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# An Integrated Multi-Criterion Decision-Making Analysis to Rank the Pareto-Front Solutions of Time-Cost Trade-Off Problems

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**Abstract:** In the construction management planning, both the client and the contractor are interested in completing the project within the planned schedule. To achieve it, time-cost trade-off problem (TCTP) is carried-out to obtain the optimal set of time-cost alternatives. Although Pareto front solutions are not preferred to each other, the decision-maker (DM) has to choose only the best solution. The DMs are neither well-educated nor have adequate knowledge to make proper decisions. Thus, such a choice has to be made through additional preferences not included in the original formulation of the optimization problem. To better support the meta-heuristic optimization outputs, in this paper, an integration of entropy weight, simple additive weighting (SAW), and technique for order of preference by similarity to ideal solution (TOPSIS) are modelled to solve the MCDM problem, while the Teaching Learning Based Optimization (TLBO) algorithm is applied to solve the proposed multi-objective decision-making model. In the proposed model, the weights selections are done objectively to demonstrate the variation on the rankings of MCDM approaches. While the entropy technique served to determine the weight of the criteria from the original matrix data objectively and the TOPSIS method is employed to rank the alternative Pareto front solutions. The proposed methodology is utilized to rank a set of Pareto optimal solutions of well-known optimization problems. The obtained results are compared against the rankings provided by (SAW) approach to investigate the efficiency of the proposed model. Results demonstrate that the present model can be a favorable decision-making model for the decision-makers.

**Keywords:** Time-cost-trade-off Problems, Pareto-front, Construction Management, TOPSIS

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## 1. Introduction

In construction management field, optimization is a very useful tool to meet the desired objectives under the given constraints. In this field, somewhere in the region of every real-world problem includes simultaneous optimization of often competing objectives. Both the client as well as the contractor tend to have the objectives e.g. time, cost or other targets efficiently be accomplished on schedule. Among all the construction resources time and cost are the most essential key project parameters. Thus, the planners have to execute analysis to compromise between time and cost of a project which is known as time cost trade-off problem (TCTP) in the literature. In the multi-criteria decision analysis, the ranking

problem has been evolved throughout the previous 20 years. In any case, there are no general methodologies. Each technique involves its individual principle and standards. The most favorable approach is to present its own priority. One of the fundamental and most straightforward multi-criteria decision making procedures is sum of the weight calculation model. In this method, a weight is appointed to every rule to exhibit its significance.

Another significant basic decision-making is the analytic hierarchy process (AHP). It is expected that AHP to be one of the MCDM approach, and Thomas L. Saaty is the one by whom mathematical theory was firstly developed. Interactions

among the objectives, basic criteria, sub-criteria, and alternatives of the problem is demonstrated by this method. Besides, qualitative and quantitative variables together considering the priorities of the group or individual in decision making process is evaluated using mathematical methods. Moreover, in a decision problem, evaluates the plenty of options based on more than one criteria, together with qualitative ones, if any, and ranks the most crucial options according to their importance. AHP proposes a process for measuring the consistency of these comparisons. With the resulting Consistency Ratio (CR), it is possible to test the consistency of the found priority vector and thus pairwise comparisons between factors.

The technique for order preference by similarity to the ideal solution (TOPSIS) was first proposed by Hwang and Yoon [1]. TOPSIS method assigns the priori weights that are specified beforehand by the decision-maker. The core of the ranking for this method lies in the distance of alternatives to the ideal and anti-ideal solutions. An alternative that is “closer to ideal” and “farther from anti-ideal” holds a higher ranking. However, TOPSIS produces an inconsistent ranking between the “closer to ideal” and “farther from anti-ideal.” Many authors have used TOPSIS as a decision-making method e.g., Chen, [2]; Gumus, [3] and Yong, [4].

In the outranking methods, the alternatives are ranked according to a pairwise comparison, and if enough evidence exists to judge if alternative  $a$  is more preferable than alternative  $b$ , then it is said that alternative  $a$  outranks the  $b$ . ELECTRE Roy [5] and PROMETHEE are based on this approach of ranking.

Reviewing literature Multi-objective optimization can be integrated with MDCM to solve multi-criteria decision making problems to facilitate the process for the decision makers. To this end, two general approaches are basically available Chaudhuri and Deb [6]. In the first approach Bazargan-Lari [7-9] multiobjective optimization is first used to obtain the set of Pareto-optimal solutions and then MCDM methods are used to select the compromise solution.

Chaudhuri and Deb [6] proposed a novel approach to combine MCDM and multi-objective optimization that allows investigation of the different regions of the Pareto-optimal frontier first and then searching through these regions as many times as required to satisfy the decision makers.

Eirgash and Dede [10] utilized the TLBO algorithm in which multi teachers are assigned to various student groups and with the adaptive teaching factor. In this study, MAWA is used as approach incorporated with the algorithm. Furthermore, 18 and 63 activity projects were used as an empirical examples to obtain successful Pareto front solutions. Comparing to the existing literature studies, the current study obtained better results. In addition, it achieves the global optimum results in 18-activity projects and obtained optimum results in 63-activity projects.

Toğan and Eirgash [11] proposed the TLBO-MAWA model for optimization and tested the performance of the method on projects with 7, 18 and 63 activities. As a result, it has been found that TLBO method has achieved successful

results and has a certain potential in producing better solutions. Furthermore, it is presented that the simplicity of TLBO algorithm is the strength of the method.

Being first introduced by [12], Ng and Zhang [13] used an evolutionary-based optimization algorithm known as the ant colony (ACO) to analyze the multi-objective TCT problem. They concluded that ACO can solve TCT problem with less computational effort.

Al-Zarrad and Fonseca [14] examined the fuzzy activity-based costing method that takes into account uncertainty in time and cost values. According to the results of the analysis, the established model is easy to apply, gives better results than GA, and can increase the reliability of time-cost trade-off decisions. They stated that this would help to establish a more reliable program and reduce the risk of over budget or projects running behind the program.

It is observed that the proposed sole MAWA-TLBO algorithm is not able to find out the optimum solutions for the 18-activity and a more complex 63-activity problems [15].

By reviewing the technical literature, it can be recognized that most preferred metaheuristic algorithms to solve DTCTP problems are GA, ACO, PSO, and improved or hybrid versions of them. However, many other optimization methods were applied to the problems encountered in different engineering fields. One of them is Teaching-Learning Based Optimization (TLBO) [16]. TLBO also largely being used in trading off construction management problems. [17].

Furthermore, as construction projects become larger and more diversified, the complexity in analyzing those projects manually becomes a nightmare and the use of software packages becomes inevitable. Thereby, to assist the process well-known software of MATLAB, and Microsoft Excel computer programs were utilized for calculations. The noteworthy contribution of this study is outlined as follows. To the best of author's knowledge, ranking of Pareto optimal solutions Via integrated MCDM with the MODM in construction management has not been addressed in the literature so far.

The paper is arranged as follows. In the next section a short introduction to the Pareto optimality is represented. The main principles of the TOPSIS and the SAW methods are explained in section 3. The proposed algorithm of the trade-off ranking is given in Section 4. In Section 5, different test cases are considered. The conclusions are provided in the final section.

