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Research Article

Advancing Solutions to Higher-Degree Polynomials: A Novel Recurrence Approach via EMS's Theorem

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Abstract

Solving higher-degree polynomial equations remains a fundamental challenge in both pure and applied mathematics. While quadratic, cubic, and quartic equations have known algebraic solutions, no general radical solution exists for degree five and higher (Abel–Ruffini theorem). This work introduces a novel recurrence-based methodology for deriving exact roots of polynomials of arbitrary degree, based on Ezouidi Mourad Sultan's Theorem (EMST). Unlike traditional algebraic techniques that are often restricted to degrees four or less or rely on numerical approximations, this framework allows for the explicit determination of roots, including irrational, complex, and multiple roots, across any polynomial degree. By systematically leveraging the structure of polynomial coefficients through recursive relationships, this approach extends the capabilities of classical methods and enhances their precision. The method is demonstrated through comprehensive examples involving irreducible and high-degree polynomials of degree 8, producing exact roots in closed form. Comparative analyses with established techniques such as Cardano's method, Newton's Method, and the Rational Root Theorem highlight the advantages of this recurrence formulation, including exactness, no reliance on initial guesses, and applicability to any degree. The EMST-based methodology offers a unified pathway toward exact solutions for longstanding algebraic problems.

Keywords

Recurrence, Polynomial Roots, EMS's Theorem, Higher-degree Polynomials, Algebraic Equations, Exact Solutions

1. Introduction

Polynomial equations serve as foundational elements across various fields in mathematics, engineering, and the sciences, underpinning models in control theory, optimization, physics, and combinatorics. While explicit solutions for quadratic, cubic, and quartic equations have been established since the Renaissance, the general solution for polynomials of degree five and higher remains one of the most significant unresolved problems in algebra. The groundbreaking work of Galois and Abel proved the impossibility of solving the general polynomial of degree five or greater through radicals, revealing intrinsic algebraic limitations [1].

Classical solution methods, such as the Rational Root Theorem, Newton's iterative method, Cardano's method, and polynomial decomposition, often provide only partial or approximate solutions. The Rational Root Theorem is limited to rational roots and becomes computationally prohibitive for high-degree or irrational roots [2, 3]. Newton's method, widely employed for numerical approximation, depends heavily on initial guesses and frequently fails to accurately identify

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multiple, complex, or closely spaced roots [4-6]. Moreover, these techniques do not offer explicit algebraic formulas, restricting their application to approximate solutions [7-10].

Recent advances in algebraic and computational techniques—such as spectral theory, orthogonal polynomials, and symbolic computation—have contributed to understanding specific classes of polynomials, including Hermite, Legendre, Chebyshev, Jacobi, and q-orthogonal polynomials [11-15]. Polynomial solutions to differential equations, recurrence relations, and combinatorial interpretations have further expanded this framework [16-20]. Despite these developments, the derivation of a universal explicit formula for the roots of arbitrary-degree polynomials remains an open challenge.

Explorations involving discriminants, recursive relationships, and algebraic curves have provided partial insights, such as bounds on root locations and conditions for root localization [21-26].Nonetheless, a comprehensive, explicit, and exact solution applicable to all polynomial degrees remains a fundamental open problem in algebra.

Recurrence-based methods for solving higher-degree polynomials are presented in [27-32]. Algebraic techniques for polynomial root finding appear in [33-36]. Advanced approaches for complex root analysis are discussed in [37-40]. Laguerre equation techniques, Gibbs phenomenon analysis, cubic equation decomposition, root bounding, and general polynomial solution frameworks are covered in [41-45]. Multiple root computation schemes for high-degree equations are found in [46-50]. Numerical methods and iterative techniques for polynomial solutions are covered in [51-55]. Additional results on difference equations and singular solutions to polynomial problems are addressed in [56-61].

This work presents a recurrence-based methodology grounded in Mourad Sultan Ezouidi's Theorem (EMST). This approach exploits recursive relationships among polynomial coefficients to explicitly determine all roots—rational, irrational, complex, and multiple—for polynomials of any degree. Unlike classical algebraic or purely numerical methods, the EMST framework offers a unified, exact pathway to solutions, thereby extending the scope of traditional techniques while overcoming their inherent limitations. Through rigorous theoretical formulation and illustrative examples involving high-degree and irreducible polynomials, this recurrence approach establishes a new paradigm for the explicit and precise resolution of polynomial equations.

2. EMS's Theorem and Proof

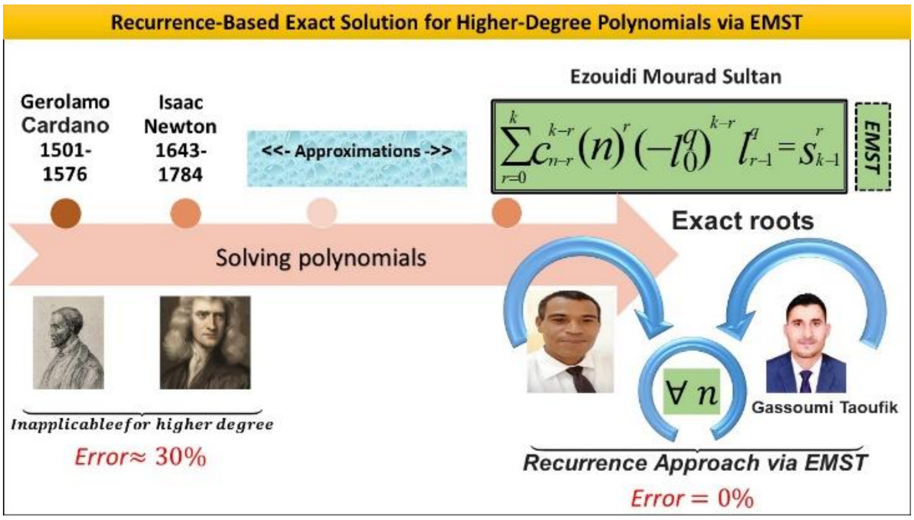
$$\text{Let } \mathrm{P(x)} = \sum_{r=0}^{n}(-1)^r l_{r-1}^q\ x^{n-k} \tag{1}$$

be a polynomial of degree n. The new coefficients of the polynomial, denoted as S_{k-1}^q , is defined as a function. l_{-1}^q , l_0^q ,l_1^q ,l_2^q ,l_3^q ,l_4^q , l_5^q ,,,,,, l_{n-2}^q , l_{n-1}^q and can be computed using the formula:

$$s_{k-1}^r = \sum_{r=0}^{k} c_{n-r}^{k-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{k-r} l_{r-1}^q \tag{2}$$

Equation (2) represents the EMS's Theorem, where l_{-1}^q , l_0^q ,l_1^q ,l_2^q ,l_3^q ,l_4^q , l_5^q ,,,,,, l_{n-2}^q , l_{n-1}^q are the terms derived from the coefficients of the polynomial, and the formula allows for the calculation of exact roots, whether they are rational, irrational, real, or complex.

Graphical abstract



Proof: Recurrence formula

Figure 1. *Graphical abstract of the EMST recurrence method for solving higher-degree polynomials.*

The proof of the EMST involves understanding how the roots of a polynomial are related to its coefficients. By using recursive method we examine the polynomial's structure and find a method to derive the exact values of the roots. This proof demonstrates that the formula provides an exact solution for all types of roots, including repeated and complex ones.

