



# Mathematical Modelling for Thermal and Mechanical Design of Shell and Tube Type Gas Cooler Used in Transcritical CO<sub>2</sub> Refrigeration System

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**Abstract:** This paper is about Mathematical Modelling for Thermal and Mechanical Design of Shell and Tube type Gas Cooler used in Transcritical CO<sub>2</sub> Refrigeration system. Transcritical refrigeration system refers to system whose condenser temperature is above critical temperature of refrigerant. To achieve it, the condenser in conventional refrigeration system is replaced by Gas Cooler where Refrigerant vapour is cooled sensibly without condensation. The gas cooler is used for cooling of refrigerant by using water as coolant. The temperature of refrigerant vapour coming out of compressor in transcritical system is more as compared to conventional refrigerant system. So gas cooler can be effectively used for heating of water. This paper describes a mathematical model that can be used in predicting the heat transfer performance of a shell and tube type Gas Cooler used in transcritical CO<sub>2</sub> refrigeration system. The model uses Kern Method of Heat exchanger design. Given the fluid inlet and outlet temperatures flow rates of fluid & fluid properties the model determines (a) the necessary heat transfer surface area, (b) Outside and inside heat transfer coefficient, (c) overall heat transfer coefficient, (d) Pressure drop on shell and tube side, (e) It also determines mechanical design parameters such as shell O. D, Shell thickness, Tube sheet thickness, Flange thickness.

**Keywords:** Gas Cooler, Transcritical, CO<sub>2</sub>, Refrigeration, Tube Sheet Thickness

## 1. Introduction

In the last few years researchers came to know about the adverse effect of refrigerants on environment i.e. global warming and ozone depletion potential as these are adverse effects government added restrictions on use of such refrigerant who causes the global warming and ozone depletion, so industries are in search of new and natural refrigerant which has low global warming and less ozone depletion potential. In this regard, CO<sub>2</sub> is one of the refrigerants, which has zero ozone depletion potential and less value of global warming potential.

CO<sub>2</sub> is having less value of critical temperature and pressure so getting less temperature range for application and restricted to 31°C to use CO<sub>2</sub> for wide range of temperature to operate the system at pressure above critical point pressure i.e. above 73 bar as the pressure of the system is above

critical point the condensation is not possible at pressure above critical pressure so in CO<sub>2</sub> transcritical Refrigeration system condenser is replaced by gas cooler.

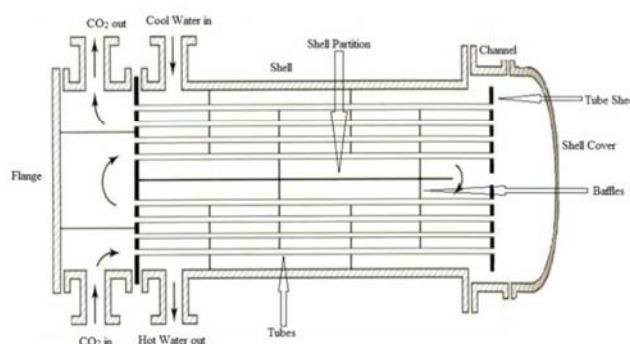


Figure 1. Constructional features of Gas Cooler.

In this paper Shell and Tube type Gas Cooler used in transcritical CO<sub>2</sub> refrigeration system is modelled using kern method as shown in figure 1. The various mathematical model used for design of shell and tube type gas cooler is explained in below section.

## 2. Method of Design

- (1) Formulation of mathematical models for thermal and mechanical Design
- (2) Calculation of different design parameters
- (3) Selection of required parameters from standard charts and tables
- (4) Final design of shell and tube gas cooler

## 3. Mathematical Models for Thermal Design

### 3.1. Heat Duty of Heat Exchanger (Q)

In the shell-and-tube condenser the water flows outside the tubes and refrigerant flows inside the tubes through the shell.

