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Adaptive finite element modeling of fatigue crack propagation

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Abstract: An adaptive finite element interactive program has been developed for fatigue crack propagation simulation under constant amplitude loading condition. The purpose of this model is on the determination of 2D crack paths and surfaces as well as on the evaluation of components Lifetimes as a part of the damage tolerant assessment. As part of a linear elastic fracture mechanics analysis, the determination of the stress intensity factor distribution is a crucial point. The fatigue crack direction and the corresponding stress-intensity factors are estimated at each small crack increment by employing the J-integral technique. The propagation is modeled by successive linear extensions, which are determined by the stress intensity factors under linear elastic fracture mechanics assumption. The stress intensity factors range history has to be recorded along the small crack increments. Upon completion of the stress intensity factors range history recording, fatigue crack propagation life of the examined specimen is predicted. Verification of the predicted fatigue life is validated with relevant experimental data and numerical results obtained by other researchers. The comparisons show that this model is capable of demonstrating the fatigue life prediction results as well as the fatigue crack path satisfactorily.

Keywords: Finite Element, Fatigue, Crack Growth, Stress Intensity Factor, Adaptive Mesh

1. Introduction

Fatigue is a localized damage process of a component produced by cyclic loading. It is the result of the cumulative process consisting of crack initiation, propagation, and final fracture of a component. During cyclic loading, localized plastic deformation may occur at the highest stress site. This plastic deformation induces permanent damage to the component and a crack develops. As the component experiences an increasing number of loading cycles, the length of the crack (damage) increases. After a certain number of cycles, the crack will cause the component to fail (separate) [1].

The fatigue life can be obtained from baseline fatigue data generated from constant-amplitude loading tests. There are three commonly used methods, the stress-life (S-N) method, the strain-life (ε-N) method, and the linear elastic fracture mechanics (LEFM) to characterize the baseline fatigue data. The LEFM method has been used in the present study.

An accurate evaluation of fracture parameters such as stress intensity factors (SIFs) becomes quite essential for the simulation based life cycle design analysis. To simulate cracked structures, a number of methods such as boundary element method [2, 3], meshfree methods [4,5,6], finite element method (FEM) [7], and finite difference method (FDM) are available. FEM has been in the forefront of numerical methods used for the simulation of fatigue fracture problems. A number of approaches have been developed in FEM over the years, which makes it as a most suited method for analyzing the asymptotic stress fields at the crack tip. However, FEM requires that the crack surface should coincide with the edge of the finite elements, i.e. a conformal mesh is needed besides special elements to handle crack tip asymptotic stresses.

This work proposes a self-adaptive user friendly model for simulating automatic fatigue crack propagation in two dimensional structural components. Moreover, the developed program has a much more flexible and portable graphical interface. The adaptive procedure provides a regular mesh refinement for the free-boundary curves (including cracks) and is based on a posteriori error estimation. An h-refinement strategy is utilized in this process.

1.2. Simulation Procedure

The automatic fatigue crack propagation is characterized by successive propagation steps performed without user interaction. Each step consists of:

1. Mesh generation
2. FE analysis of the fatigue crack propagation and storing the nodal stresses.
3. Introduction of initial crack in the model.
4. Update of geometrical model.
5. Mesh generation; refining around the crack-tip.
6. FE analysis.
7. Compute the stiffness matrix and solve the system of equations
8. Calculate the stresses components, error estimators and the stress intensity factor
9. Unstable fracture occurs or crack reaches boundary? If yes, stop.
10. Crack arrests? If yes, stop.
11. Calculation of crack propagation rate da/dN.
12. Calculation of crack propagation direction.
13. Final number of cycles reached? If yes, stop.
14. Return to Step 4.

The computational scheme of the fatigue crack propagation program and the mesh generation processes are illustrated in details by [10].

1.3. Equivalent Domain Integral Method

The J-integral was introduced by [9] to study non-linear material behavior under small scale yielding. It is a path independent contour integral defined as:

$$J = \int_c \left[Wn_1 - \sigma_{ij} n_j \frac{\partial u_i}{\partial x} \right] ds \tag{1}$$

where W is strain-energy density; σ_{ij} are stresses; u_i are the displacements corresponding to local i-axis; s is the arc length of the contour; n_j is the unit outward normal to the contour C, which is any path of vanishing radius surrounding the crack tip (Figure 1a).

