International Journal of Statistical Distributions and Applications
2023; 9(1): 24-34
http://www.sciencepublishinggroup.com/j/ijsda
doi: 10.11648/j.ijsd.20230901.13
ISSN: 2472-3487 (Print); ISSN: 2472-3509 (Online)

Analyzing Dynamic Regimes of GARCH Model on Stock Price Volatility

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To cite this article:
Rosemary Ukamaka Okafor, Josephine Nneamaka Onyeka-Ubaka. Analyzing Dynamic Regimes of GARCH Model on Stock Price Volatility. *International Journal of Statistical Distributions and Applications*. Vol. 9, No. 1, 2023, pp. 24-34. doi: 10.11648/j.ijsd.20230901.13

Received: January 20, 2023; **Accepted**: February 16, 2023; **Published**: February 27, 2023

Abstract: As a result of volatility dynamics, investors and other stakeholders in businesses and industries have difficulty making financial decisions. Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are the most widely applied in the analysis of financial derivatives volatility. Volatility persistence is a common issue when analyzing stock prices, making it cumbersome for GARCH models. The GARCH model is transformed into the Makov switching GARCH model to check for dynamics in volatility persistence. Markov Regime-Switching GARCH (MSGARCH) models permit the conditional mean and variance to change across regimes over time. The Markov switching GARCH models incorporate the regime variables in the parameter space, making it viable for the parameters to be estimated by the maximum likelihood estimation method, unlike the classical GARCH models. Zenith Bank plc's daily closing stock prices, a top-tier stock on the Nigerian Stock Exchange market, are fitted using the GARCH and MSGARCH models. The comparison between the MSGARCH model and the classical GARCH model was verified using the AIC and BIC metrics as well as the one with the maximum log likelihood estimates. The outcome suggests that MSGARCH model performs better than the single-regime GARCH model and that it yields significantly better out of-sample volatility forecasts. The results will aid the stakeholders to leverage and mitigate risks in their investment on the selected stocks.

Keywords: Volatility, GARCH Process, Regime-Switching, Markov Process, Maximum Likelihood Estimation

1. Introduction

As demonstrated by numerous authors in the past, including Bollerslev, Engle, and Nelson, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models can be used to model changes in variance or volatility over time when modeling time series, particularly in financial markets [1]. GARCH models support time-dependent volatility changes, such as increasing and declining volatility in time series. GARCH models frequently exhibit high conditional variance persistence. The nearly integrated behavior of the conditional variance demonstrated by some financial data, according to Diebold, F. X., Lamoureux, C. G., Lastrapes, W. D. and other authors, may result from structural changes in the variance process not taken into account by traditional GARCH models [2, 3]. In addition, Mikosch, T. and Starica, C. demonstrates that GARCH effect occurs when using GARCH models on samples that exhibit structural changes in the unconditional variance [4]. These findings indicate that the estimates may suffer from a significant upward bias in the persistence parameter if the form of the conditional variance is fixed and relatively rigid for the entire sample period. Therefore, allowing the parameters to change over time may be a better choice for volatility modeling. Volatility in stock prices or other financial indexes is the degree of fluctuation over time. Factors that trigger financial volatility are demand, supply, economic policy, company earnings and many others.

The Markov regime-switching model is a widely used method for allowing modifications in the data generation process. Schwert, G. W. worked with a model which switches between high and low variance returns, controlled by a two-state Markov process [5]. For stock price volatility, the Markov process is sufficient to control the switches between regimes with different variances. See Franses, P. H., Van Dijk, D.; Haas, M., Mittnik, S., and Paolella, M. S. for more details on Markov-switching models [6, 7].

The construction of the likelihood function of MSGARCH is made possible by using a dynamic model where the unobservable state variables are part of the model parameters. As demonstrated by Gray, S. F., the switching GARCH model does not place any constraints on the GARCH parameters [8]. In real-life data, according to Chung-Ming Kuan., it is possible to postulate the residuals as independent and identically distributed random variables with t(n)-distribution instead of always assuming conditional normality [9].

Classical time series models assume that a single set of parameters is sufficient to model a given data over time, but real-life data does not always support this presumption. Real-life time series may exhibit different statistics, like mean and variance, across different periods, hence the need to utilize regime-switching models. Regime-switching models estimate data as residing in various recurrent regimes or states. These models permit the characteristics of time series data, including means, variances, and model parameters, to change across the states. Regime-switching models also assume that at any given period, the series may be in any of the regimes and may transit to a different one with a certain probability. To better capture the actual behavior of real-life data, it may be better to use regime-changing models than single-regime models. See also Klaassen, F. J. G. M. [10].

Generalization of the GARCH model by allowing for regimes with volatility levels to avoid excessive GARCH effects in volatile periods was adopted. To estimate a model that permits parameter regime switching, the Markov-switching GARCH model that allows for a different persistence in the conditional variance of each regime was utilized. A 2-regime Markov chain was used to investigate whether the inclusion of states aids in resolving the issue with GARCH forecasts. Finally, compared with the single-regime GARCH, the allowance of regimes and dynamics improved shock persistence. We applied this approach to a time series of Zenith Bank plc stock prices traded daily on the Nigerian stock exchange. The bank was chosen because of it's rating in the industry as well as the volume traded daily on the Nigerian stock market.

2. Models

In this section are reviews of the standard one-regime GARCH volatility model, the Markov Switching model, and the Markov-Switching GARCH model.

2.1. GARCH Model

Bollerslev, T. extended the ARCH model to produce the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) [11]. He achieved that by including a moving average component to the autoregressive component. In particular, the model incorporates lag variance terms and residual lag errors from a mean process. Including a moving average component enables the model to capture changes in the time-dependent and conditional variance over time. See Jason B.. for more details [12]. Therefore, the GARCH model introduces a new parameter "p" that describes the number of lag variance terms, where q is the number of residual lag errors. In essence, the GARCH model uses values of the past squared observations and past variances to model the variance at time t as thus:

$$\varepsilon_t = \sigma_t v_t \tag{1}$$

$$\sigma_t{}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2 \tag{2}$$

Where $\alpha_0 > 0$; $\alpha_i \geq 0$, i=1,2,...,q and $\beta_j \geq 0$, j=1,2,...,p

In the GARCH (p, q) model, the variance forecast takes the weighted average of both past square errors and historical values. In real-life data applications, β_1 component is often used in the analysis because it clearly interprets the specific volatility processes of the model.

Thus, equation (2) becomes:

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \, \varepsilon_{t-1}^2 + \beta_1 \, \sigma_{t-1}^2 \tag{3}$$

2.2. Markov Switching Model

Markov-switching model assumes that the unobserved states are accounted for by an underlying stochastic process known as the Markov chain. Markov-chain is used to describe how data falls in unobserved regimes, and one of its properties is that future states are dependent only on the present state. Markov-switching models capture the non-linearity and asymmetries of data behavior in various regimes. See Bauwens, L., Preminger, A., Rombouts, J. V. K.; Lin, C. C., Hung, M. W. and Kuan, C. M.; Sachin, Date for more reading [13-15].

