International Journal of Statistics and Actuarial Science
2017; 1(2): 40-45
http://www.sciencepublishinggroup.com/j/ijsas
doi: 10.11648/j.ijsas.20170102.12

Second-Order Asymptotic Expansion for the Ruin Probability of the Sparre Andersen Risk Process with Reinsurance and Stronger Semiexponential Claims

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To cite this article:
Rovshan Aliyev. Second-Order Asymptotic Expansion for the Ruin Probability of the Sparre Andersen Risk Process with Reinsurance and Stronger Semiexponential Claims. *International Journal of Statistics and Actuarial Science*. Vol. 1, No. 2, 2017, pp. 40-45.
doi: 10.11648/j.ijsas.20170102.12

Received: March 11, 2017; **Accepted**: March 28, 2017; **Published**: April 14, 2017

Abstract: In this study the Sparre Andersen risk process with reinsurance is considered. The second-order asymptotic expansion for the ruin probability is obtained, when the claim sizes have the strongly semiexponential distribution. Moreover, numerical examples in cases proportional reinsurance and exsess stop loss reinsurance are provided.

Keywords: Sparre Andersen Risk Process, Reinsurance, Ruin Probability, Second-Order Asymptotic Expansion, Semiexponential Distribution

1. Introduction

Consider the surplus process

$$R_t = u + ct - \sum_{i=1}^{N_t} \eta_i \,,\; t \geq 0 \tag{1}$$

where R_t, is the surplus of the insurer at time t, $u = R_0 > 0$ the initial surplus of insurance company, $c > 0$ the constant rate per unit time at which the premiums are received, $\eta_1, \eta_2, ...$ are independently and identically distributed (i.i.d.) positive random variables representing individual claim amounts. Counting process $N_t = \max\{k : \xi_1 + ... + \xi_k \leq t\}$ denotes the number of claims up to time t, where the claim inter-arrival times or times between claims $\xi_i, i \geq 1$ are assumed i.i.d. positive random variables. Further, we assume that the sequences $\{\xi_i\}, i \geq 1$ and $\{\eta_i\}, i \geq 1$ are independent. Suppose that, also $c > m_1/\mu_1$, so that ruin is not certain to occur, where $m_1 = E\eta_1$ and $\mu_1 = E\xi_1$. To provide this condition it is suggested that $c = (1+\rho) m_1/\mu_1$, where $\rho > 0$ is safety loading coefficient. When inter-claim times $\xi_i, i \geq 1$ are have an exponential distribution with mean $1/\lambda$, which is equivalent to that, N_t has a Poisson distribution with parameter λt, in this case (1) is called classical risk process or Cramer-Lundberg model in actuarial literature. In case, when inter-claim times $\xi_i, i \geq 1$ have an arbitrary distribution on $[0, \infty)$, in other words, N_t an ordinary renewal process, (1) is called a Sparre Andersen risk process. There are exists in literature studies, where some important problems connected with Sparre Andersen risk process were solved (see, for example, Asmussen S. (2000), Albrecher H., Claramunt M. M., Mármol M. (2006), Aleškeviciene A., Leipus R., Šiaulys J. (2009), Aliyev R. T., Jafarova V. (2009), Gerber H. U., Shiu E. W. (2005), Hald M., Schmidli H. (2004), Li S., Garrido J. (2004), Li S., Dickson D. C. M. (2006), Luo S., Taksar M., Tsoi A. (2008), Schmidli H. (2002)).

The ultimate ruin probability $\psi(u)$, which is the main global characteristic of the renewal risk model, is given by

$$\psi(u) = P\{\inf_{t \geq 0} R_t < 0 \big| R_0 = u\}.$$

Note that in the classical studies major role in the study of the probability of ruin is the so-called adjustment coefficient or the Lundberg coefficient. Adjustment coefficient is defined as the positive solution of the characteristic equation with respect to r:

$$M_\eta(r) = 1 + c\mu_1 r \tag{2}$$

where

$$M_\eta(r) \equiv E\left(e^{r\eta}\right) = \int_0^\infty e^{rx} dF_\eta(x)$$

Note that a special place in the study of the probability of ruin takes the case of large claims. Note that, for the modelling of large claims is used with heavy-tailed distributions. In the case of heavy tails $M_\eta(r) \equiv E(e^{r\eta}) = \infty$. Hence, the characteristic equation (2) becomes meaningless. So in this case a new approach is required.

