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Proposition of a New High Speed and High Efficiency Control Method for Plural Eigen Frequencies by Changing Topology

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Abstract: We have developed many types of transport boxes by origami-operation and space filling operation. But it has not been solved that the fruits, vegetables, strawberries, cells, blood and a bottle of liquor are damaged, broken during transportation. It is the greatest factor in this situation that there is a danger vibration frequency band where these are easy to scratch and are prone to death. If there are eigen frequencies within this danger frequency band, it needs that those eigen frequencies within this danger frequency band are moved out of the band. But it is difficult to apply the existing topology optimization methods using homogenization method or density method to control plural eigen frequencies for two reasons. One reason is that even if homogenization method or density method is used, finally after convergence, holes are made on finite elements of which the size of the homogenizing element or the thickness is smaller than the threshold by keeping the rest the original size. In such processing, there is a possibility that the converged solution again deviates from the convergence value. Another reason is that it is difficult to control plural eigen frequencies simultaneously because some eigen frequencies go up and some ones go down no matter where it is reinforced although displacement at any point goes down in static problem. Although in such way, it is very difficult to control plural eigen frequencies, here we propose a new high precision and high efficiency method for controlling plural eigen frequencies simultaneously using the kinetic energy density and the strain energy density.

Keywords: Origami Engineering, Transportation Box, Topology Optimization, Density Method, Index of Generalized Eigen Frequency, Kinetic Energy Density, Strain Energy Density, Danger Frequency Band

1. Introduction

Fruits and vegetables, blood, induced pluripotent stem (iPS) cells, bottles of liquor, wine have frequency bands that are susceptible to damage. By designing a box such that the box system with the items does not have the eigen frequencies within these frequency bands, the items can be safely transported. When the car is running, if the vibration between 5 Hz and 10 Hz is large, passengers are likely to get car sickness. Therefore, automobile companies design seats such that the eigen frequencies of the floor structure system do not lie in this danger frequency band. Therefore, it is necessary in product development and manufacturing to control the eigen frequencies. The frequency band in question is called here the danger frequency band, and if the eigen frequency of the vibration system is in the danger frequency band, it is redesigned to move those eigen frequencies out of the band. In this study, we will conduct a basic study on the problem of moving the eigen frequencies out of the band to solve the above-mentioned problems. To expel the eigen frequencies from the danger frequency band, for example, it is redesigned that some eigen frequencies to be higher and some eigen frequencies to be lower.

In general, it is difficult even with a slight movement of the eigen frequency by dimensional optimization and the birth of topology optimization method was expected. But topology optimization method was not easy to realize.

Finally, in 1988, topology optimization method using homogenization method was developed [1]. Since then, a great deal of researches has been done up to the present day [2]. By making the mesh infinitely fine, mathematically the convergence of the solution is guaranteed [3]. This caused that the mesh was considerably finely tuned for a while after its birth in 1988 and it was difficult to use mathematical optimization methods on computers at that time. And the use of the optimality criterion method which requires less computer resources became mainstream. For this reason, the application to dynamic topology optimization was initially difficult, but the reason of this difficulty has also been indicated [4]. For the homogenization topology optimization method, it is actually difficult to make the elements infinitely small, and elements of finite size are used. And the size of the homogenizing elements within each element is normalized between 0 and 1, and after the optimization routine, for example, holes are made on the part below the threshold size. Therefore, one of the authors, Hagiwara, thought that if such a use that deviated from the original theory was allowed, it would be permissible to apply mathematical optimization methods to a dynamic problem with elements of a size large enough in the computers of the time. When this was actually applied, results comparable to the ones by optimality criterion method with much more precise meshes were obtained [5]. Moreover comparison of using plate thickness as a design variable and using homogenization indicated that there was no big difference [6]. Given this result and for simplicity, nowadays, the so-called density method is used for the topology optimization where plate thickness and density are selected as design variables. Since the paper [1], one of the authors, Hagiwara himself, has published several topology optimization papers related to vibration and noise [7-13]. As far as the problem "Simultaneously handle multiple eigen frequencies, including lowering some eigen frequencies and increasing other eigen frequencies", it is highly doubtful whether the series of methods that have been developed is applicable. One is that although it does not converge even after specified number of iterations of optimization, it is not sure if there is a solution to the problem. Furthermore, for example, when element thicknesses are used as design variables, even if the desired eigen frequencies are obtained by optimization, In the conventional method, a certain threshold value is determined, holes are made in the parts below it, and the rest is applied to the current plate thickness, and if this is executed, it will deviate from the goal value. This way it is still not easy to bring it up to a concrete design. In the eigenvalue problem involving stiffness and mass, if the plate thickness of an arbitrary part is increased, some eigen frequencies increase and some decrease, and it is difficult to blindly apply the topology optimization method.

