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The Pulling Mode of Hunting and the Root Cause of Corrugation

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Abstract: This paper deals with a newly discovered train hunting mode--- the pulling mode. As the name suggests, the pulling mode is caused by pulling. Compared with the well-known tapered wheel tread mode, we can see that the pulling mode is another train hunting mode. Thus, there are two hunting modes discovered for train movements. The starting point and hunting characteristics of the pulling mode are discussed. Just like the tapered wheel tread mode, the pulling mode has also two degrees of freedom. The pulling mode is generally and easily observed from the movement of a trailer on the road. The mechanism caused the pulling mode in a trailer-car system is first analyzed. In normal operations, lateral displacement (disturbance) to a trailer-car is inevitable no matter how good quality is the system. The pulling power will act to straighten the system at the same time to overshoot to cause hunting. That is why this hunting mode is called the pulling mode. Thus, the existence of the pulling mode is proven. Stable hunting motions can be viewed as the harmonic vibration. Hunting motion curves for the car and trailer were established respectively. The two curves have a difference of π in phase angle, and a difference in amplitudes; however, the period and the wave length are the same. The characteristics of the pulling mode, such as the sway center and the sway momentum etc., are demonstrated mainly by using a trailer-car system because it makes the hunting mode easy to understand. The first degree of freedom of wheelset hunting is the culprit of rail corrugations. The wave length of corrugations was calculated. With understanding the principle of the pulling mode, the knowledge can be applied to solve the corrugation problem caused by the pulling mode. Proper weight distributions within a vehicle are analyzed to try to reduce severer train hunting. Power distribution can be a factor to cause the polling mode hunting. Some other measures can also be taken to reduce the pulling mode hunting. A hunting eliminator can be developed to eliminate hunting totally.

Keywords: Train Hunting Modes, The Pulling Mode, The Tapered Wheel Tread Mode, Degree of Freedom, Wave Length of Corrugation, Vehicle Sway, Hunting Alleviation And Elimination

1. Introduction

Train hunting becomes an operation problem as the train speed increases but train hunting has only partially been understood by the railroad industry so far. Although much effort has been made to alleviate train hunting, train hunting is still a problem for the railroad industry today. It is clear that with the understanding of hunting by the scientists and researchers at the present time is only half way towards a nearly total understanding. Therefore, hunting problems cannot be solved correctly. It is true that scientists have long identified tapered wheel treads as the root cause of hunting, and that a thorough study on tapered wheel tread hunting has been performed by the author [1, 2]. However, there are two train hunting modes. One is the tapered tread mode which is caused by rolling radius difference (RRD), and the other is the pulling mode (named by the author) which is caused by the pulling and the train motion mechanism. The two modes will overlap to enlarge train hunting while fighting each other from the two modes to reduce hunting would be rare. So, it is impossible for the scientists and engineers to deal with hunting effectively and correctly if they do not see the whole picture of hunting.

It is easy to see and feel the hunting motion of a tapered wheel tread but it is not easy to see the pulling mode of train hunting. That is because the pulling mode is not very obvious and researchers only focus on the wheelset while researching on train hunting. Actually, the pulling mode was first visualized and identified when a four-wheel trailer towing by a tractor on a country road was observed from behind by the author. The hunting, or sway motion, of the trailer was so severe and obvious from visualization. However, the hunting motion has nothing to do with tapered wheel tread or RRD. It must be a mode of hunting other than the tapered wheel tread mode. Therefore, it was named as pulling mode.

Literature review was performed on the train hunting research. It was found that train hunting research was mostly based on the some imaginary theories which are obvious in violation of Newton's law and can be proved to be wrong, such as the creepage theory etc. [1, 8, 9, 15]. However, some hunting analyses were performed only based on the facts and the laws of mechanical engineering, and the fundamental principles of tapered wheel tread hunting have been established in the creative research [1, 2]. But only the tapered tread mode has been known so far, not the pulling mode.

Lab testing was performed on a four-wheel (two axle) trailer. It was found that the trailer would not be able to go in a straight line on a flat surface; no matter how good is the trailer or the environmental conditions. That is to say, the path of the trailer is always a curve. It is therefore to predict that a two-wheel (one axle) trailer will certainly go in a periodic motion.

