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Non-fragile Fault-Tolerant Control with Gain Perturbation for a Linear Uncertain Time-Delayed System

Lingyan Hu

College of Information Engineering, Dalian University, Dalian, China

Email address:
hulingyan@dlu.edu.cn, hingy16@163.com

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Abstract: Non-fragile fault-tolerant controller is designed for a class of linear uncertain time-delayed systems. Two cases of controller perturbation are investigated in view of different application conditions. One case is the controller gain attenuation, the other is random uncertainty caused by noise possibly existing in an industrial field. The controller to be designed can guarantee systems to have robust fault-tolerant capability and non-fragility. Sufficient conditions for the existence of such controllers are derived basing on Lyapunov stability theory and LMI method. A numerical example of a beam reheating furnace is given. And computation results demonstrate the effectiveness of the proposed algorithm.

Keywords: Fault-Tolerant Control, Non-fragility, Uncertain Time-Delayed System, Linear Matrix Inequality (LMI)

1. Introduction

In industrial engineering systems, due to a huge number of instruments, communication components and interfaces, different types of malfunction or improper behavior may result in the unexpected variations in normal operations. The final effects of these phenomena are usually represented by some sensor or actuator faults [1]. The faults may deteriorate a system performance and even cause loss of effectiveness in stability. Therefore, it is important to design a controller which can guarantee a system to have robust fault-tolerant capability. That means whenever sensor or actuator faults appear, the system can maintain stable performance close to desirable requirements.

During the past two decades, a lot of achievements have been reported for the fault-tolerant control (FTC) aiming at different types of systems. They can mainly be classified into two groups, namely the passive FTC techniques and the active ones [2-3]. In a passive FTCs [3-7], the controller generally has a fixed structure. There is no controller switching during a system running process. The control laws allow a system to cope with the fault presence within a certain margin. Theoretically, passive FTC takes the fault appearence as a system perturbation. For the active FTC approaches, they can reconfigure a controller by using the real-time information coming from the fault detection and diagnosis (FDD) module. Some self-adaptive adjustments in a closed-loop system are carried out when a fault appears in order to achieve a control objective with a minimum performance degradation [1-3, 8-10].

In addition, because of equipment aging and possible random noise in power circuits, it's also necessary to investigate the non-fragility of a fault-tolerant controller as it can improve the system robust stability. There are already a number of research results aiming at the non-fragile controller design in recent years. In [11], non-fragile H∞ controller assumed to have multiplicative gain variations is designed for a linear time-invariant system. The controller is given in terms of symmetric positive-definite solutions of algebraic Riccati inequalities. In [12], a corresponding non-fragile adaptive fault-tolerant control scheme is proposed for the longitudinal dynamics of an airbreathing hypersonic vehicle. Two adaptive laws are employed to estimate a minimum value of an actuation effectiveness factor and the upper bound under external disturbances. In [13], a non-fragile observer-based control is studied for continuous systems. Two types of uncertainties which perturb the gains of control and observer are investigated separately. In [14-15], non-fragile guaranteed cost-control controller problem is studied based on the LMI method combined with the Lyapunov theory. They take an uncertain system into account. Non-fragile controllers are given by the feasible solutions of specific LMIs.

For the above research work, seldom literatures consider both of the fault-tolerant capability and non-fragility during the design of robust state feedback controller. In the present work, a class of linear uncertain time-delayed systems are taken into account. The gain of fault-tolerant controller is assumed to have uncertain perturbation which are divided into two cases, namely gain attenuation and random noise disturbance. Therefore the controller to be designed has non-fragility under above two cases. The LMI method and Lyapunov theory are used to derive the control law which can maitain the closed-loop system asymptotically stable under any admissible fault cases and controller perturbation. Basically, it's the passive FTC approach and its real-time amount of computation is low that makes easy implementation in an engineering system. It's also an extending research of literature [7] which aims at the design of non-fragile memoryless and time-delayed fault-tolerant controllers. This paper focuses on gain perturbation of the FTC controller.

For the main content of the paper organized as follows: in section 2, a system model, lemmas and assumptions are given. The computation and proof are presented for the non-fragile FTC control. In section 4, a numerical example is given to verify the proposed method. Finally, some remarks are included in section 5.

