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Dominant Metric Dimension of Star Fan Graphs

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Abstract: A dominating set of a connected graph $G = (V, E)$ is a subset D of $V(G)$ such that every vertex of G is either in D or adjacent to a vertex in D. A resolving set of a connected graph G is a subset D of $V(G)$ such that each vertex v of G has a unique representation with respect to D. A dominant resolving set of a graph G is a subset D of $V(G)$ that resolves all vertices of G and dominates G. The dominant metric dimension, $Ddim(G)$, is the cardinality of the smallest such set. The dominant metric dimension of a graph combines metric dimension (location identification) with domination (coverage), making it ideal for sensor networks that require both unique positioning and full coverage. Similarly, a dominant edge resolving set of a connected graph G is a vertex subset D of V that resolves all edges and is a covering of G. Star fan graphs are composite graph structures formed by attaching fan graphs to the pendant vertices of a star graph, creating hierarchical networks that are useful for modeling hub-and-spoke systems with additional path structures. This paper computes the dominant metric dimension and dominant edge metric dimension of star fan graphs.

Keywords: Dominant Metric Dimension, Dominant Edge Metric Dimension, Star Fan Graph

1. Introduction

For a simple connected graph G, a subset D of V(G) is a dominating set if, for each vertex x not in D, x is adjacent to at least one vertex in D. The domination number denoted by $\gamma(G)$ is the order of the smallest such set. The concept of domination in graphs was introduced by Claude Berge [1]. The literature on domination has been surveyed in detail in two eminent books by Haynes et al. [8, 9]. For two vertices $u, v \in V(G)$, the distance between u and v denoted by $d(u, v)$ is the length of the shortest $u - v$ path. For an ordered subset $S = \{v_1, v_2, ..., v_k\}$ of V(G), the metric representation of a vertex u of G with respect to S is $d(u/S) = (d(u, v_1), d(u, v_2)...d(u, v_k))$. If every pair of distinct vertices $u, v \in V(G)$ satisfies that $d(u/S) \neq d(v/S)$, then the set S is called a resolving set of G. That is for every pair (u, v) of $V(G)$, there exists a vertex $x \in S$ such that $d(u, x) \neq d(v, x)$. The smallest possible cardinality of a resolving set is called a metric basis of G. The cardinality of a metric basis is the metric dimension of G, denoted as $Dim(G)$. The concept of metric dimension was introduced by Slater in [13] in connection with uniquely identifying the position of a vertex in a network. Harary and Melter [7] discovered the same concept independently. Many applications of the metric dimension of graphs have appeared in [11]. Applications related to image processing and pattern recognition, as well as connections between the game Mastermind and coin weighing, are presented in [4]

A set is called a dominant resolving set of G if it is a resolving set and is a dominating set of G. The smallest possible cardinality of such a set is called a dominant metric basis of G and its cardinality is the dominant metric dimension of G, denoted by $Ddim(G)$. This was introduced by Brigham et.al in [2]. Susilovati et al. [14] proved that for any connected graph G, $max\{\gamma(G), Dim(G)\} \leq Ddim(G) \leq min\{\gamma(G) + Dim(G), n - 1\}$. They have also computed the dominant metric dimension of some special classes of graphs and the joint and comb products of certain graphs.

A covering of a graph G is a subset K of V(G) such that every edge of G has at least one end in K. If $e = uv$ is any edge of G, then $d(e, x) = min\{d(u, x), d(v, x)\}$. For an ordered subset $S = \{v_1, v_2, ..., v_k\}$ of V(G), the metric representation of an edge e of G with respect to S is $d(e/S) = (d(e, v_1), d(e, v_2)...d(e, v_k))$. If every pair of distinct edges $e, f \in E$ satisfies that $d(e/S) \neq d(f/S)$, then the set S is called an edge resolving set of G. That is for every pair (e, f) of $E(G)$, there exists a vertex $x \in S$ such that $d(e, x) \neq d(f, x)$. The smallest possible cardinality of such a set is called an edge metric basis of G and its cardinality is the dominant edge metric dimension of G, denoted as $Edim(G)$. This concept was introduced by A. Kelenc et al. in [10].

