changes in motion (Fig. 4, 5) and velocity (Fig. 6, 7) of the input links in the ideal (Fig. 4,a; 5,a; 6,a; 7,a) and real (Fig. 4,b; 5,b; 6,b; 7,b) mechanisms.

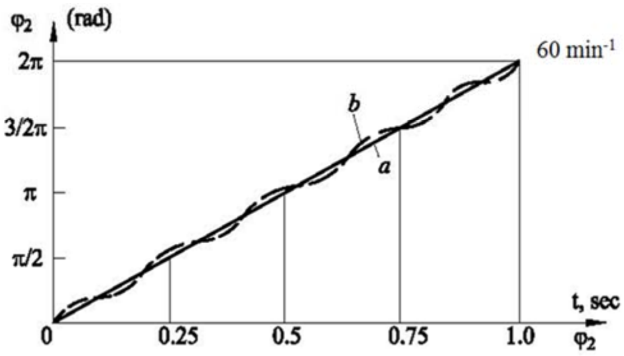

Figure 4. *The graphs of changes in motion of the input link (crank).*

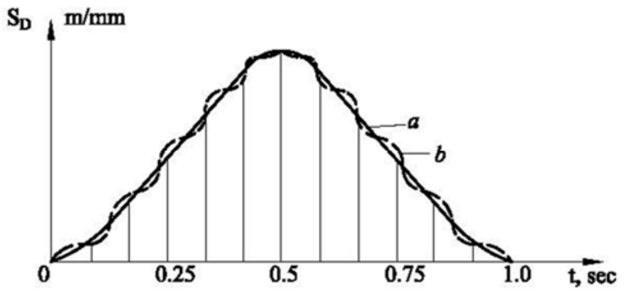

Figure 5. *The graphs of changes in motion of the input link (a sliding block).*

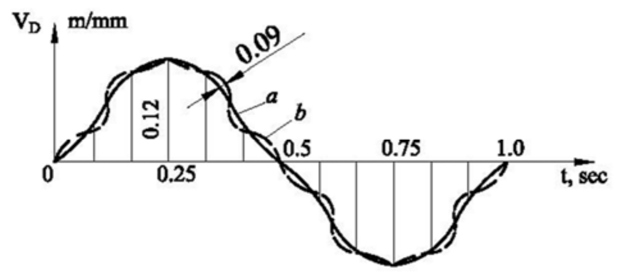

Figure 6. *The graphs of changes in the velocity of the input link (a sliding block).*

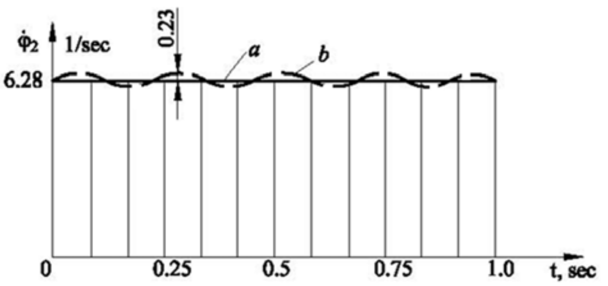

Figure 7. *The graphs of changes in the velocity of the input link (crank).*

Thus and so, by solving the differential equations (37) and (38), there have been obtained the displacements and velocities of the input links of a crank and a sliding block.

Comparative analysis of the results gives reason to conclude that a planar five-link hinged RRRRT type mechanism with two degrees of freedom must be studied in both kinematics and dynamics, which allow for assessing performance of the ideal and real mechanisms.

3. Conclusion

Based on the analysis of the planar five-link hinged RRRRT-type mechanisms with two degrees of freedom, which are used in the special-purpose devices, there have been determined the positions, velocities and accelerations of the input links and their points. There have been determined kinetic energy, the reduced moment and the reduced force of a mechanism. Also, there have been derived the non-linear differential equations of second kind, which allow for determining the laws of motion, velocity and acceleration of the input links of a mechanism. The example considered was the solution of dynamic problems for the case, when all the links of a mechanism are rigid. The results, which are given in graphs, have allowed for making a comparative analysis of the ideal and real mechanisms.

References

[1] N. Davitashvili, "Theory of hinged-lever mechanisms with two degrees of freedom", Tbilisi: Georgian Committee of IFToMM, 2009. – 372 p. (In Russian).

[2] V. V. Dobrovolski, "Theory of mechanisms with two or more degrees of freedom" in Works of the Moscow Machine Tool Design Institute. Moscow, 1939, Vol. 4, pp. 41-76. (In Russian)

[3] R. Beier, "Kinematisch–Getriebedynamisches Praktikum", Berlin: Springer. 1960. -354 p.

[4] B. M. Abramov, "The Dynamics of the hinged mechanisms with account for friction" Kharkov: Kharkov University. 1960. -150 p. (in Russian).

[5] S. N. Kozhevnikov, "The dynamics of a two-degree-of-freedom mechanisms" in Modern problems of the Theory of Machines and Mechanisms. Moscow, 1965, pp. 211-222. (InRussian).

[6] V. A. Zinovyev, A. P. Bessonov, "Foundations of the dynamics of machinery". Moscow: Mashinostroyenie, 1964. -240 p. (In Russian).

[7] A. P. Bessonov, V. A. Ponomaryov, "On the dynamics of one new scheme of a two-degree-of-freedom mechanism" in Collection of scientific works of the Chelyabinsk Polytechnic Institute, Chelyabinsk. 1974, №142, pp. 210-214 (In Russian).

[8] A. G. Ovakimov, "The dynamics of the 2-DOF mechanisms in the form of particle dynamics" in The dynamics of the machines. Moscow: Mashinovedenie 1969, pp. 265-276. (In Russian).

[9] N. Davitashvili, O. Gelashvili, "On some problems of synthesis of special five-bar hinged mechanisms with two degrees of freedom" in International Scientific Journal of Mechanical Engineering and applications. 2014, № 2(6), pp. 104-110.

[10] N. Davitashvili, O. Gelashvili, "Analysis of the special five-bar mechanisms with two dwells of output links" in International Scientific Journal of IFToMM "Problems of Mechanics". Tbilisi, 2016, № 3(64), pp.5-20.

[11] N. Davitashvili, G. Namgaladze, "Equation of couple curve of planer five-bar hinged RRRRT type mechanism with two degrees of freedom and its synthesis of desired conditions" in International Scientific Journal of IFToMM "Problems of Mechanics". Tbilisi, 2016, № 4(65), pp. 5-14.