2. Methodology and Results

2.1. System Description and Preliminaries

Consider a linear uncertain system with state and control time-delayed as follows

$$\begin{aligned}\dot{x}(t) &= \left(A+\Delta A(t)\right)x(t)+\left(A_d+\Delta A_d(t)\right)x(t-h_a)\\ &\quad +Bu(t)+B_d u(t-h_b)\end{aligned} \tag{1}$$

where $x(t)\in R^n$ is a state vector and $u(t)\in R^m$ is a control input. And h_a , h_b denote the state and input constant time-delays separately. $A\in R^{n\times n}$, $A_d\in R^{n\times n}$, $B\in R^{n\times m}$ are basic matrices with appropriate dimensions. $\Delta A(t)$, $\Delta A_d(t)$ represent time-varying parameter uncertainties. assumed to have the following form:

$$\begin{bmatrix}\Delta A & \Delta A_d\end{bmatrix} = DH(t)\begin{bmatrix}E & E_d\end{bmatrix} \tag{2}$$

where D, E and E_d are constant matrices with appropriate dimensions, and $H(t)$ is an unkown, real and possibly time-varying matrix with Lebesgue-measurable elements and satisfying

$$H^T(t)H(t)\le I, \quad \forall t \tag{3}$$

The initial condition of $x(t)$ is given by

$$x(t)=\varphi(t) \quad t\in[-h,0], h=\max\{h_a, h_b\} \tag{4}$$

where $\varphi(t)$ is a continuously differentiable function with $t\in[-h,0]$.

Next, an assumption of a fault model and some lemmas are introduced which will be used in the algorithm derivation later on.

Assumption 1: Taking the sensor fault cases for example, according to the practical sensor characteristics in engineering systems, its working states can be classfied into four types shown in Table 1:

Table 1. *Fault Mode.*

Fault cases	**The i-th Sensor State:** s_i
Normal	1
Value Reduced	$0<s_i<1$
Value Magnified	$\sigma>s_i>1$
Outage	$s_i=\sigma$

s_i denotes the i-th sensor working state, n represents the quantity of sensors existing in a system. Then the following fault model is given by

$$S = diag(s_1,s_2,\cdots s_n) \ 0\le s_i\le\sigma,\sigma\ge 1 \tag{5}$$

where $s_i=1$ represents the ith sensor is normal. And $0<s_i<1$ denotes the ith sensor measured value is less than a normal value, accordingly $\sigma>s_i>1$ represents the current value is greater than the normal value. If a sensor open circuit fault occurs, the sensor output will give out a maximum value which is denoted by $s_i=\sigma$.

Lemma 1 [16]: (*Schur complement*) For a given symmetric matrix $Z=\begin{bmatrix}Z_{11} & Z_{12}\\ * & Z_{22}\end{bmatrix}$, where $Z_{11}\in R^{r\times r}$, then the following inequalities are equivalent

$$Z<0; \tag{6}$$

$$Z_{11}<0\,, Z_{22}-Z_{12}^T Z_{11}^{-1} Z_{12}<0; \tag{7}$$

$$Z_{22}<0\,, Z_{11}-Z_{12} Z_{22}^{-1} Z_{12}^T<0 \tag{8}$$

Lemma 2 [16]: Let M , N , L and H be real matrices with appropriate dimensions and $H^T H\le I$, $\varepsilon>0$, then the following inequalities hold

$$2M^{\mathbf{T}}HL\le M^{\mathbf{T}}M+L^{\mathbf{T}}L; \tag{9}$$

$$M^{\mathbf{T}}N+N^T M\le\varepsilon M^{\mathbf{T}}M+\varepsilon^{-1}N^{\mathbf{T}}N; \tag{10}$$

$$\pm 2L^T N\le\varepsilon L^T L+\varepsilon^{-1}N^T N \tag{11}$$

2.2. Non-frangile State Feedback Controller

Consider system (1). If matrices A, A_d , B are controllable and measurable, the following non-frangile state feedback controller is employed:

$$u(t)=(K+\Delta K)x(t) \tag{12}$$

where K is a gain matrix and ΔK represents a controller perturbation.

Considering different industrial conditions, take two cases of controller perturbation ΔK into account in the paper.