## 2. Pareto Optimality

The employed multi-objective TLBO algorithm can find out the Pareto front solution which provides flexibility to planners and decision makers in making efficient time-cost decisions. By and large, the Pareto front solutions are the non-dominated solutions in multi-objective optimization which are not preferable compared to one another.

The domination concept defined as: design A dominates design B if it is better in at least one criterion and not worse in all other objectives. The non-dominated set of the entire feasible decision space is called the Pareto-optimal set. The

boundary defined by the set of all point mapped from the Pareto optimal set is called the Pareto optimal front. Comparison steps of non-dominating sorting so called Pareto-optimal solution is the following:

For two solutions A and B, A is non-dominated if:

Cost (A)  $\leq$  cost (B) & Duration (A) < duration (B)

Or

Cost (A) < cost (B) & Duration (A)  $\leq$  duration (B)

Sorting based on duration

If (Cost (A)=Cost (B) & Duration (A)=duration (B)

{Remove (B)}

Else if (Cost (A)  $\leq$  cost (B)) & Duration (A) < duration (B)

Or

Cost (A) < Cost (B)) & Duration (A)  $\leq$  duration (B))

{Remove (B)}

Else

{Remove (A)}

It can be sorted based on the total cost as well. But it doesn't make a big difference which sorting we use.

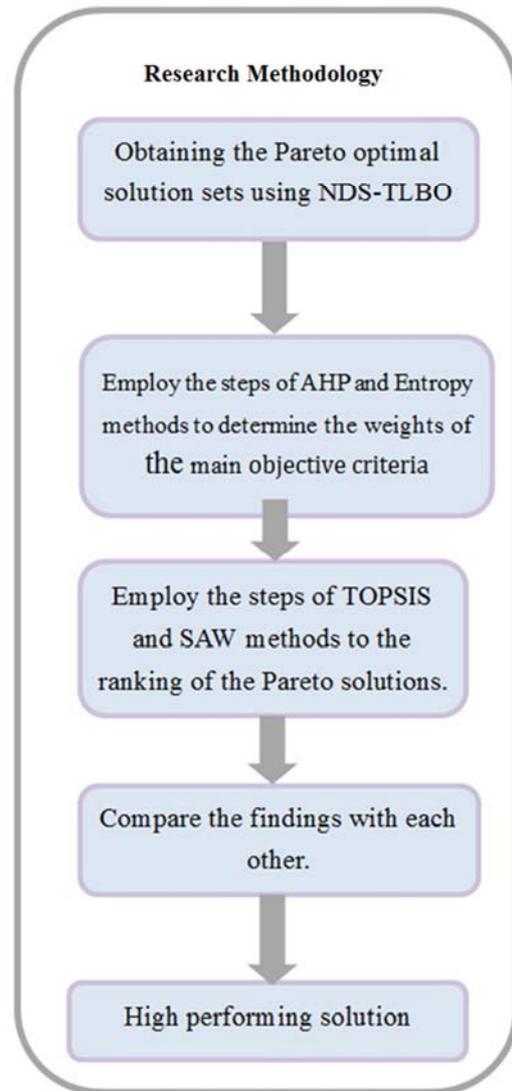
### 3. Research Methodology

The main objectives of this study is to propose an integrated approach, which uses the MCDM techniques into the body of TLBO algorithm for the selection of Pareto-optimal solutions in TCTP problems, and to compare the findings of TOPSIS with SAW techniques. In four basic steps the process performed in this study can be explained: (1) Obtaining the Pareto optimal solution sets using NDS-TLBO approach in MATLAB; (2) Employ the steps Entropy methods to determine the weights of the main objective criteria; (3) Employ the steps of TOPSIS and SAW methods to the ranking of the Pareto optimal solutions and (4) comparing the findings with each other (see Figure 1). A brief explanation of TOPSIS, and SAW methods are given in the following subsections.

Alternative weights creation options are taken as in Table 1 in order to further verify the effect of weights on the MCDM approaches. O1 Index indicates that 0.999 is the weight for the time criteria and 0.001 is the weight for the cost criteria which is obtained using entropy weight approach. To put it simply, these entropy weight values meaning that the time is more significant than the cost to the contractors. Similarly, rest of the indices are given in the same way by the decision maker's preference to observe the effect of the weights on criteria.

**Table 1.** Alternative weights created by the decision maker's preference for the MCDM approaches.

Indices	Weights for time – criteria	Weights for cost – criteria
O1	0.999	0.001
O2	0.001	0.999
O3	0.5	0.5



**Figure 1.** The basic steps of the Pareto-front selection process.

#### 3.1. Shannon's Entropy Method

In the AHP method and other similar evaluation methods, determination of criteria weights is critical. One issue about the weight assignment is that the calculation contains only the information of the individual indicator and thus ignores the relationship among other objectives. The entropy method has been proposed to solve such problems because the entropy weight objectively reflects the original data in a more comprehensive way. The steps of the process are given below:

As mentioned earlier, Shannon's entropy is a common method in achieving the weights for a MCDM problem particularly when attaining a suitable weight based on the preferences and DM experiments are found to be difficult. The original procedure of Shannon's entropy can be expressed in a series of steps:

S1: Normalize the decision matrix.

$$\text{Set } p_{ij} = \frac{x_{ij}}{\sum_{j=1}^m x_{ij}}, \quad j=1 \dots m, \quad i=1 \dots n \quad (1)$$

It is better to eliminate anomalies with various

measurement units and scales to normalize the raw data. This process transforms different scales and units among various criteria into common measurable units to allow for comparisons of different criteria.

S2: Compute entropy  $h_i$  as  $h_i = -h_0 \sum_{j=1}^m P_{ij} \ln P_{ij}$ ,  $i=1 \dots n$ , where  $h_0$  is the entropy constant and is equal to  $(\ln m)-1$ , and  $p_{ij}$  in  $p_{ij}$  is defined as 0 if  $p_{ij}=0$ .

S3: Set  $d_i = 1 - h_i$ ,  $i=1 \dots n$  as the degree of diversification.

S4: Set  $w_i = \frac{d_i}{\sum_{s=1}^n d_s}$ ,  $i=1 \dots n$  as the degree of importance of attribute  $i$ . (2)

This approach is frequently being used in widespread in order to obtain the weights objectively.

**3.2. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)**

TOPSIS is a very common technique in the field of multi-criteria decision making which was first proposed by Hwang and Yoon [1]. The TOPSIS method is one of the multi-criteria decision-making methods. The TOPSIS method can be applied directly to the data without qualitative conversion. The TOPSIS method involves the ranking of the other projects based on the most ideal project for each criterion in the set of options to be evaluated. The method is based on the assumption that the alternative to be selected should be the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. With this method, alternatives are compared according to certain criteria and by evaluating the distances between the maximum and minimum values that the criteria can take to the ideal solution and the non-ideal solution. Then the most similar alternative is chosen. The TOPSIS technique attempts to rank the alternatives based on two parameters; (a) minimum distance from the positive ideal solution; (b) farthest distance from the negative ideal solution. The TOPSIS technique has been widely used in many fields, e.g., management of supply chain industrial robotic system selection, the optimal green supplier selection procedure. In this study, the TOPSIS technique is integrated to the body of TLBO algorithm in order to rank the Pareto-optimal solutions obtained from multiobjective optimization techniques. The TOPSIS method follows the following steps.