In this direction we mention Embrechts P., Veraverbeke N. (1982), Baltrunas A. (1999), Aleškeviciene et al. (2009) etc. In study Embrechts P., Veraverbeke N. (1982) was obtained asymptotic equivalence as $u \to \infty$ for ruin probability $\psi(u)$, when equilibrium function $F^e(u) = \frac{1}{m_1}\int_0^u \overline{F}_{\eta_1}(t)dt, u \geq 0$ belong to the class of subexponentional distributions (see, definition 2 in section 2):

$$\psi(u) \sim \frac{1}{\rho}\overline{F}^e(u) \tag{3}$$

where

$$\rho = \frac{c\mu_1}{m_1} - 1$$

In study Baltrunas A. (1999) the rate of convergence for asymptotic relation (3) was obtained, when F_{η_1} belongs to a subclass of subexponential distributions and ξ_i, $i \geq 1$ have an exponential distribution. Based on results of paper Borovkov A. (2002), in study Aleškeviciene A., Leipus R., Šiaulys J. (2009) the second-order behavior in relation (3) is investigated, in the case claim size distribution belong to the strongly semiexponensional class.

It is known that insurance companies also insure their risks to another company. This type of insurance is called reinsurance. Basically there are some types of reinsurance contracts: proportional reinsurance, excess of loss reinsurance and excess stop loss reinsurance. Each type of reinsurance are described by a function $h(x)$, when describes the amount paid by the insurance company in the event of a claim value x and $0 \leq h(x) \leq x$ (see, for example, Dickson D. C., Waters H. R. (1996), Dickson D. C., Waters H. R. (1997), Dickson D. (2005), p. 190-207.

1. *Proportional reinsurance.* If a transferor company itself satisfies a certain fraction $0 < \beta \leq 1$ of each claim, and the remaining share $1-\beta$ reinsurance company, then this kind is called a proportional reinsurance. Parameter β is called the retention limit. In this case, the loss of the transmission company is $\beta\eta_i$ for i th claim. For the proportional reinsurance $h(\eta_1) = \beta\eta_1$, $0 < \beta \leq 1$ and. $\overline{F}_{h(\eta_1)}(x) = \overline{F}_{(\eta_1)}(x/\beta)$.

2. *Excess stop loss reinsurance.* In this case, reinsurance company pays claims exceeding a certain level M, and in order to insure themselves against large losses, to identify some of the upper level L. In this case $h(\eta_1) = \eta_1 - \min\{\max\{\eta_1 - M; 0\}, L\}$. It is not difficult to determine function $F_{h(\eta_1)}(x)$. Distribution function of $h(\eta_1)$ is

$$\overline{F}_{h(\eta_1)}(x) = P\{h(\eta_1) \leq x\} = P_1 + P_2 + P_3$$

where

$$P_1 = P\{h(\eta_1) \leq x, \eta_1 \leq M\}$$
$$= P\{\eta_1 \leq x, \eta_1 \leq M\} = \begin{cases} F_{\eta_1}(x), x < M \\ F_{\eta_1}(M), x \geq M \end{cases}$$

$$P_2 = P\{h(\eta_1) \leq x, M < \eta_1 \leq M + L\}$$
$$= P\{M \leq x, M < \eta_1 \leq M + L\}$$
$$= \begin{cases} 0, x < M \\ F_{\eta_1}(M+L) - F_{\eta_1}(M), x \geq M \end{cases}$$

$$P_3 = P\{h(\eta_1) \leq x, \eta_1 > M + L\}$$
$$= P\{\eta_1 - L \leq x, \eta_1 > M + L\}$$
$$= \begin{cases} 0, x < M \\ F_{\eta_1}(M+L) - F_{\eta_1}(M-L), x \geq M \end{cases}$$