Heat duty of Heat Exchanger (Q) by performing Energy Balance

$$\text{Heat Duty (Q)} = \dot{m}_w \times C_p \times (T_{w2} - T_{w1})$$

By considering uncertainty in heat load multiply Heat Duty by 1.25 to compensate Uncertainty of Heat Duty Due change inlet temperature of Cold fluid or Hot Fluid

$$\text{Corrected Heat Duty (Q}_{\text{cor}}) = 1.25 \times Q$$

Where  $\dot{m}_w$  is mass flow rate of water (kg/s) and  $C_p$  is Specific Heat of water (J/kg°C),  $T_{w1}$  and  $T_{w2}$  are respectively inlet and outlet temperature of water.

### 3.2. Log-Mean Temperature Difference (LMTD)

The log mean temperature difference  $\Delta T_m$  for counter current flow is determined by:

$$\text{LMTD} = \left( \frac{\Delta T_1 \Delta T_2}{\ln \left( \frac{\Delta T_1}{\Delta T_2} \right)} \right)$$

Where  $\Delta T_1 = (T_{c1} - T_{w2})$  &  $\Delta T_2 = (T_{c2} - T_{w1})$ ,  $T_{w1}$  and  $T_{w2}$  are respectively the inlet and outlet temperature for water,  $T_{c1}$  and  $T_{c2}$  are the inlet and outlet temperature for CO<sub>2</sub> respectively

### 3.3. Correction Factor

In design the heat exchangers, a correction factor is applied to the log mean temperature difference (LMTD) to allow for the departure from true counter current flow to determine the true temperature difference. Correction factor for LMTD can be calculated from the Standard Graph Available in Textbook D. Q Kern Process Heat Transfer. Before going to calculation of correction factor one must decide the Number of shell passes & Number of tube passes

and two factors i.e. 'R' and 'S', they can be defined as follows-

$$R = \left( \frac{T_{w1} - T_{w2}}{T_{c2} - T_{c1}} \right)$$

$$S = \left( \frac{T_{c2} - T_{c1}}{T_{w1} - T_{c1}} \right)$$

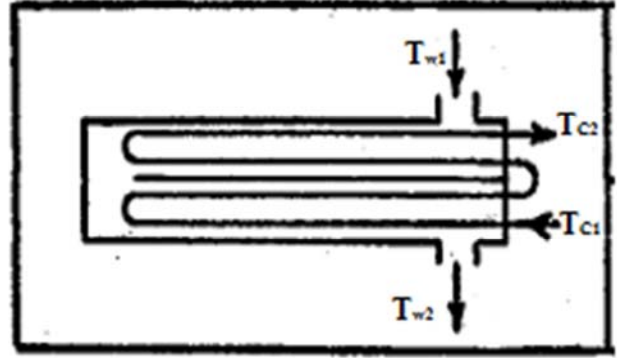


Figure 2. Flow arrangement.

The value of correction factor obtained from graph using  $R = 0.711$   $S = 0.789$  and 2 shell passes and 4 tube passes is 0.78

### 3.4. Heat Transfer Area

The heat transfer area (A) of the shell-and-tube condenser is computed by:

$$Q = U_{\text{ass}} \times A \times \text{LMTD} \times F_t$$

$$\therefore A = \left( \frac{Q}{U_{\text{ass}} \times \text{LMTD} \times F_t} \right)$$

Where  $F_t$  is LMTD correction factor and  $U_{\text{ass}}$  is assumed overall heat transfer coefficient ( $\text{W/m}^2\text{k}$ ).

### 3.5. Number of Tubes (Nt)

The Number of tubes required for required surface area is given by:

$$N_t = \left( \frac{A}{\pi \times D_o \times L} \right)$$

Where  $D_o$  is outside diameter of tube (meter) and  $L$  is length of tube (meter).