A dominant edge resolving set of a graph is a subset $V(G)$ that covers all the edge of G and is an edge resolving set of G. The minimum cardinality of such a set is the dominant edge metric dimension of the graph, denoted by $D_edim(G)$. M.Tavakoli et al.in [15] introduced the concept of dominant edge metric dimension of graphs and studied its basic properties. The NP hardness of computing the dominant edge metric dimension of connected graphs is also proved in their paper.

A fan graph F_n is the join of a single vertex u_1 and a path P_n on n vertices, say $u_{1,1}, u_{1,2}, ..., u_{1,n}$, where u_1 is adjacent to $u_{1,1}, u_{1,2}, ..., u_{1,n}$. A star graph S_m is isomorphic to $K_{1,m}(m \geq 3)$ and has one central vertex adjacent to m leaves. The Star fan graph was first introduced by Bustan in 2018 [3]. The Star fan graph, $S(F_n, m)$ attaches a fan F_n to each leaf of the star $S_m, m, n \geq 3$. It is a graph with vertex set $V(S(F_n, m)) = \{u_0\} \cup \{u_i/i = 1, 2, ..., m\} \cup \{u_{i,j}/i = 1, 2, ..., m, j = 1, 2, ..., n\}$ and edge set $E(S(F_n, m)) = \{u_0u_i/i = 1, 2, ..., m\} \cup \{u_iu_{i,j}/i = 1, 2, ..., m, j = 1, 2, ..., n\} \cup \{u_{i,j}u_{i,j+1}/i = 1, 2, ..., m, j = 1, 2, ..., n-1\}$. Star fan graphs are widely used as interconnection networks in parallel and distributed computing systems. Network science needs visibility and distinguishability for monitoring. Visibility comes from domination, and distinguishability comes from metric dimension and this is the motivation of this paper.

2. Dominant Metric Dimension of Fan Graphs

The metric dimension of F_n is determined in [5] and their result is given below.

Theorem 2.1. For all $n \geq 7$, the metric dimension of the fan graph F_n is $Dim(F_n) = \lfloor \frac{2n+2}{5} \rfloor$

$$= \begin{cases} \frac{2n-2}{5} & \text{if } n \equiv 1(\text{mod } 5) \\ \frac{2n-1}{5} & \text{if } n \equiv 3(\text{mod } 5) \\ \frac{2n}{5} & \text{if } n \equiv 0(\text{mod } 5) \\ \frac{2n+1}{5} & \text{if } n \equiv 2(\text{mod } 5) \\ \frac{2n+2}{5} & \text{if } n \equiv 4(\text{mod } 5) \end{cases}$$

Proof: Case 1:$n \equiv 1(mod5)$

It has been proved in [12] that $S = \{u_{1,5t-3}, u_{1,5t-1}/t = 1, 2, 3, ..., \frac{n-1}{5}\}$ is a resolving set for F_n and $|S| = \frac{2(n-1)}{5}$. We can observe that every vertex of F_n except the vertex $u_{1,n}$ is dominated by the set S. Therefore, the set $D_1 = S \cup \{u_1\}$ is a dominating and resolving set of F_n and $|D_1| = \frac{2n+3}{5}$. This implies $Ddim(F_n) \leq \frac{2n+3}{5}$

To prove $Ddim(F_n) \geq \frac{2n+3}{5}$, we consider any set S' with cardinality $\frac{2n+3}{5} - 1$ and prove that S' is either not a

Definition 2.1. The set of vertices adjacent to the vertex v of G is called the open neighborhood of v is denoted by $N(v)$. $N(v) \cup \{v\}$ is called the closed neighborhood of v is denoted by $N[v]$.