Consequently,

$$F_{h(\eta_1)}(x) = P\{h(\eta_1) \leq x\} = \begin{cases} F_{\eta_1}(x), x < M \\ F_{\eta_1}(x+L), x \geq M \end{cases}$$

Tail function of $F_{h(\eta_1)}(x)$ is

$$\overline{F}_{h(\eta_1)}(x) = P\{h(\eta_1) > x\} = \begin{cases} \overline{F}_{\eta_1}(x), x < M \\ \overline{F}_{\eta_1}(x+L), x \geq M \end{cases}$$

Now suppose that the insurer effects reinsurance and that the amount paid by the insurer when the i th claim η_i, occurs is $h(\eta_i)$, where $0 \leq h(\eta_i) \leq \eta_i$. We will assume throughout that reinsurance premiums are calculated with a loading factor ρ_h, where $\rho_h \geq \rho$. Then assuming that reinsurance premium are paid continuously, the insurer's surplus at time t, is denoted by R_t^*,

$$R_t^* = u + c^* t - \sum_{i=1}^{N_t} h(\eta_i),\ t \geq 0 \tag{4}$$

where

$$c^{*}=c-c_{h}=(1+\rho)\frac{m_{1}}{\mu_{1}}-(1+\rho_{h})E\left(\eta_{1}-h(\eta_{1})\right)\frac{1}{\mu_{1}} \quad (5)$$

The purpose of this paper is to investigate of the ultimate ruin probability

$$\psi_{h}(u)=P\{\inf_{t\geq 0}R_{t}^{*}<0\,\big|\,R_{0}^{*}=u\}$$

2. Main Result

Let us introduce some classes of distribution functions (see, Borovkov A. (2002)).

Definition 1. Distribution function F on $[0,\infty)$ belongs to the class L of distributions with long tails, if for any fixed $y\geq 0$ as $u\to\infty$

$$\overline{F}(u+y)\sim\overline{F}(u)$$

Definition 2. Distribution F on $[0,\infty)$ is called subexponential, and denoted $F\in S$, if $\overline{F}(x)=1-F(x)>0$ for all $x>0$ and

$$\overline{F^{*(2)}}(x)\equiv\overline{F*F}(x)\sim 2\overline{F}(x), \text{ as } x\to\infty$$

Where

$$\overline{F^{*(2)}}(x)=1-F^{*(2)}(x)=1-\int_{0}^{x}F(x-t)F(t)dt$$

Distribution function F centered on $(-\infty,\infty)$ and belongs to the class S or L, if the function $F_{+}(u)=F(u)I_{\{u\geq 0\}}$ belongs to the corresponding class, where $I_{\{u\geq 0\}}$ is indicator of the set $\{u\geq 0\}$.

Consider the following class of functions (see, Borovkov A. (2002)):

Definition 3. Distribution function F belongs to the class Se of semiexponential distributions, if

$$\overline{F}(x)=e^{-Q(x)},$$

where $Q(x)=x^{\alpha}L(x)$, $0\leq\alpha\leq 1$ and $L(x)$ a slowly varying function at infinity and $L(x)\to 0$, as $x\to\infty$, if $\alpha=1$. Furthermore,

1) $Q(x+\Delta)-Q(x)\sim\dfrac{\alpha Q(x)\Delta}{x}$ as $x\to\infty$, $\dfrac{Q(x)\Delta}{x}>\varepsilon$ for every $\varepsilon>0$;

2) $Q(x+\Delta)-Q(x)=o(1)$ as $x\to\infty$, $\dfrac{Q(x)\Delta}{x}\to 0$.

Definition 4. Distribution function F belongs to the class Se_{*} of strongly semiexponential distributions, if F is semiexponential with parameter $0<\alpha<1$.