The mechanical properties of the pulling mode hunting, such as hunting wave-length; hunting frequency etc. will be analyzed just as the tapered tread mode. A two-wheel (one axle) trailer was chosen to do the analysis because one-axle trailer can concisely show the key characteristics of the pulling mode and can be understood very easily.

The knowledge of pulling mode to of train hunting is going to be used in the project to eliminate corrugation and noise, while the knowledge is also very essential to vehicle and truck design. However, this knowledge has not been understood and available to the designers and manufacturers of the railroad industry.

2. The Observation of the Pulling Mode

For a long freight train with cylindrical wheel tread, the caboose or the last vehicle is the place where train hunting can usually be observed [4, 5]. This is clear evidence that the pulling mode of hunting does exist. The author happened to follow a four-wheel trailer pulling by a tractor on a country road. It was also noticed that the trailer had a severe hunting motion. In order to understand the rolling behavior of a trailer, a small model of a four-wheel trailer was set to roll freely on an incline, as shown in Figure 1. It was found that the trailer simply could not go in a straight line. It is not because the trailer was not in high quality in manufacturing and assembling. It is the inherited property of the trailer not to go in a straight line due to the vibrations and road unevenness etc.

Whenever a trailer is pulled or pushed, it will go hunting because its freedom to go sideways is unconstrained, or minimum constrained by the tire/road friction to be exactly. There are other outside factors will cause a train vehicle to sway, such as the wind or the air pressure from a train passing by. That is to say, the pulling mode is inevitable.

Figure 1. *Free rolling 4-wheel trailer.*

2.1. How the Pulling Mode Generates

One-axle trailers have the same hunting characteristics as two-axle trailers, but one-axle trailers are simple and easy in calculation and demonstration. So a one-axle trailer will be used to illustrate how trailer hunting is initiated. Figure 2. shows a trailer pulling by a car. If a lateral disturbance was given to the system, hunting of the fleet will begin, as shown in the video.

Figure 2. *Hunting of trailer.*

Hunting motion of the trailer can be modeled and analyzed as shown in Figure 3. Arrows 1, 2 and 3 (numbers inside the cycle) represent the directions that the trailer is pulled or steered. The direction of pulling is alternated continuously from direction 1 to 2; back to 1, and then 1 to 3; back to 1. In other worlds, the trailer is steered to the left, then to the right, and then back to the center to finish a cycle of hunting. The amplitude of hunting, A, is directly proportional to the angle of trailer sway. While wave length of hunting, L, is related to the yaw rate/time of trailer and velocity V.

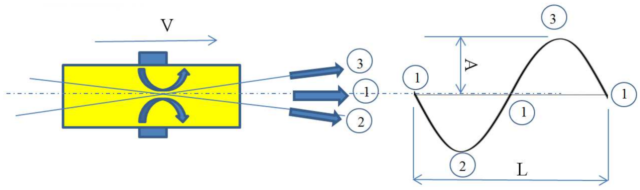

Figure 3. *Hunting motion.*

In the reality, a trailer is usually connected with a car and is pulled by a car. Therefore, the next step is to analyze a trailer pulling by a car. Figure 4 shows a trailer pulling by a car and moving with velocity V. If for some reasons, the fleet was subject to a lateral deflection δ via swaying, the hunting motion will begin. What can be seen is that the trailer and car swaying back and forth.