Definition 2.2. A vertex $v \in V(G)$ is called a lonely vertex with respect to a set S, if $N[v] \not\subset S$

Observation 1: $d(u_1, u_{1,i}) = 1$ for all i and

$$d(u_{1,j}, u_{1,i}) = \begin{cases} 1 & \text{if } u_{1,j} \text{ is a neighbor of } u_{1,i} \\ 2 & \text{otherwise} \end{cases}$$

Observation 2: The vertex u_1 cannot resolve any pair of vertices of F_n

Lemma 2.1. A set S of vertices resolves F_n if and only if
(a) there is at most one lonely vertex with respect to the set S
(b) any set of 5 consecutive vertices of P_n will contain at least two vertices of S.

Proof: Suppose S is a resolving set of F_n. To prove(a), suppose there exist two lonely vertices in $V(G) - S$ say $x, y \in V(P_n)$. $d(x, u) = d(y, u) = 2$ for all $u \in S$. Therefore, S is not a resolving set of F_n. To prove (b), suppose the contrary.
Case(1): There is a set of five consecutive vertices of the path which does not contain any vertex of S say $\{u_{1,j+1}, u_{1,j+2}, ..., u_{1,j+5}\}, 0 \leq j \leq n-5$. Then, $d(u_{1,j+2}/S) = d(u_{1,j+3}/S)$.

Case(2): There is a set of five consecutive vertices of the path which contains exactly one vertex of S. Let that set of vertices be $\{u_{1,j+1}, u_{1,j+2}, ..., u_{1,j+5}\}, 0 \leq j \leq n-5$ and the vertex of S be $u_{1,j+k}, 1 \leq k \leq 5$. If $1 < k < 5$, then $d(u_{1,j+k+1}/S) = d(u_{1,j+k-1}/S)$. If k=1, then $d(u_{1,j+3}/S) = d(u_{1,j+4}/S)$. If k=5, then $d(u_{1,j+3}/S) = d(u_{1,j+2}/S)$.

In both the cases, S is not a resolving set of F_n.

Conversely assume that a set S satisfies the two conditions (a) and (b) given above. To prove S is a resolving set of F_n. Consider any two vertices x and y of F_n which are not in S.
Case(1): One of x and y is a lonely vertex, say x. Then, $d(x, u) = 2$ for all $u \in S$ and $d(y, v) = 1$ for some $v \in S$, since y is not a lonely vertex.

Case(2): Both x and y are not lonely vertices. Then, there exist $x_1 \in S$ such that $x_1 \in N(x)$ and $x_1 \notin N(y)$. This implies $d(x/S) \neq d(y/S)$. Suppose $x_1 \in N(x)$ and $N(y)$. Since the set S satisfies (b), there exists a vertex of S say y_1, in the neighborhood of x or y other than x_1. Assume the vertex $y_1 \in N(y) \cap S$ and $y_1 \notin N(x)$. $d(x, y_1) \neq 1 = d(y, y_1)$. This implies S is a resolving set of F_n.

Theorem 2.2. For all $n \geq 7$, the dominant metric dimension of the fan graph F_n is

$$\text{Ddim}(F_n) = \begin{cases} \lceil \frac{2n+2}{5} \rceil & \text{if } n \equiv 1, 3 \pmod 5 \\ \lfloor \frac{2n+2}{5} \rfloor & \text{if } n \equiv 0, 2, 4 \pmod 5 \end{cases}$$

dominating set or not a resolving set of F_n. We consider the following two subcases

Sub case 1: S' contains u_1. We can find a set with 5 consecutive vertices such that either there is no vertex of S' or it contain exactly one vertex of S'. Using Lemma 2, S' is not a resolving set.

Sub case 2: S' does not contain u_1.This implies S' contains only vertices of the path. By Lemma 2.6, the only possible resolving sets of F_n with cardinality $\frac{2(n-1)}{5}$ are either $S' = \{u_{1,5t}, u_{1,5t-2}/t = 1,2,...,\frac{n-1}{5}\}$ or $S' = \{u_{1,5t-1}, u_{1,5t-3}/t = 1,2,...,\frac{n-1}{5}\}$. But these two sets are not dominating sets of F_n. (In the first set, $u_{1,1}$ is not dominated by any of the vertices of S'. In the second set, $u_{1,n}$ is not dominated by any of the vertices of S').This implies $Ddim(F_n) \geq \frac{2n+3}{5}$ and hence, $Ddim(F_n) = \frac{2n+3}{5} = \lceil \frac{2n+2}{5} \rceil$. A dominant metric basis for F_n is given by

$$D_i = \left\{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n-1}{5}\right\} \cup \{u_1\} \tag{1}$$