2.2.1. Case 1 with K-dependent Controller Perturbation

In this case, consider the controller perturbation ΔK caused by controller component aging or equipment performance degradation. ΔK is K-dependent. It's given by the following form

$$\Delta K = \delta K,\ \ \delta = diag[\delta_1 \quad \delta_2 \quad \cdots \quad \delta_j \quad \cdots \quad \delta_m]$$

where δ is a perturbation coefficient matrix with $|\delta_j| \le \bar{\delta}_j < 1,\ \ (j = 1,2,\cdots,m)$, $\bar{\delta}_j$ is the upper bound of parameter perturbation.

Consider sensor fault cases with $S = diag(s_1, s_2, \cdots s_m)$, system equation (1) can be written by

$$\begin{aligned}\dot{x}(t) &= [A + DHE + BKS + B\delta KS]x(t) \\ &+ (B_d KS + B\delta KS)x(t-h_b) \\ &+ (A_d + DHE_d)x(t-h_a)\end{aligned} \tag{13}$$

Then the following stability criteria can be obtained.

Theorem 1: Consider any admissible sensor fault. Suppose the scalars $\lambda_i > 0 (i = 1,2)$ are given. The closed-loop system (1) is asymptotically stable if there exist a matrix K with appropriate dimension and a symmetric positive-definite matrix P satisfying with the following LMI

$$\begin{bmatrix} \Sigma & X^T & (1+\delta)S^T Y^T & \sqrt{2}D & X^T E^T & X^T E_d^T & A_d & B_d \\ * & -\lambda_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\lambda_2^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\lambda_1 & 0 \\ * & * & * & * & * & * & * & -\lambda_2 \end{bmatrix} < 0 \tag{14}$$

where $X = P^{-1}$, $K = Y \cdot X^{-1}$, $\Sigma = 2AX + 2BYS + 2B_d\delta Y$

Proof: Choose a Lyapunov-Krasovskii candidate to be

$$\begin{aligned}V(x,t) &= x^T(t)Px(t) + \int_{t-h_a}^{t} x^T(s)Q_1 x(s)ds \\ &+ \int_{t-h_b}^{t} x^T(s)Q_2 x(s)ds\end{aligned} \tag{15}$$

If defining $V_1(x,t) = x^T(t)Px(t)$, then calculate the derivative of $V_1(x,t)$ along the solutions of closed system (13) and yield

$$\begin{aligned}\dot{V}_1(x,t) &= x^T(t)(A + BKS + B\delta K + DHE)^T Px(t) \\ &+ x^T(t)P(A + BKS + B\delta K + DHE)x(t) \\ &+ x^T(t-h_b)(B_d KS + B_d\delta KS)^T Px(t) \\ &+ x^T(t)P(B_d KS + B_d\delta KS)x(t-h_b) \\ &+ x^T(t-h_a)(A_d + DHE_d)^T Px(t) \\ &+ x^T(t)P(A_d + DHE_d)x(t-h_a)\end{aligned} \tag{16}$$

Basing on lemma 2

$$\begin{aligned}&2x^T(t)PDHEx(t) \\ &\le x^T(t)PDD^T Px(t) + x^T(t)E^T Ex(t)\end{aligned} \tag{17}$$

$$\begin{aligned}&2x^T(t)PA_d x(t-h_a) \\ &\le \lambda_1^{-1} x^T(t)PA_d A_d^T Px(t) + \lambda_1 x^T(t-h_a)x(t-h_a)\end{aligned} \tag{18}$$

$$\begin{aligned}&2x^T(t)PDHE_d x(t-h_a) \\ &\le x^T(t)PDD^T Px(t) + x^T(t-h_a)E_d^T E_d x(t-h_a)\end{aligned} \tag{19}$$

$$\begin{aligned}&2x^T(t)PB_d(1+\delta)KSx(t-h_b) \\ &\le \lambda_2^{-1} x^T(t)PB_d B_d^T Px(t) + \lambda_2(1+\delta)^2 x^T(t-h_b)SK^T KSx(t-h_b)\end{aligned} \tag{20}$$