Examples for strongly semiexponential distributions such as the Weibull distribution and Benktandera II-type tails which are as follows, respectively:

$$\overline{F}(x)=e^{-\lambda x^{\alpha}},\ x\geq 0,\ \lambda>0,\ 0<\alpha<1;$$

$$\overline{F}(x)=x^{\alpha-1}\exp\left\{\frac{\lambda}{\alpha}-\frac{\lambda}{\alpha}x^{\alpha}\right\},\ x\geq 0,\ \lambda>0,\ 0<\alpha<1.$$

In paper, Borovkov A. (2002) was proved the following theorem:

Theorem 1 (Borovkov). Let $X_{1},X_{2},\ldots$ independent and identically distributed random variables with nonarithmetic distribution function $F_{X_{1}}\in Se_{*}$ and $EX_{1}<0$, $EX_{1}^{2}<\infty$. Then for as $u\to\infty$

$$P\left\{\sup_{n\geq 0}\sum_{k=1}^{n}X_{k}>u\right\}=-\frac{1}{EX_{1}}\int_{u}^{\infty}\overline{F}_{X_{1}}(t)dt+a\overline{F}_{X_{1}}(u)+o(\overline{F}_{X_{1}}(u)) \quad (6)$$

Where

$$a=\frac{EX_{1}^{2}}{2\left(EX_{1}\right)^{2}}-\frac{b}{EX_{1}},\ b=E\left(\sup_{n\geq 0}\sum_{k=1}^{n}X_{k}\right)=\sum_{n=1}^{\infty}\frac{1}{n}\int_{0}^{\infty}tdF_{X_{1}}^{*(n)}(t),$$

$F_{X_{1}}^{*(n)}$ - n vold convolution of function of $F_{X_{1}}(x)$ with itself.

Let us give following lemmas from Foss S., Korshunov D., Zachary S. (2009) and Aleškeviciene A., Leipus R., Šiaulys J. (2009).

Lemma 1. Let $F\in L$ centered on $[0,\infty)$ and G centered on $(-\infty,0]$. Then when $u\to\infty$, $\overline{F*G}(u)\sim\overline{F}(u)$.

Lemma 2. Let Z nonnegative random variable with $F_{Z}\in L$ and Y nonnegative random variable, not depending of Z, such that $EZ<\infty$. Then when $u\to\infty$

$$\int_{u}^{\infty}\overline{F}_{Z-Y}(t)dt=\int_{u}^{\infty}\overline{F}_{Z}(t)dt-EY\overline{F}_{Z}(u)+o(\overline{F}_{Z}(u)).$$

The main result of this paper can be formulated in the form of the following theorem.

Theorem 2. Let the sequences $\{\xi_{i}\}, i\geq 1$ and $\{\eta_{i}\}, i\geq 1$ be two independent sequences of random variables, such that variables in each sequence are independent and identically distributed. Moreover, $F_{\eta_{1}}\in Se_{*}$, $m_{2}=E\eta_{1}^{2}<\infty$ and $\mu_{2}=E\xi_{1}^{2}<\infty$. Then as $u\to\infty$

$$\psi_{h}(u)=\frac{1}{e_{1}}\int_{u}^{\infty}\overline{F}_{h(\eta_{1})}(t)dt+a_{h}\overline{F}_{h(\eta_{1})}(u)+o(\overline{F}_{h(\eta_{1})}(u)) \quad (7)$$

Where

$$a_{h}=\frac{e_{2}}{2e_{1}^{2}}+\frac{b_{h}}{e_{1}}-\frac{c^{*}\mu_{1}}{e_{1}}$$

$$e_{k}=E\left(c^{*}\xi_{1}-h(\eta_{1})\right)^{k},\ k=1,2$$

$$b_h = E\left(\sup_{n\geq 0}\sum_{k=1}^{n}(h(\eta_k) - c^*\xi_k)\right) = \sum_{n=1}^{\infty}\frac{1}{n}\int_0^{\infty} t dF^{*(n)}_{h(\eta_1)-c^*\xi_1}(t)$$

c^* defined from (5).