Case 2: $n \equiv 3$ (mod 5)

For $n \equiv 3$ (mod 5), it has been proved in [12] that $S = \{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n-3}{5}\} \cup \{u_{1,n}\}$ is a resolving set of F_n and $|S| = \frac{2n-1}{5}$. We can observe that every vertex of F_n except the vertex $u_{1,n-2}$ is dominated by the set S. Therefore, the set $D_i = S \cup \{u_1\}$ as a dominant resolving set of F_n and $|D_i| = \frac{2n+4}{5}$. This implies $Ddim(F_n) \leq \frac{2n+4}{5}$. As in the previous case, we can prove $Ddim(F_n) \geq \frac{2n+4}{5}$ and hence, $Ddim(F_n) = \frac{2n+4}{5} = \lceil \frac{2n+2}{5} \rceil$. A dominant metric basis for F_n is given by

$$D_i = \left\{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n-3}{5}\right\} \cup \{u_1, u_{1,n}\} \tag{2}$$

Case 3: $n \equiv 0,2,4 (mod 5)$

Let $n \equiv 0(mod 5)$ A metric basis for F_n is given in [12], $|D_i| = \frac{2n}{5}$ and $D_i = \left\{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n}{5}\right\}$. We observe that this is also a dominating set of F_n. A dominant metric basis for F_n is given by

$$D_i = \left\{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n}{5}\right\} \tag{3}$$

Let $n \equiv 2(mod 5)$ A metric basis for F_n is given in [12], $|D_i| = \frac{2n+1}{5}$ and $D_i = \left\{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n-2}{5}\right\} \cup \{u_{1,n}\}$. We observe that this is also a dominating set of F_n. A dominant metric basis for F_n is given by

$$D_i = \left\{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n-2}{5}\right\} \cup \{u_{1,n}\} \tag{4}$$

Let $n \equiv 4(mod 5)$. A metric basis for F_n is given in [12]then $|D_i| = \frac{2n+2}{5}$. We observe that this is also a dominating set of F_n. A dominant metric basis for F_n is given by

$$D_i = \left\{u_{1,5t-3}, u_{1,5t-1}/t = 1,2,...,\frac{n+1}{5}\right\} \tag{5}$$

In all the above cases, $Ddim(F_n) \leq \lfloor \frac{2n+2}{5} \rfloor$. We know $Ddim(G) \geq max\{Dim(G), \gamma(G)\}$ [6] for any graph G. Therefore $Ddim(F_n) \geq Dim(F_n) = \lfloor \frac{2n+2}{5} \rfloor$ from theorem 2.1.

Hence,

$$Ddim(F_n) = \begin{cases} \lceil \frac{2n+2}{5} \rceil & \text{if } n \equiv 1,3 (\text{mod } 5) \\ \lfloor \frac{2n+2}{5} \rfloor & \text{if } n \equiv 0,2,4 (\text{mod } 5) \end{cases}$$

3. Dominant Metric Dimension of Star Fan Graph

Definition 3.1. A subgraph H of a graph G is called an isometric subgraph of G if $d_H(u,v) = d_G(u,v)$ for all $u, v \in V(H)$.