If the dynamic pattern of stock prices is represented as an unobservable state-dependent process, then the standard 2-state Markov switching model is given as:

$$y_t = \mu_{st} + \sigma_{st}\varepsilon_t, \text{ where } \varepsilon_t \sim N(0,1) \tag{4}$$

such that

$$y_t = \mu_1 + \sigma_1\varepsilon_t \quad \text{when } S_t = 1 \tag{5}$$

$$y_t = \mu_2 + \sigma_2\varepsilon_t \quad \text{when } S_t = 2. \tag{6}$$

where y_t is the log return of stock price at period t, S_t= k is the period in state k, and ε_t is the white noise error term.

The unobserved state variable S_t {1,2,...k} controls the mean, μ and variance, σ^2 of the Markov process. The transition of the state variables from one regime to another is dependent on the transition probabilities.

Let's take k = 2 for the analysis, where S_t= 1 is the downward shift of the stock price, and S_t= 2 is the upward shift of the stock price.

Since the state variables S_t follow the first-order Markov process on a finite set $S = \{1, \ldots, k\}$, with transition probabilities $\{P_{ij} = P(S_t = j \mid S_{t-1} = i)\}$ and the state probability distribution $\{\pi_t\}$.

Then, the probability of state variables of S_t is:

$$P_{ij} = Pr\,(S_t = j \mid S_{t-1} = i) \qquad i, j = 1,2 \tag{7}$$

and

$$\sum_{i=1}^{2} P_{ij} = 1 \tag{8}$$

where P_{ij}= Pr (S_t= j |S_{t-1}= i) is the probability that the process is in state j at time t given that it resided in i at the previous period. The transition probability in matrix form is

$$P = \begin{bmatrix} P_{11} & 1 - P_{11} \\ 1 - P_{22} & P_{22} \end{bmatrix} \quad (9)$$

The unconditional probability distribution of S_t is the state probability distribution of the Markov variable S_t at each time step t and is denoted by π_t . The unconditional probability distribution always adds up to one as with other probability distributions. The unconditional probability of the process being in state j at time t can be represented as π_{jt}.

2.3. Markov-Switching GARCH Model

The regime-switching GARCH model allows all the GARCH parameters to switch and does not impose any constraints on the parameters; thus, it is dynamic. Recall also that in practice, instead of assuming conditional normality, it is possible to postulate as i.i.d. random variables with t(n)-distribution. Therefore, to estimate a model that permits regime switching in the parameters, the Markov-switching GARCH (MSGARCH) model, a conception of the GARCH model that permits distinct persistence in the conditional variance of each regime is applied. The Markov-Switching GARCH model, therefore, involves estimating the optimal values of these variables: the predicted mean, the conditional variance, the regime-switching process, and the model's distribution. See also Gray, S.F.; Mohd, Azizi Amin Nunian; Siti, Meriam Zahari and Sarifah, Radiah Shariff [16, 17].

The predicted mean equation can be modeled as a constant mean using any of the AR, ARMA, or ARIMA components; see Onyeka-Ubaka, J. N., Ogundeji, R. K., Okafor, R. U. [18].

The conditional variance of the model can be expressed as:

$$\sigma_t^2 = \alpha_{0_{st}} + \alpha_{1_{st}}\varepsilon^2{}_{t-1} + \beta_{1_{st}}\sigma^2{}_{t-1} \quad (10)$$

where $S_t \in \{1, 2, \ldots k\}$ and $\varepsilon_t = Y_t - \mu_{st}$. The conditional variance depends on the observable Ω_{t-1}, the current regime S_t that determines all the parameters, and also on all past states S_{t-1} . Therefore, the optimal parameters can be decomposed into three components:

$$\theta_t{}^{(i)} = (\mu_t{}^{(i)}, \sigma_t{}^{2(i)}, \upsilon_t{}^{(i)}) \quad (11)$$

where $\mu_t{}^{(i)}$= E $[r_t|\Omega_{t-1}, S_t = i]$ is the conditional mean, $\sigma_t{}^{2(i)}$= Var $(r_t|\Omega_{t-1})$ the conditional variance, and $\upsilon_t{}^{(i}$ the shape parameter of the conditional distribution.

The pivot feature of MSGARCH models is the ability for some or all of their parameters to switch across different regimes, controlled by a state variable S_t. This state variable is assumed to follow a first-order Markov chain with a transition probability, P_{ij} = Pr(S_t = j| S_{t-1} = i). The state transition probability contains the probabilities of transition to the next state which are conditional upon the current state.

Thus, to apply the regime-switching GARCH model that allows for regime-switching in the parameters, let $S_t \in \{1, 2, \ldots, k\}$ be the unobserved state variable for each t, { P_{ij}= P(S_t= j|S_{t-1}= i)}is the transition probability, and Ω_{t-1} is the information set available at time t.

Then, the Markov switching GARCH model can be written as:

$$r_t|(S_t = k, \Omega_{t-1}) = \begin{cases} f(\theta_t{}^{(i)}) & when\ P(P_{1,t}) \\ f(\theta_t{}^{(j)}) & when\ P(1 - P_{1,t}) \end{cases} \quad (12)$$

where f (θ_t) is the conditional distribution, ($\theta_t{}^{(i)}$) is the vector of parameters in the ith regime, $P_{1,t}$= Pr [S_t=1 |Ω_{t-1}] is the state probability distribution, Ω_{t-1} is the information set up until (t - 1) of the Markov process, S_t = k is the current state and r_t is the stock price returns at time t. r_t is transformed to stationarity with zero mean and is uncorrelated before using for the analysis.

3. Methodology

The first step in creating a volatility model for an asset return series in the GARCH model is to define a mean equation by determining whether the data is serially dependent. If they are, we build an econometric model (such as the ARMA or ARIMA) for the return series to eliminate any linear dependence. Afterward, the residuals of the mean equation is used to test for ARCH effects. If the ARCH effects are statistically significant, a volatility model is specified and then a joint estimation of the mean and volatility equations is performed. See more details at Caporale, G.M.; Zekokh, T.; Yuehchao Wu and Remya Kannan [19, 20]. The parameters of the models can be estimated using Maximum Likelihood (MLE). In modeling volatility, choosing an appropriate model from various suitable models is essential. Identifying the best model in time series analysis is crucial in getting the best result. The model selection principle is a criterion to assess if the fitted model suggests an optimum balancing between parsimony and goodness of fit. This study utilized the frequently used model selection principle, the Akaike Information Criterion (AIC), to analyze the fitted models. Also used for th ecomparison is the maximum likelihood estimation method. Hence, the best model has a lower AIC value and the highest log-likelihood metrics.