1. Step: Vector normalization. The matrix is normalized by taking the square root of the sum of the points or properties of the criteria in the decision matrix. This process is done with the following formula.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad i=1, 2, 3 \dots m \text{ (projects)}, j=1, 2, 3 \dots n \text{ (Criteria)} \quad (3)$$

2. Step: Weighting normalized values. The elements of the normalized decision matrix are weighted according to the importance given to the criteria (creation of a weighted normalized decision matrix). The normalized version of the

10-point scale is used for weighting. This is done as follows.

$V_{ij} = w_j * r_{ij}$  where  $w_j$  is the weight of the  $j$ th criterion.

3. Step: Finding the positive and negative ideal solutions  $A^+$  and  $A^-$  and A-ideal points are defined: Here the maximum and minimum values are determined in each column in the weighted matrix.

The ideal solution consists of the best performance values of the weighted normalized decision matrix; the negative ideal solution consists of the worst values.

$A^+$  (ideal solution) and  $A^-$  (negative ideal solution) are calculated by the following equations.

$A^+ = \{v_1^+, v_2^+ \dots v_n^+\}$  (maximum values)

$A^- = \{v_1^-, v_2^- \dots v_n^-\}$  (minimum values)

$$A^+ = \{(\max v_{ij} / j \in J), (\max v_{ij} / j \in J^+)\} \quad (4)$$

$$A^- = \{(\max v_{ij} / j \in J), (\max v_{ij} / j \in J^+)\} \quad (5)$$

4. Step: Calculation of the Euclidean distance ( $S_i^+$ ,  $S_i^-$ ) of the options to the ideal solutions The distance of the  $J$  alternative from the ideal solution is the ideal separation ( $S_i^+$ ) and the distance from the negative ideal solution to the negative ideal separation ( $S_i^-$ ) is calculated using the following formulas.

$$S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad (6)$$

$$S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2} \quad (7)$$

5. Step: Calculation of the similarities of the options to the positive ideal solution ( $C_i^+$ ) The  $C_i^+$  (Proximity to Ideal Solution) is calculated by the following formula.

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^+} \quad (8)$$

**3.3. Simple Additive Weighting (SAW)**

It is believed that one of the largely preferred MCDM methods is SAW method. It is based on the weighted average evaluation of the attributes. In the SAW method, each one of the attributes is given a certain weight and each alternative is specified with respect to the corresponding attribute. By multiplying the scaled value, calculation of an evaluation score is carried out for each alternative. SAW involves the following steps. Obtain the decision matrix.

Obtain the normalized decision

$$r_{ij} = \left\{ \begin{array}{ll} \frac{x_{ij}}{\max x_{ij}}, & i = 1, \dots, m; \quad j = 1, \dots, n \\ \frac{\min x_{ij}}{x_{ij}}, & i = 1, \dots, m; \quad j = 1, \dots, n \end{array} \right\} \quad (9)$$

Where  $x_{ij} / \max x_{ij}$  is used for positive criteria and  $\min x_{ij} / x_{ij}$  is used for negative criteria. Here,  $x_{ij}$  is the criterion value,  $\max x_{ij}$  is the maximum value for each positive

criterion,  $\min x_{ij}$  is the minimum value for each negative criterion, and  $r_{ij}$  is the normalized value.

Get the weighted score for each alternative

$$A_j = \sum_{i=1}^m w_j r_{ij}, \quad i=1, \dots, m. \quad (10)$$

Here,  $x_{ij}$  is the score of alternative  $i$  to criteria  $j$  and  $w_j$  is the weight of criteria  $j$ . Rank the obtained scores.

## 4. Multi-Objective Optimization Techniques

A time-cost trade-off problem (TCTP) is indeed a multi-objective programming problem that can be unravelled by three distinct methods. In the first method, one seeks the satisfactory solution from the non-dominated solutions based on the experiences and knowledge of decision makers, whereas the determination of the non-inferior solutions is a bit more sophisticated and complicated. The second converts the multi-objective problem to a single-objective problem, and then utilizing a single-objective optimization approach to find the satisfactory solution which is known as weighted method. The final approach utilizes a multi-objective optimization approach to find the satisfactory solution. The method utilized in this paper belongs to the first category, which provides a satisfactory solution, and also determines Pareto-front solution that is beneficial for the further decision making process.

### 4.1. Time-Cost Trade-Off Optimization

The main goal of a discrete TCT optimization problem is to determine a set of time-cost alternatives which provide an optimal balance between the time and cost for project scheduling under the specific conditions. The TCT analysis is implemented to meet the project deadline for a project with a fixed deadline or for a project which is running behind schedule. As mentioned above, TCTP mainly concentrates on selecting appropriate options for every activity to obtain the objective of time and cost of a project. The objective of time of a project can be calculated according to Equations (11) – (14).

$$ES_0 = 0 \quad (\text{the subscript } 0 \text{ represent zero}) \quad (11)$$

$$ES_j = \max_{i \in p_j} \{EF_i\} \quad j = 1, \dots, n+1 \quad (12)$$

$$EF_i = ES_i + t_i^{(k)} x_i^{(k)} \quad i = 0, \dots, n+1 \quad (13)$$

$$T = \sum_i^k t_i^k x_i^k \quad (14)$$

Where,  $T$  is the total time duration of the project and maximization of which is one of the objectives of TCTP. It represents the complete time of critical activities placed on the

critical path of the project activity network.  $ES_j$  and  $EF_j$  are earliest start time and earliest finish time, respectively;  $p_j$  is immediate predecessor of activity  $j$ ;  $t_i(k)$  is duration of activity  $i$  for the  $k$ th option; and  $x_i(k)$  is index variable of activity  $i$ . If  $x_i(k)=1$ , then activity  $i$  performs the  $k$ th option, while  $x_i(k)=0$  means not. The sum of index variables of all options should be equal to 1. Activity 0 ( $n+1$ ) is the only dummy activity.

The total cost of a project composes of direct cost and indirect cost. Sum of direct cost of all activities within a project network gives the direct cost. Besides, indirect cost depends on the project duration. Thus, indirect cost increases as the finishing date of a project is getting longer. Afterwards, Equations. (15) – (17) are applied to calculate the total cost of a project.

$$DC = \sum_i^k DC_i^k x_i^k + t_i \times ic_i^k \quad (15)$$

$$IC = T \times ICR \quad (16)$$

$$C = DC + IC \quad (17)$$

Where  $DC$  and  $IC$ , respectively, are the total direct and indirect costs of a project;  $C$  is the total cost of a project;  $dc_i(k)$  shows the direct cost of activity  $i$  under the  $k$ th option; and  $ICR$  is the indirect cost rate of a project.

### 4.2. Non-dominating Sorting Approach

In this study, to overcome the deficiency of modified adaptive weight approach (MAWA), an effective and more promising non-dominating sorting (NDS) concept is employed to obtain the Pareto optimal solutions of time cost trade-off problems. In contrast to MAWA approach, there is no unique solution provided by NDS approach, but Pareto front solutions are produced and selected by comparing two solutions to each other. This NDS approach seeks the satisfactory solution from the non-dominated solutions depending on the experience and knowledge of decision-makers. The employed multi-objective TLBO algorithm can find out the Pareto front solution which provides flexibility to planners and decision makers in making efficient time-cost decisions. The concept of the Pareto front solution is the commonly accepted tool for comparing two solutions in multi-objective optimization that have no unified criterion with respect to optima. In addition to this, to develop a flexible time-cost trade-off (TCT) model, critical path method (CPM) scheduling in MATLAB to be used for applying multi-objective TLBO optimization engine and then integrating with the MCDM approaches to rank the obtained solutions. The results reveal that NDS-TLBO is more effective as compared to other state-of-the-art algorithms. The flowchart of the process is given in Figure 2.