Let

P= $\{P(n)\ \ \exists n_0\ \ \in(0,1,,,n-1)\ /\ \ P(n_0)\ \ \Longrightarrow)P(n-1\}$ is true

For (k=0), application of EMST Equation (2) yields:

$$s_{-1}^{r}=\sum_{r=0}^{0}c_{n-r}^{0-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{0-r}l_{r-1}^{q}=c_n^{-1+1}=c_n^0=1 \tag{3}$$

For (k=1):

$$s_0^r=\sum_{r=0}^{1}c_{n-r}^{1-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{1-r}l_{r-1}^{q}=c_n^1(n)^0(-l_0^q)^1l_{-1}^q+c_n^0(n)^1(-l_0^q)^0l_0^q=-c_n^1l_0^ql_{-1}^q+nl_0^q$$

As l_{-1}^{q}=1 then

$$s_0^r=\sum_{r=0}^{1}c_{n-r}^{1-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{1-r}l_{r-1}^{q}=-c_n^1l_0^ql_{-1}^q+nl_0^q \tag{4}$$

For (k=2):

$$s_1^r=\sum_{r=0}^{2}c_{n-r}^{2-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{2-r}l_{r-1}^{q}=c_n^2(n)^0(l_0^q)^2l_{-1}^q-c_{n-1}^1(n)^1(l_0^q)^1l_0^q+n^2l_1^q=(c_n^2-nc_{n-1}^1)(l_0^q)^2+n^2l_1^q$$

$$=-c_1^1c_n^2)(l_0^q)^2+n^2l_1^q$$

When l_{-1}^{q}=1, it follows that

$$\mathrm{s}_1^{\mathrm{r}}=\sum_{r=0}^{2}c_{n-r}^{2-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{2-r}l_{r-1}^{q}=(c_n^2-nc_{n-1}^1)(l_0^q)^2+n^2l_1^q=-c_1^1c_n^2)(l_0^q)^2+n^2l_1^q$$

i.e

$$\mathrm{s}_1^{\mathrm{r}}=-c_1^1c_n^2)(l_0^q)^2+n^2l_1^q \tag{5}$$

For r (k=3):

$$s_2^r=\sum_{r=0}^{3}c_{n-r}^{3-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{3-r}l_{r-1}^{q}=-c_n^3(n)^0(l_0^q)^3l_{-1}^q+c_{n-1}^2(n)^1(l_0^q)^2l_0^q-c_{n-2}^1(n)^2(l_0^q)^1l_1^q+n^3l_2^q$$

$$=-c_n^3(l_0^q)^3+nc_{n-1}^2(l_0^q)^3-n^2c_{n-2}^1(l_0^q)^1l_1^q+n^3l_2^q=(-c_n^3+nc_{n-1}^2)(l_0^q)^3-n^2c_{n-2}^1(l_0^q)^1l_1^q+n^3l_2^q$$

i.e

$$s_2^r=c_n^2c_n^3(l_0^q)^3-n^2c_{n-2}^1(l_0^q)^1l_1^q+n^3l_2^q \tag{6}$$

For (k=4):

$$s_3^r=\sum_{r=0}^{4}c_{n-r}^{4-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{4-r}l_{r-1}^{q}=c_n^4(n)^0(l_0^q)^4l_{-1}^q-c_{n-1}^3(n)^1(l_0^q)^3l_0^q+c_{n-2}^2(n)^2(l_0^q)^2l_1^q$$

$$-c_{n-3}^1(n)^3(l_0^q)^1l_2^q+n^4l_3^q$$

$$=(c_n^4-nc_{n-1}^3)(l_0^q)^4+c_{n-2}^2(n)^2(l_0^q)^2l_1^q-c_{n-3}^1(n)^3(l_0^q)^1l_2^q+n^4l_3^q$$

$$=-c_3^1c_n^3(l_0^q)^4+c_{n-2}^2(n)^2(l_0^q)^2l_1^q-c_{n-3}^1(n)^3(l_0^q)^1l_2^q+n^4l_3^q$$

$$s_3^r=-c_3^1c_n^4(l_0^q)^4+c_{n-2}^2(n)^2(l_0^q)^2l_1^q-c_{n-3}^1(n)^3(l_0^q)^1l_2^q+n^4l_3^q \tag{7}$$

For (k=5)

$$s_4^r=\sum_{r=0}^{5}c_{n-r}^{5-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{5-r}l_{r-1}^{q}=-c_n^5(n)^0(l_0^q)^5l_{-1}^q+c_{n-1}^4(n)^1(l_0^q)^4l_0^q-c_{n-2}^3(n)^2(l_0^q)^3l_1^q+c_{n-3}^2(n)^3(l_0^q)^2l_2^q$$

$$- \ c_{n-3}^{1}(n)^{4}(l_{0}^{q})^{1}l_{3}^{q} + n^{5}l_{4}^{q}$$

$$= (-c_{n}^{5} + nc_{n-1}^{4})(l_{0}^{q})^{5} - c_{n-2}^{3}(n)^{2}(l_{0}^{q})^{3}l_{1}^{q} + c_{n-3}^{2}(n)^{3}(l_{0}^{q})^{2}l_{2}^{q}$$

$$- \ c_{n-3}^{1}(n)^{4}(l_{0}^{q})^{1}l_{3}^{q} + n^{5}l_{4}^{q}$$

$$s_{4}^{r} = c_{4}^{1}c_{n}^{5}(l_{0}^{q})^{5} - c_{n-2}^{3}(n)^{2}(l_{0}^{q})^{3}l_{1}^{q} + c_{n-3}^{2}(n)^{3}(l_{0}^{q})^{2}l_{2}^{q} - \ c_{n-4}^{1}(n)^{4}(l_{0}^{q})^{1}l_{3}^{q} + n^{5}l_{4}^{q} \tag{8}$$

Assume that the property $\{P(n) \ \exists n_0 \ \in(0,1,,,n) \,/\, \ P(n_0) \ \Longrightarrow P(n) \ \ is \ \ true\} \rightarrow$ holds
The objective is to demonstrate that $\{P(n) \ \exists n_0 \ \in(0,1,,,n) \,/\, \ P(n_0) \ \Longrightarrow P(n) \ \ is \ \ true\}$ remains valid at order n+1

$$\{P(n) \ \exists n_0 \ \in(0,1,,,n+1) \,/\, \ P(n_0) \ \Longrightarrow P(n) \ \ is \ \ true\}$$

For (k=n+1):

$$s_{n}^{r} = \sum_{r=0}^{n+1} c_{n-r}^{n+1-r}(\mathrm{n})^{\mathrm{r}}(-l_{0}^{q})^{n+1-r}l_{r-1}^{q} = 0 \ \ \forall \ \ k \notin [0,1,,,,,n] \tag{9}$$

$\{P(n) \ \exists n_0 \ \in(0,1,,,n+1) \,/\, \ P(n_0) \ \Longrightarrow P(n+1) \ \ is \ \ true\}$ that is to say that

$$\{P(n) \ \exists n_0 \ \in(0,1,,,n+1) \,/\, \ P(n_0) \ \Longrightarrow P(n+1) \ \ is \ \ true\}$$

$$\Longrightarrow \{P(n) \ \exists n_0 \ \in(0,1,,,n+1) \,/\, \ P(n_0) \ \Longrightarrow P(n+1) \ \ is \ \ true\}$$

$$\{P(n) \ \exists n_0 \ \in(0,1,,,n+1) \,/\, \ P(n_0) \ \Longrightarrow P(n) \ \ is \ \ true\}$$

is strongly verified.

3. Methods Comparison and Advantages of the Ezouidi Mourad Sultan's Theorem (EMST)

Traditional methods for finding polynomial roots, such as Newton's Method and the Rational Root Theorem, each have significant limitations. Newton's Method, while widely used for numerical approximation, relies heavily on the choice of initial guess and may fail to converge or yield inaccurate results, especially for high-degree polynomials or when roots are closely spaced, complex, or repeated. Moreover, it provides only approximate values, making it unsuitable for cases where exact solutions are required. On the other hand, the Rational Root Theorem can only identify possible rational roots, offering no means to determine irrational or complex roots, and becomes inefficient for polynomials of higher degree due to the explosion in the number of candidate roots. Additionally, it cannot detect or quantify repeated roots, further limiting its applicability in comprehensive root-finding for general polynomials.

Unlike Newton's Method and the Rational Root Theorem, the Ezouidi Mourad Sultan's Theorem (EMST) presents a powerful and unified approach for solving polynomial equations. The EMST discriminant formula enables the determination of exact roots for all types of polynomials, including those with irrational, complex, or repeated roots. Its general applicability extends to polynomials of any degree, even those well beyond the reach of classical methods, such as equations of the eighteenth degree and higher. Unlike iterative methods, EMST does not depend on initial guesses or approximations; instead, it derives all roots directly from the structure and coefficients of the polynomial. Moreover, it systematically identifies both distinct and repeated roots, as well as complex solutions, making it a comprehensive and robust solution method for the full spectrum of polynomial equations.

4. Application and Verification of the EMST on Polynomials of Degree 8

The effectiveness of the EMST is demonstrated by analyzing an eighth-degree polynomial equation, with exact solutions obtained via EMST systematically compared to the approximate results produced by Newton's Method and the Rational Root Theorem.