Proof. It is easy to see that for the proportional and excess stop loss reinsurance types the condition $F_{\eta_1} \in Se_*$ provides $F_{h(\eta_1)} \in Se_*$. Consequently, can be used the scheme of the proof of Theorem 1 from Aleškeviciene A., Leipus R., Šiaulys J. (2009). Since, $F_{h(\eta_1)} \in Se_* \subset S \subset L$ and ξ_1 is positive random variable, according to Lemma 1, as $u \to \infty$ can be obtained:

$$\overline{F}_{h(\eta_1)-c^*\xi_1}(u) \sim \overline{F}_{h(\eta_1)}(u) \tag{8}$$

On the other hand $F_{h(\eta_1)-c^*\xi_1} \in Se_*$, according to the definition of the class of strongly semiexponential distributions.

It is not difficult to see that $E\left(h(\eta_1) - c^*\xi_1\right) < 0$, or $c^*\mu_1 > Eh(\eta_1)$.

Indeed, from (5) can be obtained:

$$\begin{aligned}\mu_1 c^* &= (1+\rho)m_1 - (1+\rho_h)E\left(\eta_1 - h(\eta_1)\right) \\ &= (\rho - \rho_h)m_1 + (1+\rho_h)Eh(\eta_1)\end{aligned}$$

On the other hand by definition $0 \leq h(\eta_i) \leq \eta_i$. Therefore, $Eh(\eta_1) \leq m_1$. Consequently, taking into account $\rho_h \geq \rho$ can be obtained:

$$\mu_1 c^* = (\rho - \rho_h)m_1 + (1+\rho_h)Eh(\eta_1) > (1+\rho)Eh(\eta_1) > Eh(\eta_1)$$

Since, $h(\eta_1) - c^*\xi_1$ have non-arifmetic distribution and $E\left(h(\eta_1) - c^*\xi_1\right) < 0$, so using Theorem 1 can be obtained:

$$\psi_h(u) = P\{\inf_{t\geq 0} R_t^* < 0 \,|\, R_0^* = u\} = P\left\{\sup_{n\geq 0}\sum_{k=1}^{n}\left(h(\eta_k) - c^*\xi_k\right) > u\right\} =$$

$$= -\frac{1}{E\left(h(\eta_1) - c^*\xi_1\right)}\int_u^{\infty}\overline{F}_{h(\eta_1)-c^*\xi_1}(t)dt + \left(\frac{e_2}{2e_1^2} + \frac{b_h}{e_1}\right)\overline{F}_{h(\eta_1)-c^*\xi_1}(u) + o(\overline{F}_{h(\eta_1)-c^*\xi}(u)) =$$

$$= \frac{1}{e_1}\int_u^{\infty}\overline{F}_{h(\eta_1)-c^*\xi_1}(t)dt + \left(\frac{e_2}{2e_1^2} + \frac{b_h}{e_1}\right)\overline{F}_{h(\eta_1)}(u) + o(\overline{F}_{h(\eta_1)}(u)) \tag{9}$$

where

$$e_k = E\left(c^*\xi_1 - h(\eta_1)\right)^k, \; k = 1, 2,$$

$$b_h = E\left(\sup_{n\geq 0}\sum_{k=1}^{n}(h(\eta_k) - c^*\xi_k)\right).$$

Applying Lemma 2 to the integral $\int_u^{\infty}\overline{F}_{h(\eta_1)-c^*\xi_1}(t)dt$ can be obtained:

$$\int_u^{\infty}\overline{F}_{h(\eta_1)-c^*\xi_1}(t)dt = \int_u^{\infty}\overline{F}_{h(\eta_1)}(t)dt - c^*E\xi_1\overline{F}_{h(\eta_1)}(u) + o(\overline{F}_{h(\eta_1)}(u)). \tag{10}$$

Taking into account (10) in (9) statement of Theorem 2 can be obtained.