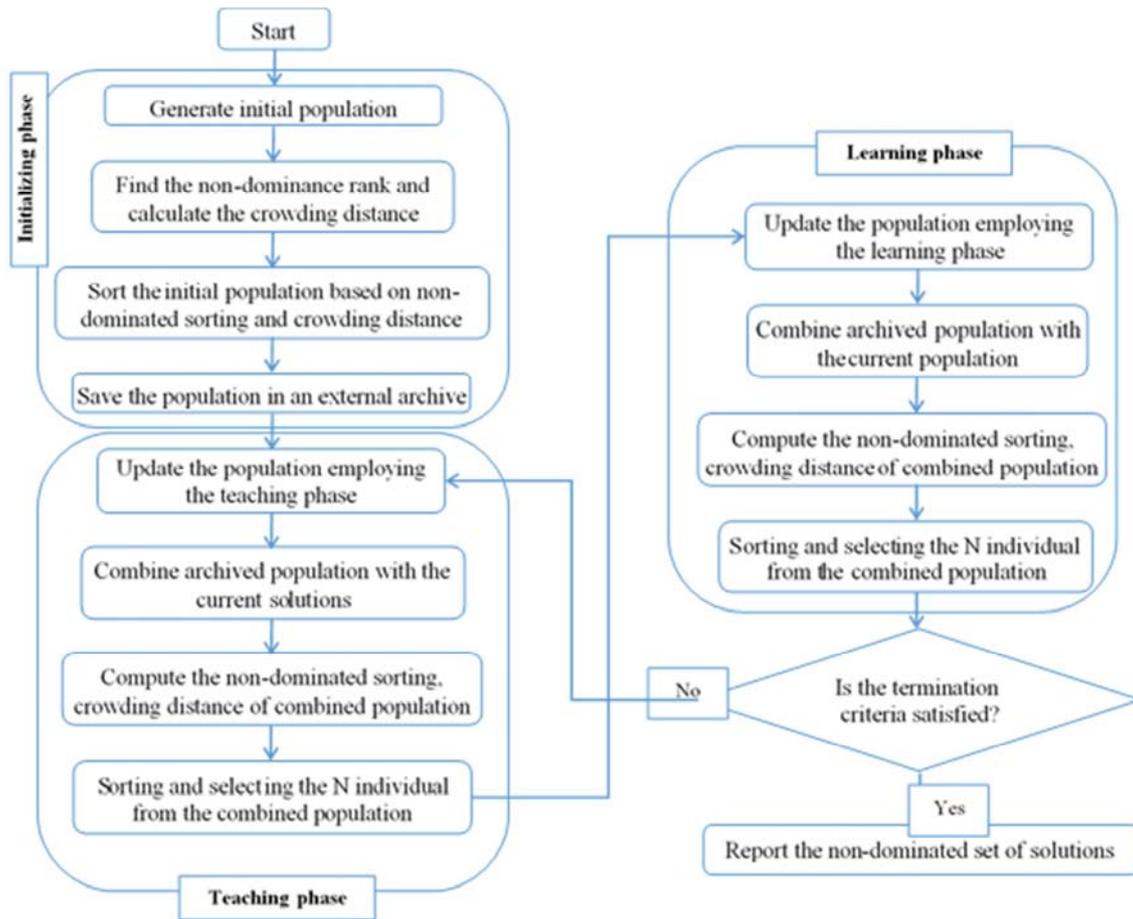


Figure 2. Flowchart of the NDS-TLBO algorithm for TCTP.

## 5. Description of the Benchmark Case Problem

### 5.1. Small-Scale Test Problem

To verify and demonstrate the efficiency of the proposed model to integrate the MCDM methods, namely, Entropy, SAW and TOPSIS, into TCTPs, a small-scale project activity network consisting of 18 activities, first proposed by [18], was

adapted. Furthermore, medium scale 63 activities Benchmark Case problems taken from the technical literature is investigated. The utilized algorithm was executed in MATLAB environment and implemented on a personal computer having Intel (R) Core (TM) i3 CPU 2.40 GHz and 3GB RAM. The activity on the node network diagram of the case study is illustrated in Figure 3. The corresponding time and cost for each mode of activities are listed in Table 2.

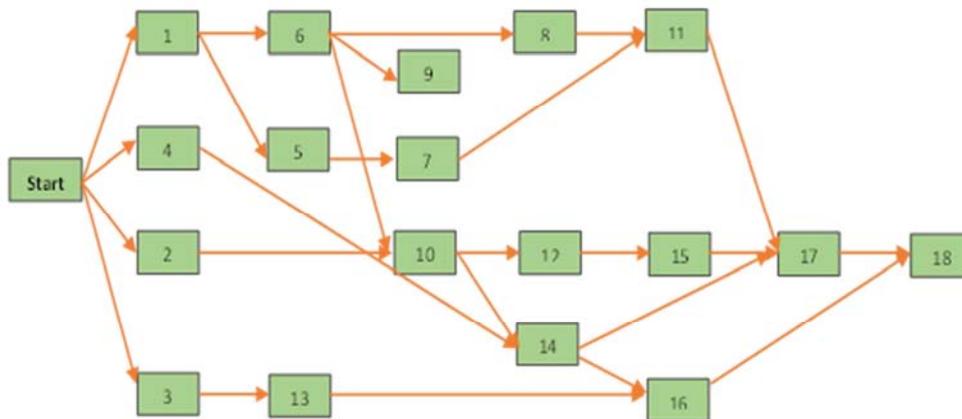


Figure 3. Network configuration for the model project of 18 activities.

Table 2. Options for 18 activities project with five modes.

Activities		5 Methods of Construction - Normal to Crash									
Activity Number	Precedent Activity	Option / Mode1		Option / Mode 2		Option / Mode 3		Option/Mode4		Option/Mode5	
		Dur (day)	Direct Cost	Dur (day)	Direct Cost	Dur (day)	Direct Cost	Dur (day)	Direct Cost	Dur (day)	DirectCost
1	-	14	2400	15	2150	16	2400	21	1500	24	1200
2	-	15	300	18	2400	20	1900	23	1500	25	1000
3	-	15	4500	22	4000	33	1800				
4	-	12	45000	16	35000	20	3200				
5	1	22	20000	24	17500	28	30000	30	10000		
6	1	14	40000	18	32000	24	15000				
7	5	9	30000	15	24000	18	18000				
8	6	14	220	15	21	16	22000	21		24	
9	6	15	300	18	240	20	200	23	208	25	120
10	2, 6	15	450	22	400	33	180		150		100
11	7, 8	12	450	16	350	20	320				
12	5, 9, 10	22	2000	24	1750	28	1500	30			
13	3	14	4000	18	3200	24	1800				
14	4, 10	9	3000	15	2400	18	2200				
15	12	12	4500	16	3500						
16	13, 14	20	3000	22	2000	24	1750	28	1500	30	1000
17	11, 14, 15	14	4000	18	3200	24	1800				1200
18	16, 17	9	3000	15	2400	18	2200				1000

The obtained ranking value using TOPSIS as well as SAW approaches adapting the entropy weight values is tabulated in table 3. It simply means that the time is more valuable than the cost of the project. In this case, contractor or any other

decision makers are giving more and more importance to the timespan of the project. And also, Pareto optimal solutions of 18 activities problem obtained by NDS-TLBO algorithm is graphically presented in Figure 4.