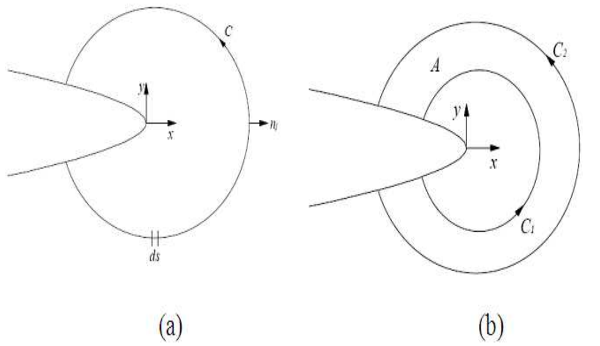


Figure 1. *(a) Arbitrary contour surrounding the crack tip; (b) Area to be employed to calculate the J-integral*

The Equivalent Domain Integral Method replaces the integration along the contour with one over a finite size domain by the divergence theorem. This domain integration is more convenient for finite element analyses. For two-dimensional problems, the contour integral is replaced an area integral (Figure 2b). Then, equation (1) is rewritten as:

$$J_k = -\int_A \left[W\frac{\partial q}{\partial x} - \sigma_{ij}\frac{\partial u_i}{\partial x}\frac{\partial q}{\partial x} \right] dA - \int_A \left[\frac{\partial W}{\partial x} - \frac{\partial}{\partial x}\left(\sigma_{ij}\frac{\partial u_i}{\partial x} \right) \right] q dA - \int_s t_i \frac{\partial u_i}{\partial x} q ds \tag{2}$$

In the linear elastic analysis, the J-integral definition considers a balance of mechanical energy for a translation in front of the crack along the x-axis. In the case of either pure Mode I or pure Mode II, equation (2) allows calculation of the stress intensity factors K_I or K_{II} Nevertheless, in the mixed mode condition this equation alone does not allow K_I and K_{II} to be calculated separately. In this case, invariant integrals are used. Usually, the integrals defined by [10] are employed:

$$J_k = -\int_A \left[W\frac{\partial q}{\partial x_k} - \sigma_{ij}\frac{\partial u_i}{\partial x_k}\frac{\partial q}{\partial x_j} \right] dA - \int_A \left[W\frac{\partial q}{\partial x_k} - \sigma_{ij}\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_k} \right) \right] q dA - \int_s t_i \frac{\partial u_i}{\partial x_k} q ds \tag{3}$$

where k is an index for local crack tip axis (x, y). These integrals were introduced initially for small deformation11 and were extended by [11] for finite deformation.

The stress intensity factors can be obtained by two possible ways. The first approach is through relationships between the J-integral and the stress intensity factors. These relations are:

$$J_1 = \frac{\kappa+1}{8\mu}(K_I^2 + K_{II}^2)$$
$$J_2 = -\frac{\kappa+1}{4\mu} K_I K_{II} \tag{4}$$

Then the relations between the stress intensity factors and J_1, J_2 are:

$$K_I = 0.5\sqrt{\frac{8\mu}{\kappa+1}}(\sqrt{J_1 - J_2} + \sqrt{J_1 + J_2})$$
$$K_I = 0.5\sqrt{\frac{8\mu}{\kappa+1}}(\sqrt{J_1 - J_2} - \sqrt{J_1 + J_2}) \tag{5}$$

2. Fatigue Crack Propagation Analysis

In order to simulate fatigue crack propagation under linear elastic condition, the crack path direction must be determined. There are several methods use to predict the direction of crack trajectory such as the maximum circumferential stress theory, the maximum energy release rate theory and the minimum strain energy density theory. Bittencourt et al. [12] have shown that, if the crack orientation is allowed to change in automatic fracture simulation, the three criteria provide basically the same numerical results, since the maximum circumferential stress criterion is the simplest, presenting a closed form solution, it is briefly described below.