Therefore, we try to develop a new method of controlling the eigen frequencies by returning to the origin of the vibration theory, in which the eigen frequency is determined by the equivalent stiffness and the equivalent mass. In chapter 2, it is handled two cases: convergent and not convergent for controlling plural eigen frequencies non- including in the danger frequency band using conventional density topology optimization techniques. In chapter 3, the proposed method is applied to the same two cases as in chapter 2 and it is indicated that solutions can be obtained very easily for the converged problem in chapter 2, and it is clarified why it cannot be converged for the non-convergent problem in chapter 2. In chapter 4, it is summarized the results.

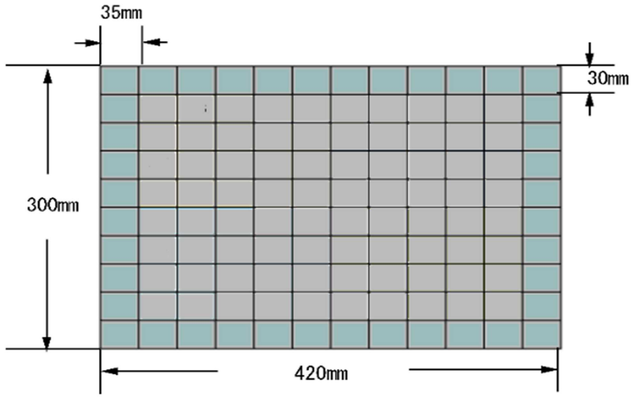

Figure 1. *FEM model for test piece: 120 elements using vertical and horizontal symmetry. Eigen frequencies from 7th to 10th under free-free condition: 8.14Hz, 9.38Hz, 19.14Hz, 19.34Hz.*

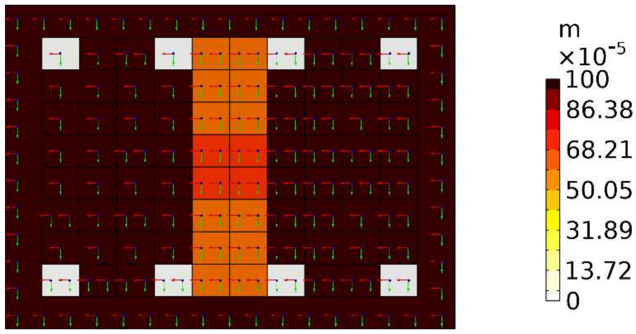

Figure 2. *Relationship between objective function and number of iterations to keep all eigen frequencies out of danger frequency band (from 7Hz to 17Hz). As a result, the goal was achieved.*

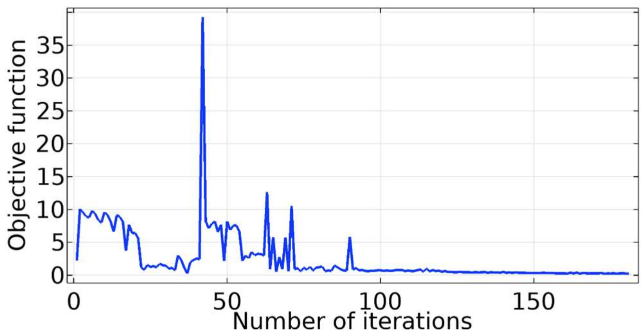

Figure 3. *Left: Shape obtained by optimization simulation. Navy blue: 1.0mm, White: 0.1mm, eigen frequencies (7^{th}, 8^{th}, 9^{th}, 10^{th}) = (5.67Hz, 6.91Hz, 17.23Hz, 17.34Hz). Right: Making holes less than 0.3mm, Eigen frequencies (7^{th}, 8^{th}, 9^{th}, 10^{th}) = (7.33Hz, 7.53Hz, 18.00Hz, 18.14Hz).*

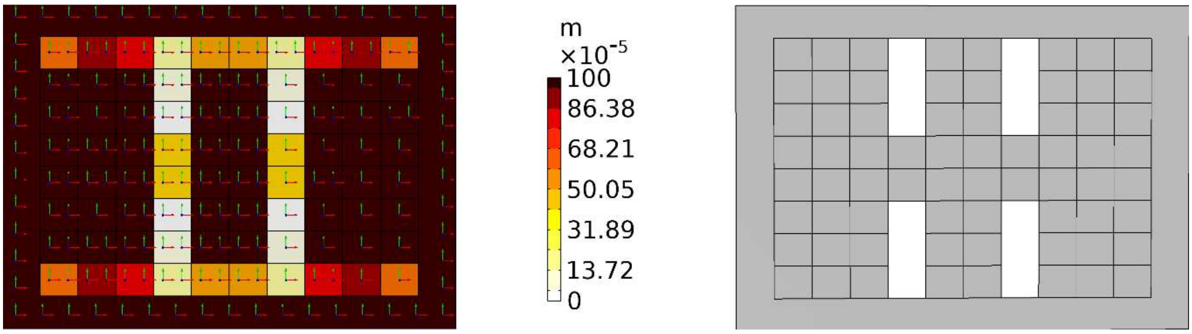

Figure 4. *Relationship between objective function and number of iterations to keep all eigen frequencies out of danger frequency band (from 8Hz to 20Hz) As a result, the goal was not achieved.*