If defing

$$Q_1 = \lambda_1 I + E_d^T E_d$$

$$Q_2 = \lambda_2(1+\delta)^2 SK^T KS$$

Using (16)-(20), calculate the derivative of $V(x,t)$ and yield

$$\begin{aligned}\dot{V}(x,t) &= 2x^T(t)P\dot{x}(t) + x^T(t)Q_1 x(t) - x^T(t-h_a)Q_1 x(t-h_a) \\ &+ x^T(t)Q_2 x(t) - x^T(t-h_b)Q_2 x(t-h_b)\end{aligned}$$

$$\begin{aligned}\dot{V}(x,t) &\le x^T(t)[2PA + 2PBKS + 2PB_d\delta K + 2PDD^T P \\ &+ \lambda_1 I + \lambda_2(1+\delta)^2 SK^T KS + E^T E + E_d^T E_d \\ &+ \lambda_1^{-1} PA_d A_d^T P + \lambda_2^{-1} PB_d B_d^T P]x(t) \\ &= x^T(t)\Delta x(t)\end{aligned} \tag{21}$$

If defing

$$\begin{aligned}\Theta = {} & 2PA+2PBKS+2PB_d\delta K+\lambda_1 I \\ & +\lambda_2(1+\delta)^2 SK^T KS+2PDD^T P \\ & +E^T E+E_d^T E_d+\lambda_1^{-1}PA_d A_d^T P \\ & +\lambda_2^{-1}PB_d B_d^T P\end{aligned} \tag{22}$$

According to Lyapunov-Krasovskii stability theory, it can be drawn that the closed-loop system (13) is asymptotically stable with $\Theta<0$. Now multiplying P^{-1} on both sides of inequality $\Theta<0$ and defining $X=P^{-1}, Y=KX$, it can be obtained

$$\begin{aligned}\tilde{\Theta} = {} & 2AX+2BYS+2B_d\delta Y+\lambda_1 X^T X+\lambda_2(1+\delta)^2 S^T Y^T YS \\ & +2DD^T+(EX)^T EX+(E_d X)^T E_d X \\ & +\lambda_1^{-1}A_d A_d^T+\lambda_2^{-1}B_d B_d^T \\ & <0\end{aligned} \tag{23}$$

Basing on lemma 1, it can be found that inequality (23) is equivalent with (14), which completes the proof.

Remark 1: It can be found the stabilization criterion is dependent with δ which denotes the controller perturtation in a system. Thus we can obtain the permissible perturtation value by solving the inequality (14). On the other side, the stability condition is delay-independent by choosing a proper Lyapunov-Krasovskii functional.

Now taking an actuator failure into account, the following fault actuator matrix is given

$$G=diag(g_1,g_2,\cdots g_m)\ \ 0\le g_i\le\sigma,\sigma\ge 1 \tag{24}$$

G has a similar structure with sensor fault model shown in Table 1. m represents the actuator quantity in a system. Accordingly a closed-loop system equation with actuator fault cases can be given by

$$\begin{aligned}\dot{x}(t) = {} & [A+DHE+BGK+BG\delta K]x(t) \\ & +(B_d GK+BG\delta K)x(t-h_b) \\ & +(A_d+DHE_d)x(t-h_a)\end{aligned} \tag{25}$$

It can be noticed equation (25) has similar structure with (13). After calculating, we obtain the stabilization condition under actuator fault cases given as corollary 1.

Corollary 1: Consider any admissible actuaor fault. Suppose positive scalars $\gamma_i>0(i=1,2)$ are known. The closed-loop system (25) is asymptotically if there exist a matrix K with appropriate dimension and a symmetric positive-definite matrix P satisfying with the following LMI

$$\begin{bmatrix} \tilde{\Sigma} & X^T & (1+\delta)G^T Y^T & \sqrt{2}D & X^T E^T & X^T E_d^T & A_d & B_d \\ * & -\gamma_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma_2^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\gamma_1 & 0 \\ * & * & * & * & * & * & * & -\gamma_2 \end{bmatrix}<0 \tag{26}$$

where $X=P^{-1}$, $K=Y\cdot X^{-1}, \tilde{\Sigma}=2AX+2BGY+2B_d\delta Y$

Remark 2: It can be found that LMI structure of a stabilization condition under an actuator fault case is the same with (11) if the quantity of actuators is the same with sensors' in one system. Thus a control law derived in the sensor fault case is also fit for an actuator fault case.