The maximum circumferential stress theory [13] asserts that, for isotropic materials under mixed-mode loading, the crack will propagate in a direction normal to maximum tangential tensile stress. In polar coordinates, the tangential stress is given by

$$\sigma_\theta = \frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[K_I\cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] \tag{6}$$

The direction normal to the maximum tangential stress can be obtained by solving $d\sigma_\theta / d\theta = 0$ for θ. The nontrivial solution is given by:

$$K_I\sin\theta + K_{II}\left(3\cos\theta - 1\right) = 0 \tag{7}$$

which can be solved as:

$$\theta = \pm\cos^{-1}\left\{\frac{3K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right\} \tag{8}$$

Since fatigue is a cyclic dissipation of energy, in the form of hysteretic loops, which are related to a cumulative damage process, the elapsed time for damage is expressed in terms of the number of fatigue cycles (N). The control parameter that is used to evaluate this process is the rate of crack growth per cycle (da/dN). Hence, da/dN depends on the applied stress intensity factor range and N is the well-known fatigue life term. For crack initiation, the threshold stress intensity factor and threshold stress range are associated as:

$$\Delta K_{th} = f\,\Delta\sigma_{th}\sqrt{\pi a} \tag{9}$$

where f is a function of geometry and loading and $\Delta\sigma_{th}$ is analogous to fatigue limit. This equation indicated that if $\Delta\sigma < \Delta\sigma_{th}$ crack growth does not occur. Practically, during the implementation we use the equivalent $\Delta K_{eq} \geq \Delta K_{th}$ as the condition for crack to propagate.

According to this criterion, the equivalent mode I stress intensity factor is obtained as:

$$K_{Ieq} = K_I\,cos^3(\theta/2) - 3K_{II}\,cos^2(\theta/2)\,sin(\theta/2) \tag{10}$$

To model the stable crack propagation, we use the generalized Paris' law:

$$\frac{da}{dN} = C\,(\Delta K_{Ieq})^m \tag{11}$$

where C and m are the material properties, a is the crack length, N is the number of loading cycles and ΔK_{Ieq} is obtained by equation (10) by substituting ΔK_I and ΔK_{II} to K_I and K_{II}. Then, the number of cycles N_{if} for crack propagation from the initial crack length a_i to the final crack length aif can be integrated as:

$$N_{if} = \int_{a_i}^{a_f} \frac{1}{C\left(\Delta K_{Ieq}\right)^m}\,da \tag{12}$$

The developed program has safety features to automatically stop the calculation if, during any loading event, it detects that: (i) $K_{Ieq,max} = K_{Ic}$; (ii) the crack has reached its maximum specified size; (iii) one of the borders of the piece is reached by the crack front.

3. Numerical Results and Analysis

3.1. Center Cracked Tension (CCT) Specimen with Two Holes

A rectangular nickel plate (320 mm length, 70 mm width, and 5 mm thickness) with a small two holes as shown in Figure (2a) is subjected to a cyclic loading with displacement amplitude 0.5 mm per 0.05 s and the mean displacement is 1 mm. The material properties are E = 177 GPa, ν = 0.3, m = 4.127 and C = 2.635×10^{-10}. The final adaptive mesh for the first step before crack growth is shown in Figure (2b) as well as the enlargements of the holes area is shown in Figure (3) including the stress distribution.

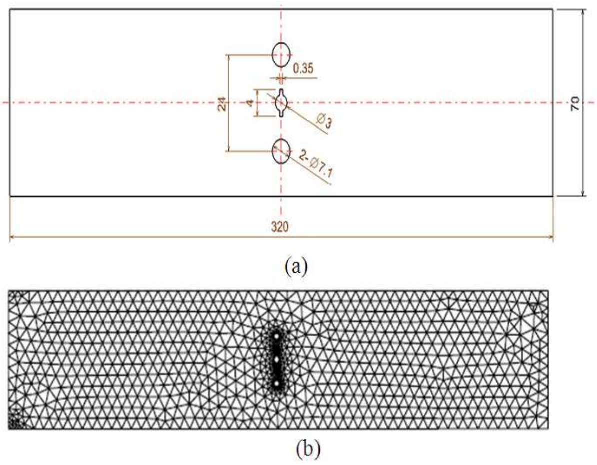


Figure 2. (a) Center cracked tension (CCT) specimen with two holes, (b) Final adaptive mesh for the first step