4.1. Recurrence Approach via EMS's Theorem: First Example Consider the Following Polynomial

$$\mathrm{P(x)}{=}x^{8} - 16x^{7} + \frac{7204}{8^{2}}x^{6} - \frac{232832}{8^{3}}x^{5} + \frac{4726206}{8^{4}}x^{4} - \frac{61697920}{8^{5}}x^{3} + \frac{505838740}{8^{6}}x^{2} - \frac{2381353600}{8^{7}}x + \frac{4928741769}{8^{8}} = 0 \tag{10}$$

$$\mathrm{P(x)}{=}x^8 - l_0^q x^7 + l_1^q x^6 - l_2^q x^5 + l_3^q x^4 \ - l_4^q x^3 \ + l_5^q x^2 \ - l_6^q x + l_7^q = 0 \tag{11}$$

Let's focus on this polynomial to the power eight. To find out the roots of this polynomial we will determine the values of s_r^q relative respectively to l_r^q *where r belongs to the set of numbers* (-1, 0, 1, 2, 3, 4, 5, 6, 7)

$$\mathrm{l}_0^\mathrm{q} = 16, \mathrm{l}_1^\mathrm{q} = \frac{7204}{8^2} \quad \mathrm{l}_2^\mathrm{q} = \frac{232832}{8^3}, \mathrm{l}_3^\mathrm{q} = \frac{4726206}{8^4}, \mathrm{l}_4^\mathrm{q} = \frac{61697920}{8^5} \quad \mathrm{l}_5^\mathrm{q} = \frac{505838740}{8^6} \quad \mathrm{l}_6^\mathrm{q} = \frac{2381353600}{8^7} \mathrm{l}_7^\mathrm{q} = \frac{4928741769}{8^8} \tag{12}$$

Using Ezouidi Mourad Sultan (EMST)

$$s_{k-1}^r = \sum_{r=0}^k c_{n-r}^{k-r} (\mathrm{n})^\mathrm{r} (-l_0^q)^{k-r} l_{r-1}^q \tag{13}$$

For k=0 we write

$$s_{-1}^r = \sum_{r=0}^0 c_{n-r}^{-r} (\mathrm{n})^\mathrm{r} (-l_0^q)^{-r} l_{r-1}^q = c_n^{-1+1} = c_n^0 = 1 \tag{14}$$

For k=1 we write

$$s_0^r = \sum_{r=0}^1 c_{n-r}^{1-r} (\mathrm{n})^\mathrm{r} (-l_0^q)^{1-r} l_{r-1}^q = c_n^1 (n)^0 (-l_0^q)^1 l_{-1}^q + c_n^0 (n)^1 (-l_0^q)^0 l_0^q = -c_n^1 l_0^q l_{-1}^q + n l_0^q = (n-n) l_0^q = 0 \tag{15}$$

$$s_0^r = -c_8^1 (16) + 8(16) = -128 + 128 = 0 \tag{16}$$

For k=2 we find

$$s_1^r = \sum_{r=0}^2 c_{n-r}^{2-r} (\mathrm{n})^\mathrm{r} (-l_0^q)^{2-r} l_{r-1}^q = c_n^2 (n)^0 (l_0^q)^2 l_{-1}^q - c_{n-1}^1 (n)^1 (l_0^q)^1 l_0^q + n^2 l_1^q = (c_n^2 - n c_{n-1}^1)(l_0^q)^2 + n^2 l_1^q$$

$$= -c_1^1 c_n^2)(l_0^q)^2 + n^2 l_1^q \tag{17}$$

$$s_1^r = -c_1^1 c_8^2 (16)^2 + 8^2 (\tfrac{7204}{8^2}) = 36 \tag{18}$$

For k=3 we evaluate

$$s_2^r = \sum_{r=0}^3 c_{n-r}^{3-r} (\mathrm{n})^\mathrm{r} (-l_0^q)^{3-r} l_{r-1}^q = -c_n^3 (n)^0 (l_0^q)^3 l_{-1}^q + c_{n-1}^2 (n)^1 (l_0^q)^2 l_0^q - c_{n-2}^1 (n)^2 (l_0^q)^1 l_1^q + n^3 l_2^q$$

$$= -c_n^3 (l_0^q)^3 + n c_{n-1}^2 (l_0^q)^3 - n^2 c_{n-2}^1 (l_0^q)^1 l_1^q + n^3 l_2^q = (-c_n^3 + n c_{n-1}^2)(l_0^q)^3 - n^2 c_{n-2}^1 (l_0^q)^1 l_1^q + n^3 l_2^q$$

$$= c_n^2 c_n^3 (l_0^q)^3 - n^2 c_{n-2}^1 (l_0^q)^1 l_1^q + n^3 l_2^q \tag{19}$$

$$\mathrm{s}_2^\mathrm{r} = c_2^1 c_8^3 (16)^3 - c_6^1 (8)^2 (16)^1 (\tfrac{7204}{8^2}) + 8^3 (\tfrac{232832}{8^3}) = 0 \tag{20}$$

For k=4 we write

$$s_3^r = \sum_{r=0}^4 c_{n-r}^{4-r} (\mathrm{n})^\mathrm{r} (-l_0^q)^{4-r} l_{r-1}^q = c_n^4 (n)^0 (l_0^q)^4 l_{-1}^q - c_{n-1}^3 (n)^1 (l_0^q)^3 l_0^q + c_{n-2}^2 (n)^2 (l_0^q)^2 l_1^q - c_{n-3}^1 (n)^3 (l_0^q)^1 l_2^q + n^4 l_3^q$$

$$= (c_n^4 - n c_{n-1}^3)(l_0^q)^4 + c_{n-2}^2 (n)^2 (l_0^q)^2 l_1^q - c_{n-3}^1 (n)^3 (l_0^q)^1 l_2^q + n^4 l_3^q$$

$$= -c_3^1 c_n^3 (l_0^q)^4 + c_{n-2}^2 (n)^2 (l_0^q)^2 l_1^q - c_{n-3}^1 (n)^3 (l_0^q)^1 l_2^q + n^4 l_3^q$$

$$s_3^r = -c_3^1 c_n^4 (l_0^q)^4 + c_{n-2}^2 (n)^2 (l_0^q)^2 l_1^q - c_{n-3}^1 (n)^3 (l_0^q)^1 l_2^q + n^4 l_3^q \tag{21}$$

$$\mathrm{s}_3^\mathrm{r} = -c_3^1 c_8^4 (16)^4 + c_6^2 (8)^2 (16)^2 (\tfrac{7204}{8^2}) - c_5^1 (8)^3 16)^1 (\tfrac{232832}{8^3}) + (8)^4 (\tfrac{4726206}{8^4} = 446 \tag{22}$$

For k=5 we obtain

$$s_4^r = \sum_{r=0}^5 c_{n-r}^{5-r} (\mathrm{n})^\mathrm{r} (-l_0^q)^{5-r} l_{r-1}^q = -c_n^5 (n)^0 (l_0^q)^5 l_{-1}^q + c_{n-1}^4 (n)^1 (l_0^q)^4 l_0^q - c_{n-2}^3 (n)^2 (l_0^q)^3 l_1^q + c_{n-3}^2 (n)^3 (l_0^q)^2 l_2^q -$$

$$c_{n-3}^{1}(n)^{4}(l_0^q)^{1}l_3^q + n^5 l_4^q$$

$$= (-c_n^5 + nc_{n-1}^4)(l_0^q)^5 - c_{n-2}^3(n)^2(l_0^q)^3 l_1^q + c_{n-3}^2(n)^3(l_0^q)^2 l_2^q - \quad c_{n-3}^1(n)^4(l_0^q)^1 l_3^q + n^5 l_4^q$$

$$s_4^r = c_4^1 c_n^5(l_0^q)^5 - c_{n-2}^3(n)^2(l_0^q)^3 l_1^q + c_{n-3}^2(n)^3(l_0^q)^2 l_2^q - \quad c_{n-4}^1(n)^4(l_0^q)^1 l_3^q + n^5 l_4^q \tag{23}$$

$$\mathrm{s}_4^{\mathrm{r}} = c_4^1 c_8^5(16)^5 - c_6^3(8)^2(16)^3(\tfrac{7204}{8^2}) + c_5^2(8)^3(16)^2(\tfrac{232832}{8^3}) - c_4^1(8)^4 16)^1(\tfrac{4726206}{8^4}) + (8)^5(\tfrac{61697920}{8^5}) = 0 \tag{24}$$