This completed the proof of Theorem 2.

Remark. In the particular case *h*(*x*)=*x*, i.e. when there is without reinsurance, Theorem 2 implies Theorem 1 of paper Aleškeviciene A., Leipus R., Šiaulys J. (2009).

Corollary 1. Let conditions of Theorem 2 be satisfied and ξ_1 has an exponential distribution with a tail $\overline{F}_{\xi}(x) = e^{-x}, \; x \geq 0$. Then as $u \to \infty$

$$\psi_h(u) = \tilde{\psi}_h(u) + o(\overline{F}_{h(\eta_1)}(u)) \tag{11}$$

where

$$\tilde{\psi}_h(u) = \frac{1}{e_1}\int_u^{\infty}\overline{F}_{h(\eta_1)}(t)dt + \frac{Eh^2(\eta_1)}{e_1^2}\overline{F}_{h(\eta_1)}(u)$$

Really, in this case, the constant b_h in the asymptotic expansion (9) can be explicitly computed using the formula Pollaczek-Khinchin (see, Aleškeviciene A., Leipus R., Šiaulys J. (2009)). Therefore, coefficient is $a_h = \frac{Eh^2(\eta_1)}{e_1^2}$ and from (7) can be obtained (11).

3. Numerical Examples

In this section we can consider the following numerical examples.

Example 1. Let ξ_1 has an exponential distribution with a tail $\overline{F}_{\xi}(x) = e^{-x}, \; x \geq 0$ and $h(x) = x$, i.e. without reinsurance. In this case $\mu_1 = E\xi_1 = 1$ and using (11) and Corollary 1 of study Aleškeviciene A., Leipus R., Šiaulys J. (2009) as $u \to \infty$ can be obtained:

$$\psi(u) = \tilde{\psi}(u) + o(\overline{F}_{\eta_1}(u)) \tag{12}$$

where

$$\tilde{\psi}(u) = \frac{1}{e_1}\int_u^{\infty} \overline{F}_{\eta_1}(t)dt + \frac{E\eta_1^2}{e_1^2}\overline{F}_{\eta_1}(u)$$

$$e_1 \equiv E\left(c^*\xi_1 - h(\eta_1)\right) = E\left(c\xi_1 - \eta_1\right) = c - m_1 = \rho m_1$$

Let also the random variable η_1 has Weibull distribution with tail $\overline{F}_{\eta_1}(x) = e^{-x^{\frac{1}{3}}}$, $x \geq 0$ and $\rho = 0,7$.

The results of calculations values of function $\tilde{\psi}(u)$ at different values of the initial capital u are given in the following table:

Table 1. ($\mu_1 = 1$; $m_1 = 6$; $\rho = 0,7$; $c = 10,2$).

μ_1	100	200	250	300	350	400	450	500
$\tilde{\psi}(u)$	0.6197	0.2165	0.1462	0.1038	0.0764	0.0578	0.0447	0.0352

Example 2 (Proportional reinsurance). Under conditions of Example 1 consider proportional reinsurance. Let $\rho = 0,7$ and the relative insurance premium reinsurance company be $\rho_h = 0,81$. It is known that retention limit must satisfy the inequality (see, Dickson D. (2005), p. 199):

$$\beta > 1 - \frac{\rho}{\rho_h}$$

In our example $\beta > 0{,}1358$. Let $\beta = 1/2$. Consequently, we can draw the following tables for the function $\tilde{\psi}(u)$ from (11) for different values of the initial capital u:

Table 2. ($\mu_1 = 1$; $m_1 = 6$; $\rho = 0,7$; $\rho_h = 0,81; \beta = 0,7; c* = 6,942$).