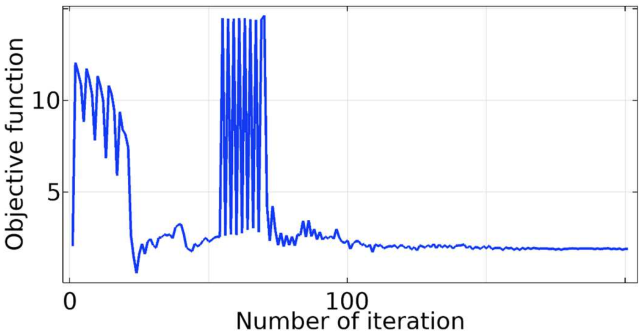

Figure 5. *Shape with plate thickness distribution obtained by optimization simulation. to avoid 8-20Hz. As a result, eigen frequencies (7th, 8th, 9th, 10th) =(7.46Hz, 7.79Hz, 17.99Hz, 18.80Hz).*

2. Application of Topology Optimization Method Using Conventional Density Method

The goal of this research is the design of transport boxes that safely carry fruits and vegetables such as strawberries, cells, and blood. But here the object is to certify the proposed method and so the model is a rectangular flat plate with the size of 420 mm × 300 mm, thickness of 1 mm as indicated in Figure 1. The material of the plate is cardboard, which has a density of 256.9 kg/m^3, Young's modulus of 0.664 GPa, and Poisson's ratio of 0.34. As indicated in the same figure, the flat plate that divided into 120 (30 mm long and 35 mm wide per piece). By modal analysis under unconstrained conditions, first to sixth orders are rigid body modes and the seventh to twelfth orders are 8.14 Hz, 9.38 Hz, 19.14 Hz, 19.34 Hz, 23.85 Hz, and 28.20 Hz.

2.1. Application of Density Topology Optimization Method to the Problem with Two Eigen Frequencies in the Danger Frequency Band

Here, by using the density topology optimization method, we deal with the problem where the band between 7.0 Hz and 17.0 Hz is considered to be a danger frequency band with the above-described model. Because the seventh and eighth eigen frequencies are in this danger frequency band, we consider to decrease both of these eigen frequencies to 7.0 Hz or less while the other eigen frequencies are kept at their current values. The objective function of the optimization is the generalized eigenvalue index [4] indicated in Eq. (1),

$$f_x = f_0^* + \left(\sum_{i=1}^{m} W_i (f_i - f_{0i})^n / \sum_{i=1}^{m} W_i\right)^{1/n} \tag{1}$$

f_i (i = 1~4) are the target values 6.0Hz, 6.5Hz, 21.5Hz, and 22.0Hz of the seventh to tenth eigen frequencies, f_0*=0 Hz, n = 2, and m is the number of eigen frequencies to consider, here m = 4. W_i(i=1~4) are weights. The density topology optimization is performed as follows. (1) The design variables are the plate thickness of each element except the plate thicknesses of the elements in surrounding part so that the outer shape does not change by optimization calculation. (2) As indicated in Figure 1, the elements are divided into right and left, up and down symmetry, and is also set so that symmetry is maintained. The thickness of each element shall be 0 mm in the lower limit and 1.0 mm in the upper limit. (3) The convergent condition is the value of Eq.(1) is 0.1 or less. (4) Modal analysis using the finite element method and optimization by linear approximation method [14] are both with COMSOL Multiphysics [15]. (5) Weight reduction is given as a constraint function. Here, the total weight of the cardboard in the shape of Figure 1 is 32.4 g, and the weight corresponding to 40 elements of the surrounding part is 10.8 g so that the lower limit is 11 g, and the upper limit is 90% of current weight, 29.16 g.

Here it is set the target values of seventh to tenth eigen frequencies. The seventh and eighth eigen frequencies within the danger frequency band, shall be changed to 6.6 Hz and 6.8 Hz. Although the ninth and tenth orders are higher than 17 Hz, that is, these eigen frequencies are out of danger frequency band in the current state, these eigen frequencies may be lower than 17Hz during the optimization process so that it is also set the target eigen frequencies of the nineth and tenth eigen frequencies to 19.14 Hz, 19.34 Hz, or more respectively. Since the eleventh eigen frequency is 23.85 Hz, that is, away from 17 Hz, it starts by not considering the eleventh or higher eigen frequencies. The above optimization has been performed. With W= [1,1,1,1], the repeat number 180 or later, the seventh to tenth eigen frequencies converge to 7.21 Hz, 7.51 Hz, 17.81 Hz, and 18.44 Hz respectively, that is, it does not converge to the target frequencies.