2.2.2. Case 2 with K-independent Controller Perturbation

Consider the controller perturbation caused by random noise because of fluctuations in power system or communication circuits. ΔK is K-independent. It's supposed to have the following form

$$\Delta K=DH(t)E_K$$

where E_K is a specific constant matrix, D and $H(t)$ have the same state values with (2). Then system equation (1) can be written by

$$\begin{aligned}\dot{x}(t) = {} & [(A+\Delta A(t))+B(K+DHE_K)]x(t) \\ & +(A_d+\Delta A_d(t))x(t-h_a) \\ & +B_d(K+DHE_K)x(t-h_b)\end{aligned} \tag{27}$$

If considering the actuator fault cases with $G=diag(g_1,g_2,\cdots g_m)$. The closed-loop system equation (27) is changed to

$$\begin{aligned}\dot{x}(t) = {} & \left(A+BGK+BGDHE_K+DHE\right)x(t) \\ & +(B_d GK+B_d GDHE_K)x\left(t-h_b\right) \\ & +(A_d+DHE_d)x(t-h_a)\end{aligned} \tag{28}$$

Then the following stability criteria can be deduced.

Theorem 2: Suppose that the scalars $\alpha_i>0(i=1,2)$ are known. The closed-loop system (28) is asymptotically stable with any possible actuaor fault if there exist a matrix K with appropriate dimension and a symmetric positive-definite matrix P satisfying with the following LMI

$$\begin{bmatrix} \Omega & \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 \\ * & -\Lambda_1 & 0 & 0 & 0 \\ * & * & -\Lambda_2 & 0 & 0 \\ * & * & * & -\Lambda_3 & 0 \\ * & * & * & * & -\Lambda_4 \end{bmatrix} < 0 \tag{29}$$

where

$$X = P^{-1},\ K = Y \cdot X^{-1}$$
$$\Omega = 2AX + 2BGY$$

$$\begin{aligned} \Gamma_1 &= [\sqrt{2}D \quad X^T E^T \quad X^T E_d^T \quad \sqrt{2} X^T E_K^T] \\ \Gamma_2 &= [X^T \quad Y^T G^T] \\ \Gamma_3 &= [A_d \quad B_d] \\ \Gamma_4 &= [BD \quad B_d GD] \end{aligned}$$

$$\begin{aligned} \Lambda_1 &= diag(I, I, I, I) \\ \Lambda_2 &= diag(\alpha_1^{-1} I, \alpha_2^{-1} I) \\ \Lambda_3 &= diag(\alpha_1 I, \alpha_2 I) \\ \Lambda_4 &= diag(I, I) \end{aligned}$$

Proof: Choose the same Lyapunov-Krasovskii candidate as (15) and define $V_1(x,t) = x^T(t)Px(t)$. Then calculate the derivative of $V_1(x,t)$ along the closed system (28) and yield

$$\begin{aligned} \dot{V}_1(x,t) &= 2x^T(t)PAx(t) + 2x^T(t)PBGKx(t) \\ &+ 2x^T(t)PBDHE_K x(t) + 2x^T(t)PDHEx(t) \\ &+ 2x^T(t)P(B_d GK + B_d GDHE_K)x(t-h_b) \\ &+ 2x^T(t)P(A_d + DHE_d)x(t-h_a) \end{aligned} \tag{30}$$

According to lemma 2, the following inequalities hold

$$\begin{aligned} &2x^T(t)PBDHE_K x(t) \\ &\le x^T(t)PBDD^T B^T Px(t) + x^T(t)E_K^T E_K x(t) \end{aligned} \tag{31}$$

$$\begin{aligned} &2x^T(t)PDHEx(t) \\ &\le x^T(t)PDD^T Px(t) + x^T(t)E^T Ex(t), \end{aligned} \tag{32}$$

$$\begin{aligned} &2x^T(t)PB_d GKx(t-h_b) \\ &\le \alpha_2^{-1} x^T(t)PB_d B_d^T Px(t) + \alpha_2 x^T(t-h_b)K^T G^T GKx(t-h_b) \end{aligned} \tag{33}$$

$$\begin{aligned} &2x^T(t)PA_d x(t-h_a) \\ &\le \alpha_1^{-1} x^T(t)PA_d A_d^T Px(t) + \alpha_1 x^T(t-h_a)x(t-h_a) \end{aligned} \tag{34}$$

$$\begin{aligned} &2x^T(t)PDHE_d x(t-h_a) \\ &\le x^T(t)PDD^T Px(t) + x^T(t-h_a)E_d^T E_d x(t-h_a) \end{aligned} \tag{35}$$

$$\begin{aligned} &2x^T(t)PB_d GDHE_K x(t-h_b) \\ &\le x^T(t)PB_d GDD^T G^T B_d^T Px(t) + x^T(t-h_b)E_K^T E_K x(t-h_b) \end{aligned} \tag{36}$$