For k=6 we write

$$s_5^r = \textstyle\sum_{r=0}^{6} c_{n-r}^{6-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{6-r} l_{r-1}^q = c_n^6(n)^0(l_0^q)^6 l_{-1}^q - c_{n-1}^5(n)^1(l_0^q)^5 l_0^q + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + c_{n-4}^2(n)^4(l_0^q)^2 l_3^q \ -c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q$$

$$= (c_n^6 - nc_{n-1}^5)(l_0^q)^6 + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + c_{n-4}^2(n)^4(l_0^q)^2 l_3^q - c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q$$

$$= c_5^1 c_n^6)(l_0^q)^6 + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + c_{n-4}^2(n)^4(l_0^q)^2 l_3^q - c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q$$

$$s_5^r = -c_5^1 c_n^6(l_0^q)^6 + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + c_{n-4}^2(n)^4(l_0^q)^2 l_3^q - c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q \tag{25}$$

$$\mathrm{s}_5^{\mathrm{r}} = -c_5^1 c_8^6(16)^6 + c_6^4(8)^2(16)^4(\tfrac{7204}{8^2}) - c_5^3(8)^3(16)^3(\tfrac{232832}{8^3}) + c_4^2(8)^4 16)^2(\tfrac{4726206}{8^4}) - c_3^1(8)^5 16)^1(\tfrac{61697920}{8^5})$$

$$+(8)^6(\tfrac{505838740}{8^6}) = 2196 \tag{26}$$

For k=7 we write

$$s_6^r = \textstyle\sum_{r=0}^{7} c_{n-r}^{7-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{7-r} l_{r-1}^q = -c_n^7(n)^0(l_0^q)^7 l_{-1}^q + c_{n-1}^6(n)^1(l_0^q)^6 l_0^q - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - c_{n-4}^3(n)^4(l_0^q)^3 l_3^q$$

$$+c_{n-5}^2(n)^5(l_0^q)^2 l_4^q \ -c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q$$

$$= (-c_n^7 + nc_{n-1}^6)(l_0^q)^7 l_{-1}^q - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - c_{n-4}^3(n)^4(l_0^q)^3 l_3^q + c_{n-5}^2(n)^5(l_0^q)^2 l_4^q$$

$$-c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q$$

$$= c_6^1 c_n^7(l_0^q)^7 - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - c_{n-4}^3(n)^4(l_0^q)^3 l_3^q + c_{n-5}^2(n)^5(l_0^q)^2 l_4^q - c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q$$

$$s_6^r = c_6^1 c_n^7(l_0^q)^7 - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - c_{n-4}^3(n)^4(l_0^q)^3 l_3^q + c_{n-5}^2(n)^5(l_0^q)^2 l_4^q - c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q \tag{27}$$

$$\mathrm{s}_6^{\mathrm{r}} = c_6^1 c_8^7(16)^7 - c_6^5(8)^2(16)^5(\tfrac{7204}{8^2}) + c_5^4(8)^3(16)^4(\tfrac{232832}{8^3}) - c_4^3(8)^4 16)^3(\tfrac{4726206}{8^4}) + c_3^2(8)^5 16)^2(\tfrac{61697920}{8^5})$$

$$-c_2^1(8)^6 16)^1(\tfrac{505838740}{8^6}) + (8)^7(\tfrac{2381353600}{8^7}) = 0 \tag{28}$$

For k=8 we write

$$s_7^r = \textstyle\sum_{r=0}^{8} c_{n-r}^{8-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{8-r} l_{r-1}^q = c_n^8(n)^0(l_0^q)^8 l_{-1}^q - c_{n-1}^7(n)^1(l_0^q)^7 l_0^q + c_{n-2}^6(n)^2(l_0^q)^6 l_1^q - c_{n-3}^5(n)^3(l_0^q)^5 l_2^q + c_{n-4}^4(n)^4(l_0^q)^4 l_3^q \ -c_{n-5}^3(n)^5(l_0^q)^3 l_4^q \ +c_{n-6}^2(n)^6(l_0^q)^2 l_5^q - c_{n-7}^1(n)^7(l_0^q)^1 l_6^q + n^8 l_7^q$$

$$= (c_n^8 - nc_{n-1}^7)(l_0^q)^8 + c_{n-2}^6(n)^2(l_0^q)^6 l_1^q - c_{n-3}^5(n)^3(l_0^q)^5 l_2^q + c_{n-4}^4(n)^4(l_0^q)^4 l_3^q - c_{n-5}^3(n)^5(l_0^q)^3 l_4^q$$

$$+c_{n-6}^2(n)^6(l_0^q)^2 l_5^q - c_{n-7}^1(n)^7(l_0^q)^1 l_6^q + n^8 l_7^q$$

$$= -c_7^1 c_n^8(l_0^q)^8 + c_{n-2}^6(n)^2(l_0^q)^6 l_1^q - c_{n-3}^5(n)^3(l_0^q)^5 l_2^q + c_{n-4}^4(n)^4(l_0^q)^4 l_3^q - c_{n-5}^3(n)^5(l_0^q)^3 l_4^q$$

$$+c_{n-6}^{2}(n)^{6}(l_0^q)^2 l_5^q - c_{n-7}^{1}(n)^{7}(l_0^q)^1 l_6^q + n^8 l_7^q$$

$$s_7^r = -c_7^1 c_n^8 (l_0^q)^8 + c_{n-2}^{6}(n)^2(l_0^q)^6 l_1^q - c_{n-3}^{5}(n)^3(l_0^q)^5 l_2^q + c_{n-4}^{4}(n)^4(l_0^q)^4 l_3^q - c_{n-5}^{3}(n)^5(l_0^q)^3 l_4^q + c_{n-6}^{2}(n)^6(l_0^q)^2 l_5^q$$

$$-c_{n-7}^{1}(n)^7(l_0^q)^1 l_6^q + n^8 l_7^q \quad (29)$$

$$\mathrm{s}_7^\mathrm{r} = -c_7^1 c_8^8 (16)^8 + c_6^6 (8)^2 (16)^6 (\tfrac{7204}{8^2}) - c_5^5 (8)^3 (16)^5 (\tfrac{232832}{8^3}) + c_4^4 (8)^4 16)^4 (\tfrac{4726206}{8^4}) - c_3^3 (8)^5 16)^3 (\tfrac{61697920}{8^5})$$

$$+c_2^2 (8)^6 16)^2 (\tfrac{505838740}{8^6}) - c_1^1 (8)^7 (16)^1 (\tfrac{2381353600}{8^7}) + (8)^8 (\tfrac{4928741769}{8^8}) = 3465 \quad (30)$$

In other words $\mathrm{s}_0^\mathrm{r} = \mathrm{s}_2^\mathrm{r} = \mathrm{s}_4^\mathrm{r} = \mathrm{s}_6^\mathrm{r} = 0$
Except $\mathrm{s}_1^\mathrm{r} = 36$ $\mathrm{s}_3^\mathrm{r} = 446$ $\mathrm{s}_5^\mathrm{r} = 2196$ $\mathrm{s}_7^\mathrm{r} = 3465$

$$\mathrm{P(x)} = \mathrm{x}^8 - \mathrm{s}_0^\mathrm{r}\mathrm{x}^7 + \mathrm{s}_1^\mathrm{r}\mathrm{x}^6 - \mathrm{s}_2^\mathrm{r}\mathrm{x}^5 + \mathrm{s}_3^\mathrm{r}\mathrm{x}^4 - \mathrm{s}_4^\mathrm{r}\mathrm{x}^3 + \mathrm{s}_5^\mathrm{r}\mathrm{x}^2 - \mathrm{s}_6^\mathrm{r}\mathrm{x} + \mathrm{s}_7^\mathrm{r} = 0 \quad (31)$$

$$= \mathrm{x}^8 + \mathrm{s}_1^\mathrm{r}\mathrm{x}^6 + \mathrm{s}_3^\mathrm{r}\mathrm{x}^4 + \mathrm{s}_5^\mathrm{r}\mathrm{x}^2 + \mathrm{s}_7^\mathrm{r} = 0 \quad (32)$$

$$= \mathrm{x}^8 + 36\mathrm{x}^6 + 446\mathrm{x}^4 + 2196\mathrm{x}^2 + 3465 = 0 \ \ \text{let} \ \ \mathrm{x}^2 = X \quad (33)$$

$$= \mathrm{X}^4 + 36\mathrm{X}^3 + 446\mathrm{X}^2 + 2196\mathrm{X}^1 + 3465 = 0 \quad (34)$$