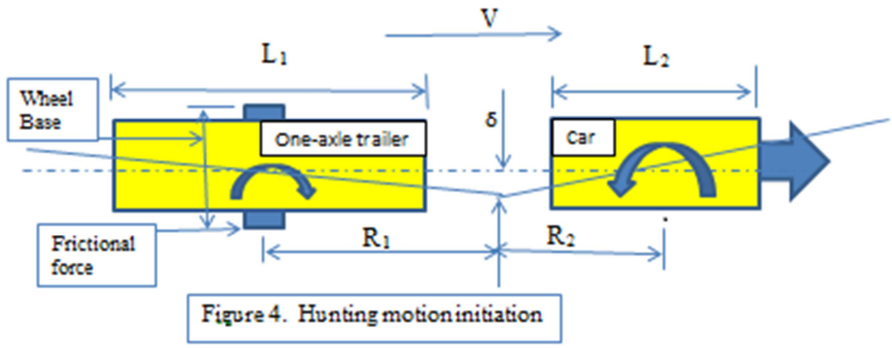

Figure 4. *Hunting motion initiation.*

It is easy to see that the fleet will go back to its neutral state under the pulling force from the car. But why the fleet sways pass the neutral state? That is because the pulling force at this state is the maximum force needed to recover to the neutral state (Note that small pulling forces will not cause hunting). See Figure 5. However, less and less recovering energy will be consumed as the system sways to the neutral state, and the system will continue to sway to the other side to consume all the recovering energy. As the same token, next round of system sway will repeat itself to the no end. However, the magnitude of the sway to the other side determines the magnitude of the hunting motion of the trailer. The magnitude has three possible options: 1). The magnitude of hunting is less than that of the previous hunting; hunting will die out, disappear soon, 2). The magnitude of hunting is equal to that of the previous hunting; hunting will be stable and 3). The magnitude of hunting is larger than that of the previous hunting; hunting will be unstable. See Figure 6.

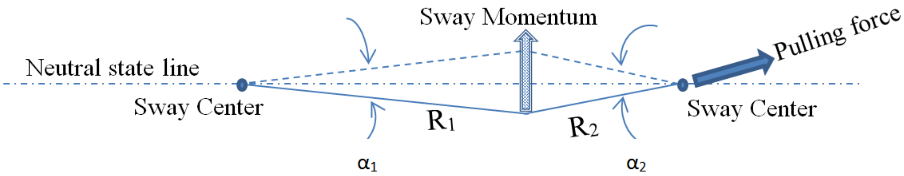

Figure 5. *Maximum sway energy state.*

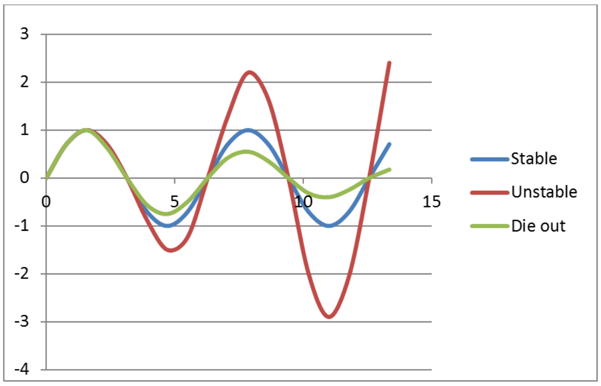

Figure 6. *Three states of pulling mode hunting.*

3. The Sway Center

Train hunting by the pulling mode can be treated as the sway motion of the vehicles from engineering point of view. It can be observed that there is a sway center when a vehicle sways, although there is no clear definition for the sway center in mechanical engineering. At least we can define that sway centers are the centers of the trailer and car sway, or the centers of yaw motion. Every vehicle has its own center of sway, and it should be the geometry center of the vehicle. Meanwhile, sway centers may not be necessary the same as the center of gravity.

We can further assume that a sway is stable if a vehicle sways around its sway center. Then, an unstable sway is that a vehicle does not sway around its sway center. From the observation of the trailer hunting, as shown in Figure 2, one can see that the trailer does not sway around its sway center while at severe hunting. That means the trailer enters the stage of unstable sway. It can be seen that the sway center of the trailer is at the hinge point of the trailer with car. At the same time, the trailer acts like a pendulum from the mechanical point of view. That should be the base for dealing with unstable sway, but in this paper, only the stable sway will be studied.