Although ninth and tenth orders reach the target ones, seventh and eighth orders do not reach. On reconsideration with W= [2,2,1,1], the repeat number 160 or later, the seventh to tenth eigen frequencies converge to 6.56 Hz, 6.93 Hz, 16.97 Hz, and 17.46 Hz respectively. That is seventh, eighth and tenth orders reach the target values but ninth order reaches lower than 17 Hz in the danger frequency band.

After repeated trial and error, the repeat number 185 or later, the seventh to tenth orders converge to 5.67 Hz, 6.91 Hz, 17.23 Hz, and 17.34 Hz respectively with W= [1,10,10,1] where the target eigen frequencies of seventh to tenth orders are 5.5 Hz, 6.74, 17.50 Hz and 17.70 Hz. This means that although the convergence condition is not satisfied, all of the eigen frequencies are out of the danger frequency band. Figure 2 is a graph of the transition of the generalized eigenvalue index until the end, where the vertical axis is the value of Eq.(1) and the horizontal axis is repeating counts. The weight is reduced to 85% of current weight, 27.6g. Although the calculation time is 40 minutes, it has finally arrived after more than 10 trials and errors, and so this gives us the impression that the application of the conventional method to practical design is very difficult. The shape after optimization is indicated on the left side of Figure 3, and the dark blue color is the thickest, 1 mm, and the thinnest white part is 0.1mm. Many papers end here. However, it is not realistic for our ultimate goal to use the structure for actual safe shipping box design because the plate thickness is not uniform. Based on the result on the left of Figure 3, for example, conventional methods recommend to make holes on the plate where the plate thickness is 0.3 mm or less. This result is on the right of Figure 3. This structure gives 7.33 Hz, 7.53 Hz, 18.00 Hz and 18.14 Hz of seventh to tenth eigen frequencies under the same unconstrained conditions. This deviates from the goal again. And so even if the goal can be achieved by optimization, it is not easy to bring it to a concrete design.

2.2. Application of Density Topology Optimization Method to the Problem with Four Eigen Frequencies in the Danger Frequency Band

When the danger frequency band is set from 8.0 Hz to 20.0 Hz, in addition to 8.14 Hz of the seventh order and 9.38 Hz of the eighth order, 19.14 Hz of the ninth order and 19.34 Hz of the tenth order are also issues within the danger frequency band. As in the previous section, first, the objective function uses Eq.(1), the target frequencies of the seventh to tenth orders are 5.6 Hz, 6.8 Hz, 20.5 Hz, and 20.7 Hz, respectively with W = [1, 1, 1, 1], and the thicknesses of all elements except the surrounding part are the design variables, with lower limit of 0 mm and upper limit of 1 mm under the same end condition as in the previous section. It converges at iteration number 190. Figure 4 indicates the transition of Eq.(1) up to this point. The shape at that time is indicated in Figure 5. Eigen frequencies from the seventh to the tenth orders are 7.46 Hz, 7.89 Hz, 17.99 Hz, and 18.80 Hz. That is, it can not reach the goal where no eigen frequencies are from 8 Hz to 20 Hz. Below, we conducted the same trial and error as in the previous section, but the result did not reach the goal.

As described above, in the conventional density topology optimization method, when convergence is difficult, it is unclear why, and even if convergence is possible, it is difficult to incorporate it into a practical design with the thickness distribution. And to brake this, if you make holes on the plate where the plate thickness is below the threshold, it deviates from the goal. In this way, obtaining actual design specifications is extremely difficult with the conventional methods. In the next chapter, we propose a new method to solve these problems.

3. Proposition of Topology Optimization Method Using Energy Density

The newly proposed topology optimization method using energy density is as follows. First, the energy density distribution of each eigen frequency mode to be moved is investigated, and the position of the spring part with large strain energy density distribution and the mass part having large kinetic energy density are grasped for each eigenmode. If you want to lower the eigen frequency, holes are provided on the spring part or reinforcements are provided on the mass part. If you want to increase it, holes are provided on the mass part or reinforcements are provided on the spring part. The eigen frequency is controlled by providing holes or reinforcements, but from the viewpoints of weight reduction and work efficiency, the work of providing holes is first made.

Each eigen angular frequency is given by Eq. (2).

$$\omega_n = \sqrt{\frac{k_n}{m_n}} \tag{2}$$

Where ω_n, k_n, and m_n of Eq.(2), are nth order eigen angular frequency, equivalent stiffness, and equivalent mass. If the equivalent stiffness becomes smaller by providing holes on the spring part, or if the equivalent mass is increased by providing reinforcements on the mass part, the eigen angular frequency decreases. Conversely, when it is necessary to increase the eigen angular frequency, holes are provided on the mass part and/or reinforcements are provided on the spring part. This is the basic idea of this newly proposed method.

