American Journal of Applied Mathematics
2026, Vol. 14, No. 3, pp. 168–173
https://doi.org/10.11648/j.ajam.20261403.17

Research Article

Radial Radio Sequences of Perfect Matching-deleted and Minimum Edge Covering-deleted Subgraphs of the Complete Graph K_n

Vimalajenifer Selvaraj[*]

Department of Data Science, Ayya Nadar Janaki Ammal College, Sivakasi, India

Abstract

Graph labeling is an important and rapidly developing area in graph theory due to its numerous applications in communication networks, frequency assignment, channel allocation, and coding theory. Among the various labeling methods, radial radio labeling has gained attention because of its connection with distance-based constraints and graph structure. In this paper, we study the radial radio labeling of certain connected subgraphs derived from complete graphs. We introduce the concepts of radial radio number and radial radio sequence and examine their behavior for specific graph classes obtained from complete graphs through edge deletions. The main objective of this work is to determine the radial radio sequence of connected graphs formed by deleting a perfect matching and an edge covering from the complete graph. Using fundamental graph-theoretic techniques and structural analysis, we derive exact values for the radial radio sequence of these graph families. The obtained results provide insight into the influence of graph structure and neighborhood properties on radial radio labeling. This study extends existing research on distance-based graph labeling and contributes to a better understanding of how structural modifications in graphs affect labeling parameters, which may further support applications in communication network optimization and interference reduction problems.

Keywords

Radio Number, Radial Radio Number, Radius, Diameter, Radial Radio Sequence

1. Introduction

Throughout this paper, we only consider the simple, connected and undirected graphs. The *closed neighbourhood* of a vertex v is a subset of vertices denoted by $N\ [v]$ and is defined by $N[v] = \{v, u:\ uv \in E(G)\}$. The induced subgraph induced by the closed neighbourhood of v is denoted as $< N\ [v]\ >$ and is defined by $< N\ [v]\ >= (N\ [v], E(< N\ [v]\ >))$, where $E(< N[v] >) = \{vx:\ vx \in E(G), x \in N\ [v]\ \} \cup \{uw:\ u, w \in N[v] and\ \ uw \in E(G)\}$. A *clique*, S, is a subset of $V(G)$ with maximum number of vertices such that $< S >$ is complete. The *clique number*, ω or $\omega(G)$, is the number of vertices in the clique of G.

A *perfect matching* in a graph G is a set M, of edges such that every vertex in G is incident to exactly one edge in M. Equivalently, M is a matching of size $\frac{|V(G)|}{2}$. Note that, a perfect matching exists only when the graph G has even number of vertices.

An *edge cover* or *edge covering* of a graph G is a set C of edges such that every vertex in G is incident to at least one

---

[*] Correspondence: Vimalajenifer Selvaraj (vimalajenima430@gmail.com)

Received: 12 May 2026; **Accepted:** 26 May 2026; **Published:** 27 June 2026

edge in C. In other words, the union of end points of the edges in C covers the entire vertex set V(G). An edge cover is said to be *minimum edge cover* if it has the smallest of possible cardinality among all edge covers of G.

The *distance* between any two distinct vertices u and v is the length of the shortest uv −path in G and is denoted by $d(u,v)$. The *eccentricity* of a vertex v, $e(v)$, is the distance between v and the vertex farthest from v. The minimum eccentricity among all vertices is called as the *radius* of G and is denoted by *rad(G)*. The maximum eccentricity among the vertices is called as the *diameter* of G and is denoted by *diam(G)*.

The study of graph labeling has emerged as a significant area within graph theory, offering insights into combinatorial properties and their applications in network design, communication systems, and optimization problems. The concept labeling was introduced by Rosa, et. Al. [12]. The detailed study of Frequency Assignment Problem [7] by Chartrand et. Al. [6] leads to the new concept of radio k coloring.

A function $\varsigma\colon\ V(\mathrm{G}) \to N$ is said to be a *radial radio labeling*, if it satisfies the following condition: for any two distinct vertices u and v,

$$d(u,v) + |\varsigma(u) + \varsigma(v)| \geq 1 + rad(\mathrm{G}) \qquad (1)$$

where $d(u,v)$ is the shortest distance between u and v. The inequality (1) is known as the *radial radio condition* of G. The *span* of a radial radio labeling, ς, is the largest integer in the range of ς, and is denoted by $span(\varsigma)$. The minimum span taken over all possible radial radio labelings of G is known as the *radial radio number* of G.

Motivated by the concept, $\mu_1(v) - rr$ sequence introduce by Selvam et. Al., [3], the concept radial radio sequence is defined in the following manner: The non-decreasing sequence, $(rr(< N[u] >))_{u\in V(G)}$, is called as the *radial radio sequence* of a graph G, where $rr(< N[u] >)$ is the radial radio number of the induced subgraph induced by the closed neighbourhood of the vertex u. Radial radio sequences of product of some standard graphs have been determined in [4].

The following lemma provides the necessary and sufficient condition for a function to be a radial radio labeling of a graph with radius 1.