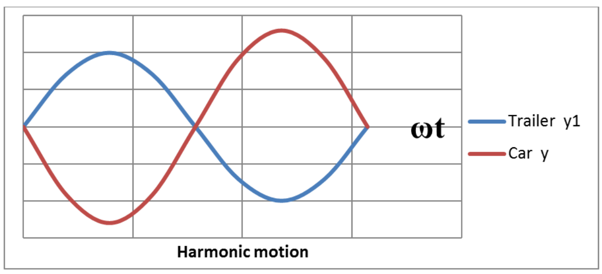

Figure 7. *Hunting curves---car and trailer.*

3.1. The Hunting Motion Curves

The stable hunting curve can be treated as a harmonic vibration curve. Let's make the hunting curve for the car has the form:

$$y = A \sin(\omega t) \quad (1)$$

where A denotes the amplitude of the vibration and ω the circular frequency with ωT = 2π (T the period). In this trailer-car system, the car has a shorter sway radius ($R_2 < R_1$) or a larger sway angle because the car is shorter in length. So the car will get its maximum sway, that is A = δ. See Figures 4 & 5.

Due to the lengths of the vehicles are different, the sway angle (about the sway center) for vehicles are different. When the car reaches its maximum lateral displacement (δ), the trailer does not because the trailer has a smaller sway angle; actually, the trailer is in the way to its maximum lateral displacement (not reach the maximum yet). At the same time, when the car reaches its maximum lateral displacement, the car will sway to the other side, and the trailer has to sway to the other side because the car and trailer are connected together. Thus, the amplitude of the car's sway is larger than that of trailer's. The hunting curve for the trailer has the form:

$$y_1 = A_1 \sin(\omega t - \pi) \quad (2)$$

where A_1 denotes the amplitude of the vibration for the trailer ($A_1 < \delta$) and the trailer hunting has phase angle difference of π.

Let V be the velocity of the train and L be the wave length of hunting. We have

$$\omega = 2\pi(V/L) \text{ and } L = 2\pi(V/\omega).$$

From observation, we can see that the trailer and car must have the same wave length because they are connected and move with the same velocity, therefore cover the same distance longitudinally at the same time.

4. The Sway Momentum

The sway momentum (SM for short) can be viewed as the energy or the force produced by the pulling force to sway the vehicle [10, 11]. As shown in Figures 4 & 5, the pulling mode of hunting is caused by a deflection δ while the pulling forces trying to recuperate the deflection. It is the SM created by the polling power that acts to recover the deflection. The velocity to recover the deflection δ can be calculate as:

$$V_1 = 4\delta/T = 4\delta\ V/L \quad (3)$$

This equation states that the deflection δ is recovered in a quarter of the period, T/4. Then, according to the definition of angular momentum [10], we have the sway momentum for the whole system

$$SM = I_1 *(\omega_1) + I_2 *(\omega_2) = I_1*(V_1/ R_1) + I_2*(V_1/ R_2) \quad (4)$$

where I_1 and I_2 are the moment of inertia for the trailer and the car respectively. It is important to point out that this away momentum is the maximum energy which the trailer system will have to provide to have a sway motion. This energy consumes the pulling power to cost the system efficiency. In other words, sway motion costs additional energy. Ideally, sway momentum should be made smaller. However, if the maximum energy is not large enough to recuperate the deflection δ, for instance, velocity too small, the sway motion will not happen. On the other hand, if the sway momentum is too large to recuperate the deflection δ, for instance, velocity too large or bad weight distributions, the sway motion will be unstable [14].

4.1. The Anti-sway Momentum

The anti-sway momentum (ASM) is the ability of a vehicle to act against the sway motion. Due to the fact that two vehicles are connected together, ASM is provided by the two vehicles. For the trailer,

$$ASM = (\text{frictional force})*(WB)*(T/2)$$
$$= (\text{frictional force})*(WB)*(L/2V) \quad (5)$$

where WB denotes the wheel base. The anti-sway momentum for the car can be calculated in the same way and added up together to obtain the anti-sway momentum for the whole system.

It is important to realize that the anti-sway momentum is very beneficial to the stability of the system while the sway momentum is detrimental to the stability. Every effort should be made to enlarge the ASM and to decrease the SM.

4.2. Central Axle Versus Car-End Axle (One Axle vs. Two-Axle)

A vehicle can have an axle installed in the center or at the car-end. See Figure 2. For a vehicle to fight the pulling hunting better, axles must be installed at the car-end. This can be easily seen by examining how the friction between the wheel and the ground acts against the vehicle's sway motion.