3.1. Application to the Problem That Can Be Solved by Conventional Density Topology Optimization Method

As indicated in the previous chapter, the solution is not easily obtained by using the conventional density topology optimization method. In this section, referring to the energy density, it is delt with the problem that the danger frequency band is between 7 Hz and 17 Hz, which is the same as that obtained the goal value after trial and error in section 2.1. When the modal value analysis of the flat plate of Figure 1 is performed under unconstrained conditions, the eigen frequencies from seventh to twelfth orders are 8.14 Hz, 9.38 Hz, 19.14 Hz, 19.34 Hz, 23.85 Hz, and 28.20 Hz as described above. The seventh and eighth eigen frequencies are within the danger frequency band between 7 Hz and 17 Hz, and we first aim to lower the seventh and eighth orders. To lower the seventh and eighth orders, it is considered at first that the eleventh order does not move into the danger frequency band but there is also a concern that the ninth and tenth orders may enter the danger frequency band. The left of Figure 6 indicates the strain energy and kinetic energy density distributions from the seventh to the tenth orders. The red part is the part with high energy density and the dark blue is low. The area of the seventh order spring part encompasses the eighth order spring part. Therefore, as indicated on the left of Figure 6, first, it is considered to provide a hole in the area indicated by the circle on the eighth spring distribution figure. Here, the concern is whether the ninth and tenth eigen frequencies do not transition to 17 Hz or less. When viewed from the ninth order energy distributions, since the spring is superior to the mass on this hole, it is predicted that the ninth eigen frequency will decrease due to the providing this hole. And although in the case of the tenth order, it is not so clear as in the case of the ninth order, it is predicted that the tenth eigen frequency will also decrease because the spring is superior to the mass on this hole part. The right model of Figure 6 with a hole in the area indicated by the circle gives the eigen frequencies of the seventh to ninth orders are 5.49 Hz, 7.17 Hz, 16.26 Hz, and 18.01 Hz under unconstrained conditions, that is, seventh order meets the goal but eighth order still does not meet the condition. The ninth and tenth orders decrease as expected, especially the ninth order decreases down to the danger frequency band lower than 17 Hz or less. Due to the above, it is necessary to lower the eighth order and increase the ninth order. The energy densities of Figure 6 are indicated on the left of Figure 7. By focusing on the ninth order kinetic energy density distribution, a hole is provided in the part indicated by a circle on the ninth kinetic distribution. The center coordinate of the hole is (210, 150) and the radius is 90 mm. Considering the influences of this hole on other modes, it is predicted that the seventh eigen frequency will increase slightly because the mass is superior to the spring on this hole part and the eighth eigen frequency will decrease because the spring is superior to the mass on the hole part. Furthermore, the tenth order is predicted to decrease because the spring is superior to the mass on the hole part. The eigen frequencies with the hole on the right side of Figure 7 under unconstrained conditions are 6.09 Hz, 6.40 Hz, 17.26 Hz, and 17.37 Hz, that is, all are changed in the predicted direction and the results reach the goal. The weight of the model on the right side of Figure 7 obtained here is 76% of the initial weight. Thus, according to the proposed method, compared to the conventional topology optimization method it is indicated that the target result can be obtained interactively and overwhelmingly in a short time and more accurately, while predicting the direction of each eigen frequency change. By combining this with the surface response optimization method [16], for example, it is possible to set more strictly the goal frequency such as 6.5 Hz compared to 7.0Hz or less this time after adding the minimum weight condition but this will be discussed in the following report.

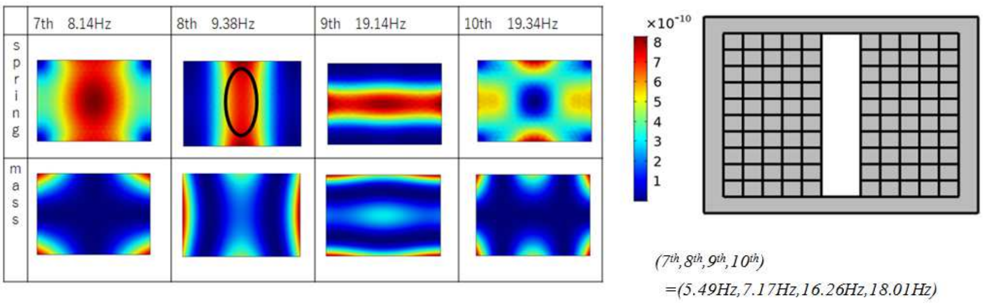

Figure 6. *Top row: Eigen frequencies of the plate model in Figure 1.*

Upper left: Spring part extraction from strain energy density distribution of the plate model in Figure 1,
Lower left: Mass part extraction from kinematic energy density distribution of the plate model in Figure 1,
Right: The plate model generated by making a hole on the blue ○ part of the eighth strain energy distribution of the plate in Figure 1.