$$s_1^r = -c_1^1 c_n^2)(l_0^q)^2 + n^2 l_1^q \quad (35)$$

$$s_1^r = -c_1^1 c_4^2 (-36)^2 + 4^2(446) = -640 \quad (36)$$

$$s_2^r = c_2^1 c_n^3 (l_0^q)^3 - c_{n-2}^{1}(n)^2(l_0^q)^1 l_1^q + n^3 l_2^q \quad (37)$$

$$s_2^r = c_2^1 c_4^3 (-36)^3 - c_2^1 (4)^2 (-36)^1 (446) + 4^3(-2196) = 0 \quad (38)$$

$$s_3^r = -c_3^1 c_n^4 (l_0^q)^4 + c_{n-2}^{2}(n)^2(l_0^q)^2 l_1^q - c_{n-3}^{1}(n)^3(l_0^q)^1 l_2^q + n^4 l_3^q \quad (39)$$

$$s_3^r = -c_3^1 c_n^4 (-36)^3 + (4)^2(-36)^2(446) - 4^3(-36)(-2196) + 4^4(3465) = 36864 \quad (40)$$

$$\rightarrow \mathrm{P(x)} = \mathrm{x}^4 + \mathrm{s}_1^\mathrm{r}\mathrm{x}^2 + \mathrm{s}_3^\mathrm{r} = 0 \quad \rightarrow \mathrm{P(x)} = \mathrm{x}^4 - 640\mathrm{x}^2 + 36864 = 0 \quad (41)$$

$$\mathrm{s}_1^{\mathrm{q}\prime} = -\mathrm{s}_1^{\mathrm{q}^2} + 4\mathrm{s}_3^\mathrm{q} = -(-640)^2 + 4 \times 36864 = -262144 \rightarrow -\mathrm{s}_1^{\mathrm{q}'} = 262144 = 512^2 \quad (42)$$

$$x^2 = \frac{-\mathrm{s}_1^\mathrm{q} + \sqrt{-\mathrm{s}_1^{\mathrm{q}'}}}{2} = \frac{640+\sqrt{512^2}}{2} = \frac{640+512}{2} = 576 = 24^2 \rightarrow \mathrm{x}_{\mathrm{k}\epsilon(0,1,)} = \pm 24 \quad (43)$$

$$\alpha_{\mathrm{k}\epsilon(0,1,)} = \frac{\mathrm{l}_0^\mathrm{q} + \mathrm{x}_{\mathrm{k}\epsilon(0,1)}}{4} = \frac{-36\pm 24}{4} = -3 \ \ \text{or} - 15 \quad (44)$$

$$x^2 = \frac{-\mathrm{s}_1^\mathrm{q} - \sqrt{-\mathrm{s}_1^{\mathrm{q}'}}}{2} = \frac{640-\sqrt{512^2}}{2} = \frac{640-512}{2} = 64 = 8^2 \rightarrow \mathrm{x}_{\mathrm{k}\epsilon(0,1,)} = \pm 8 \quad (45)$$

$$\alpha_{\mathrm{k}\epsilon(0,1,)} = \frac{\mathrm{l}_0^\mathrm{q} + \mathrm{x}_{\mathrm{k}\epsilon(0,1)}}{4} = \frac{-36\pm 8}{4} = -7 \ \ \text{or} - 11 \quad (46)$$

In other words

$$\mathrm{P(x)} = \mathrm{x}^8 - \mathrm{s}_0^\mathrm{r}\mathrm{x}^7 + \mathrm{s}_1^\mathrm{r}\mathrm{x}^6 - \mathrm{s}_2^\mathrm{r}\mathrm{x}^5 + \mathrm{s}_3^\mathrm{r}\mathrm{x}^4 - \mathrm{s}_4^\mathrm{r}\mathrm{x}^3 + \mathrm{s}_5^\mathrm{r}\mathrm{x}^2 - \mathrm{s}_6^\mathrm{r}\mathrm{x} + \mathrm{s}_7^\mathrm{r} = 0 \quad (47)$$

$$= \mathrm{x}^8 + \mathrm{s}_1^\mathrm{r}\mathrm{x}^6 + \mathrm{s}_3^\mathrm{r}\mathrm{x}^4 + \mathrm{s}_5^\mathrm{r}\mathrm{x}^2 + \mathrm{s}_7^\mathrm{r} = 0 \quad (48)$$

$$= x^8 + 36x^6 + 446x^4 + 2196x^2 + 3465 = 0 \quad \text{let} \quad x^2 = X \tag{49}$$

$$= X^4 + 36X^3 + 446X^2 + 2196X^1 + 3465 = 0 \tag{50}$$

$$x^2 = X = -3 \quad or - 7 \quad or - 11 \quad or - 15 \quad \rightarrow x = \pm\sqrt{3}i \quad or \quad \pm\sqrt{7}ior \quad or \quad \pm\sqrt{11}i \quad or \quad or \quad \pm\sqrt{15}i \tag{51}$$

$$\alpha_k = \frac{l_0^q + x_k}{8} = \frac{l_0^q \pm \sqrt{X_k}}{8} = \frac{16 \pm i\sqrt{3}}{8} \quad or \quad \frac{16 \pm i\sqrt{7}}{8} \quad or \quad \frac{16 \pm i\sqrt{11}}{8} or \quad \frac{16 \pm i\sqrt{15}}{8} \tag{52}$$

4.2. Recurrence Approach via EMS's Theorem: Second Example

Consider the following 8th polynomial

$$P(x) = x^8 - 16x^7 + \frac{7132}{8^2}x^6 - \frac{225920}{8^3}x^5 + \frac{4449726}{8^4}x^4 - \frac{55799680}{8^5}x^3 + \frac{435055468}{8^6}x^2 - \frac{1928228224}{8^7} \quad x + \frac{3719657865}{8^8} = 0$$

$$P(x) = x^8 - l_0^q x^7 + l_1^q x^6 - l_2^q x^5 + l_3^q x^4 - l_4^q x^3 + l_5^q x^2 - l_6^q x + l_7^q = 0 \tag{53}$$

Let's focus on this polynomial to the power eight. To find out the roots of this polynomial we will determine the values

$$l_0^q = 16, l_1^q = \frac{7132}{8^2} \quad l_2^q = \frac{225920}{8^3}, l_3^q = \frac{4449726}{8^4}, l_4^q = \frac{55799680}{8^5} \quad l_5^q = \frac{435055468}{8^6} l_6^q = \frac{1928228224}{8^7} l_7^q = \frac{3719657865}{8^8} \tag{54}$$

Using Ezouidi Mourad Sultan's Theorem (EMST)

$$s_{k-1}^r = \sum_{r=0}^{k} c_{n-r}^{k-r} (n)^r (-l_0^q)^{k-r} l_{r-1}^q \tag{55}$$

For k=0 we write

$$s_{-1}^r = \sum_{r=0}^{0} c_{n-r}^{-r} (n)^r (-l_0^q)^{-r} l_{r-1}^q = c_n^{-1+1} = c_n^0 = 1 \tag{56}$$

For k=1 we write

$$s_0^r = \sum_{r=0}^{1} c_{n-r}^{1-r} (n)^r (-l_0^q)^{1-r} l_{r-1}^q = c_n^1 (n)^0 (-l_0^q)^1 l_{-1}^q + c_n^0 (n)^1 (-l_0^q)^0 l_0^q = -c_n^1 l_0^q l_{-1}^q + n l_0^q = (n-n) l_0^q = 0 \tag{57}$$

$$s_0^r = -c_8^1 (16) + 8(16) = -128 + 128 = 0 \tag{58}$$

For k=2 we find

$$s_1^r = \sum_{r=0}^{2} c_{n-r}^{2-r} (n)^r (-l_0^q)^{2-r} l_{r-1}^q = c_n^2 (n)^0 (l_0^q)^2 l_{-1}^q - c_{n-1}^1 (n)^1 (l_0^q)^1 l_0^q + n^2 l_1^q = (c_n^2 - n c_{n-1}^1)(l_0^q)^2 + n^2 l_1^q = -c_1^1 c_n^2)(l_0^q)^2 + n^2 l_1^q \tag{59}$$

$$s_1^r = -c_1^1 c_8^2 (16)^2 + 8^2 (\tfrac{7132}{8^2}) = -36 \tag{60}$$

For k=3 we evaluate

$$s_2^r = \sum_{r=0}^{3} c_{n-r}^{3-r} (n)^r (-l_0^q)^{3-r} l_{r-1}^q = -c_n^3 (n)^0 (l_0^q)^3 l_{-1}^q + c_{n-1}^2 (n)^1 (l_0^q)^2 l_0^q - c_{n-2}^1 (n)^2 (l_0^q)^1 l_1^q + n^3 l_2^q$$

$$= -c_n^3 (l_0^q)^3 + n c_{n-1}^2 (l_0^q)^3 - n^2 c_{n-2}^1 (l_0^q)^1 l_1^q + n^3 l_2^q = (-c_n^3 + n c_{n-1}^2)(l_0^q)^3 - n^2 c_{n-2}^1 (l_0^q)^1 l_1^q + n^3 l_2^q$$

$$= c_n^2 c_n^3 (l_0^q)^3 - n^2 c_{n-2}^1 (l_0^q)^1 l_1^q + n^3 l_2^q \tag{61}$$

$$s_2^r = c_2^1 c_8^3 (16)^3 - c_6^1 (8)^2 (16)^1 (\tfrac{7132}{8^2}) + 8^3 (\tfrac{225920}{8^3}) = 0 \tag{62}$$