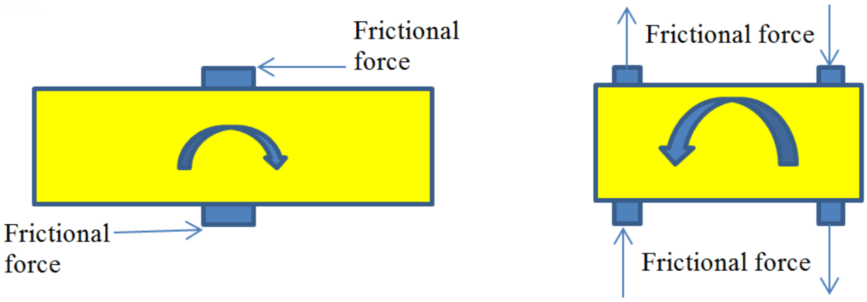

Figure 8. *One axle vs. two axles.*

The frictional force always acts against the sway motion of the vehicle as the result of an anti-rotation torque formed by the frictional forces. A larger anti-rotation torque is usually created with axles installed at the car-end, especially for the long vehicle as in some trains.

Since frictional forces are critical to the pulling hunting, it is beneficial to increase the friction force of a vehicle as far as anti-rotation torque is concerned. Increasing the weight for a vehicle will increase the frictional forces, so that the vehicle will be more stable to pulling hunting. However, as will be demonstrated, weight distribution will affect the stability of vehicles.

5. Vehicle Interaction

Vehicles must interact with each other because they are connected, and vehicle interaction is a favorable factor to the stability of vehicles. Depending on vehicle configurations, the general observation is that hunting is mostly seen; or severe, in the very last vehicle of a train, not the vehicles in between. This is because there is no vehicle interaction at the end of the last vehicle since it is the last vehicle. However, it should be pointed out that the last vehicle is a driven (not driving) vehicle in the observation.

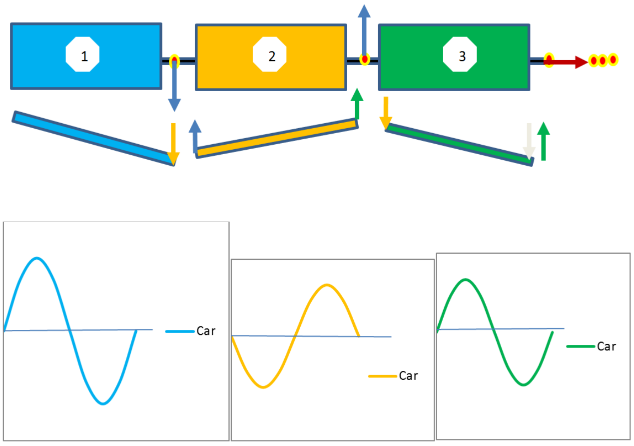

Figure 9. *Vehicle interaction and hunting curves.*

When a train hunting under the pulling mode, only the last vehicle (vehicle 1) is subject to one lateral force (at the front end); all other vehicles are subject two lateral forces at both ends. That is, a vehicle is tugged laterally by the vehicle before while tugging laterally the vehicle behind. From Newton's law of action and reaction, when a vehicle tugs at the other vehicle, the other vehicle will have a reaction tug to the vehicle. See Figure 9. Vehicle 2 (yellow in color) have one action force and one reaction force (yellow in color), so does vehicle 3.

It can be seen that the reaction tug force always acts against vehicle sway to damp out vehicle hunting. The last vehicle does not have a vehicle behind to damp out the pulling hunting, so the last vehicle must have a larger hunting motion than the rest of the vehicles.

5.1. Power Distribution in a Train

We have just finished the analysis of vehicle interaction, but with all vehicles without power. That is, all vehicles are driven. Assuming we have a train with all axels powered, then a new analysis on vehicle interaction is needed.