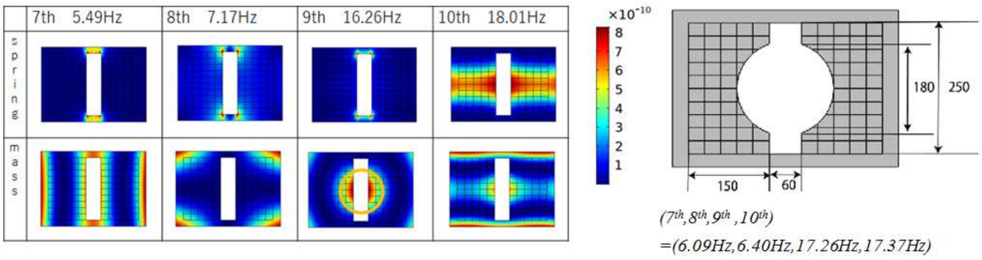

Figure 7. *Top row: Eigen frequencies of the plate model in Figure 6 right.*

Upper left: Spring part extraction from strain energy density distribution of the plate model in Figure 6 right,
Lower left: Mass part extraction from kinematic energy density distribution of the plate model in Figure 6 right,
Right: The plate model generated by making a hole on the yellow ○ part of the ninth dynamic energy distribution of the plate in Figure 6 right.

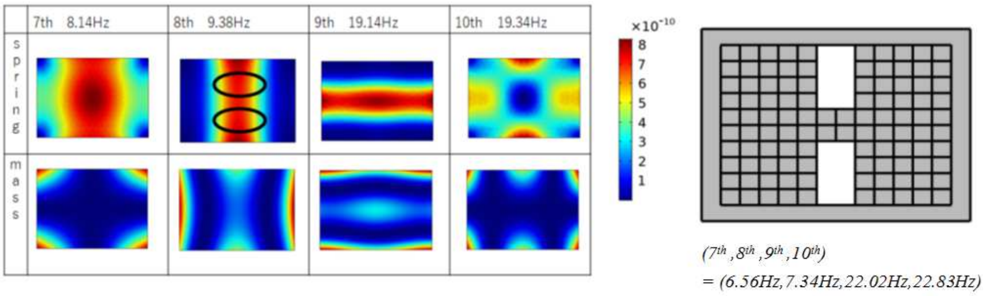

Figure 8. *Top row: Eigen frequencies of the plate model in Figure 1.*

Upper left: Spring part extraction from strain energy density distribution of the plate model in Figure 1,
Lower left: Mass part extraction from kinematic energy density distribution of the plate model in Figure 1,
Right: The plate model generated by making a hole on the ○ part of the 8th strain energy distribution in Figure 1.

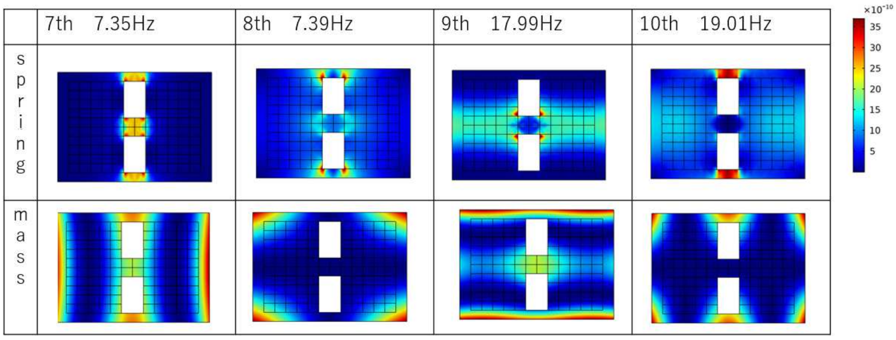

Figure 9. *Top row: Eigen frequencies of the plate model in Figure 8 right.*

Upper: Spring part extraction from strain energy density distribution of the plate model in Figure 8 right,
Lower: Mass part extraction from kinematic energy density distribution of the plate model in Figure 8 right.

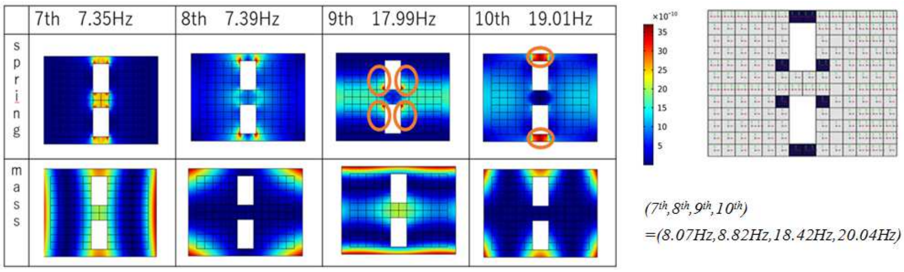

Figure 10. *Top row: Eigen frequencies of the plate model in Figure 8 right.*

Upper left: Spring part extraction from strain energy density distribution of the plate model in Figure 8 right,
Lower left: Mass part extraction from kinetic energy density distribution of the plate model in Figure 8 right,
Right: The plate model generated by reinforcing (doubling the plate thickness) on the ○ parts of 9^{th} and 10^{th} strain energy density distributions.