For k=4 we write

$$s_3^r = \sum_{r=0}^{4} c_{n-r}^{4-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{4-r} l_{r-1}^q = c_n^4(n)^0(l_0^q)^4 l_{-1}^q - c_{n-1}^3(n)^1(l_0^q)^3 l_0^q + c_{n-2}^2(n)^2(l_0^q)^2 l_1^q - c_{n-3}^1(n)^3(l_0^q)^1 l_2^q + n^4 l_3^q$$

$$= (c_n^4 - nc_{n-1}^3)(l_0^q)^4 + c_{n-2}^2(n)^2(l_0^q)^2 l_1^q - c_{n-3}^1(n)^3(l_0^q)^1 l_2^q + n^4 l_3^q$$

$$= -c_3^1 c_n^3(l_0^q)^4 + c_{n-2}^2(n)^2(l_0^q)^2 l_1^q - c_{n-3}^1(n)^3(l_0^q)^1 l_2^q + n^4 l_3^q$$

$$s_3^r = -c_3^1 c_n^4(l_0^q)^4 + c_{n-2}^2(n)^2(l_0^q)^2 l_1^q - c_{n-3}^1(n)^3(l_0^q)^1 l_2^q + n^4 l_3^q \tag{63}$$

$$\mathrm{s}_3^{\mathrm{r}} = -c_3^1 c_8^4(16)^4 + c_6^2(8)^2(16)^2(\tfrac{7132}{8^2}) - c_5^1(8)^3 16)^1(\tfrac{225920}{8^3}) + (8)^4(\tfrac{4449726}{8^4} = 446 \tag{64}$$

For k=5 we obtain

$$s_4^r = \sum_{r=0}^{5} c_{n-r}^{5-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{5-r} l_{r-1}^q = -c_n^5(n)^0(l_0^q)^5 l_{-1}^q + c_{n-1}^4(n)^1(l_0^q)^4 l_0^q - c_{n-2}^3(n)^2(l_0^q)^3 l_1^q + c_{n-3}^2(n)^3(l_0^q)^2 l_2^q$$

$$- \quad c_{n-3}^1(n)^4(l_0^q)^1 l_3^q + n^5 l_4^q$$

$$= (-c_n^5 + nc_{n-1}^4)(l_0^q)^5 - c_{n-2}^3(n)^2(l_0^q)^3 l_1^q + c_{n-3}^2(n)^3(l_0^q)^2 l_2^q \quad - \quad c_{n-3}^1(n)^4(l_0^q)^1 l_3^q + n^5 l_4^q \quad s_4^r$$

$$= c_4^1 c_n^5(l_0^q)^5 - c_{n-2}^3(n)^2(l_0^q)^3 l_1^q + c_{n-3}^2(n)^3(l_0^q)^2 l_2^q - \quad c_{n-4}^1(n)^4(l_0^q)^1 l_3^q + n^5 l_4^q \tag{65}$$

$$\mathrm{s}_4^{\mathrm{r}} = c_4^1 c_8^5(16)^5 - c_6^3(8)^2(16)^3(\tfrac{7132}{8^2}) + c_5^2(8)^3(16)^2(\tfrac{225920}{8^3}) - c_4^1(8)^4 16)^1(\tfrac{4449726}{8^4}) + (8)^5(\tfrac{55799680}{8^5}) = 0 \tag{66}$$

For k=6 we write

$$s_5^r = \sum_{r=0}^{6} c_{n-r}^{6-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{6-r} l_{r-1}^q = c_n^6(n)^0(l_0^q)^6 l_{-1}^q - c_{n-1}^5(n)^1(l_0^q)^5 l_0^q + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + \\ c_{n-4}^2(n)^4(l_0^q)^2 l_3^q \ -c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q$$

$$= (c_n^6 - nc_{n-1}^5)(l_0^q)^6 + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + c_{n-4}^2(n)^4(l_0^q)^2 l_3^q - c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q$$

$$= c_5^1 c_n^6)(l_0^q)^6 + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + c_{n-4}^2(n)^4(l_0^q)^2 l_3^q - c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q$$

$$s_5^r = -c_5^1 c_n^6(l_0^q)^6 + c_{n-2}^4(n)^2(l_0^q)^4 l_1^q - c_{n-3}^3(n)^3(l_0^q)^3 l_2^q + c_{n-4}^2(n)^4(l_0^q)^2 l_3^q - c_{n-5}^1(n)^5(l_0^q)^1 l_4^q + n^6 l_5^q \tag{67}$$

$$\mathrm{s}_5^{\mathrm{r}} = -c_5^1 c_8^6(16)^6 + c_6^4(8)^2(16)^4(\tfrac{7132}{8^2}) - c_5^3(8)^3(16)^3(\tfrac{225920}{8^3}) + c_4^2(8)^4 16)^2(\tfrac{4449726}{8^4}) - c_3^1(8)^5 16)^1(\tfrac{55799680}{8^5})$$

$$+(8)^6(\tfrac{435055468}{8^6}) = -2196 \tag{68}$$

For k=7 we write

$$s_6^r = \sum_{r=0}^{7} c_{n-r}^{7-r}(\mathrm{n})^{\mathrm{r}}(-l_0^q)^{7-r} l_{r-1}^q = -c_n^7(n)^0(l_0^q)^7 l_{-1}^q + c_{n-1}^6(n)^1(l_0^q)^6 l_0^q - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - \\ c_{n-4}^3(n)^4(l_0^q)^3 l_3^q + c_{n-5}^2(n)^5(l_0^q)^2 l_4^q \ -c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q$$

$$= (-c_n^7 + nc_{n-1}^6)(l_0^q)^7 l_{-1}^q - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - c_{n-4}^3(n)^4(l_0^q)^3 l_3^q + c_{n-5}^2(n)^5(l_0^q)^2 l_4^q$$

$$-c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q$$

$$= c_6^1 c_n^7(l_0^q)^7 - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - c_{n-4}^3(n)^4(l_0^q)^3 l_3^q + c_{n-5}^2(n)^5(l_0^q)^2 l_4^q$$

$$-c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q$$

$$s_6^r = c_6^1 c_n^7(l_0^q)^7 - c_{n-2}^5(n)^2(l_0^q)^5 l_1^q + c_{n-3}^4(n)^3(l_0^q)^4 l_2^q - c_{n-4}^3(n)^4(l_0^q)^3 l_3^q + c_{n-5}^2(n)^5(l_0^q)^2 l_4^q - c_{n-6}^1(n)^6(l_0^q)^1 l_5^q + n^7 l_6^q \tag{69}$$

$$\mathrm{s}_6^{\mathrm{r}} = c_6^1 c_8^7(16)^7 - c_6^5(8)^2(16)^5(\tfrac{7132}{8^2}) + c_5^4(8)^3(16)^4(\tfrac{225920}{8^3}) - c_4^3(8)^4 16)^3(\tfrac{4449726}{8^4}) + c_3^2(8)^5 16)^2(\tfrac{55799680}{8^5})$$

$$-c_2^1(8)^6 16)^1\left(\frac{435055468}{8^6}\right)+(8)^7\left(\frac{1928228224}{8^7}\right)=0 \tag{70}$$