### 3.6. Inside Heat Transfer Coefficient

Inside Heat transfer Coefficient Using siedler-Tate equation is given by:

$$j_H = \left( \frac{h_i \times D_i}{K_c} \right) \times \left( \frac{\mu \times C_{pc}}{K_c} \right)^{-\frac{1}{3}} \times \left( \frac{\mu}{\mu_w} \right)^{-0.14}$$

$j_H$  can be calculated from Standard graph by knowing the Reynolds number below graph shows the value  $j_H = 70$  for Reynolds Number = 19339.98

Where  $h_i$  is inside heat transfer coefficient ( $\text{W/m}^2\text{k}$ ), ( $\mu_w$ ) is

Dynamic viscosity at wall temperature (N. S/m<sup>2</sup>),  $\mu$  is Dynamic viscosity at bulk mean temperature (N. S/m<sup>2</sup>),  $C_{pc}$  is Specific Heat of CO<sub>2</sub> (J/kg<sup>0</sup>K),  $K_c$  is Thermal Conductivity of CO<sub>2</sub> (W/m<sup>0</sup>K) and  $D_i$  is inside diameter of tube.

### 3.7. Shell Side Heat Transfer Coefficient $h_o$

For calculation of  $h_o$  first one has to calculate Re for Shell side fluid for that purpose one must know the Mass velocity (Gs) of shell side fluid, Cross flow area ( $a_c$ ).

For calculating Mass velocity equivalent diameter of Shell (De) is required.

Equivalent diameter of Shell (De) =  $\left( \frac{4 \times (Pt)^2 - \frac{\pi}{4} \times Do^2}{\pi \times Do} \right)$  for square pitch

Tube clearance (C) = (Pt – Do)

Shell side cross flow area ( $a_s$ ) =  $\left( \frac{C \times B \times D_s}{P_t} \right)$

Mass Velocity (Gs) =  $\left( \frac{\dot{m}_w}{a_s \times \rho_w} \right)$

Reynolds number for shell side fluid (Re)<sub>s</sub> =  $\left( \frac{G_s \times D_e}{\nu_w} \right)$

For calculation of outside heat Transfer Coefficient ( $h_o$ ) sieder-Tate equation is used (Ref. D. Q Kern Process Heat transfer Text book)

$$j_H = \left( \frac{h_o \times D_e}{K_w} \right) \times \left( \frac{\mu \times C_{pw}}{k_w} \right)^{-\frac{1}{3}} \times \left( \frac{\mu}{\mu_w} \right)^{-0.14}$$

Where  $K_w$  is thermal conductivity of water (w/m<sup>0</sup>C),  $h_o$  is outside heat transfer coefficient,  $G_s$  is Mass flow Velocity (m/sec),  $a_c$  is cross flow area, Pt is pitch of tube (meter), B is baffle spacing (meter),  $D_s$  is shell inside diameter (meter),  $\rho_w$  is density of water (kg/m<sup>3</sup>) and  $\nu_w$  is Kinematic Viscosity (m<sup>2</sup>/s).

### 3.8. Overall Heat Transfer Coefficient

Overall heat transfer coefficient U based on inside surface area depends on the tube inside diameter, tube outside diameter, tube side convective coefficient, shell side convective coefficient, tube side fouling resistance, shell side fouling resistance, and tube material is given by:

$$\frac{1}{UA_i} = \left( \frac{1}{A_i \times h_i} \right) + \left( \frac{1}{A_o \times h_o} \right)$$

To determine % error between assumed value of overall heat transfer coefficient  $U_{ass}$  and Actual value of overall Heat transfer coefficient  $U_{act}$

$$\% \text{ Error} = \left( \frac{U_{ass} - U_{act}}{U_{ass}} \right)$$

### 3.9. Pressure Drop

a) Tube side pressure drop

It consists of two type of pressure losses i.e. frictional pressure drop ( $\Delta P$ )<sub>t</sub> due to friction between fluid and tube wall and return losses ( $\Delta P$ )<sub>rt</sub> due to change in flow direction of the tube side fluid

Pressure drop due to friction ( $\Delta P$ )<sub>t</sub> =  $\left( \frac{f \times L \times (G_t)^2 \times N_p \times \rho_c}{D_i \times (S. G)_c \times \phi_t} \right)$

Friction factor (f) is calculated from standard graph available corresponding to Reynolds number.