For the Markov switching models, the method of maximization of log-likelihood is by taking partial derivatives of the log-likelihood w.r.t. to each parameter P_{11}, P_{22}, $\beta_0{}^\wedge{}_1$, $\beta_0{}^\wedge{}_2$, σ^2, setting them to zero, and solving the resulting system of equations using any optimization algorithm.

In MSGARCH model, for a 2-regime switching, the transition probabilities in transition matrix form are represented as follows:

$$P = \begin{bmatrix} P_{11} & 1 - P_{11} \\ 1 - P_{22} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (13)$$

4. Results and Discussion

The data used for the work is the daily historical closing prices in Nigerian Naira of Zenith Bank Nigeria PLC stock, extracted from the Nigerian Stock Exchange website. The duration is March 1, 2012, to March 29, 2022, with 2494 observations. Some of the observations fell into the Covid era but the effect on the market was negligible since the devastation of the virus was not huge on the Nigerian market unlike in the western countries. Therefore, no consideration was given to that in this study. First, using the rugarch package for the Garch model process in R Studio, followed by the R package MSGARCH, which implements a comprehensive set of functionalities for Markov-switching GARCH and a mixture of GARCH models. [21]. The model's conditional distribution is verified using the normal distribution and Student t-distribution for the GARCH process. Student t distribution fits better for the sGARCH(1,1) model. Therefore, in the MSGARCH model analysis, the Student t distribution was adopted. Finally, the performance of the models is estimated using the AIC and BIC metrics and the log-likelihood.

4.1. Exploratory Analysis of the Data

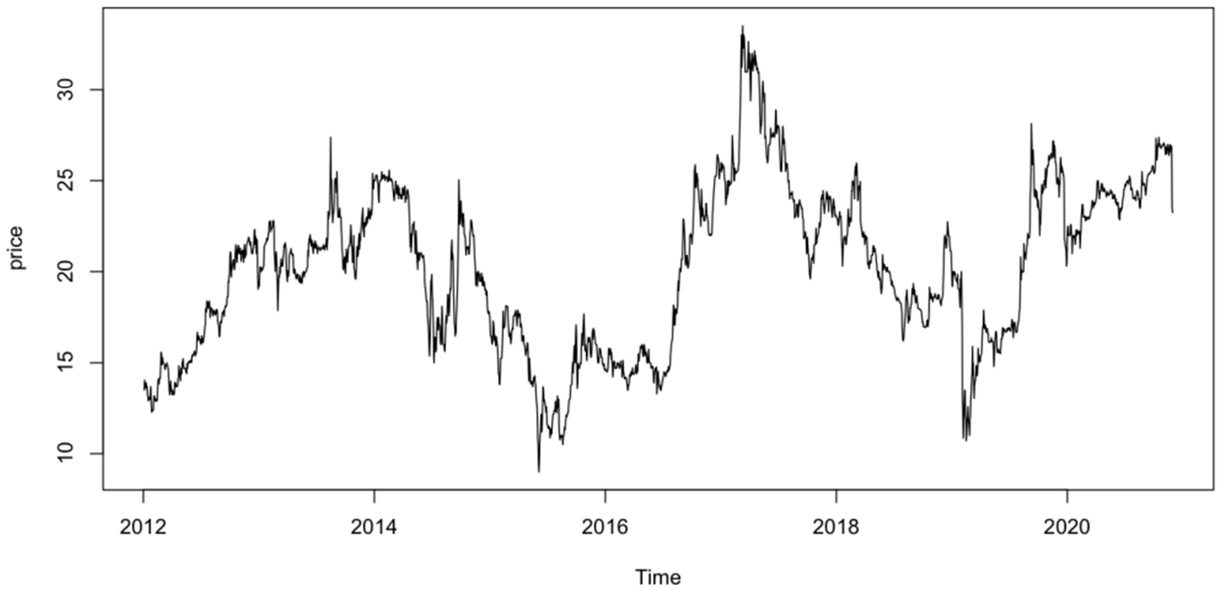

Figure 1. *Time plot of Daily Zenith Stock Prices (2012-2022).*

The time plot of the data appears to be non-stationary with an upward and downward trend, which indicates changes in variation over time, as shown in Figure 1. To further check for the stationarity of the time series, let’s examined the ACF and PACF plots.