If defing

$$Q_1 = \alpha_1 I + E_d^T E_d$$

$$Q_2 = \alpha_2 K^T G^T GK + E_K^T E_K$$

According to (30)-(36), the derivative of $V(x,t)$ can be calculated

$$\begin{aligned} \dot{V}(x,t) \le x^T(t)[&2PA + 2PBGK + \alpha_1 I + \alpha_2 K^T G^T GK \\ &+ 2PDD^T P + E^T E + PBDD^T B^T P \\ &+ 2E_K^T E_K + E_d^T E_d + \alpha_1^{-1} PA_d A_d^T P \\ &+ \alpha_2^{-1} PB_d B_d^T P + PB_d GDD^T G^T B_d^T P]x(t) \end{aligned} \tag{37}$$

Then define

$$\begin{aligned} \Delta = &2PA + 2PBGK + \alpha_1 I + \alpha_2 K^T G^T GK \\ &+ 2PDD^T P + E^T E + PBDD^T B^T P \\ &+ 2E_K^T E_K + E_d^T E_d + \alpha_1^{-1} PA_d A_d^T P \\ &+ \alpha_2^{-1} PB_d B_d^T P + PB_d GDD^T G^T B_d^T P \end{aligned} \tag{38}$$

Similarly, basing on Lyapunov-Krasovskii stability theory, the closed-loop system (28) is asymptotically stable with $\Delta < 0$. Now let $X = P^{-1}, Y = KX$, multiplying P^{-1} on both sides of inequality (38), yield

$$\begin{aligned} \Delta^{\bullet} = &2AX + 2BGY + 2DD^T + X^T E^T EX + X^T E_d^T E_d X \\ &+ 2X^T E_K^T E_K X + \alpha_1 X^T X + \alpha_2 Y^T G^T GY + \alpha_1^{-1} A_d A_d^T \\ &+ \alpha_2^{-1} B_d B_d^T + BDD^T B^T + B_d GDD^T G^T B_d^T \\ &< 0 \end{aligned} \tag{39}$$

Basing on lemma 1 (Schur complement), it can be found that inequality (39) is equivalent with (29), which completes the proof.

Remark 3: From the above result, it can be found that the stabilization criterion is dependent with D, E_K which represents the state feedback controller perturtation in a system. It's also delay-independent.

If taking the sensor failure into account, use the same fault model as case 1. Accordingly the closed-loop system equation can be written as

$$\begin{aligned} \dot{x}(t) = &[A + BKS + BDHE_K S + DHE]x(t) \\ &+ (B_d KS + B_d DHE_K S)x(t-h_b) \\ &+ (A_d + DHE_d)x(t-h_a) \end{aligned} \tag{40}$$

Then the stabilization criterion can be given with corollary 2.

Corollary 2: Consider any admissible sensor fault. Suppose that positive scalars $\beta_i > 0 (i = 1, 2)$ are known. The closed-loop system (40) is asymptotically stable if there exist the matrix K with appropriate dimension and symmetric positive-definite matrix P satisfying the following LMI

$$\begin{bmatrix} \tilde{\Omega} & \tilde{\Gamma}_1 & \tilde{\Gamma}_2 & \tilde{\Gamma}_3 & \tilde{\Gamma}_4 \\ * & -\tilde{\Lambda}_1 & 0 & 0 & 0 \\ * & * & -\tilde{\Lambda}_2 & 0 & 0 \\ * & * & * & -\tilde{\Lambda}_3 & 0 \\ * & * & * & * & -\tilde{\Lambda}_4 \end{bmatrix} < 0 \qquad (41)$$