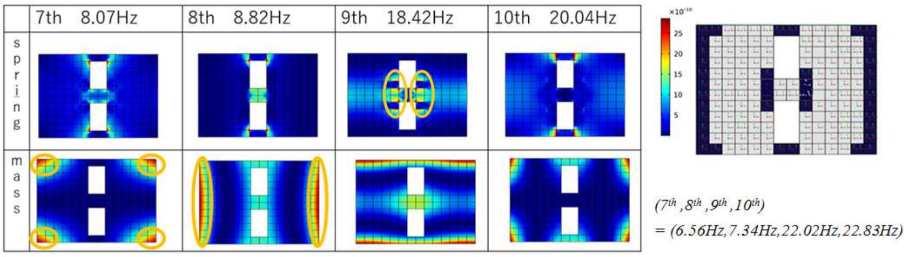

Figure 11. *Top row: eigen frequencies of the plate model in Figure 10 right.*

Upper left: Spring part extraction from strain energy density distribution of the plate model in Figure 10 right,
Lower left: Mass part extraction from kinematic energy density distribution of the plate model in Figure 10 right,
Right:: The plate model generated by reinforcing (doubling the plate thickness) on the ○ parts of 9^{th} and 10^{th} kinematic energy density distributions of the plate model in Figure 10 right.

3.2. Application to the Problem That Cannot Be Solved by Conventional Density Topology Optimization Method

In this section, we try to apply this proposed method to the problem with the frequency band between 8 Hz and 20 Hz as a danger frequency band which can not be coped with by the conventional method. Here again, the seventh to tenth eigen frequencies obtained under unconstrained conditions with the flat plate model of Figure 1 are 8.14 Hz, 9.38 Hz, 19.14 Hz, and 19.34 Hz. Because these eigen frequencies exist in the danger frequency band, it is necessary to move these four eigen frequencies so that the seventh and the eighth eigen frequencies are below 8 Hz, on the other hand, the ninth and tenth order eigen frequencies are moved into 20 Hz or more. We first try to achieve the goal by changing topology by installing holes, as per basic premise. The left side of Figure 8 is the kinetic and strain energy density distributions of the seventh to tenth orders of Figure 1 flat plate. As indicated in section 3.1, both of the seventh and eighth orders have prominent spring parts, and since the seventh order spring part includes the eighth order spring part, it is considered that a hole is installed with reference to the eighth order spring part. However, both of the ninth and the tenth orders have more superior spring part to the mass part on the hole area and so it is expected that both of the ninth and tenth orders will go down, further away from the goal. Although we want to increase the ninth and tenth eigen frequencies by installing holes, it is not possible because there are mass parts only on the surrounding part that cannot be added to design variables. Therefore, at this stage, we abandoning to increase the ninth and the tenth eigen frequencies and a hole is installed in the circled part to lower the seventh and eighth eigen frequencies. The right side model of Figure 8 gives 7.35 Hz, 7.39 Hz, 17.99 Hz and 19.01 Hz of the 7^{th} to 10^{th} eigen frequencies, that is, seventh and eighth orders reach the goal, but both of the ninth and tenth orders decrease as expected. As indicated in Figure 9, since there is no mass part that can be made holes from the kinetic energy density distribution in this state, it is difficult to deal with further. It can be seen that it is difficult to achieve the task by raising the eigen frequencies of the ninth and tenth orders any further with only the method of making holes. From the above, since the goal for eigen frequencies is set not to be achieved even with this proposed method, it was not possible to apply the conventional density method as indicated in section 2.2. It is possible to use this new method for a judgment whether the problem is set appropriately by the conventional method. Even if the goal cannot be achieved by installing holes alone, there is a possibility that the goal can be achieved by placing reinforcement, so in the next section, we will consider the control of the eigen frequencies by including reinforcement.