Table 3. Comparison of Pareto fronts located for 18-activity problem using NDS-TLBO.

Dur. (days)	Zhang [13]	NDS-TLBO (This paper)	$S_i^+$	$S_i^-$	$C_i$	TOPSIS Ranking	Sum of the Row	SAW Ranking
100	287720	283320	10.103	152.315	0.938	2	0.9091818	4
101	284020	279820	9.801	149.268	0.938	3	0.9182513	3
104	280020	276320	9.376	146.984	0.940	1	0.9454844	2
110	273720	271270	12.032	144.711	0.923	4	0.9999575	1
Pop. Size	50	40						
Num. of iterations	500	100				Entropy Weights	$C_i=0.999$ $C_c=0.001$	
NFE	25000	8040						

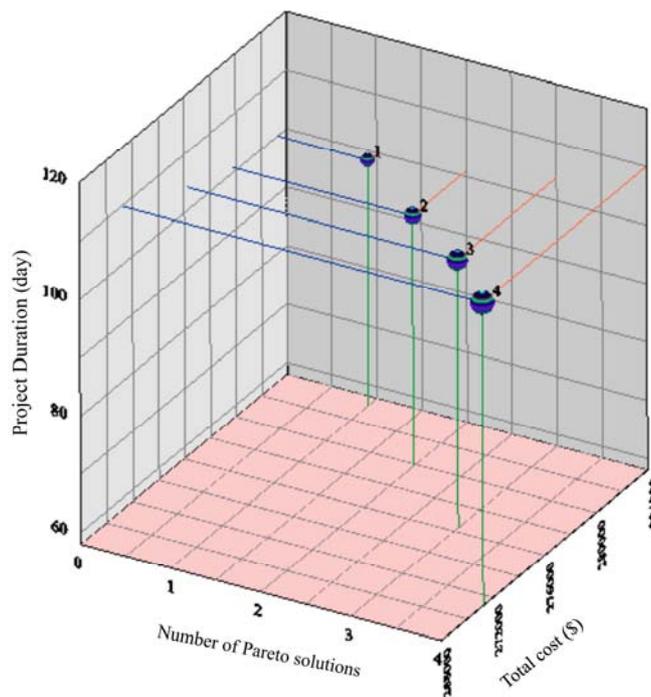


Figure 4. Pareto optimal solutions of 18 activities problem obtained by NDS-TLBO algorithm.

Table 4. Options selected and solution generated for 18-activity TCTP problem with five modes.

PF Sol	Proj. Time (day)	Project Cost (\$)	Selected duration of the corresponding activity (days)																	
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	100	283320	14	25	33	20	28	14	18	24	15	15	16	22	24	18	12	30	14	9
2	101	279820	14	25	33	20	30	14	18	24	15	15	16	22	24	18	12	30	14	9
3	104	276320	14	25	33	20	30	18	18	24	15	15	16	22	24	18	12	30	14	9
4	110	271270	14	25	33	20	30	24	18	24	15	15	20	22	24	18	12	30	14	9

Table 4 shows the options selected and solution generated for 18-activity TCTP problem. And obtained rankings using TOPSIS as well as SAW approaches considering the optional weights are given in Tables 5 and 6. It simply means in both the cases the cost is more valuable than the time of the project. Therefore, contractors or any other decision makers are paying more attention to the cost of the project.

Table 5. Result matrix of TOPSIS method using different weights.

Sr. No	NDS-TLBO		Different weight preferences			TOPSIS Ranking	Equal weight			TOPSIS Ranking
			$C_i=0.001$	$C_c=0.999$	$C_i$		$C_i=0.5$	$C_c=0.5$	$C_i$	
	Dur	Cost	$S_i^+$	$S_i^-$	$C_i$		$S_i^+$	$S_i^-$	$C_i$	
1	100	283320	0.010	144372.8	0.999	1	5.057	72258.70	0.999	1
2	101	279820	3544.99	140827.8	0.975	3	1774.277	70484.42	0.975	2
3	104	276320	7045.92	137326.9	0.951	2	3526.490	68732.21	0.951	4
4	110	271270	12019.60	132353.2	0.917	4	6015.819	66242.88	0.917	3

Table 6. Result matrix of SAW method using different weights.

NDS-TLBO		Sum of the Row	SAW Ranking	Sum of the Row	SAW Ranking
Dur	Cost	Different weight preferences		Equal weight	
		$C_i=0.001$	$C_c=0.999$	$C_i=0.5$	$C_c=0.5$
100	283320	0.9999091	1	0.9545455	3
101	279820	0.9875770	2	0.9529141	4
104	276320	0.9752631	3	0.9603738	2
110	271270	0.9575111	4	0.9787343	1

5.2. Medium-Scale Test Problem

A medium-scale project with 63 activities taken from the literature is examined as a second test project to exhibit the performance of the proposed NDS-TLBO. Subsequently, the

obtained Pareto-optimal solutions are ranked to have a clear guidance for the contractors. The activity-on-node diagram for the project is presented in Figure 5, and the time-cost optional modes are given in Table 7.

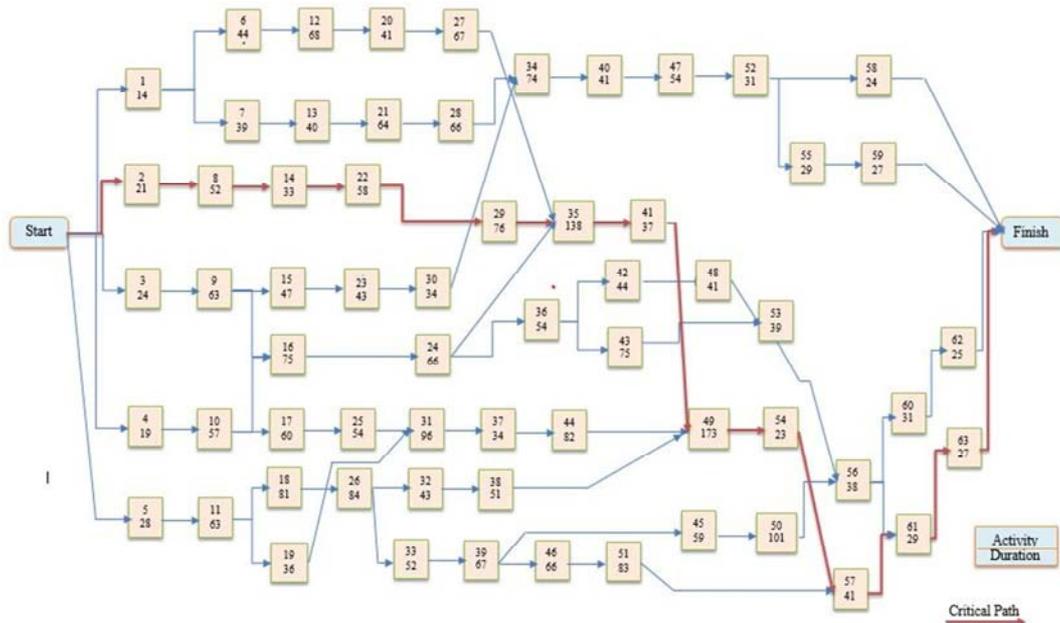


Figure 5. Network representation of the 63 activities project.