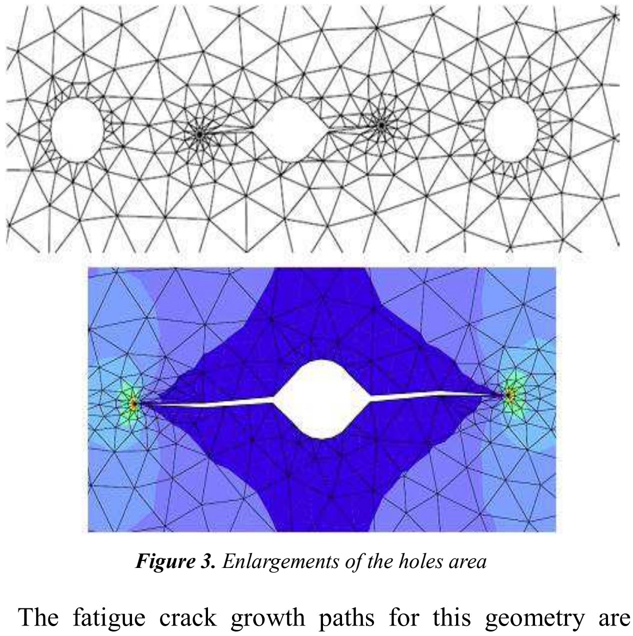
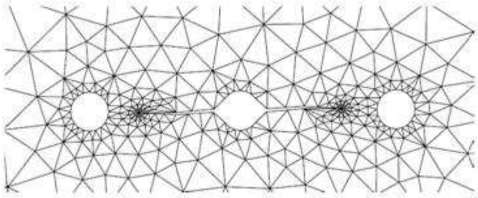
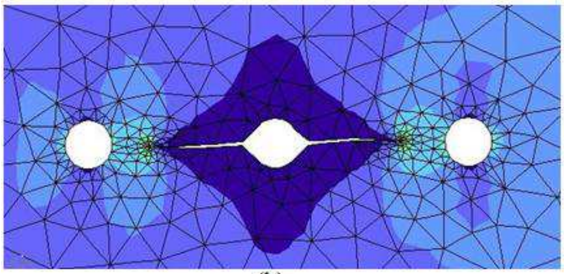


Figure 3. Enlargements of the holes area

The fatigue crack growth paths for this geometry are presented in Figure 4a and the maximum principal stress distribution is also presented in Figure 3b. It can be concluded that, the effect of holes at the crack propagation direction is that the hole sucks crack even though crack propagation direction is changed.



(a)



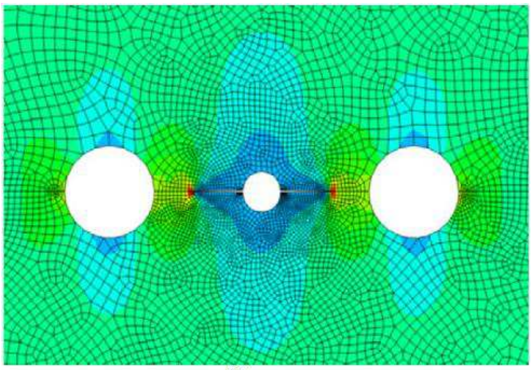
(b)

Figure 4. *(a) Fatigue crack growth path (b) maximum principal stress distribution*

The present study has been compared to the experimental and numerical study presented by Cho et al.[14] which is shown in Figures 5a and 5b. The comparison shows an excellent agreement in both numerical and experimental results.

(a)

Figure 5. *(a) Experimental results obtained by Cho et al. [14]*



(b)

Figure 5. *(b) numerical results obtained by Cho et al. [14]*

Figure 6 shows the comparison stress intensity factor between the finite element solution in the present study and experimental calculation and numerical simulation obtained by Cho et al. [14]. These results show that the present study was closer to the experimental results compared to the finite element results obtained by [14].