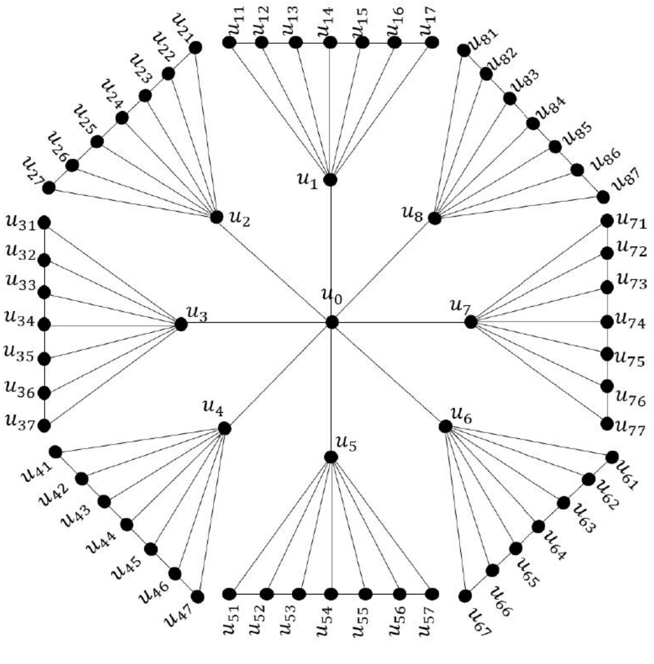

Figure 1. $S(F_7, 8)$.

Theorem 3.1. Let $G = S(F_n, m)$ be a star fan graph. For all m, and for all $n \geq 7$,

$$Ddim(G) = \begin{cases} m\lceil \frac{2n+2}{5} \rceil & \text{if } n \equiv 1,3 \text{ (mod 5)} \\ m\lfloor \frac{2n+2}{5} \rfloor + 1 & \text{if } n \equiv 0,2,4 \text{ (mod 5)} \end{cases}$$

Proof: Let G_i be the fan graph induced by $\{u_i, u_{i,1}, ..., u_{i,n}\}$ for $i = 1,2,...,m$. Each G_i is an isometric subgraph of G.

Case 1: $n \equiv 1,3$(mod 5). Let $D = \bigcup_{i=1}^{m} D_i$ and $|D| = m\lceil \frac{2n+2}{5} \rceil$ where D_i is the dominant metric basis of G_i mentioned in equations (1) and (2) of theorem 2.7.

Claim: D is a dominant metric basis of G.

To prove

1. Every pair of distinct vertices $u, v \in G$ satisfies that $d(u/D) \neq d(v/D)$

2. There exists some vertex $x \in D$ which dominates all vertices of G.
   We consider the following three cases:

   (a) $u, v \in G_i$: Since D_i is the dominant metric basis of G_i, there exist $x \in D_i$ such that $d(x, u) \neq d(x, v)$. This implies $d(u/D_i) \neq d(v/D_i)$

   (b) $u \in G_i, v \in G_j$ and $i \neq j$: Since D_i is the dominant metric basis of G_i, there exist $x \in D_i$ such that $d(x, u) < d(x, v)$. This implies $d(u/D_i) \neq d(v/D_i)$.

   (c) $u = u_0, v = u_{i,j}, 1 \leq i \leq m, 1 \leq j \leq n$: Since D_i is the dominant metric basis of G_i, there exist $x \in D_i$ such that$d(x, u) > d(x, v)$.

Since D_i is the dominant metric basis of G_i, there exist vertices in D_i that dominate vertices of G_i for all $i = 1, 2, ..., m$. The vertex u_0 is dominated by any $u_i : i = 1, 2, ..., m$. Hence D is a dominant resolving set of G, and

$$Ddim(G) \leq m \left\lceil \frac{2n+2}{5} \right\rceil .$$

Since each G_i is an isometric subgraph of G, every vertex of the dominant metric basis of G_i must be included in the dominant metric basis of G. Therefore, $\lceil \frac{2n+2}{5} \rceil$ vertices from each G_i are needed to form the dominant metric basis of G. This implies

$$Ddim(G) \geq m \left\lceil \frac{2n+2}{5} \right\rceil .$$

Case 2: $n \equiv 0, 2, 4(\text{mod } 5)$

For $n \equiv 0(\text{mod } 5)$, $D_i = \{u_{i,5t-3}, u_{i,5t-1}/t = 1, 2, ..., \frac{n}{5}\}$ $n \equiv 2(\text{mod } 5)$, $D_i = \{u_{i,5t-3}, u_{i,5t-1}/t = 1, 2, ..., \frac{n-2}{5}\} \cup \{u_{i,n}\}$ $n \equiv 4(\text{mod } 5)$, $D_i = \{u_{i,5t-3}, u_{i,5t-1}/t = 1, 2, ..., \frac{n+1}{5}\}$ as given in equations (3) (4) and (5) of theorem 2.7 are the dominant metric bases of F_n. Let $D = \bigcup_{i=1}^{m} D_i \cup \{u_0\}$. $|D| = m\lceil \frac{2n+2}{5} \rceil + 1$