Lemma

Let G be a connected graph with $rad(G) = 1$. A function $\varsigma\colon\ V(G) \to N$ is a radial radio labelling of G if and only if $|\varsigma(u) + \varsigma(v)| \geq 1$ whenever uv is an edge.

Proof

If $rad(\mathrm{G}) = 1$, then the radial radio condition becomes

$$d(\mu,v) + |\varsigma(\mu) + \varsigma(v)| \geq 1 + rad(\mathrm{G})$$

$$\Rightarrow d(\mu,v) + |\varsigma(\mu) + \varsigma(v)| \geq 2$$

$$\Rightarrow |\varsigma(\mu) + \varsigma(v)| \geq 2 - d(\mu,v)$$

$$\Rightarrow |\varsigma(\mu) + \varsigma(v)| \geq 2 - d(\mu,v)$$

$$\Rightarrow |\varsigma(\mu) + \varsigma(v)| \geq \begin{cases} 1, if & d(\mu,v) = 1 \\ 0, if & d(\mu,v) = 2 \end{cases}$$

This completes the proof.

Remark

To verify the radial radio condition for a graph with radius 1, by above lemma, it is enough to show that the label difference between adjacent vertices is at least 1 and the label difference between non-adjacent vertices is at least 0.

Equivalently, the non-adjacent vertices may receive the same positive integers as labels.

The following theorem is useful in establishing the results of this paper.

Theorem A: For any simple connected graph G, $rr(G) \geq \omega(G)$, where $\omega(G)$ is the clique number of G. [2].

For further basic concepts, one may refer [5] and [8]. For more results on radio number and the radial radio number, one can refer [1, 9-11] and [13, 15]. In this research paper, we study the radial radio sequence of the subgraphs of the complete graph K_n which are obtained by deleting some of the subgraphs such as perfect matching and minimum edge covering. The radial radio numbers of these graphs are already determined in [14].

2. Radial Radio Sequence of K_{2n}-M, n ≥ 2

The graph $K_{2n} - \mathrm{M}$ is obtained by deleting the perfect matching, M, from the complete graph K_{2n}. Let $\mathrm{V}(K_{2n}) = \{\alpha_i\colon\ 1 \leq i \leq n\}$ and let $M = \{\alpha_{2i-1}\alpha_{2i}\colon\ 1 \leq i \leq n\}$. After deleting M, we observe the following:

a) $V(K_{2n} - M) = V(K_{2n})$
b) $E(K_{2n} - M) = E(K_{2n}) - M$
c) $\deg(\boldsymbol{\alpha}_i) = 2n - 2$, for each i, $1 \leq i \leq 2n$.
d) $rad(K_{2n} - M) = 2$

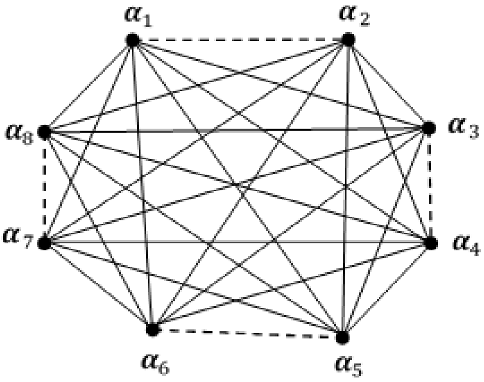


Figure 1. $K_8 - M$.

For, the graph $\mathrm{K}_8 - \mathrm{M}$ is illustrated in Figure 1.

Theorem 2.1 provides the radial radio sequence of the graph $\mathrm{K}_{2n} - \mathrm{M}$.

Theorem 2.1

The radial radio sequence of $K_{2n} - M$ is $(\underbrace{n, n, n, \dots, n}_{2n\ \text{times}})$, for $n \geq 3$.

Proof

We have $N[\alpha_i] = \{\alpha_i\colon\ 1 \leq i \neq i+1 \leq 2n-1\}$ and $N[\alpha_{2n}] = V(K_{2n} - M) - \{\alpha_{2n-1}\}$. We observe that, the induced subgraphs induced by $N[\alpha_i]$ and $N[\alpha_j]$, $1 \leq i \neq j \leq 2n$ are isomorphic. This forces that, $rr(< N[\alpha_i] >) = rr(< N[\alpha_j] >)$, $1 \leq i \neq j \leq 2n$. So that, it is enough to find radial radio number of $N[\alpha_i]$, for some, i, $1 \leq i \leq 2n$. Without loss of generality, assume that, $i = 1$.

We note that, K_n is a clique in $< N[\alpha_1] >$ and so, by Theorem A,

$$rr(< N[\alpha_1] >) \geq n \tag{2}$$

Define $\zeta\colon\ N[\alpha_1] \to \{1,2,3,\dots\}$ such that $\zeta(\alpha_1) = 1$; $\zeta(\alpha_{2i-1}) = \zeta(\alpha_{2i}) = i, 2 \leq i \leq n$.