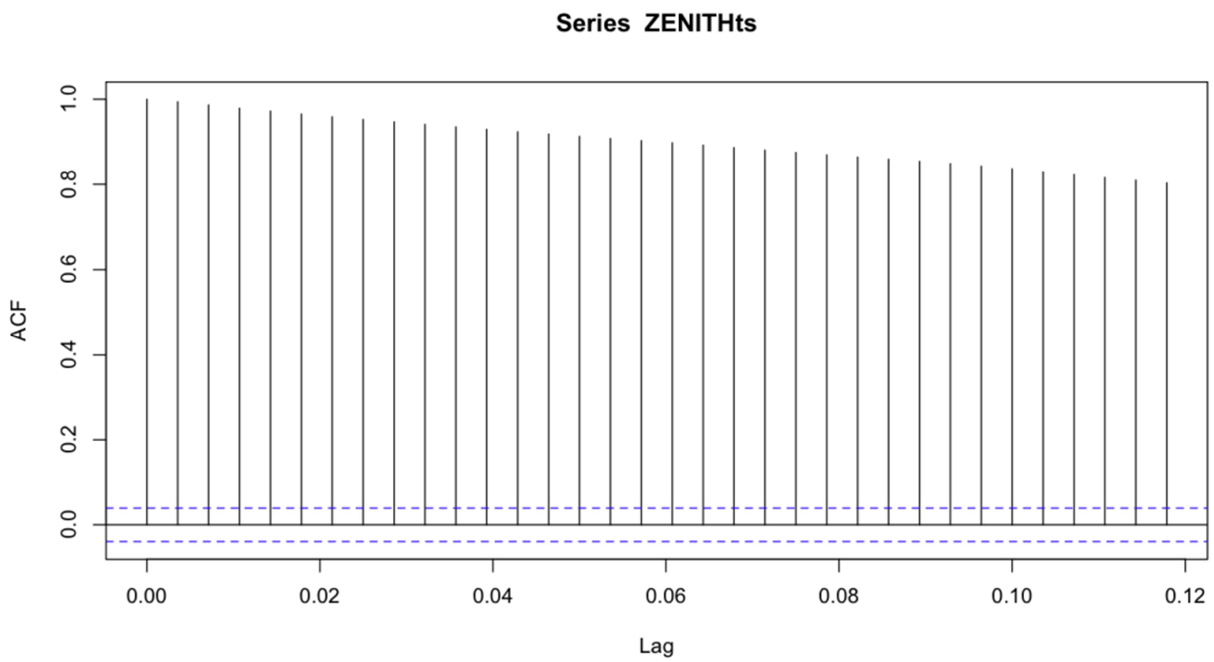

Figure 2. *ACF Plot of Daily Zenith Stock Prices (2012-2022).*

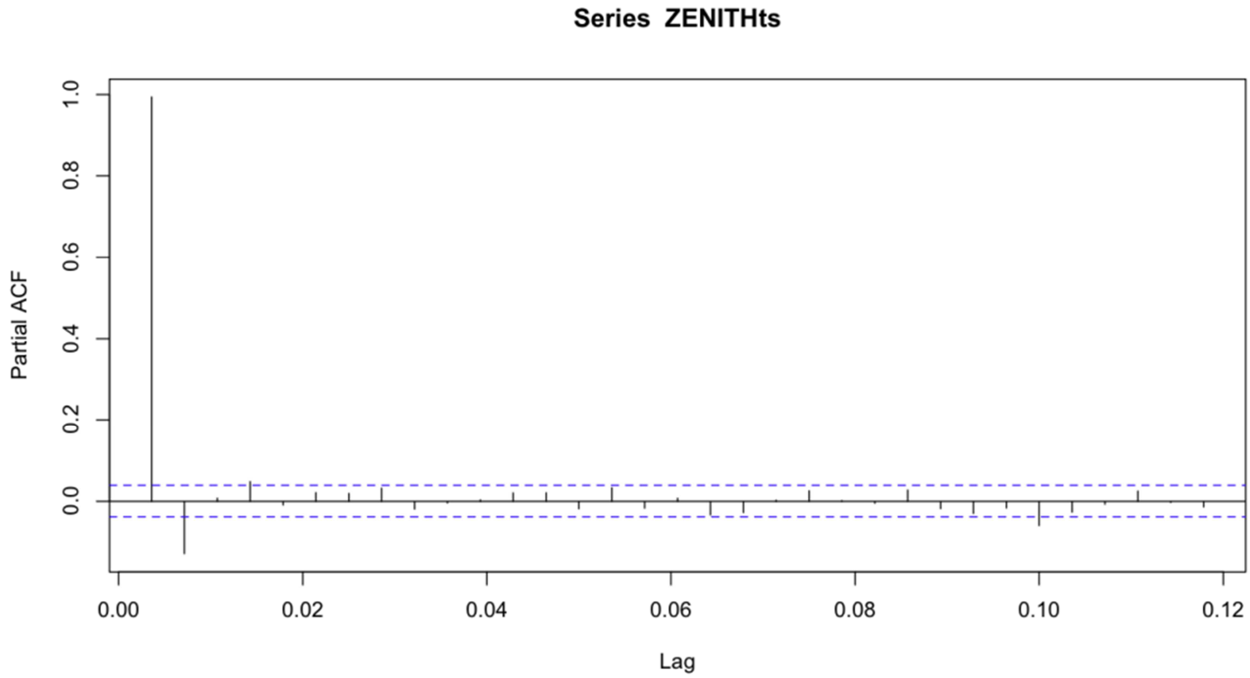

Figure 3. *PACF Plot of Daily Zenith Stock Prices (2012-2022).*

Figures 2 and 3 are the Autocorrelation Function (ACF) and Partial autocorrelation Function (PACF) plots of Zenith bank daily stock prices. The ACF plot decays to zero slowly, indicating that the shock affects the process which also confirms non-stationarity. Therefore, to achieve time series stationarity, the data is transformed to obtain log return of the stock prices.

Figure 4 is the time series plot of the Zenith stock price return series, and Figure 5 is the time plot of the Zenith stock price squared return. The log returns distribution also show non-normality.

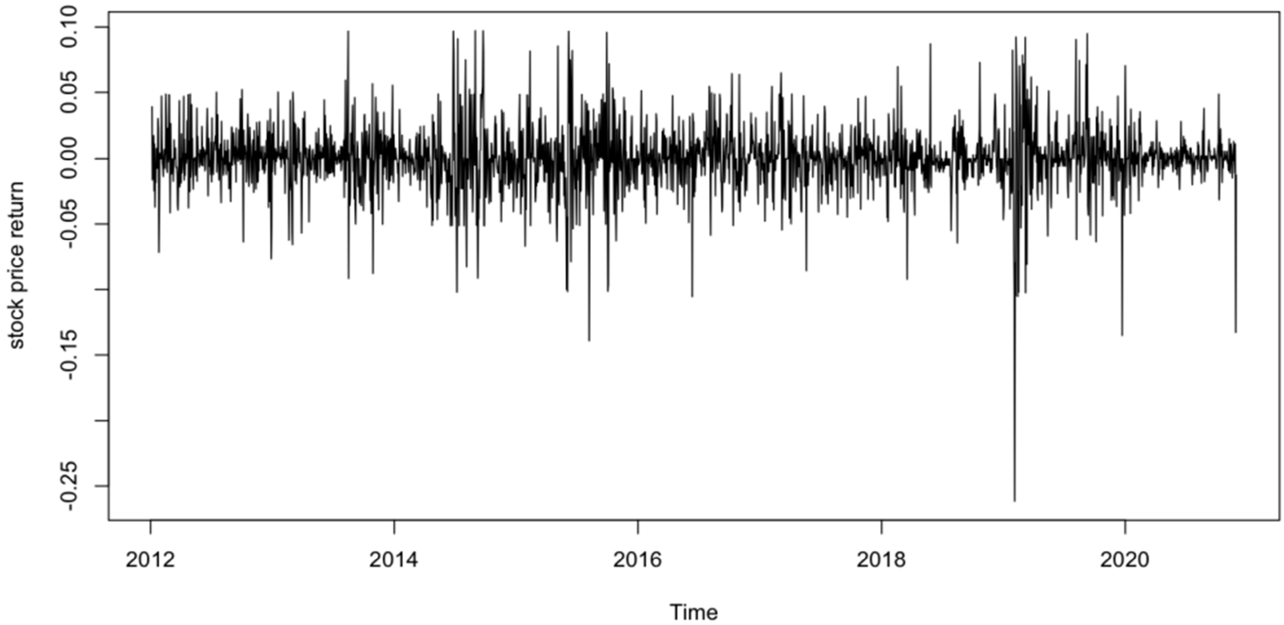

Figure 4. *Plot of Daily Zenith Stock Price Return (2012-2022).*

Table 1. *Descriptive Statistics of Zenith Stock* (y_t) *and Zenith Returns* $r_t = log\ (\frac{y_t}{y_{t-1}})$.