Where $X = P^{-1}$, $K = Y \cdot X^{-1}$, $\tilde{\Omega} = 2AX+2BYS$

$$\tilde{\Gamma}_1 = [\sqrt{2}D \quad X^T E^T \quad X^T E_d^T \quad X^T E_K^T \quad X^T S^T E_K^T]$$
$$\tilde{\Gamma}_2 = [X^T \quad Y^T S]$$
$$\tilde{\Gamma}_3 = [A_d \quad B_d]$$
$$\tilde{\Gamma}_4 = [BD \quad B_d D]$$

$$\tilde{\Lambda}_1 = diag(I, I, I, I, I)$$
$$\tilde{\Lambda}_2 = diag(\beta_1^{-1} I, \beta_2^{-1} I)$$
$$\tilde{\Lambda}_3 = diag(\beta_1 I, \beta_2 I)$$
$$\tilde{\Lambda}_4 = diag(I, I)$$

Remark 4: From the corollary 2, it can also be verified that the inequality structure under actuator faults is consistent with sensor's if the numbers of sensors and actuators are the same in a system.

If a special feedback controller with $\Delta K = 0$ is considered, it can be written by $u(t) = Kx(t)$. Then the control law that maintains the system (1) asymptotically stable is given by corollary 3.

Corollary 3: Suppose a sensor fault matrix S and positive scalar $\zeta_i > 0 (i = 1,2)$ are known. The system (1) with control law $u(t) = Kx(t)$ has robust asymptotic stability if there exist matrix K with appropriate dimension and symmetric positive-definite matrix P satisfying the following LMI

$$\begin{bmatrix} \Sigma & X^T & Y^T S & \sqrt{2}D & X^T E^T & X^T E_d^T & A_d & B_d \\ * & -\zeta_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\zeta_2^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\zeta_1 & 0 \\ * & * & * & * & * & * & * & -\zeta_2 \end{bmatrix} < 0 \qquad (42)$$

where $\Sigma = 2AX+2BYS$, $K = YX^{-1}$.

Referring to the derived experience, the stabilization condition under actuator fault cases can also be derived.

For above stabilization conditions, we can obtain the solutions by calculating LMIs with the help of Matlab software.

3. Application Example

Taking a *Heat Zone I* of a beam reheating furnace for example with technological parameters shown in Table 2.

Table 2. *Technological parameters.*

Furnace parameters	Description
Flat burner power	260kw
Flat burner number	6
Billet component	QBe_2
Beam step velocity	$0.01 m/s$
Initial temperature	350°C
Billet regulation (Length*Width*Height)	5000× 430 ×210 (mm)
Heat zone I dimension (Length*Width*Height)	7000×5700×1600 (mm)

Then the state space model of *Heat Zone I* is given by

$$\begin{cases} \dot{x}_{heat_1}(t) = A_{heat_1} x(t) + B_{heat_1} u(t), \\ y_{heat_1}(t) = C_{heat_1} x(t) + D_{heat_1} u(t) \end{cases}$$

Using data collected in the control process, the basic parameter values of A_{heat_1}, B_{heat_1}, C_{heat_1} and D_{heat_1} can be identified

$$A_{heat_1} = \begin{bmatrix} -1.5625 & -0.3608 \\ 1 & 0 \end{bmatrix}, B_{heat_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_{heat_1} = \begin{bmatrix} 0 & 3.698 \end{bmatrix}, \qquad D_{heat_1} = \begin{bmatrix} 0 \end{bmatrix}$$

If considering the time delay and all the possible uncertainty caused by gas flow fluctuation or the billet capacity change in a system, then the state equation can be written by

$$\dot{x}_{heat_1}(t) = (A_{heat_1} + \Delta A_{heat_1}(t))x(t) + (A_{heat_{1d}} + \Delta A_{heat_{1d}}(t))$$
$$\cdot x(t-h_a) + B_{heat_1} u(t) + B_{heat_{1d}} u(t-h_b)$$

where $\Delta A_{heat_1}(t)$, $\Delta A_{heat_{1d}}(t)$ are parameter uncertainties which are assumed to comply with (2), and we employ the following matrix

$$A_{heat_{1d}} = \begin{bmatrix} -0.25 & 0.15 \\ 0.15 & 0 \end{bmatrix}, D = \begin{bmatrix} 0.20 & 0.15 \\ 0.15 & 0.20 \end{bmatrix},$$

$$E_{heat_1} = \begin{bmatrix} 0.15 & 0.10 \\ 0.10 & 0.15 \end{bmatrix}, \quad B_{heat_{1d}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad E_{heat_{1d}} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

In the present work, there are two thermocouples in heat zone I. Taking the sensor faults into consideration, it's found that most fault cases are loss of effectiveness with $0 < s_i < 1$. With the help of Matlab software, we can solve the LMIs and obtain the solutions under different controller modes with sensor failure.