Return loss ( $\Delta P$ )<sub>rt</sub> =  $(2 \times N_p - 1.5) \left( \frac{(G_t)^2 \times \rho_c}{(S. G)_c} \right)$

Total pressure drop in tubes =  $[(\Delta P)_t + (\Delta P)_{rt}]$

Where  $f_t$  is friction factor for tubes,  $G_t$  is tube side Mass Velocity (m/s),  $N_p$  is number of tube passes,  $\rho_c$  is density of CO<sub>2</sub> (kg/m<sup>3</sup>), S. G is Specific Gravity of water and  $\phi_t = \left( \frac{\mu}{\mu_w} \right)$ .

b) Shell side pressure drop

Pressure Drop due to friction

$$(\Delta P)_s = \left( \frac{f \times D_s \times (G_s)^2 (N_b + 1) \times \rho_w}{D_e \times (S. G)_w \times \phi_s} \right)$$

Friction factor (f) calculated from standard graph available corresponding to Reynolds number.

Return losses due to change in flow direction of shell side fluid

$$(\Delta P)_{rs} = (2 \times N_p - 1.5) \left( \frac{(G_s)^2 \times \rho_w}{(S. G)_w} \right)$$

Total pressure drop in Shell =  $[(\Delta P)_s + (\Delta P)_{rs}]$

## 4. Mathematical Models for Mechanical Design

Shell Thickness ( $t_s$ )

The shell thickness ( $t_s$ ) can be calculated from the equation below based on the maximum allowable stress and corrected for joint efficiency

$$t_s = \left( \frac{P \times D_s}{f \times J - 0.6 P} \right) + C_a$$

O. D of Shell ( $D_{so}$ ) = I. D of Shell + 2 x thickness of shell  
= 254 + 2 x 35 = 324 mm

324 is selected as nominal diameter of Shell from standard IS 2844-1967

Where P is Design pressure (N/mm<sup>2</sup>), f is Maximum allowable stress of the material used for construction (N/mm<sup>2</sup>), J is Joint efficiency (usually varies from 0.7 to 0.9) and  $C_a$  corrosion allowance.

Tube Sheet Thickness

The minimum tube-sheet thickness (TEMA standard) to 'resist bending' can be calculated by,

$$t_{ts} = \frac{F \times G_p}{3} \times \left( \sqrt{\frac{P}{K \times f}} \right)$$

Where f is for fixed tube sheet,  $G_p$  is diameter over which pressure is acting ( for fixed tube sheet it is equal to shell inside diameter ), P is shell pressure (N/mm<sup>2</sup>) and K is Mean ligament efficiency.

For square pitch K is given by:

$$K = 1 - \left( \frac{0.785}{\left( \frac{PT}{do} \right)} \right)$$

For 19 mm outside diameter of tube the minimum tube sheet thickness is 15 mm, as calculated tube sheet thickness

is greater than 15 mm design is satisfied.

#### Gasket Design

A preliminary estimation of gaskets is done using following expression:

Residual gasket force=Gasket seating force-(Hydrostatic pressure force)

The residual gasket force should be greater than that required to prevent the leakage of the internal fluid. This condition results the final expression in the form of

$$\left(\frac{D_{OG}}{D_{IG}}\right) = \left(\sqrt{\frac{Y - P \times m}{Y - P (m + 1)}}\right)$$

Where m is gasket factor for Flat iron jacketed asbestos (m

= 3.75), Y is Minimum Design seating stress (for Flat iron jacketed asbestos Y=52.48 N/mm<sup>2</sup>), D<sub>OG</sub> is Outside diameter of Gasket (mm), D<sub>IG</sub> is inside diameter of Gasket (mm)

Gasket Width (N) = (D<sub>OG</sub> - D<sub>IG</sub>) / 2

Mean Gasket Diameter (G) = (D<sub>OG</sub> + D<sub>IG</sub>) / 2

Basic Gasket Seating Width (b<sub>o</sub>) = (N/2)

Effective Gasket Seating Width (b) = 0.5 √b<sub>o</sub>

## 5. Results

By using the mathematical models mentioned above the different parameters are calculated and tabulated in subsequent tables. For calculation of parameters some parameters are directly taken from standard tables and standard graphs.