Table 7. Data for the project with 63 activities.

Activity num.	Precedent Activity	Option / Mode 1		Option / Mode 2		Option / Mode 3		Option / Mode 4		Option / Mode 5	
		Dur (days)	Cost (\$)								
1	-	14	3700	12	4250	10	5400	9	6250		
2	-	21	11250	18	14800	17	16200	15	19650		
3	-	24	22450	22	24900	19	27950	17	31650		
4	-	19	17800	17	19400	15	21600	-			
5	-	28	31180	26	34200	23	38250	21	41400		
6	1	44	54260	42	58450	38	63225	35	68150		
7	1	39	47600	36	50750	33	54800	30	59750		
8	2	52	62140	47	69700	44	72600	39	81750		
9	3	63	72750	59	79450	55	86250	51	91500	49	99500
10	4	57	66500	53	70250	50	75800	46	80750	41	86450
11	5	63	83100	59	89450	55	97800	50	104250	45	112400
12	6	68	75500	62	82000	58	87500	53	91800	49	96550
13	7	40	34250	37	38500	33	43950	31	48750		
14	8	33	52750	30	58450	27	63400	25	66250		
15	9	47	38140	40	41500	35	47650	32	54100		
16	9, 10	75	94600	70	101250	66	112750	61	124500	57	132850
17	10	60	78450	55	84500	49	91250	47	94640		
18	10, 11	81	127150	73	143250	66	154600	47	161900		
19	11	36	82500	34	94800	30	101700	-			
20	12	41	48350	37	53250	34	59450	32	66800		
21	13	64	85250	60	92600	57	99800	53	107500	49	113750
22	14	58	74250	53	79100	50	86700	47	91500	42	97400
23	15	43	66450	41	69800	37	75800	33	81400	30	88450
24	16	66	72500	62	78500	58	83700	53	89350	49	96400
25	17	54	66650	50	70100	47	74800	43	79500	40	86800
26	18	84	93500	79	102500	73	111250	68	119750	62	128500
27	20	67	78500	60	86450	57	89100	56	91500	53	94750
28	21	66	85000	63	89750	60	92500	58	96800	54	100500
29	22	76	92700	71	98500	67	104600	64	109900	60	115600
30	23	34	27500	32	29800	29	31750	27	33800	26	36200
31	19, 25	96	145000	89	154800	83	168650	77	179500	72	189100
32	26	43	43150	40	48300	37	51450	35	54600	33	61450
33	26	52	61250	49	64350	44	68750	41	74500	38	79500
34	28, 30	74	89250	71	93800	66	99750	62	105100	57	114250
35	24, 27, 29	138	183000	126	201500	115	238000	103	283750	98	297500
36	24	54	47500	49	50750	42	56800	38	62750	33	68250
37	31	34	22500	32	24100	29	26750	27	29800	24	31600
38	32	51	61250	47	65800	44	71250	41	76500	38	80400
39	33	67	81150	61	87600	57	92100	52	97450	49	102800
40	34	41	45250	39	48400	36	51200	33	54700	31	58200
41	35	37	17500	31	21200	27	26850	23	32300		
42	36	44	36400	41	39750	38	42800	32	48300	30	50250
43	36	75	66800	69	71200	63	76400	59	81300	54	86200
44	37	82	102750	76	109500	70	127000	66	136800	63	146000
45	39	59	847500	55	91400	51	101300	47	126500	43	142750
46	39	66	94250	63	99500	59	108250	55	118500	50	136000
47	40	54	73500	51	78500	47	83600	44	88700	41	93400
48	42	41	36750	39	39800	37	43800	34	48500	31	53950
49	38, 41, 44	173	267500	159	289700	147	312000	138	352500	121	397750
50	45	101	47800	74	61300	63	76800	49	91500		
51	46	83	84600	77	93650	72	98500	65	104600	61	113200
52	47	31	23150	28	27600	26	29800	24	32750	21	35200
53	43, 48	39	31500	36	34250	33	37800	29	41250	26	44600
54	49	23	16500	22	17800	21	19750	20	21200	18	24300
55	52, 53	29	23400	27	25250	26	26900	24	29400	22	32500
56	50, 53	38	41250	35	44650	33	47800	31	51400	29	55450
57	51, 54	41	37800	38	41250	35	45600	32	49750	30	53400
58	52	24	12500	22	13600	20	15250	18	16800	16	19450
59	55	27	34600	24	37500	22	41250	19	46750	17	50750
60	56	31	28500	29	30500	27	33250	25	38000	21	43800
61	56, 57	29	22500	27	24750	25	27250	22	29800	20	33500
62	60	25	38750	23	41200	21	44750	19	49800	17	51100
63	61	27	9500	26	9700	25	10100	24	10800	22	12700

The results of the NDS-MTLBO for medium networks indicate that the proposed algorithm normally provides adequate

optimal and near-optimal solutions for the TCTP. Subsequently, obtained rankings using TOPSIS as well as SAW approaches considering the entropy weights are illustrated in Table 8. To put it simply, rankings obtained using these entropy weights indicating that the time is more significant than the cost to the contractors. Graphical representations of the Pareto front solution of the problem is given in Figure 6.

Table 8. Analysis results of 63b-Activity project for the Case (IC=\$3500).

Pareto Front	Bettemir		This study		Obtained Entropy Weights			TOPSIS Ranking	Sum of the Row	SAW Ranking
	NDS-GA		NDS-TLBO		$C_c=0.999$	$C_c=0.001$	$C_i$			
	Dur	Cost	Dur	Cost	$S_i^+$	$S_i^-$				
PF1	617	6462580	612	6192140	5.8655	17.041	0.74394	2	0.9919031	4
PF2	651	6411540	617	6184820	9.51946	19.637	0.67350	4	0.9999976	2
PF3	647	6442440	590	6188690	18.5612	3.4809	0.15792	10	0.9562818	7
PF4	639	6420500	588	6195910	18.5710	7.1205	0.27715	7	0.9530447	9
PF5	648	6447900	591	6191490	17.3680	5.2324	0.23151	8	0.9579014	6
PF6	627	6433810	586	6196840	19.7309	7.6010	0.27810	6	0.9498066	10
PF7	618	6439240	592	6189140	17.3032	4.6157	0.21058	9	0.9595201	5
PF8	623	6449790	589	6199870	17.7812	9.6987	0.35293	5	0.9546645	8
PF9	630	6443805	617	6187390	7.89552	19.704	0.71393	3	0.9999980	1
PF10	629	6450065	616	6190570	5.92089	19.332	0.76554	1	0.9983794	3
Pop. Size	-	-	180	-	Num. of iterations	-	450	-	-	-