μ_1	100	200	250	300	350	400	450	500
$\tilde{\psi}(u)$	0.4153	0.1266	0.0812	0.055	0.0389	0.0284	0.0212	0.0162

Table 3. ($\mu_1 = 1$; $m_1 = 6$; $\rho = 0,7$; $\rho_h = 0,81; \beta = 0,5; c* = 4,77$).

μ_1	100	200	250	300	350	400	450	500
$\tilde{\psi}(u)$	0.2829	0.0742	0.045	0.029	0.0196	0.0138	0.0099	0.0073

Table 4. ($\mu_1 = 1$; $m_1 = 6$; $\rho = 0,7$; $\rho_h = 0,81; \beta = 0,2; c* = 1,512$).

μ_1	100	200	250	300	350	400	450	500
$\tilde{\psi}(u)$	0.1613	0.0241	0.0118	0.0064	0.0037	0.0022	0.0014	0.0009

Example 3 **(**Exsess stop loss reinsurance**).** Under conditions of Example 1 consider exsess stop loss reinsurance. Let $\rho = 0,7;$ and $\rho_h = 0,81$.

It is not difficult to see, that

$$Eh(\eta_1) = \int_0^{\infty} \overline{F}_{h(\eta_1)}(x)dx = \int_0^{M} \overline{F}_{h(\eta_1)}(x)dx + \int_M^{\infty} \overline{F}_{h(\eta_1)}(x+L)dx$$

$$= \int_0^{M} \overline{F}_{h(\eta_1)}(x)dx + \int_{M+L}^{\infty} \overline{F}_{h(\eta_1)}(x)dx$$

$$Eh^2(\eta_1) = 2\int_0^{\infty} x\overline{F}_{h(\eta_1)}(x)dx = 2\int_0^{M} x\overline{F}_{h(\eta_1)}(x)dx + 2\int_M^{\infty} x\overline{F}_{h(\eta_1)}(x+L)dx =$$

$$= 2\int_0^{M} x\overline{F}_{h(\eta_1)}(x)dx + 2\int_{M+L}^{\infty} x\overline{F}_{h(\eta_1)}(x)dx - 2L\int_{M+L}^{\infty} \overline{F}_{h(\eta_1)}(x)dx,$$

Consequently, we can draw the following tables for the function from (11) for different values of the initial capital u and parameters M and L:

Table 5. ($\mu_1 = 1$; $m_1 = 6$; $\rho = 0,7$; $\rho_h = 0,81;$ $M = 50; L = 30; c* = 9,202$).

μ_1	100	200	250	300	350	400	450	500
$\tilde{\psi}(u)$	0,45	0,1802	0,126	0,0917	0,0688	0,0529	0,0415	0,033

Table 6. ($\mu_1 = 1$; $m_1 = 6$; $\rho = 0,7$; $\rho_h = 0,81;$ $M = 50; L = 40; c* = 8,9786$).

μ_1	100	200	250	300	350	400	450	500
$\tilde{\psi}(u)$	0,4064	0,1688	0,1192	0,0874	0,066	0,051	0,0401	0,0321

Table 7. ($\mu_1 = 1$; $m_1 = 6$; $\rho = 0,7$; $\rho_h = 0,81;$ $M = 50; L = 50; c* = 8,79$).

μ_1	100	200	250	300	350	400	450	500
$\tilde{\psi}(u)$	0,3681	0,158	0,1126	0,0832	0,0632	0,049	0,0387	0,031

Note: If the transmission company will pay all claims up to a certain limit M, and the claims exceeding the limit M is suing to reinsurance companies. If this rule applies to each claim, then this type of reinsurance is called the excess of loss reinsurance. Parameter M is called the limit of retention. For the excess loss reinsurance $h(x) = \min\{x, M\}$.

It is not difficult to see that in this case

$$F_{h(\eta_1)}(x) = F_{\eta_M}(x) = \begin{cases} F_{\eta_1}(x), & x < M \\ 1, & x \geq M \end{cases}.$$

Therefore, $F_{h(\eta_1)} \notin Se_*$. Consequently, main result of present study Theorem 2 does not include excess of loss reinsurance.