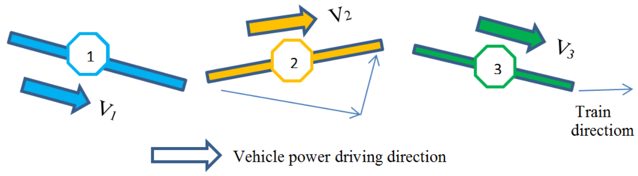

Figure 10. *Difference in vehicle driving directions.*

The power system in a vehicle is always driving the vehicle forward. However, when a train is in hunting, the power system will still driving the vehicle forward but the direction of driving is different from adjacent vehicles because vehicles sway. It is reasonable to assume that all vehicles in the train have the same velocity, that is V_1=V_2=V_3. It is immediately from the drawing above that V_1 is larger than component of V_2 in V_1 direction. Thus, vehicle 1 will push vehicle 2 laterally to sway more, so does vehicle 2 to vehicle 3. Actually, every vehicle in the train will push the vehicle before it laterally.

From the demonstration of vehicle driving direction, we can see that power distributions in a vehicle will also cause train hunting when the train in curving (Figure 11). The key is that all vehicles in curving have their own driving directions. Thus, every vehicle in the train will push (or drag) the vehicle before (or behind) it laterally to cause hunting as shown previously.

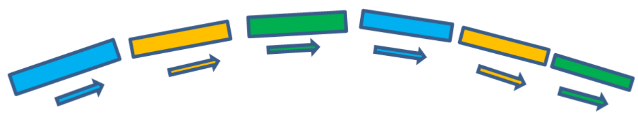

Figure 11. *Curving---vehicle driving directions.*

5.2. Weight Distribution in a Vehicle

It was shown previously that heavier weight more stable the vehicle as far as friction is concerned. Furthermore, from the trailer experiment, we can see that weight distribution in a vehicle will significantly affect the hunting stability of the vehicle. There are two typical scenarios that need to be considered: 1). Weights in the center and 2). Weights at either end of the vehicle. Note that weights at either end will have the same effect on vehicle hunting. So, here only the weight at the left end will be analyzed.

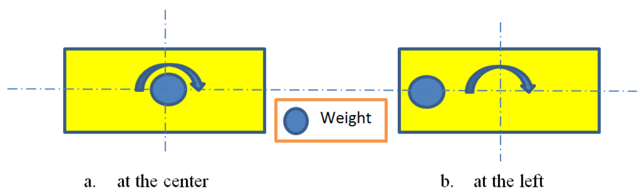

Figure 12. *Weight locations on vehicle.*

In order to do a dynamic analysis, the vehicle-weight system can be idealized to obtain the mechanical model, as shown. (m: mass of the weight. M: mass of the vehicle.).

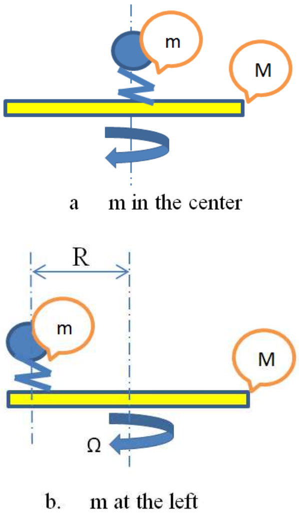

Figure 13. *Weight-vehicle model, M, m, mass.*

Studying weight distributions on a moving vehicle has to deal with momentum [10], a subject of physics. Momentum is a vector reflecting the level of forces or energy in a system. Angular momentum p is defined as

$$p = I\,\Omega \qquad (6)$$

where Ω represents the angular velocity of the vehicle in yaw motion, and I the moment of inertia. Recall the sway momentum we demonstrated before, this angular momentum is the sway momentum. For the weight m at the left and m in the center, the moment of inertia has the following relationship

$$I_b = I_a + mR^2$$

with R being the distance from the mass to the center of rotation as shown.

It is obvious that mass m in the center (Figure 13) has the least sway momentum while mass m at the left has the largest momentum due to the distance R. When the vehicle sways, the larger sway momentum the worse the stability. Important thing to do is reduce the sway momentum of the vehicle while hunting, because larger sway momentum means more or severer hunting.