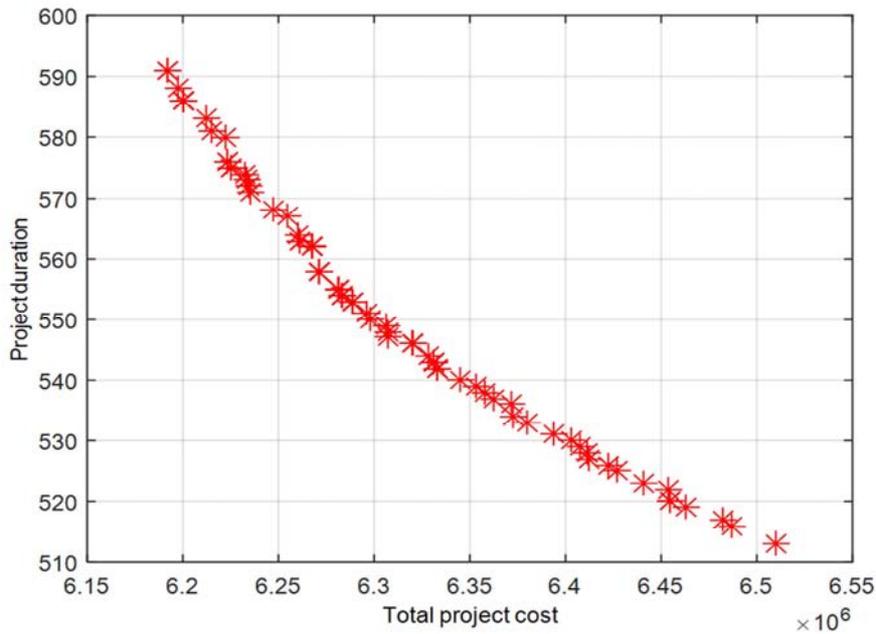


Figure 6. Pareto front solutions of 63b problem obtained by NDS-TLBO algorithm.

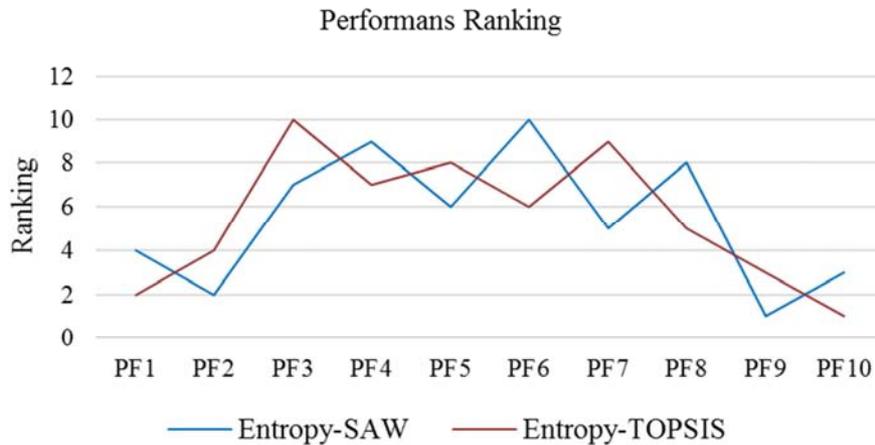


Figure 7. Performance ranking of the Entropy-TOPSIS and Entropy-SAW.

Considering Figure 7, it can be stated that PF9 is the compromised solutions in case of SAW and PF10 alternative was found to be a compromise solution in case of TOPSIS approach. It simply indicates that the contractor can pick up either PF9 or PF10. Tables 9 and 10 demonstrate the result matrix of the obtained rankings using TOPSIS as well as SAW approaches considering the optional weights. It expresses that in both the cases the cost is more valuable than the time of the project. Therefore, contractors or any other decision makers are paying more attention to the cost of the project.

Table 9. Result matrix of TOPSIS method using different weights.

Sr. No	NDS-TLBO		Different weight preferences			TOPSIS Ranking	Equal weight			TOPSIS Ranking
			$C_c=0.001$	$C_c=0.999$	$C_i$		$C_c=0.5$	$C_c=0.5$		
			$S_i^+$	$S_i^-$			$S_i^+$	$S_i^-$	$C_i$	
PF1	612	6192140	4887.39	4622.54	0.486	4	489.238	462.953	0.486	4
PF2	617	6184820	9509.94	0.01965	2.07E-06	10	951.947	17.691	0.018	10
PF3	590	6188690	7066.74	2443.20	0.256	8	707.550	244.575	0.257	8
PF4	588	6195910	2504.52	7005.42	0.736	3	251.251	701.244	0.736	3
PF5	591	6191490	5298.09	4211.85	0.442	5	530.549	421.616	0.443	5
PF6	586	6196840	1916.48	7593.46	0.798	2	192.654	760.106	0.798	2
PF7	592	6189140	6782.54	2727.4	0.286	7	679.085	273.034	0.287	7
PF8	589	6199870	0.01779	9509.94	0.999	1	16.019	951.948	0.983	1
PF9	617	6187390	7887.62	1622.32	0.170	9	789.552	163.355	0.171	9
PF10	616	6190570	5879.30	3630.64	0.381	6	588.520	363.830	0.382	6

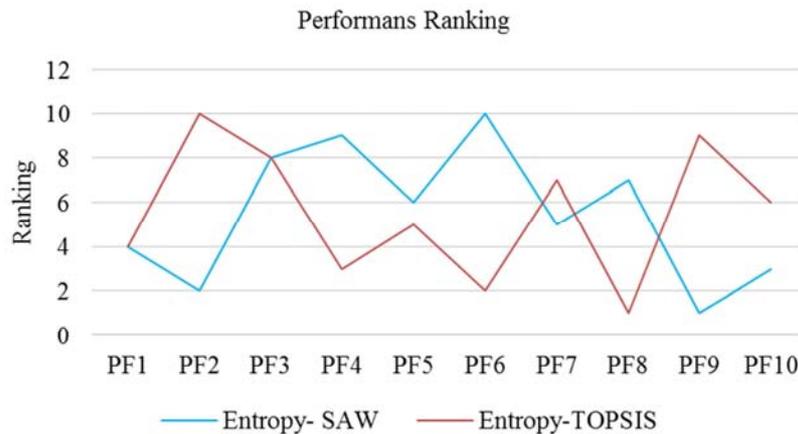


Figure 8. Performance ranking of the TOPSIS and SAW with equal weights.

As presented in Figure 8, PF8 was ranked as the first in the TOPSIS method while PF9 was ranked as the first in the SAW method. It is to mention that the adapted weight for both the approaches are equal. And the Pareto compromising solution remained same in case of SAW approach, however, it has changed in case of TOPSIS approach and become alternative PF8.

Table 10. Result matrix of SAW method using different weights.