Statistics	**Zenith Stock (y_t)**	**Zenith Returns (r_t)**
Observations	2494. 00000	2493.00000
Minimum	9.00000	-0.261480
Maximum	33.51000	0.097222

Statistics	Zenith Stock (y_t)	Zenith Returns (r_t)
Mean	20.174439	0.000218
Median	20.55500	0.00000
Standard deviation	4.447583	0.024175
Skewness	0.067143	-0.730154
Kurtosis	-0.538133	9.406979

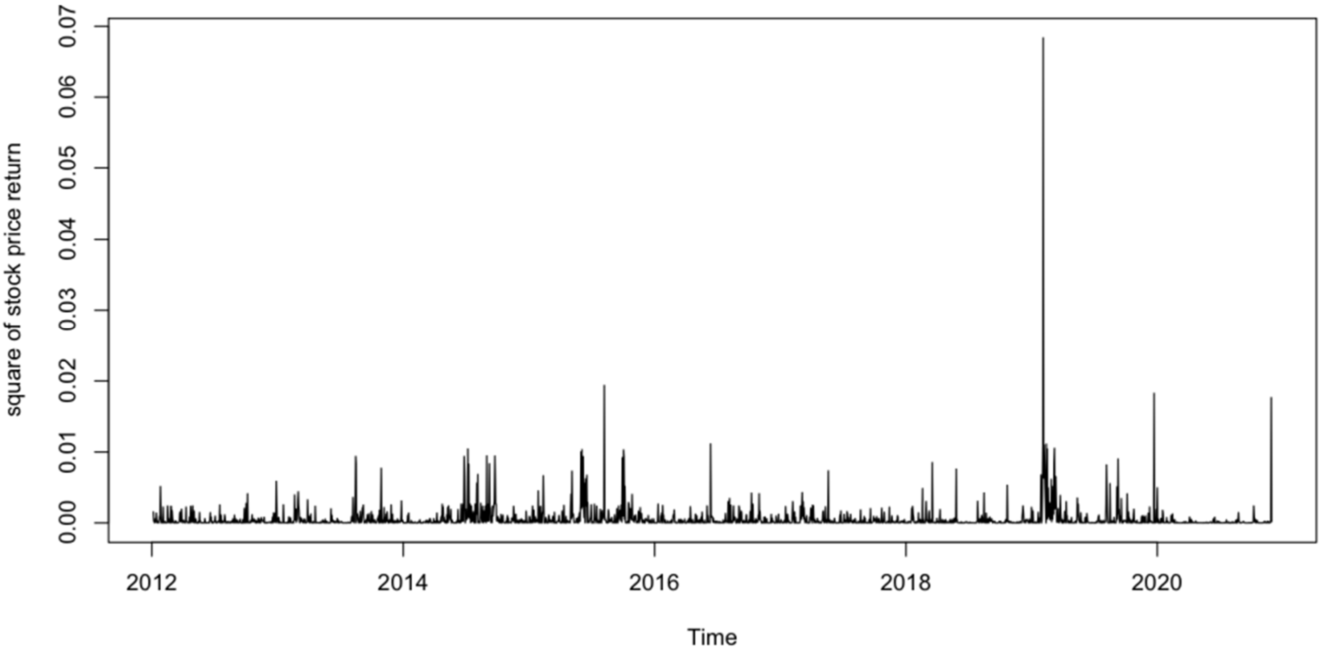

Figure 5. *Plot of Daily Zenith Stock Price Squared Return (2012-2022).*

The descriptive statistics in Table 1 show that the mean of the return series was constant and nearly zero. The ACF plot in Figure 6 also confirms that the log stock price returns are uncorrelated for the time series. However, the ACF plot of stock price return squared in Figure 7 shows correlation.