Table 1. Input parameters.

Sr. no	Input parameters	Value
1	Inlet temperature of cold fluid (Water) in °C	23°C
2	Outlet temperature of Cold fluid (Water) in °C	55°C
3	Inlet temperature of Hot fluid (Carbon Dioxide) in °C	80°C
4	Outlet temperature of Hot fluid (Carbon Dioxide) in °C	35°C
5	Pressure in Gas cooler	90 bars
6	Mass flow rate of water(mw)	0.0389kg/s

Table 2. Fluid properties.

Fluid properties of hot fluid	
Properties of Hot Fluid (Carbon Dioxide at mean bulk Temperature of 57.5°C and at working pressure of 90 bars)	
1. Density (ρ <sub>c</sub> )	247.8 kg/m <sup>3</sup>
2. Specific Heat (C <sub>pc</sub> )	2707 J/kg°C
3. Thermal Conductivity (K <sub>c</sub> )	0.03629 W/m°C
4. Dynamic viscosity (μ <sub>c</sub> )	21.86x10 <sup>-6</sup> N. S/m <sup>2</sup>
5. Kinematic Viscosity (ν <sub>c</sub> )	0.0886 x 10 <sup>-6</sup> m <sup>2</sup> /s
6. Prandtl number (Pr) <sub>c</sub>	1.62
7. Specific Gravity (S. G) <sub>c</sub>	0.2478
Fluid properties of Cold Fluid	
Properties of Cold Fluid (Water at mean bulk Temperature of 39°C)	
Density (ρ <sub>w</sub> )	992.22 kg/m <sup>3</sup>
Specific Heat (C <sub>pw</sub> )	4179 J/kg°C
Thermal Conductivity (K <sub>w</sub> )	0.631 W/m°C
Dynamic viscosity (μ <sub>w</sub> )	0.000652 N. S/m <sup>2</sup>
Kinematic Viscosity (ν <sub>w</sub> )	0.6581 x 10 <sup>-6</sup> m <sup>2</sup> /s
Prandtl number (Pr) <sub>w</sub>	4.16
Specific Gravity (S. G) <sub>w</sub>	1

Table 3. Parameters calculated using mathematical modelling.

Sr. No	Calculated parameters	Symbol	units	Numerical value
1	Heat duty	Q	watts	5202
2	Corrected heat duty	Q <sub>c</sub>	watts	6502
3	Mass flow rate of CO <sub>2</sub>	m <sub>c</sub>	Kg/sec	0.0534
4	LMTD	LMTD	-	17.71
5	Correction factor for LMTD	F <sub>t</sub>	-	0.78
6	Heat transfer area	A <sub>s</sub>	m <sup>2</sup>	2.35
7	No of tubes	N <sub>t</sub>	-	40
8	Tube side velocity	V	m/s	0.1056
9	Reynolds no for tube side fluid	(Re) <sub>t</sub>	-	19339.98
10	Inside heat transfer coefficient	h <sub>i</sub>	w/m <sup>2</sup> k	185.67
11	Equivalent diameter of shell	D <sub>e</sub>	mm	24.08
12	Shell side cross flow area	A <sub>s</sub>	m <sup>2</sup>	0.0136
13	Mass flow Velocity	G <sub>s</sub>	m/sec	0.00287
14	Reynolds no for shell side fluid	(Re) <sub>s</sub>	-	1109.10
15	Outside heat transfer coefficient	h <sub>o</sub>	w/m <sup>2</sup> k	725.33
16	Overall heat transfer coefficient	U	w/m <sup>2</sup> k	152.30