4. Conclusion

In present paper the Sparre Andersen risk process with reinsurance is considered. Second-order asymptotic expansion for the ruin probability is obtained, when claim sizes have the strongly semiexponential distribution. The obtained result shows that, this asymptotic expansion true also for the proportional reinsurance and excess stop loss reinsurance types, but is not satisfied for the excess loss reinsurance. Numerical examples provided in paper show that, as are insurance effect on the probability of ruin. Note that, since, in most cases there are no analytic expressions available for the deficit distribution and its moments at the time of ruin in future studies can be obtain asymptotic expansions for these characteristics using methods of present study.

Acknowledgement

The author express his thanks to Professor T. A. Khaniyev, TOBB University of Economics and Technology (Turkey), for his supports and valuable advices.

References

[1] Albrecher H., Claramunt M. M., Mármol M. (2006). On the distribution of dividend payments in a Sparre Andersen model with generalized Erlang (*n*) interclaim times. Insurance: Mathematics and Economics 37, pp. 324–334.

[2] Aleškeviciene A., Leipus R., Šiaulys J. (2009). Second-order asymptotics of ruin probabilities for semiexponential claims. Lithuanian Mathematical Journal, 49, 4, pp. 364–371.

[3] Aliyev R. T., Jafarova V. (2009). On the moments of the Sparre Andersen surplus process and it's average value. 13th International Congress on Insurance: Mathematics and Economics, Istanbul, Turkey, p. 39.

[4] Asmussen S. (2000). Ruin Probabilities. World Scientific, 382 p.

[5] Baltrunas A. (1999). Second-order asymptotics for the ruin probability in the case of very large claims. Sib. Math. J., 40, pp. 1034–1043.

[6] Borovkov A. A. (2002). On subexponential distributions and asymptotics of the distribution of the maximum of sequential sums. Sib. Math. J., 43, pp. 995–1022.

[7] Dickson D. C., Waters H. R. (1996). Reinsurance and ruin. Insurance: Mathematics and Economics, 19, 1, pp. 61-80.

[8] Dickson D. C., Waters H. R. (1997). Relative reinsurance retention levels. ASTIN Bulletin, 27, 2, pp. 207–227. ASTIN BULLETIN, Vol. 27, No. 2. 1997, pp. 207-227.

[9] Dickson D. Proportional Reinsurance. Encyclopedia of Actuarial Science 2006.

[10] Dickson D. Insurance Risk and Ruin, Cambridge University Press, 2005, 229 p.

[11] Embrechts P., Veraverbeke N. (1982). Estimates for the probability of ruin with special emphasis on the possibility of large claims. Insurance: Mathematics and Economics, 1, pp. 55–72.

[12] Foss S., Korshunov D., Zachary S. (2009). Convolutions of long-tailed and subexponential distributions, Preprint available at http://www.math.nsc.ru/LBRT/v1/foss

[13] Gerber H. U., Shiu E. W. (2005). The Time Value of Ruin in a Sparre Andersen Model. North American Actuarial Journal, 9 (2), pp. 49–84.

[14] Hald M., Schmidli H. (2004). On the maximisation of the adjustment coefficient under proportional reinsurance. ASTIN Bulletin, 34, pp. 75-83.

[15] Li S., Garrido J. (2004). On ruin for the Erlang (*n*) risk process, Insurance: Mathematics and Economics 34 (3), pp. 391–408.

[16] Li S., Dickson D. C. M. (2006). The maximum surplus before ruin in an Erlang (*n*) risk process and related problems, Insurance: Mathematics & Economics 38 (3), pp. 529–539.

[17] Luo S., Taksar M., Tsoi A. (2008). On reinsurance and investment for large insurance portfolios. Insurance: Mathematics and Economics, 42, 1, pp. 434–444.

[18] Schmidli H. (2002). On minimizing the ruin probability by investment and reinsurance. Ann. Appl. Probab. 12, 3, pp. 890-907.