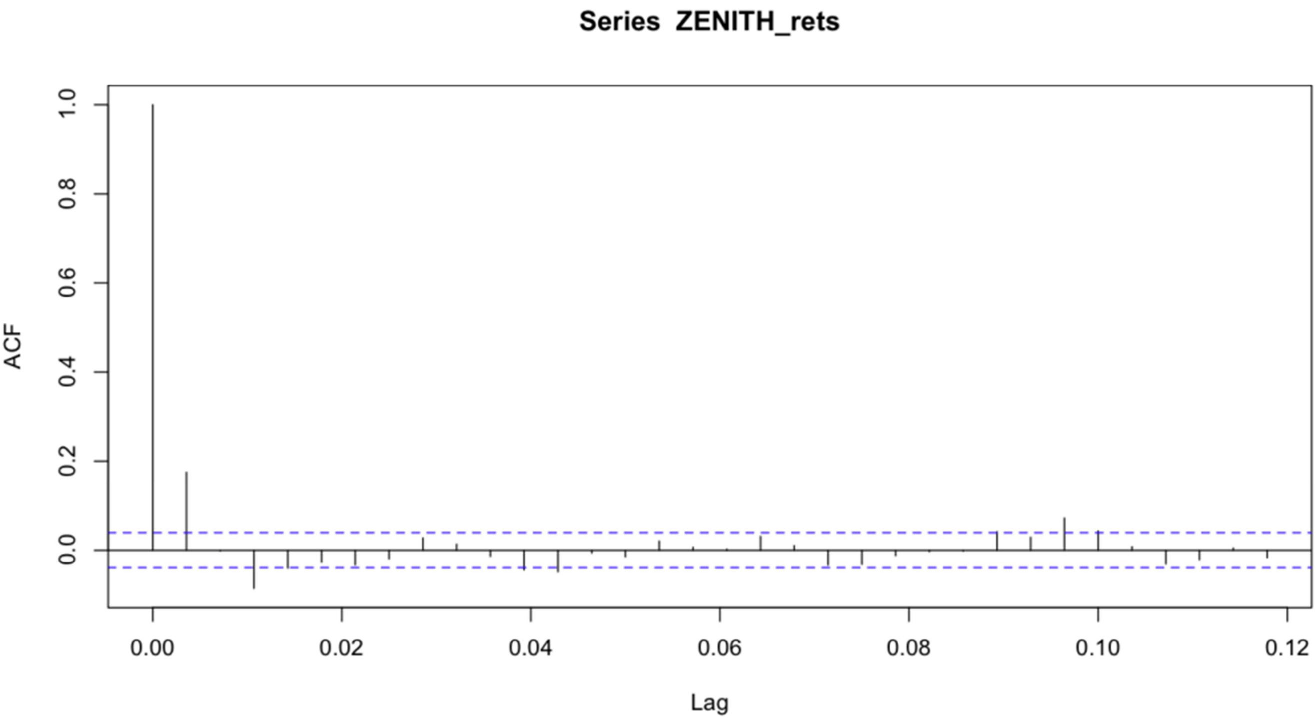

Figure 6. *ACF Plot of Zenith Stock Prices Returns (2012-2022).*

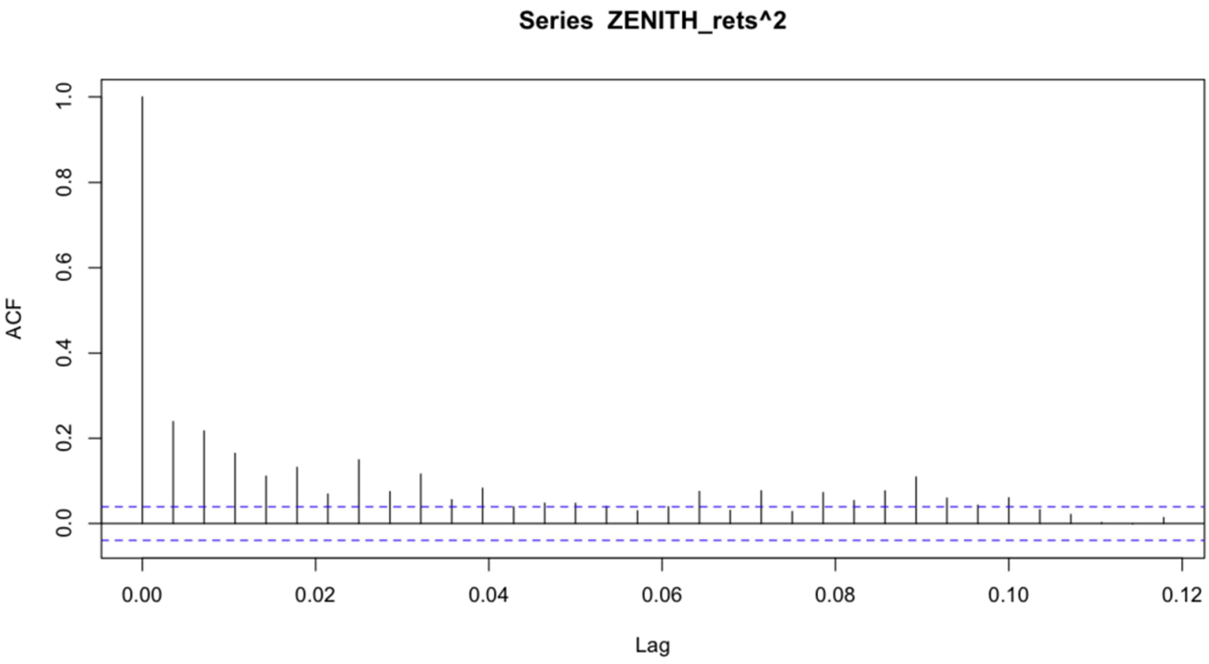

Figure 7. *ACF Plot of Zenith Stock Returns Squared (2012-2022).*

For the residual diagnostic check, the Ljung Box test was used to check the independence of the stock return price residuals. The Ljung Box tests from the results show that the log returns of Zenith stock prices are not correlated as the p-values are > 0.05. Hence, the default null hypothesis of no autocorrelation cannot be rejected. From the Ljung Box tests on the squared values of the stock price returns and the absolute values of the stock price returns, the ARCH effect on the log-returns of the stock prices was confirmed, hence the conclusion that there are ARCH effects and that the volatility can be modeled. For the parameter of the mean equation to be used for the MSGARCH model, the ARIMA model was used to select the optimal parameters. Using the "auto.arima" command in R Studio, the best model is ARIMA (2, 1, 1) with AIC as 3174.51. The auto.arima function generally picks the ARIMA(p, d, q) with the lowest AIC metric.

4.1.1. GARCH Process

To determine the order of the GARCH Model, the plots of the PACF of the log return of the stock prices, and the squared log return of the stock prices were checked. Figure 8 is the PACF of the Zenith stock price returns, while Figure 9 is the PACF plot of the squared return of Zenith stock prices.