As in case 1, we can easily prove that D is a dominant resolving set of G. Hence,

$$Ddim(G) \leq m \left\lceil \frac{2n+2}{5} \right\rceil + 1$$

Since each G_i is an isometric subgraph of G, every vertex of the dominant metric basis of G_i must be included in the dominant metric basis of G. Therefore, $\lceil \frac{2n+2}{5} \rceil$ vertices from each G_i are needed to form a dominant metric basis of G. This implies that $Ddim(G) \geq m\lceil \frac{2n+2}{5} \rceil$. As we proved in Theorem 2.7, any set without the vertex $\{u_0\}$ of cardinality $m\lceil \frac{2n+2}{5} \rceil$ is not a dominating set of G, and any set with the vertex $\{u_0\}$ of cardinality $m\lceil \frac{2n+2}{5} \rceil$ is not a resolving set of G. Hence,

$$Ddim(G) \geq m \left\lceil \frac{2n+2}{5} \right\rceil + 1$$

Hence, the theorem.

4. Edge Metric Dimension of Star Fan Graph

The edge metric dimension of F_n is determined in [10] and their result is given below.

Theorem 4.1. For all $n \geq 3$, $Edim(F_n) = n - 1$ and $S = \{u_{1,1}, u_{1,2}, ..., u_{1,n-1}\}$ is an edge metric basis of F_n.

Theorem 4.2. Let $G = S(F_n, m)$ be a star fan graph. For all $m, n > 3$, $Edim(G) = m(n - 1)$.

Proof: Let $S = \bigcup_{i=1}^{m} S_i$ where $S_i = \{u_{i,1}, u_{i,2}, ..., u_{i,n-1}\}$ and S_i is an edge metric basis of G_i.

Claim: S is an edge metric basis of G.

To prove that S resolves every pair of edge of G we consider the following five cases:

Case 1: $e, f \in E(G_i)$. Since S_i is an edge metric basis of G_i, there exists $x \in S_i$ such that $d(e, x) \neq d(f, x)$.

Case 2: $e \in E(G_i), f \in E(G_j), i \neq j$. Since S_i is an edge metric basis of G_i, there exists $x \in S_i$ such that $d(e, x) \leq 2, d(f, x) = d(f, u_j) + d(u_j, x) > 2$. Therefore, $d(e, x) \neq d(f, x)$.

Case 3: $e \in E(G_i)$, $f = u_0u_j, i \neq j$. Since S_j is an edge metric basis of G_j, there exists $x \in S_i$ such that $d(f, x) = 1, d(e, x) \geq 3$. Therefore, $d(e, x) \neq d(f, x)$.

Case 4: $e \in E(G_i)$, $f = u_0u_i$. Since $S_j, j \neq i$ is an edge metric basis of G_j, there exists $x \in S_i$ such that $d(f, x) = 2, d(e, x) > 2$. Therefore, $d(e, x) \neq d(f, x)$

Case 5: $e, f \in \{u_0u_1, u_0u_2, ..., u_0u_m\}$, say $e = u_0u_i, f = u_0u_j.j \neq i$ Since S_i is an edge metric basis of G_i, there exists $x \in S_i$ such that $d(f, x) > 1, d(e, x) = 1$. Therefore, $d(e, x) \neq d(f, x)$.

In the above cases, $Edim(G) \leq m(n - 1)$.

Since each G_i is an isometric subgraph of G, every vertex of the edge metric basis of G_i must be included in the edge metric basis of G. Therefore, $n - 1$ vertices from each G_i are needed to form an edge metric basis of G. This implies $Edim(G) \geq m(n - 1)$. Hence, the theorem.