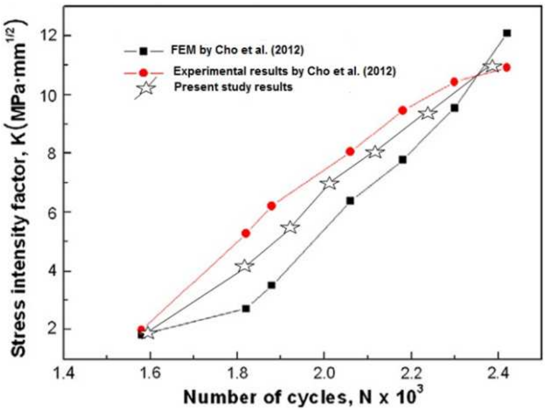


Figure 6. *Stress intensity factor ranges versus number of cycle for the nickel CCT specimen with 2 holes.*

4. Conclusion

In the present paper, a comprehensive adaptive Finite Element model for fatigue crack propagation analysis was developed using the developed source code written in FORTRAN language. The fatigue crack propagation is modeled by successive linear extensions, which are determined by the stress intensity factors obtained after a linear elastic analysis. The fatigue crack path, fatigue life and stress intensity factors along the crack length were predicted. The results of the developed program have been successfully validated through direct comparisons with the experimental and numerical results obtained by other researchers.

References

[1] Yung-Li Lee, Y.L, Pan, J., Hathaway, R. Barkey, M.E., 2005. Fatigue testing analysis theory and practice, Elsevier Butterworth–Heinemann.

[2] Portela A, Aliabadi M, Rooke D. The dual boundary element method: effective implementation for crack problem. Int J Numer Methods Eng, 1991; 33: 1269–87.

[3] Yan AM, Nguyen-Dang H. Multiple-cracked fatigue crack growth by BEM. Comput Mech 1995; 16: 273–80.

[4] Yan, X. A boundary element modeling of fatigue crack growth in a plane elastic plate. Mech Res Commun 2006; 33: 470–81.

[5] Belytschko T, Gu L, Lu YY. Fracture and crack growth by element-free Galerkin methods. Model Simul Mater Sci Eng 1994; 2: 519–34.

[6] Duflot, M., Dang, H.N. Fatigue crack growth analysis by an

enriched meshless method. J Comput Appl Math 2004; 168: 155–64.

[7] Singh, I.V., Mishra, B.K., Bhattacharya, S., Patil, R.U., The numerical simulation of fatigue crack growth using extended finite element method, 2012, International Journal of Fatigue, 36, 109–119

[8] Alshoaibi M. Abdulnaser, Finite Element Procedures for the Numerical Simulation of Fatigue Crack Propagation under Mixed Mode Loading. Structural Engineering and Mechanics. 2010. Vol. 35, No.3, pp.283-299.

[9] Rice, J.R., 1968. A path independent integral and the approximate analysis of strain concentration by notches and cracks", J. App. Mech., 35, pp.379-386.

[10] Knowles, J.K. and E. Sternberg, On a class of conservation laws in linearized and finite elastostatics", Archives for Rational Mechanics & Analysis, 1972, 44, pp.187-211.

[11] Atluri, S.N., 1982. Path-independent integrals in finite elasticity and inelasticity, with body forces, inertia, and arbitrary crack-face conditions", Engineering Fracture. Mechanics, 16, pp.341-364.

[12] Bittencourt, T.N., P.A. Wawrzynek, A.R. Ingraffea and J.L. Sousa, 1996. Quasi-automatic simulation of crack propagation for 2D LEFM problems. Engineering Fractrue Mechanics, Volume 55, pp. 321-334.

[13] Erdogan, F. and G. Sih, 1963. On the crack extension in plates under plane loading and transverse shear. Journal of Basic Engineering 85: 519–27.

[14] Cho, J.U, Xie, L., Cho, C., Lee, S., 2012. Crack propagation of CCT foam specimen under low strain rate fatigue. International Journal of Fatigue, 35, pp. 23–30