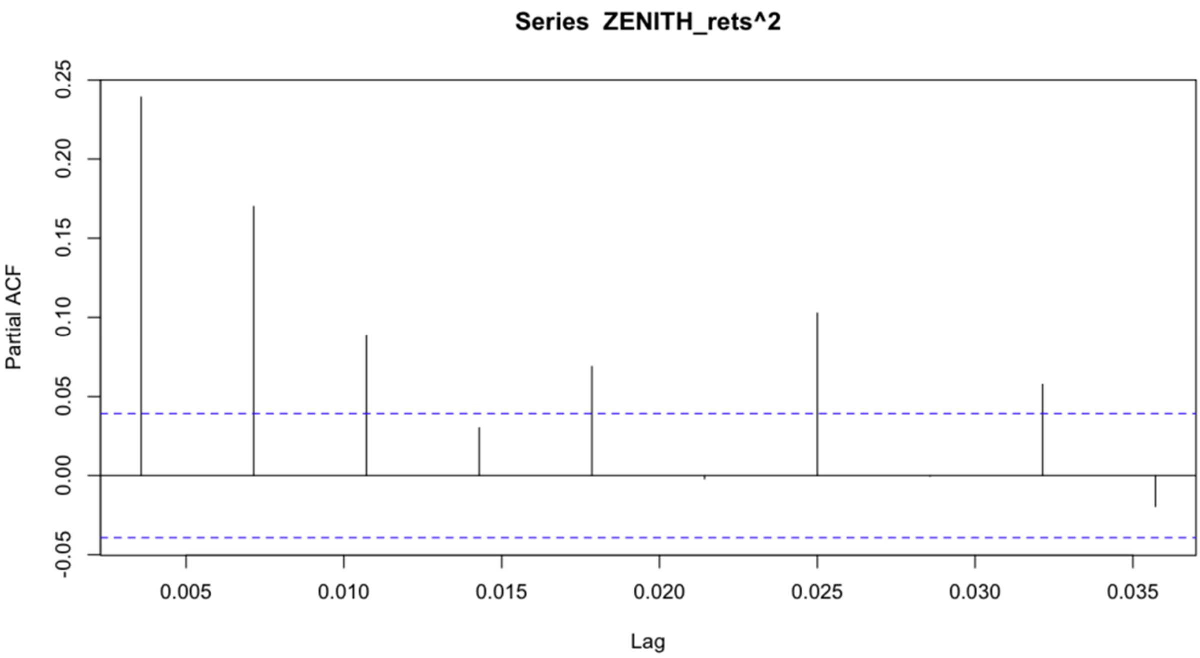

Figure 8. *PACF Plot of Zenith Stock Price Returns.*

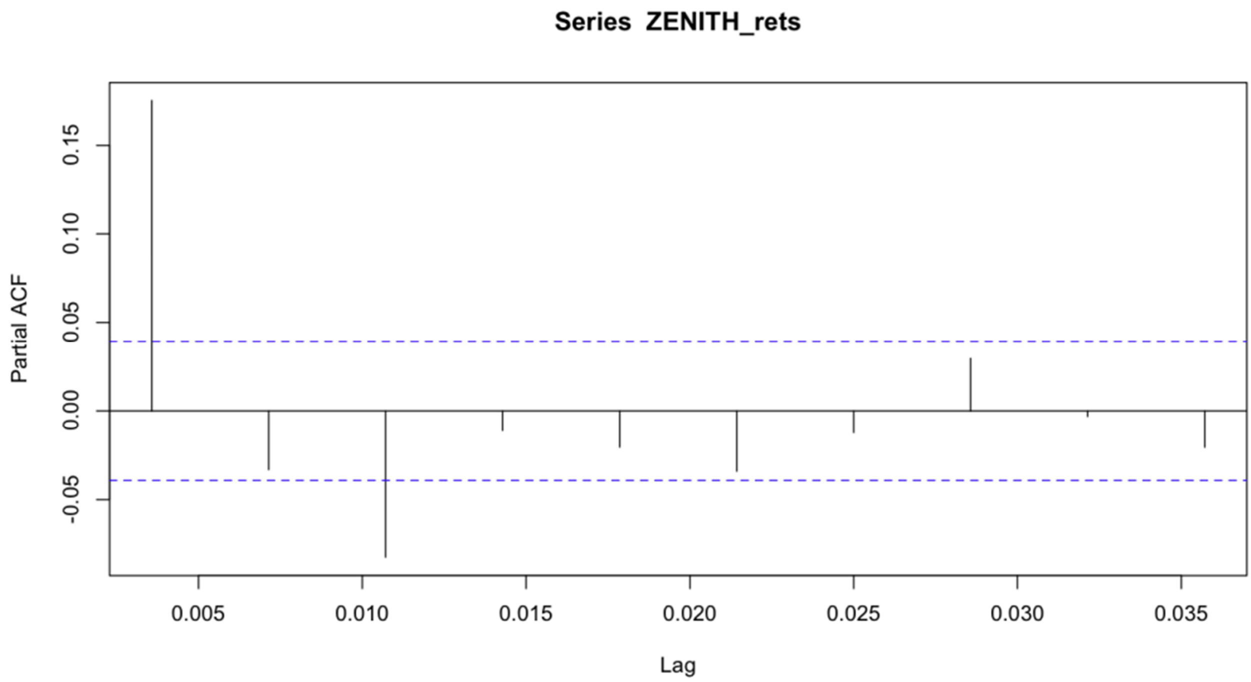

Figure 9. *PACF of Zenith Stock Return Squared.*

GARCH(1,1) model was found appropriate for the process and since a significant ARCH effects was noticed from the PACF plots, a joint estimation of the mean and volatility equations was performed where ARMA(2,1) model is fit as the mean equation and sGARCH(1,1) model is fit as the volatility equation. The conditional variance dynamics are shown in Table 2. The Log-Likelihood is 6348.738, Akaike: -5.0868, and Bayes: -5.0682.

Table 2. *The Conditional Variance Dynamics.*

Parameter	Estimate	Std. Error	*t* value	Pr(>|t|)
Mu	0.000098	0.000273	0.35952	0.719207
ar1	0.522552	0.192771	2.71074	0.006713
ar2	-0.098546	0.022894	-4.30443	0.000017
ma1	-0.426970	0.193651	-2.20484	0.027465
omega	0.000059	0.000017	3.41552	0.000637
alpha1	0.391050	0.061688	6.33918	0.000000
beta1	0.607950	0.063528	9.56984	0.000000
shape	3.197932	0.199045	16.06641	0.000000

The simple GARCH model consists of;
Mean equation: $Y_t = \mu + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$
The fitted model is thus; $Y_t = \mu_t\hat{} + \varepsilon_t$

$$Y_t = 0.000098 + \varepsilon_t$$

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$
The fitted model is thus;

$$\sigma_t^{2^\wedge} = \alpha_0 + \alpha_1 \, \varepsilon^{\wedge 2}_{t-1} + \beta_1 \, \sigma^{\wedge 2}_{t-1}$$

$$\sigma_t^{2^\wedge} = 0.000059 + 0.391050 \, \varepsilon^{\wedge 2}_{t-1} + 0.607950 \, \sigma^{\wedge 2}_{t-1}$$

4.1.2. GARCH Process Residual Diagnostic

The Ljung-Box test on squared residuals indicates no serial correlation since the p-values are > 0.05. Thus, the null hypothesis of serial correlation is not accepted, and we conclude that the residuals behave as a white noise process. The goodness-of-fit test reveals that all the p-values are > 0.05, indicating that this model adequately fits the data.

From our analysis, the best model, from the results of all the potential models that are good candidates for the GARCH model, is the sGARCH(1, 1) with student t-distribution, which is the model with the lowest AIC value of -5.0868, BIC value of-5.0682, and log-likelihood of 6348.738. Table 3 is the 9-day-ahead GARCH model forecast prediction of Zenith stock volatility returns.

Table 3. *A 9 days ahead GARCH model prediction of Zenith stock returns volatility.*

	T+1	T+2	T+3	T+4	T+5	T+6	T+7	T+8	T+9
Series	6.393e-03	4.660e-03	1.862e-03	5.701e-04	1.709e-04	8.960e-05	8.645e-05	9.282e-05	9.646e-05
Sigma	0.06715	0.06756	0.06796	0.06836	0.06876	0.06916	0.06955	0.06995	0.07033

From the volatility forecast, the sigma represents the expected conditional volatility at time (t + h) and the conditional mean at time (t + h) is represented by the series.

4.2. Markov Switching GARCH Process

Let r_t represents the log-returns of the Zenith Bank plc stock price at time *t*. The conditional variance of r_t is assume to follow a GARCH process and the unobserved state variable S_t assume to follow a first-order Markov chain with transition probability matrix P.

Thus, the Markov-Switching GARCH model is represnted as $r_t \;|(S_t = k, \Omega_{t-1}) \sim f\,(0, {\sigma_{k,t}}^2, \theta_k)$.

where k is the number of states, Ω_{t-1} is the information set available at time (t -1),$\sigma_{k,t}^2$ is the time-dependant variance, $\theta_k$is the shape parameters in the vector and f(0,$\sigma_{k,t}^2$,θ_k) is a conditional distribution such as normal, student t or generalized normal distribution, having zero mean. In this work, a Markov 2-regime-switching homogeneous MSGARCH model was considered. The sGARCH conditional variance with the student t-distribution was used to analyze the error distribution in the model. The log return time series of Zenith's daily stock prices served as the input for the MSGARCH model for estimating the volatility of the stock prices. The performance of the model was determined using the AIC and BIC metrics. Tables 4 and 5 contain the fitted parameters for regimes 1 and 2 of the model respectively. Table 6 shows the parameter estimates of the transition matrix. The Log-likelihood estimate of the Markov Switching GARCH model is 6816.9143, the AIC: -13613.8285, and BIC: -13555.6161. Table 7 is the Initial probabilities of the states and Table 8 shows the MSGARCH model's nine-day-ahead prediction of the Zenith stock price volatility forecast.