5. Dominant Edge Metric Dimension of Star Fan Graph

Theorem 5.1. For all $n > 3$, the dominant edge metric dimension of a fan graph, $D_edim(F_n) = n$

Proof: Consider the set $S = \{u_{1,2}, u_{1,3}, ..., u_{1,n}\}$ which is an edge resolving set of F_n as given in theorem 5. But S is not a covering of F_n, since there is an edge $e = u_1u_{1,1}$ whose both end points are not in S. Now consider the set $S' = \{u_1, u_{1,2}, u_{1,3}, ..., u_{1,n}\}$ and $|S'| = n$, which is an edge resolving set and a covering of F_n. This implies $D_e dim(F_n) \leq n$.

To prove $D_e dim(F_n) \geq n$, consider any set S with $n-1$ vertices. There arise two cases.

Case 1: The set S contains u_1. Then the set S cannot contain $u_{1,i}, u_{1,j}$ for some1 $\leq i,j \leq m$ Consider the edges $e = u_1u_{1,i}, f = u_1u_{1,j}$ we can see that $d(e/S) = d(f/S)$. Therefore, S is not an edge resolving set of F_n.

Case 2: The set S does not contain u_1.Then the set S contains $n-1$ vertices from the path except $u_{1,i}$ for some $i = 1, 2, ..., m$. But there is an edge $e = u_1u_{1,i}$ whose both endpoints are not in S. This set S is not a covering of F_n. Hence $D_e dim(F_n) > n-1$.

Therefore, the set is an edge resolving set and a covering of G with minimum cardinality. Hence, the theorem.

Theorem 5.2. Let $G = S(F_n, m)$ be a star fan graph. For all m and for all $n > 3, D_e dim(G) = mn$

Proof: From Theorem 5.1, we see that $S_i = \{u_i, u_{i,2}, u_{i,3}, ..., u_{i,n}\}$ is a dominant edge metric basis of G_i. Let $S = \bigcup_{i=1}^{m} S_i$.

Claim: S is the dominant edge resolving set of G.

To prove

(1) S resolves any pair of edges of G.

(2) each edge of G is incident to at least one vertex of the set S.

We consider the following cases.

1. $e, f \in E(G_i)$. Since S_i is a dominant edge metric basis of G_i, S resolves any pair of edges of G.

2. $e \in E(G_i), f \in E(G_j), i \neq j$, there exists $x \in S_i$ such that $d(e,x) \neq d(f,x)$ because S_i, S_j are the dominant edge metric basis of G_i, G_j respectively.

3. $e \in E(G_i), f = u_0u_j, i \neq j$, then there exists $x \in S_j$ such that $d(f,x) = 1, d(e,x) \geq 3$, since S_j is an edge metric basis of G_j. Therefore, $d(e,x) \neq d(f,x)$

4. $e \in E(G_i), f = u_0u_i$, then there exists $x \in S_j$ such that $d(f,x) = 2$ and $d(e,x) > 2$, since $S_j, j \neq i$ is an edge metric basis of G_j. Therefore, $d(e,x) \neq d(f,x)$

5. $e, f \in \{u_0u_1, u_0u_2, ..., u_0u_m\}, e = u_0u_i, f = u_0u_j$, there exists $x \in S_i$ such that $d(f,x) > 1, d(e,x) = 1$ since $S_i, i \neq j$ is an edge metric basis of G_i. Therefore, $d(e,x) \neq d(f,x)$.
   Since S_i is the dominant edge metric basis of G_i, there exist vertices in S_i that covers edges of G_i for all $i = 1, 2, ..., m$. The edges $\{u_0u_i / i = 1, 2, ..., m\}$ are covered by $u_i : i = 1, 2, ..., m$. Hence S is a dominant edge resolving set of G. $D_e dim(G) \leq mn$.
   Since each G_i is an isometric subgraph of G, every vertex of the dominant edge metric basis of G_i must be included in the dominant edge metric basis of G. Therefore, n vertices from each G_i are needed to form a dominant edge metric basis of G. This implies $D_e dim(G) \geq mn$. Hence, the theorem.

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Conflicts of Interest

The authors declare no conflicts of interest.

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