Consider the following sensor working states: $S_0 = diag(1,1)$ which represents both of two sensors are normal; then two states under sensor loss of effectiveness with $S_1 = diag(0.65, 0.70)$ and $S_2 = diag(0.90, 0.95)$.

(i). Non-fragile state feedback controller with ΔK satisfying with case 1

For a controller used in a beam reheat furnace, some components will get aging because of working long hours. Suppose there is a controller perturtation in heat zoneI with $\delta = diag[0.05\ \ 0.05]$. And let the given scalars be $\lambda_i = 0.25 (i = 1,2)$. After solving the LMIs composed by S_0, S_1, S_2 satisfying with (14), the following results can be obtained

$$X = \begin{bmatrix} 0.2612 & -1.1015 \\ -1.1015 & 0.2239 \end{bmatrix}, Y = \begin{bmatrix} 16.1521 & -15.9687 \\ -15.9687 & 16.1521 \end{bmatrix}$$

Thus

$$K = \begin{bmatrix} 6.5373 & -6.5153 \\ -7.8067 & 4.9289 \end{bmatrix}$$

$$P = \begin{bmatrix} -0.0469 & -0.4456 \\ -0.4456 & -0.0526 \end{bmatrix}$$

(ii). Non-fragile state feedback controller with ΔK satisfying with case 2

In view of the system equation (28), suppose there is a jump change caused by noise interference in power supply circuits. And the interference signal is represented by D and E_{hold_K} under case 2.

And let the scalars be $\alpha_i = 0.20 (i = 1,2)$, after solving the LMIs composed by S_0, S_1, S_2 satisfying with (41), yield

$$X = \begin{bmatrix} 0.3885 & -2.3526 \\ -2.3526 & 0.4652 \end{bmatrix}, Y = \begin{bmatrix} 15.7216 & -18.1282 \\ -18.1282 & 15.7216 \end{bmatrix}$$

Then

$$P = \begin{bmatrix} -0.0815 & -0.4123 \\ -0.4123 & -0.0646 \end{bmatrix}, K = \begin{bmatrix} 5.4618 & -3.7018 \\ -5.3273 & 3.8957 \end{bmatrix}$$

(iii). Memoryless state feedbank controller

Consider the control law with $\Delta K = 0$. Assuming $\zeta_i = 0.15 (i = 1,2)$, after solving the LMIs composed by S_0, S_1, S_2 satisfying with (42), we have

$$X = \begin{bmatrix} 0.1280 & -4.1225 \\ -4.1225 & 0.2249 \end{bmatrix}, \quad Y = \begin{bmatrix} 3.1295 & -6.5600 \\ -6.5600 & 5.7280 \end{bmatrix}$$

Thus

$$P = \begin{bmatrix} -0.0132 & -0.2422 \\ -0.2422 & -0.0075 \end{bmatrix}, \quad K = \begin{bmatrix} 1.6299 & -0.7085 \\ -1.4738 & 1.5455 \end{bmatrix}$$

From the above results, it can be drawn that the model system of heat zoneI in the reheating furnace can maitain robust asymptotically stability and non-fragility under above sensor working cases: S_0, S_1, S_2 and assuming controller perturtation.

The above results are only specific solutions for system (1) under different state feedback controllers with sensor loss of effectiveness. The sulotions under other cases can also be solved by LMI toolbox.

4. Conclusion

In the present work, design of non-fragile fault-tolerant controller is investigated for a linear uncertain systems with time-delays. Two cases of controller perturtation are taken into account due to component aging and noise disturbance possibly exist in engineering systems. Considering sensor or actuator fault conditions, the stablity criteria in LMI are derived separately under above two cases basing on Lyapunov stability theory. The non-fragile fault-tolerant controllers can be given by the feasible solutions of the stablity criteria. A numerical example is given to demonstrate the computation process for the algorithm. It also verifies the effectiveness of the proposed approach. However, the results are obtained under specific conditions, namely sensor loss of effetiveness, given system uncertainties and norm-bounded controller perturtation. Thus, there is still further research work to be done, such as the upper bound of FTC controller perturbation and the admissible scope of sensor failure.

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