Table 4. *Parameter Estimates for Regime 1.*

Parameter	Estimate	Std. Error	*t*-value	Pr (>|t|)
alpha0_1	0.0000	0.0000	2917.3060	<1e-16
alpha1_1	0.0000	0.0000	51.9364	<1e-16
beta_1	0.0000	0.0002	0.0241	4.904e-01
mu_1	2.1000	0.0000	63235.0455	<1e-16

The MSGARCH model is thus;

Conditional mean:,$Y_t = \mu + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$ when $S_t = 1$

The fitted model is thus: $Y_t = \mu_1\hat{} + \varepsilon_1$

$$Y_t = 2.1 + \varepsilon_1$$

Conditional variance: ${\sigma_t}^2 = \alpha_{0\ st} + \alpha_{1_{st}}\, \varepsilon_{t-1}^2 + \beta_{1_{st}}\, \sigma_{t-1}^2$

$${\sigma_t}^2 = \alpha_0 + \alpha_1\, \varepsilon_{t-1}^2 + \beta_1\, \sigma_{t-1}^2$$

The fitted model is thus: $\sigma_1^{2\hat{}} = 0 + 0 + 0$

Table 5. *Parameter Estimates for Regime 2.*

Parameter	Estimate	Std. Error	*t*-value	Pr (>|t|)
alpha0_2	0.0000	0.0000	3.3320	4.311e-04
alpha1_2	0.2800	0.0539	5.1970	1.013e-07
beta_2	0.7031	0.0555	12.6597	<1e-16
mu_2	3.9127	0.2872	13.6216	<1e-16

The MSGARCH model is thus;

Conditional mean: $Y_t = \mu + \varepsilon_t$,, $\varepsilon_t \sim N(0, \sigma^2)$ when $S_t = 2$

The fitted model is thus: $Y_t = \mu_2\hat{} + \varepsilon_2$

$$Y_t = 3.9127 + \varepsilon_2$$

Conditional variance: ${\sigma_t}^2 = \alpha_{0\ st} + \alpha_{1_{st}}\, \varepsilon_{t-1}^2 + \beta_{1_{st}}\, \sigma_{t-1}^2$

$$\sigma_2^2 = \alpha_{0_2} + \alpha_2 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2$$

The fitted model is thus: $\sigma_2^{2\hat{}} = 0 + 0.28\, \varepsilon_{t-1}^2 + 0.7031\, \sigma_{t-1}^2$

The Transition matrix is:

$$P = \begin{bmatrix} P_{11} & 1 - P_{11} \\ 1 - P_{22} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0.2003 & 0.7997 \\ 0.0839 & 0.9161 \end{bmatrix}$$

Table 6. *Parameter Estimate of the Transition Matrix.*

	Estimate	Std. Error	t value	Pr (>|t|)
P_1_1	0.2003	0.0271	7.3912	7.272e-14
P_2_1	0.0839	0.0062	3.6147	1e-16

The state transition probability contains the probabilities of transition to the next state which are conditional upon the current state. The results show that the probability of the process remaining in state 1 given that it was previously in state 1 is 0.2 and the probability of remaining in state 2 is 0.9 which shows higher pesistence in state 2. Again, the probability of moving to state 2 from state 1 is approximately 0.8 while that of moving from state 2 to 1 is 0.1 meaning that the process has a lower chance of transiting from state 2 to 1 than from state 1 to 2.

Table 7. *Initial probabilities.*

State 1 (π_1)	State 2 (π_2)
0.0949	0.9051

Table 7 is the initial probability distribution over the two states. The state probability vector $\{\pi_t\}$ contains the unconditional probability of being in a certain state at time t. For the 2-regime Markov random variable S_t, the state probability distribution π_t, is given by the following 2-element vector: $\pi_t = P(S_t=1), P(S_t = 2)$. The result shows that the probability of the Markov chain to start in state 1 is 0.0949 while the probability that the Markov chin will start in states 2 is 0.9051.

Table 8. *A 9-day ahead MSGARCH model prediction of Zenith stock returns volatility.*

	T + 1	T + 2	T + 3	T + 4	T + 5	T + 6	T + 7	T + 8	T + 9
sigma	0.05846	0.06876	0.05629	0.05532	0.05138	0.05208	0.05032	0.05066	0.05081

The results of the GARCH and MSGARCH models were compared. Both the AIC and BIC values in the MSGARCH model were smaller than those of the GARCH model. The MSGARCH model also has the maximum likelihood estimates. Therefore, the Markov Switching GARCH model is concluded to be a better choice than the standard GARCH model. The results of the 9-day volatility forecast of both models also point to these facts. The state transition matrix of the 2-state Markov process contains the probabilities of transition to the next state conditional upon the current state. These results show that the Zenith stock price return has a high duration in Regime 2, with a probability value of 0.7997, indicating that when the process is in Regime 1, the probability of switching back to Regime 2 is high. The state initial probability vector $\{\pi_t\}$ contains the unconditional likelihood of being in a particular state at time t. The prior probability for the MSGARCH is 0.0949 for Regime 1 and 0.9051 for Regime 2 in our 2-step Markov random variables.

5. Conclusion

Stock prices display extreme volatility due to many factors as established in the existing literature. Many analysts prefer the GARCH models for volatility modeling of stock prices and other financial indexes. The choice of the typical GARCH models for erratic financial indexes may result in inaccurate predictions, poor decisions, and in essence, monetary loss. Allowing regime dynamics in the classical GARCH model parameter space permits the characteristics of time series, including means, variances, and model parameters, to change across regimes. Therefore the approach that increases the model's dynamics which seems to better capture the persistence of the volatility shocks in stock price analysis was adopted. From the observed results, the inclusion of states with transition probabilities, equips regime-switching GARCH models to capture the behavior of real-world data better than standard GARCH models.

The results of the models were compared using two statistical metrics; the AIC and the Log-likelihood functions. The Akaike's information was used to select the model with the lowest metric. From the results, the AIC for the GARCH model is -5.087 while that of MSGARCH is -13613.83, making it a better model for the time series. Also, from the Log-likelihood function which was used to select the model with the maximum likelihood of the optimal parameters, the GARGH model is 6348.74 while the MSGARCH model is 6816.91 making it the model with maximum likelihood out of the two models. Looking at the p-values, the value for the mean estimate of the GARCH model is greater than 0.05 and therefore not significant while that for the mean estimate of the MSGARCH model for both states are significant. Therefore, the results confirm that using the MSGARCH model out-performs using the GARCH model in the analysis of real-life time-series data which most times exhibit varying characteristics across different periods. GARCH parameters may be optimized in the future using data stream analysis which is a natural blend and extension of practical time series analysis or, other data mining algorithms for parameter estimation may be utilized for comparison.

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