Understanding the effect of weight distributions in a vehicle is very useful in vehicle design and train management. For a passenger train, the design of the vehicle should make it possible that human being weight will be at the center of the vehicle as the first choice. While for a freight train, the laden should be placed at the center of the vehicle as possible. Thus, the effect of train hunting will be reduced for a train.

6. Wheelset Balance and Vibration Damper

Let's consider some measures to alleviate train hunting. Wheelsets play a key part in train hunting. The first thing to do is to check wheelset balance. This is a very important step to assure vehicle stability especially in high speed trains. We know that tires must be balanced before installed into an automobile. By the same token, wheelset balance must be checked before installed into a truck. However, wheelset balance is not required in the railroad industry. Furthermore, unlike the tires, wheelset balance has two parts: 1). the Center of Gravity of the wheelset is at the center and 2). Wheelset balance velocity is below the critical velocity. Critical velocity is defined as the velocity at which the wheelset begins to derail [2], and is infinite for the wheelset with cylindrical wheel tread. If the wheelset is balance and its balance velocity is under the maximum train velocity allowed, the wheelset is stable.

To reduce rail corrugation, a vibration damper should be considered for a wheelset. There is not only one frequency of wheelset vibration with many train operation speeds. The untuned viscous torsional damper is known to be effective over a wide frequency range. Different kinds of vibration dampers are applied to wheelsets in the railroad [6, 7], obviously; the untuned viscous torsional damper is the first choice for the best results. However, the cavity containing viscous fluids would make the manufacturing of the wheels complicated. Nevertheless, vibration dampers can also be considered for the truck or the vehicle besides the wheelset.

7. Interaction of the Two Hunting Modes and Hunting Eliminator

As demonstrated previously, there are two hunting modes in a moving train. The two modes will interact to each other. Usually, the two modes will interact to increase the hunting motion (Figure 14), while interacting to cancel out each other to decrease the hunting is rare (Figure 15). However, Decreasing hunting is beneficial and is what we are struggling to created.

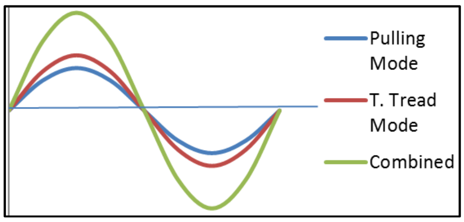

Figure 14. *Tread Mode + Pulling Mode.*

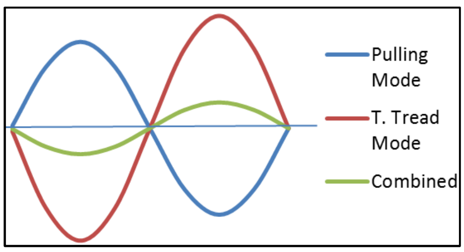

Figure 15. *Tread Mode - Pulling Mode.*

The two hunting waves overlapping will produce a larger hunting wave. Due to the fact that amplitudes of the two hunting are hard to predict and control, the two waves annulling each other usually will not eliminate the hunting wave totally. Nevertheless, under either way of interaction, the train hunting wave would be sinusoidal. That would make to create some methods to eliminate train hunting possible.

As a matter of fact, a hunting eliminator was designed and constructed to eliminate the tapered wheel-tread hunting mode [7]. Testing showed that the hunting motion of the tapered wheelset on a straight track disappeared on the device. The tapered wheelset moved almost in a straight line without sway. However, effectiveness of the hunting eliminator to train motions with the two hunting modes combined has not been proved.

8. The First Degree of Freedom of Hunting and Rail Corrugation

Just as shown in the tapered wheel tread hunting mode, wheelset hunting is a motion with two degrees of freedom [2, 3]. By the same token, the pulling mode train hunting also has two degrees of freedom. These two degrees of freedom are related, or conjugate. See Figure 16. What has been demonstrated so far is only about one degree of freedom; the second degree of freedom, while the other degree of freedom has never been mentioned. However, it is important to keep in mind that the two modes of hunting all have two degrees of freedom. The hunting curves may not be exactly sinusoidal as demonstrated due to the effect from the other degree of freedom.