Sr. No	Calculated parameters	Symbol	units	Numerical value
16	Tube side pressure drop	$\Delta p_t$	N/m <sup>2</sup>	191.74
17	Shell side pressure drop	$\Delta p_s$	N/m <sup>2</sup>	7.12
18	Shell thickness & shell O. D	$t_s$ & $D_s$	mm	35,324
19	Shell cover thickness	$t_h$	mm	25.34
20	Channel cover thickness	$t_{cc}$	mm	19
21	Tube sheet thickness	$t_{ts}$	mm	35.81
22	Nozzle thickness calculations	$t_n$	mm	3.33
23	Gasket diameter	G	mm	340
24	Gasket width	N	mm	85.4
25	Flange thickness	$t_f$	mm	129

Table 4. Parameters directly selected from standard charts for modelling.

Sr. No	Selected parameters	Description and value
1	Tube material	Stainless steel
2	Outside diameter of tube	19.05 mm
3	Wall thickness of tube	16 BWG(1.651 mm)
4	Inside diameter of tube	16.1036 mm
5	Length of tube	1000 mm
6	Type of shell & Tube H. E	Fixed tube sheet type H. E (2 shell & 4 tube passes)
7	Factor for sider tate equation for inside heat transfer coefficient( $j_H$ )	70
8	Tube pitch(Pt)	2.54 mm
9	Factor for sider tate equation for inside heat transfer coefficient ( $j_H$ )	17
10	Inside shell diameter ( $D_{si}$ )	254 mm
11	Type of baffle	Segmented baffle with 25 % cut
12	Tube side fluid	CO <sub>2</sub> Due to high pressure
13	Shell side fluid	Water
14	No of baffles ( $N_b$ )	04
15	Friction factor for tube ( $f_t$ )	0.03312
16	Friction factor for shell ( $f_s$ )	0.4752
17	Corrosion allowance (Ca)	1.5 mm carbon steel
18	Permissible stress for Carbon steel(f)	100.6 N/mm <sup>2</sup>

## 6. Conclusion

When we compare the results of shell & Tube heat exchanger used in conventional refrigeration system & transcritical CO<sub>2</sub> refrigeration system we found out that:

- (1) Design pressure is higher in Gas Cooler so the thickness of shell, Tubes, flanges, tube sheet are higher than conventional shell and tube heat exchanger.
- (2) Due to high pressure in CO<sub>2</sub> refrigeration system the material used for construction of tube is stainless steel rather than copper and brass.
- (3) The CO<sub>2</sub> gas cooler is working under higher pressure than conventional shell and tube heat exchanger.
- (4) As temperature of refrigerant vapour is higher than conventional refrigeration system gas cooler can be effectively used for water heating for domestic & industrial purpose.

## Nomenclature

$A_s$	Heat transfer area (m <sup>2</sup> )
$N_t$	Number of tubes
$Q$	Heat duty (W)
$Q_c$	Corrected heat duty (W)

$\dot{m}_c$	Mass flow rate of CO <sub>2</sub> (Kg/sec)
Re	Reynolds number
$h_i$	Inside heat transfer coefficient (W/m <sup>2</sup> k)
$h_o$	Outside heat transfer coefficient (W/m <sup>2</sup> k)
$G_s$	Mass flow Velocity (m/sec)
U	Overall heat transfer coefficient (W/m <sup>2</sup> k)
$t_s$	Shell thickness (m)
$D_s$	Shell outer diameter (m)
$t_h$	Shell cover thickness (m)
$t_{cc}$	Channel cover thickness (m)
$t_{ts}$	Tube sheet thickness (m)
$\mu_w$	Dynamic viscosity (N. S/m <sup>2</sup> )
$P_r$	Prandtl number
S. G	Specific Gravity
Subscripts	
c	CO <sub>2</sub>
w	Water
e	Equivalent
i	Inside
o	Outside
t	Tube side
s	Shell side

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