NDS-TLBO			Sum of the Row	SAW Ranking	Sum of the Row	SAW Ranking
P. Front	Dur	Cost	Different weight preferences		Equal weight	
			$C_c=0.001$	$C_c=0.999$	$C_c=0.5$	$C_c=0.5$
PF1	612	6192140	0.99874	5	0.9953247	4
PF2	617	6184820	0.99757	10	0.9987863	2
PF3	590	6188690	0.99815	8	0.9772183	8
PF4	588	6195910	0.99931	3	0.9761798	9
PF5	591	6191490	0.99860	4	0.9782545	6
PF6	586	6196840	0.99946	2	0.9746341	10
PF7	592	6189140	0.99823	7	0.9788753	5
PF8	589	6199870	0.99995	1	0.9773096	7
PF9	617	6187390	0.99798	9	0.9989935	1
PF10	616	6190570	0.99849	6	0.9984396	3

## 6. Conclusion and Recommendations

In construction projects, the time and cost of completion of the construction period is a crucial aspect. The project owner

or contractor would like to use less amounts of resources when performing the activities. Many studies have been conducted in the literature already to tackle with TCTP problems. These optimization algorithms could provide the Pareto-optimal solutions, however, they can only create the Pareto frontier at

most without any further guidance to select the best choice among possible and available alternatives. The integration of MCDM methods with multi-objective optimization methods has been discussed in other fields such as water resource management, forest management, green energy planning, and etc. Hence, this study answers specifically to the question that why not using MCDM methods in construction project scheduling in the context of TCTPs. The framework proposed here, is able to establish a successful linkage between the optimization algorithms and MCDM methods. This enables the DMs to move into a further step where the decision making process of selecting the best alternative is carried out through an understanding framework.

The obtained results show that, first of all, the MCDM methods are highly efficient in ranking the project scheduling alternatives, and the DMs can be provided with higher level of confidence to implement the selected solution in real practice. Since, multi-objective optimization techniques, such as TLBO which are used here, are able to obtain a set of non-dominated solutions, being called as the Pareto-optimal solutions, there is the need to take one more step ahead to select the best optimal solution among the achieved Pareto-optimal set of solutions.

Firstly, the proposed TLBO multi-objective optimization algorithms with NSTLBO-II based procedure to solve TCTPs were coded in MATLAB R2013a. This problem with a total of  $4.72 \times 10^9$  possible schedules is examined with a daily indirect cost of \$1500. In every 10 runs of the algorithm, exactly 105 Pareto solutions were identified; out of which 4 random Pareto solutions are picked up to be given and ranked. However, the utilized algorithm exhibits its competency and accuracy by exploring a tiny portion [ $5640/4.72 \times 10^9 = 0.00012\%$ ] of the solution space. This reveals a remarkable reduction in number of function evaluations of administered algorithm comparing the literature.

In the medium scale example problem, the NDS-TLBO searched 162,180 ( $=180 \times 450 \times 2 + 180$ ) possible different schedules, only searching a negligible portion of the solution space ( $162,180/1.4E+42$ ). Population and number of iterations are adopted as 180 and 450, respectively.

Secondly, the entropy approach is rather a straightforward MCDM method while the TOPSIS is too simple in terms of mathematical formulations. On the other hand, the SAW approach which is simpler in comparison with the TOPSIS method, has given a similar rank for the solutions. Thus, it is proposed that the TOPSIS approach can be more accurate approach in evaluating the solutions. Therefore, in TCTPs the TOPSIS approach can be efficiently used in comparison with the other methods although it is very simple.

Finally, the developed approaches can be made more efficient by employing fuzzy approach or hybrid algorithms which are more effective in avoiding local optimization. In addition, the approach can be extended with the Microsoft project interface as well as Primavera software.

## Conflict of Interest Statement

The authors declare that they have no competing interests.

## References

- [1] Hwang, C. L., Yoon, K. (1981), "Multiple attribute decision making". Methods and applications a state of the art survey". Berlin-Heidelberg: Springer-Verlag.
- [2] Chen, C. T. (2000), "Extensions of the TOPSIS for group decision-making under fuzzy environment". *Fuzzy Sets and Systems*, 114 (1): 1–9.
- [3] Gumus, A. T. "Evaluation of hazardous waste transportation firms by using a two-step fuzzy-AHP and TOPSIS methodology". *Expert Systems with Applications*, 36: 4067–4074, (2009).
- [4] Yong, D. (2006), "Plant location selection based on fuzzy TOPSIS". *The International Journal of Advanced Manufacturing Technology*, 28 (7-8): 839–844.
- [5] Roy, B. (1991) "The outranking approach and the foundations of Electre methods". *Theory and Decision*, 31 (1): 49-73.
- [6] Chaudhuri, S., & Deb, K. (2010), "An interactive evolutionary multi-objective optimization and decision making procedure". *Applied Soft Computing*, 10 (2): 496–511.
- [7] Bazargan-Lari, M. R. (2014), "An evidential reasoning approach to optimal monitoring of drinking water distribution systems for detecting deliberate contamination events". *Journal of Cleaner Production*, 78 (1): 1-14.
- [8] Monghasemi, S., Nikoo, M. R., Fasaee, M. A. K., & Adamowski, J. (2015), "A Novel Multi Criteria Decision Making Model for Optimizing Time-Cost-Quality Trade-off Problems in Construction Projects". *Expert Systems with Applications*, 42 (6): 3089-3104.
- [9] Perera, A., Attalage, R., Perera, K., & Dassanayake, V. (2013), "A hybrid tool to combine multi-objective optimization and multi-criterion decision making in designing standalone hybrid energy systems". *Applied Energy*, 107: 412-425.
- [10] Eirgash, M. A., and Dede, T. (2018). "A multi-objective improved teaching learning-based optimization algorithm for time-cost trade-off problems." *J. Constr. Eng. Manage. Innovation*, 1 (3): 118-128.
- [11] Toğan, V., and Eirgash, M. A. (2019). "Time-cost trade-off optimization of construction projects using teaching learning-based optimization." *KSCE J. Civ. Eng.*, 23 (1), 10–20.
- [12] Colomi, A., Dorigo, M., Maniezzo, V., & Trubian, M. "Ant system for job-shop scheduling". *Belgian Journal of Operations Research, Statistics and Computer Science*, 34: 39–53 (1994).
- [13] Ng, T. S., & Zhang, Y. (2008), "Optimizing construction time and cost using ant colony optimization approach". *Journal of Construction Engineering and Management*, 134 (9): 721-728.
- [14] Mohammad Ammar Al-Zarrad and Daniel Fonseca (2018) "A New Model to Improve Project Time-Cost Trade-Off in Uncertain Environments", *Contemporary Issues and Research in Operations Management*, DOI: 10.5772/intechopen.74022.
- [15] Eirgash, M. A. Pareto-Front Performance of Multiobjective Teaching Learning Based Optimization Algorithm on Time-Cost Trade-Off Optimization Problems. Master of Science Thesis, Karadeniz Technical University, (2018), Turkey.

- [16] Rao, R. V., & Patel, V. (2011). "Multi-objective optimization of combined Brayton and inverse Brayton cycles using advanced optimization algorithms". *Engineering Optimization*, 44 (8): 965-983.
- [17] Eirgash, M. A., Toğan, V., and Dede, T. (2019). "A multi-objective decision-making model based on TLBO for the time-cost trade-off problems." *Struct. Eng. Mech.*, 71 (2), 139-151.
- [18] Feng, C.-W., Liu, L., & Burns, S. A. (1997), "Using genetic algorithms to solve construction time-cost trade-off problems". *Journal of Computing in Civil Engineering*, 11 (3): 184-189.