3.3. Control of the Eigen Frequencies by Including Holes and Reinforcements

Here, again, the seventh to tenth order eigen frequencies obtained under unconstrained conditions of the flat plate model in Figure 1, are 8.14 Hz, 9.38 Hz, 19.14 Hz, and 19.34 Hz. In addition to holes, reinforcements are also attempted to move the seventh and eighth orders to 8 Hz or less, and the ninth and tenth orders to 20 Hz or more. As discussed in section 3.2, with the hole on the spring part of eighth order, the eigen frequencies of the seventh to tenth orders are 7.35 Hz, 7.39 Hz, 17.99 Hz, and 19.01 Hz. It was mentioned in the previous section that it was not possible to increase the eigen frequencies of the ninth and tenth orders any further by installing holes. We start from the examination of reinforcements of the spring parts of the ninth and tenth orders. For ease, reinforcement is done by doubling the plate thickness of the reinforcement part, that is, 2mm. Central of the ninth order in Figure 10 has a remarkable spring part. By reinforcing the central part of the plate enclosed in circles, the eigen frequency of the ninth order increases. Considering this influence on the other orders, since the seventh order has a strong spring part on this central part, the seventh eigen frequency is predicted to increase by reinforcement on the spring part of the ninth order. As for the eighth order, since it is clear that the spring part is winning, it is expected to rise steadily with the reinforcement of the ninth order. For the tenth order, since the spring part is clearly superior, it is conceivable that the goal can be obtained only with the reinforcement of the ninth order, but 1Hz is left until the target frequency. And so again it is tried to install the reinforcement on the tenth spring part where the mass part is superior to the spring one for the seventh order and so the seventh order is expected to decrease in a good direction. For the eighth order, the spring part is winning and there is a possibility that it will increase. For the ninth order, the mass is stronger, and the eigen frequency may decrease. In fact, looking at the eigen frequencies after reinforcement of the ninth and tenth orders, the eigen frequencies are 8.07 Hz, 8.82 Hz, 18.42 Hz, and 20.04 Hz, that is, as expected, the eigen frequencies of the seventh and eighth orders are higher than 8 Hz. The ninth order has not yet reached the goal, and only the tenth order has satisfied the goal. The energy density distribution at that time is indicated on the left of Figure 11. In the seventh order that needs to be lowered first, there are prominent mass parts at the four corners, and when these parts are reinforced, the eigen frequency of the seventh order decreases, and it is predicted that the eighth to tenth orders will also decrease because the mass part is superior to the spring part. Looking at the kinetic energy density distribution of the eighth order, the circled parts at both ends of the flat plate are reinforced. Since there are no mass and spring parts of the ninth and tenth orders on these reinforcement parts, even if these parts are reinforced, there is no influence on the eigen frequencies of the ninth and tenth orders. With the above two reinforcements, there is a possibility that the target frequencies of the seventh and eighth orders can be obtained. Next, looking at the strain energy density distribution of the ninth order, a spring part is recognized in the yellow circle part. Looking at other eigen frequencies, the spring part of the seventh order is superior to the mass on this part, and although the eigen frequency of the seventh order may increase with this reinforcement, it is considered that there is no problem because it is lowered by reinforcement of the eighth order mass part. For the eighth order, it is considered that the strength of the mass and the spring is almost same and so there is no change due to the reinforcement of the yellow circle part of the ninth order spring. Looking at the influence on the tenth order, since there are no spring and mass parts on this reinforcement, it is expected that the tenth eigen frequency will be almost same after this reinforcement. In such way, with reinforcements on the mass parts of the seventh and eighth orders and on the spring circle parts of the ninth order indicated on the right side of Figure 11, the eigen frequencies become 6.56 Hz, 7.34 Hz, 22.02 Hz, and 22.83 Hz, that is, each eigen frequency changes in the predicted direction and the target value is obtained. Using the proposed method in this way, it is possible to make various design specifications. In the future, we plan to conduct studies that incorporate optimal design methods such as minimum weight.

4. Conclusion

In the design and manufacturing field, there is a desire to move multiple eigen frequencies simultaneously, sometimes significantly. The most useful method that can meet this demand is considered to be topology optimization, and many studies have already been conducted. In this paper, we also attempted density topology optimization using generalized eigenvalue indices for the objective function to control multiple eigen frequencies simultaneously. However,

there was no way to judge whether the problem converges in the first place, and after trial and error, the goal could not be achieved in the end. In addition, even if it can be achieved once, the plate thickness of the structure obtained by topology optimization is distributed, and as an actual design specification, holes should be installed at places below the threshold on the current structure. This is often done, but it has also been indicated that there is a possibility that the converged solution again deviates from the convergence value. Therefore, we return to the origin of the vibration theory that each eigen frequency is determined by the equivalent stiffness and the equivalent mass. If the eigen frequency is to be lowered, it will be done by making holes on the spring part or by reinforcing the mass part and if the eigen frequency is to be increased, it will be done by making holes in the mass part or by reinforcing the spring part. According to this proposed method, it has been indicated that the goal results can be obtained more accurately because we can predict the direction of change of each eigen frequency after design change, so to speak, interactively and overwhelmingly in a short time, compared to the conventional density topology optimization. Moreover, it was indicated that the new proposed method can be used to validate the problem set in the conventional topology optimization.

In this study, we have indicated for the first time that eigen frequencies can be controlled interactively and extremely efficiently while referring to the strain and the kinetic energy density distributions, but as a future development of this technique, we are specifically considering its application to strawberries and liquor shipping boxes. That time, by combining the response surface optimization method, we intend to discuss the design under the condition of minimum weight while maintaining strength and rigidity. In addition to the development of vibration-safe transport boxes, which is the purpose here, it can be widely applied to issues such as ride comfort, maneuvering stability, interior noise of automobiles which have a frequency band that governs them. We plan to expand on such issues as well.

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