For k=8 we write

$$s_7^r=\sum_{r=0}^{8} c_{n-r}^{8-r}((\mathrm{n})^{\mathrm{r}}-l_0^q)^{8-r}l_{r-1}^q=c_n^8(n)^0(l_0^q)^8l_{-1}^q-c_{n-1}^7(n)^1(l_0^q)^7l_0^q+c_{n-2}^6(n)^2(l_0^q)^6l_1^q-c_{n-3}^5(n)^3(l_0^q)^5l_2^q+$$
$$c_{n-4}^4(n)^4(l_0^q)^4l_3^q\ -c_{n-5}^3(n)^5(l_0^q)^3l_4^q\ +c_{n-6}^2(n)^6(l_0^q)^2l_5^q-c_{n-7}^1(n)^7(l_0^q)^1l_6^q+n^8l_7^q$$

$$=(c_n^8-nc_{n-1}^7)(l_0^q)^8+c_{n-2}^6(n)^2(l_0^q)^6l_1^q-c_{n-3}^5(n)^3(l_0^q)^5l_2^q+c_{n-4}^4(n)^4(l_0^q)^4l_3^q-c_{n-5}^3(n)^5(l_0^q)^3l_4^q$$

$$+c_{n-6}^2(n)^6(l_0^q)^2l_5^q-c_{n-7}^1(n)^7(l_0^q)^1l_6^q+n^8l_7^q$$

$$=-c_7^1c_n^8(l_0^q)^8+c_{n-2}^6(n)^2(l_0^q)^6l_1^q-c_{n-3}^5(n)^3(l_0^q)^5l_2^q+c_{n-4}^4(n)^4(l_0^q)^4l_3^q-c_{n-5}^3(n)^5(l_0^q)^3l_4^q$$

$$+c_{n-6}^2(n)^6(l_0^q)^2l_5^q-c_{n-7}^1(n)^7(l_0^q)^1l_6^q+n^8l_7^q$$

$$s_7^r=-c_7^1c_n^8(l_0^q)^8+c_{n-2}^6(n)^2(l_0^q)^6l_1^q-c_{n-3}^5(n)^3(l_0^q)^5l_2^q+c_{n-4}^4(n)^4(l_0^q)^4l_3^q-c_{n-5}^3(n)^5(l_0^q)^3l_4^q+c_{n-6}^2(n)^6(l_0^q)^2l_5^q$$

$$-c_{n-7}^1(n)^7(l_0^q)^1l_6^q+n^8l_7^q \tag{71}$$

$$\mathrm{s}_7^{\mathrm{r}}=-c_7^1c_8^8(16)^8+c_6^6(8)^2(16)^6\left(\frac{7132}{8^2}\right)-c_5^5(8)^3(16)^5\left(\frac{225920}{8^3}\right)+c_4^4(8)^4 16)^4\left(\frac{4449726}{8^4}\right)-c_3^3(8)^5 16)^3\left(\frac{55799680}{8^5}\right)$$

$$+c_2^2(8)^6 16)^2\left(\frac{435055468}{8^6}\right)-c_1^1(8)^7(16)^1\left(\frac{1928228224}{8^7}\right)+(8)^8\left(\frac{3719657865}{8^8}\right)=3465 \tag{72}$$

In other words $\mathrm{s}_0^{\mathrm{r}}=\mathrm{s}_2^{\mathrm{r}}=\mathrm{s}_4^{\mathrm{r}}=\mathrm{s}_6^{\mathrm{r}}=0$
Except $\mathrm{s}_1^{\mathrm{r}}=-36$ $\mathrm{s}_3^{\mathrm{r}}=446$ $\mathrm{s}_5^{\mathrm{r}}=-2196$ $\mathrm{s}_7^{\mathrm{r}}=3465$

$$\mathrm{P(x)}=\mathrm{x}^8-\mathrm{s}_0^{\mathrm{r}}\mathrm{x}^7+\mathrm{s}_1^{\mathrm{r}}\mathrm{x}^6-\mathrm{s}_2^{\mathrm{r}}\mathrm{x}^5+\mathrm{s}_3^{\mathrm{r}}\mathrm{x}^4-\mathrm{s}_4^{\mathrm{r}}\mathrm{x}^3+\mathrm{s}_5^{\mathrm{r}}\mathrm{x}^2-\mathrm{s}_6^{\mathrm{r}}\mathrm{x}+\mathrm{s}_7^{\mathrm{r}}=0 \tag{73}$$

$$=\mathrm{x}^8+\mathrm{s}_1^{\mathrm{r}}\mathrm{x}^6+\mathrm{s}_3^{\mathrm{r}}\mathrm{x}^4+\mathrm{s}_5^{\mathrm{r}}\mathrm{x}^2+\mathrm{s}_7^{\mathrm{r}}=0 \tag{74}$$

$$=\mathrm{x}^8-36\mathrm{x}^6+446\mathrm{x}^4-2196\mathrm{x}^2+3465=0\ \ \text{let}\ \ \mathrm{x}^2=X \tag{75}$$

$$=\mathrm{X}^4-36\mathrm{X}^3+446\mathrm{X}^2-2196\mathrm{X}^1+3465=0 \tag{76}$$

$$s_1^r=-c_1^1c_n^2)(l_0^q)^2+n^2l_1^q \tag{77}$$

$$s_1^r=-c_1^1c_4^2(36)^2+4^2(446)=-640 \tag{78}$$

$$s_2^r=c_2^1c_n^3(l_0^q)^3-c_{n-2}^1(n)^2(l_0^q)^1l_1^q+n^3l_2^q \tag{79}$$

$$s_2^r=c_2^1c_4^3(36)^3-c_2^1(4)^2(36)^1(446)+4^3(2196)=0 \tag{80}$$

$$s_3^r=-c_3^1c_n^4(l_0^q)^4+c_{n-2}^2(n)^2(l_0^q)^2l_1^q-c_{n-3}^1(n)^3(l_0^q)^1l_2^q+n^4l_3^q \tag{81}$$

$$s_3^r=-c_3^1c_n^4(36)^4+(4)^2(36)^2(446)-4^3(36)(2196)+4^4(3465)=36864 \tag{82}$$

$$\rightarrow \mathrm{P(x)}=\mathrm{x}^4+\mathrm{s}_1^{\mathrm{r}}\mathrm{x}^2+\mathrm{s}_3^{\mathrm{r}}=0\quad \rightarrow \mathrm{P(x)}=\mathrm{x}^4-640\mathrm{x}^2+36864=0 \tag{83}$$

$$\mathrm{s}_1^{\mathrm{q}\prime}=-\mathrm{s}_1^{\mathrm{q}^2}+4\mathrm{s}_3^{\mathrm{q}}=-(-640)\ ^2+4\times 36864=-262144\rightarrow -\mathrm{s}_1^{\mathrm{q}'}=262144=512^2 \tag{84}$$

$$x^2=\frac{-\mathrm{s}_1^{\mathrm{q}}+\sqrt{-\mathrm{s}_1^{\mathrm{q}\prime}}}{2}=\frac{640+\sqrt{512^2}}{2}=\frac{640+512}{2}=576=24^2\rightarrow \mathrm{x}_{\mathrm{k}\epsilon(0,1,)}=\pm 24 \tag{85}$$

$$\alpha_{k\epsilon(0,1,)} = \frac{l_0^q + x_{k\epsilon(0,1)}}{4} = \frac{36\pm24}{4} = 3 \ \text{or} 15 \tag{86}$$

$$x^2 = \frac{-s_1^q - \sqrt{-s_1^{q'}}}{2} = \frac{640-\sqrt{512^2}}{2} = \frac{640-512}{2} = 64 = 8^2 \rightarrow x_{k\epsilon(0,1,)} = \pm8 \tag{87}$$

$$\alpha_{k\epsilon(0,1,)} = \frac{l_0^q + x_{k\epsilon(0,1)}}{4} = \frac{36\pm8}{4} = 7 \ \text{or} 11 \tag{88}$$

$$\mathrm{x}^2 = X = 3 \ \ or7 \ \ or11 \ \ or15 \ \ \rightarrow x = \pm\sqrt{3}or \ \ \pm\sqrt{7} \ \ or \ \ or \ \ \pm\sqrt{11} \ \ or \ \ or \ \ \pm\sqrt{15} \tag{89}$$

$$\alpha_k = \frac{l_0^q + y_k}{8} = \frac{l_0^q \pm \sqrt{x_k}}{8} = \frac{16\pm\sqrt{3}}{8} \ \ or \ \ \frac{16\pm\sqrt{7}}{8} \ \ or \ \ \frac{16\pm\sqrt{11}}{8} or \ \ \frac{16\pm\sqrt{15}}{8} \tag{90}$$