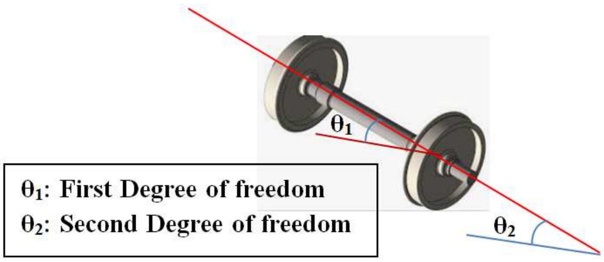

Figure 16. *Two degrees of freedom, wheelset hunting.*

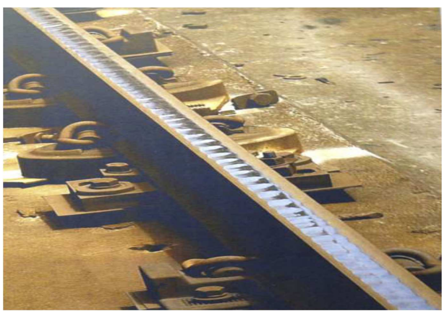

Figure 17. *Rail corrugation, copied from ref. 12.*

Rail corrugations are most common track damage from train hunting. The corrugations are usually in the form of very short wave lengths [12, 13]. This phenomenon indicates that the first degree of freedom is the culprit of rail corrugations, because the rail damage pattern (corrugation) matches with the movement of the first degree of freedom (θ_1). It is the yaw motion due to the first degree of freedom that causes the wheel tread to rub on rail to wear the rail to form the corrugations. The natural frequency of the vibration due to the first degree of freedom for a tapered wheelset was derived as [2]:

$$\Omega_n = \frac{2V}{G(r_1+r_1)}\sqrt{2r_1(r_1+r_1)} \quad \text{(Rad)} \qquad (7)$$

where G, r_2 and r_1 are the gauge width, major and minor radii of the wheel respectively. The wave length of rail corrugation is therefore computed as

$$L_c = V*T= V*(2\pi/\Omega_n) = G(r_1 + r_2)\ \pi/[2r_1(r_2-r_1)]^{0.5} \qquad (8)$$

It can be seen that the wave length of corrugation is not a function of the train velocity, while the wave length is infinite for a cylindrical wheelset (r_2=r_1). Larger gauge width G will have a larger wave length and a larger First degree vibration. At the same time, wave lengths must be affected by the factors such as the friction between rail and wheel, the support conditions of the rail, etc. The tapered wheelset mode is used here to demonstrate the physical properties of corrugation. For the pulling mode, however, the physical properties of corrugation can be calculated in the same way but with different parameters. Thus, to solve the corrugation problem, we have two steps to go.

1) Applying wheelset balance technology and vibration dampers to control or alleviate vibrations from the First degree of freedom. However, it can be seen here that wider gauge is certainly not beneficial in reducing the vibrations from the First degree of freedom.
2) Applying hunting eliminator technology to eliminate hunting from the first degree of freedom. If the first degree hunting can be reduced, the hunting from the First degree of freedom is automatically reduced.

9. Conclusion

The tapered wheel-tread hunting mode is easy to understand physically. But there exists other train hunting mode---the pulling mode, and this mode can be initiated without the tapered wheel-tread. The main cause of the pulling mode is due to vehicle vibrations caused by road conditions, environmental factors, power distribution and curving, etc. Train hunting, especially the First degree of freedom of the hunting, is connected to rail corrugations. From this point of view, a wider gauge will have more rail corrugations. Power distribution for every vehicle in a train is not a good idea. Weight distribution in a vehicle is also important in order to reduce the sway momentum. To reduce rail corrugations, performing a wheelset balance test is suggested before installing the wheelset into the train. Application of wheelset vibration damper is a way to reduce rail corrugations. The two hunting modes will usually interact to each other to enlarge the motion of train hunting. Thus, a hunting eliminator is a good option for the railroad industry to take to reduce train hunting, although the technology has been in developing. To eliminate the rail-corrugation problems, more research needs to be done for this research is only the beginning.

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