Table 1. Cardano's, Newton's, and EMST Methods for n-th Degree Polynomial solutions.

n^{th} degree	Polynomial equations	Cardano's method	Newton's method	Present work Ezouidi Mourad Sultan's Theorem (EMST)
3	$P(x) = x^3 + 86.4x + 525.312 = 0$	x_1≈−4.7669 x_2≈2.3835+10.1570i,; x_3≈2.3835−10.1570i	$x_1 \approx -4.8018$, x_2 ≈2.4009+10.202i x_3 ≈2.4009−10.202i	$4.8 \pm 6\sqrt{2.88}i$ or -4.8
5	$P(x) = x^5 - 12x^4 + \frac{1440}{5^2}x^3 - \frac{17280}{5^3}x^2 + \frac{103936}{5^4}x^1 - \frac{251904}{5^5} = 0$	Not applicable	Not applicable	$\frac{12+\sqrt[4]{256}e^{\frac{(2k+1)i\pi}{5}}}{5} = \frac{12+4e^{\frac{(2k+1)i\pi}{5}}}{5}$
6	$P(x) = x^6 - x^5 + \frac{15}{6^2}x^4 - x^3 + \frac{603}{6^4}x^2 - \frac{624}{6^5}x + \frac{7916}{6^6} = 0$	Not applicable	Not applicable	$\frac{1\pm\sqrt[3]{98-10\mathrm{i}}e^{\frac{(2k)i\pi}{3}}}{6}$
7	$P(x) = x^7 - x^6 + \frac{18}{8^2}x^5 + \frac{4}{8^3}x^4 - \frac{45}{8^4}x^3 + \frac{4}{8^5}x^2 + \frac{38}{8^6}x + \frac{12}{8^7} = 0$	Not applicable	Not applicable	$\frac{1+1}{8}; \ \frac{1\pm2}{8}; \ \frac{1\pm\sqrt{2}}{8}; \ \frac{1\pm\sqrt{3}}{8}$
8	$P(x) = x^8 - 16x^7 + \frac{7204}{8^2}x^6 - \frac{232832}{8^3}x^5 + \frac{4726206}{8^4}x^4 - \frac{61697920}{8^5}x^3 + \frac{505838740}{8^6}x^2 - \frac{2381353600}{8^7}x + \frac{4928741769}{8^8} = 0$	Not applicable	Not applicable	$\frac{16\pm\mathrm{i}\sqrt{3}}{8} \ or \ \frac{16\pm\mathrm{i}\sqrt{7}}{8} \ or \ \frac{16\pm\mathrm{i}\sqrt{11}}{8} \ or \ \frac{16\pm\mathrm{i}\sqrt{15}}{8}$
8	$\mathrm{P(x)} = x^8 - 16x^7 + \frac{7132}{8^2}x^6 - \frac{225920}{8^3}x^5 + \frac{4449726}{8^4}x^4 - \frac{55799680}{8^5}x^3 + \frac{435055468}{8^6}x^2 - \frac{1928228224}{8^7}x + \frac{3719657865}{8^8} = 0$	Not applicable	Not applicable	$\frac{16\pm\sqrt{3}}{8} \ or \ \frac{16\pm\sqrt{7}}{8} \ or \ \frac{16\pm\sqrt{11}}{8} \ or \ \frac{16\pm\sqrt{15}}{8}$

Table 1 shows that the solutions obtained using the proposed recurrence method grounded in Mourad Sultan Ezouidi's Theorem (EMST) correspond to explicit, exact roots for high-degree polynomials, including complex and irrational roots. In contrast, traditional methods such as Cardano's method and Newton's method are either inapplicable or yield only approximate solutions. For example, for the eighth-degree polynomial equations considered, the EMST approach provides precise roots expressed in radical and complex form, such as $\frac{16\pm\mathrm{i}\sqrt{3}}{8}$ or $\frac{16\pm\mathrm{i}\sqrt{7}}{8}$ or $\frac{16\pm\mathrm{i}\sqrt{11}}{8}$ or $\frac{16\pm\mathrm{i}\sqrt{15}}{8}$. Conversely, classical methods do not furnish explicit roots for these high-degree polynomials, highlighting their limitations in handling such complex cases.

This comparison underscores that EMST offers a systematic way to compute exact solutions for polynomial roots of any degree, including irreducible and highly complex cases. Traditional techniques, on the other hand, are often restricted to low-degree or specific polynomial forms, and tend to be impractical or inapplicable for higher degrees. The results demonstrate the superior applicability, precision, and completeness of the EMST-based recurrence method in solving polynomial equations explicitly, representing a significant advancement over existing approaches.

Table 2. Root-finding methodology comparison: Mean Absolute Error (MAE) analysis for EMST versus classical methods.

	Roots	$\boldsymbol{P(x) = x^3 + 86.4x + 525.312 = 0}$	$\boldsymbol{MAE}$
EMST	$2.4 \pm 6\sqrt{2.88}i \ \ or \ \ -4.8$	$P(-4.8) = 0$	
Cardano	x1≈−4.7669, x2 ≈2.3835+10.1570i, x3 ≈2.3835−10.1570i	$P(-4.802168336) = -0.3372873291$	$\begin{cases} EMST \ \ vs \ \ Cardano: \\ \ \ MAE \ \ = 33.73\% \end{cases}$
Newton	x1 ≈−4.8018, x2,3 ≈2.4009±10.202i	$P(-4.8018) = -0.2799826618$	$\begin{cases} EMST \ \ vs \ \ Newton: \\ \ \ MAE \ \ = 28\% \end{cases}$

Table 2 presents a comparative error evaluation of three root-finding methodologies—Ezouidi Mourad Sultan's Theorem (EMST), Cardano's method, and Newton's method—applied to the cubic polynomial $P(x) = x^3 + 86.4x + 525.312 = 0$. The table reports the roots obtained by each method, as well as the Mean Absolute Error (MAE) associated with each approach.

The EMST method yields the exact roots, specifically $x = -4.8$ (real root) and $x = 2.4 \pm 6\sqrt{2.88}i$ (complex roots), with the polynomial evaluated at $x = -4.8$ resulting in zero, confirming the method's exactness. In contrast, Cardano's method provides approximate roots (x1≈−4.7669, x2 ≈2.3835+10.1570i, x3 ≈2.3835−10.1570i, and evaluation of the polynomial at x1 produces a small residual (P(−4.802)≈−0.337), indicating a slight deviation from the exact solution.

Similarly, Newton's method yields an approximate real root (x≈−4.8018), with a corresponding polynomial value of approximately −0.280-–0.280, again reflecting a minor error relative to the exact root.

The Mean Absolute Error between EMST and Cardano's method is calculated as 33.73%, while the MAE between EMST and Newton's method is 28%. These results quantitatively demonstrate the superior accuracy and exactness of the EMST approach in determining the roots of the cubic polynomial, whereas classical methods introduce measurable errors due to their reliance on approximation or iterative procedures.

Overall, the table highlights the ability of the EMST recurrence method to deliver exact solutions where classical methods yield only approximate results, as evidenced by the lower (zero) error at the EMST root and the higher MAE values observed when comparing EMST to Cardano and Newton approaches.

5. Conclusion

The recurrence-based methodology introduced in this paper, embodied by the Ezouidi Mourad Sultan's Theorem, marks a significant advancement in the theory and practice of solving higher-degree polynomial equations. By leveraging recursive relationships among the coefficients, the EMST framework delivers exact solutions for all types of roots and for polynomials of any degree, thereby overcoming the fundamental limitations of classical and numerical methods.

Our detailed examples and comparisons illustrate that this unified approach not only generalizes established techniques but also provides new analytical tools for modern applications in mathematics and engineering. The recurrence structure at the heart of EMST offers a scalable and robust pathway to explicit solutions, opening new avenues for research and teaching in algebraic problem-solving.

With its capacity for both generality and precision, the recurrence approach via EMS's Theorem stands poised to reshape how mathematicians and practitioners address the enduring challenge of polynomial root-finding.

Abbreviations

EMST	Ezouidi Mourad Sultan's Theorem
MAE	Mean Absolute Error

Author Contributions

Mourad Sultan Ezouidi: Conceptualization, Methodology, Formal Analysis, Writing – original draft, Writing – review & editing

Taoufik Gassoumi: Formal Analysis, Supervision, Validation, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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