The induced subgraph $< N[\alpha_1] >$ and its corresponding labeling under ζ are presented in Figure 2.

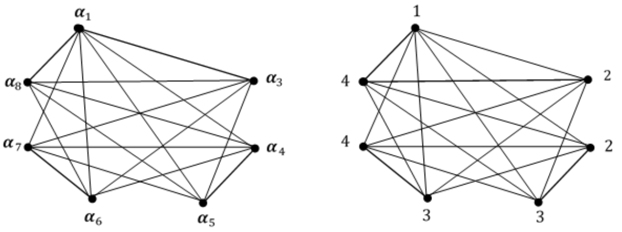


Figure 2. *a* $< N[\alpha_1] >$, *b Labeling* ζ *of* $< N[\alpha_1] >$.

Since $rad(< N[\alpha_1] >) = 1$, by Lemma, we can say that ζ is a radial radio labeling for the graph $< N[\alpha_1] >$. Hence $span(\zeta) = n$ which implies that,

$$rr(< N[\alpha_1] >) \leq n \tag{3}$$

From the inequalities (2) and (3), $rr(< N[\alpha_1] >) = n$. Thus $rr(< N[\alpha_i] >) = n$, for all i, $1 \leq i \leq n$.

3. Radial Radio Sequence of K_{2n+1}-T, Where T Is a Minimum Edge Cover of K_{2n+1}

Assume that, $V(K_{2n+1}) = \{\beta_i\colon\ 1 \leq i \leq 2n+1\}$. If we take $T = \{\beta_1\beta_{2n+1}, \beta_1\beta_2, \beta_{2i-1}\beta_{2i}\colon\ 2 \leq i \leq n\}$, then T is the minimum edge cover of K_{2n+1}. We observe that,

i) $K_{2n+1} - T$ is connected.

ii) $E(K_{2n+1} - T) = E(K_{2n+1}) - T$

iii) $rad(K_{2n+1} - T) = 2$

The graph $K_{11} - T$ is illustrated in Figure 3.

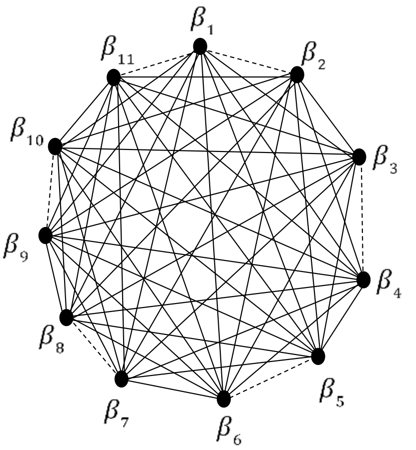


Figure 3. $K_{11} - T$.

In this section, we determine the radial radio sequence of $K_{2n+1} - T$, for $n \geq 3$.

Theorem 3.1

For $n \geq 3$, the radial radio sequence of $K_{2n+1} - T$ is $(\underbrace{n+1, n+1, \dots, n+1}_{n\ times}, n)$.

Proof

Here, we identify three types of induced subgraphs arising from the deletion of an edge covering in the graph K_{2n+1}. These types are characterized as follows:

1) Induced subgraph induced by β_1.
2) Induced subgraph induced by the vertices which are not adjacent to β_1, that is, β_2 and β_{2n+1}.
3) Induced subgraphs induced by the vertices which are adjacent to β_1.

We determine the radial radio numbers of each of these three types of induced subgraphs separately.

Case 1: $< N\ \ [\beta_1]\ \ >$

Subcase 1a:

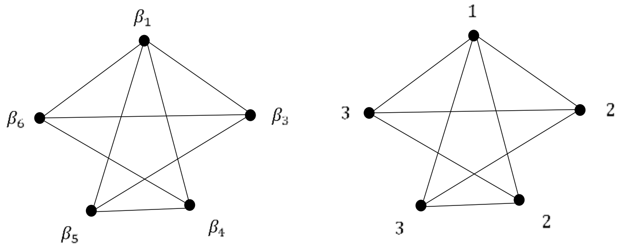


Figure 4. *a* $< N\ \ [\beta_1]\ \ >$ *in* $K_7 - T$, *4b Labeling* ξ *of* $< N\ \ [\beta_1]\ \ >$ *in* $K_7 - T$.

For $n = 3$, the sets $\{\beta_1, \beta_4, \beta_5\}$ and $\{\beta_1, \beta_3, \beta_6\}$ induce subgraphs both are isomorphic to K_3. Define $\xi\colon\ N[\beta_1] \to \{1,2,3,\dots\}$ by $\xi(\beta_1) = 1$; $\xi(\beta_3) = 2$; $\xi(\beta_6) = 3$; $\xi(\beta_4) = 2$; $\xi(\beta_5) = 3$. Figure 4 illustrates this labeling:

From Figure 4, we note that, adjacent vertices receive distinct positive integers as labels and K_3 is a clique, which proves that, $rr(<N[\beta_1]>) = 3$.

Subcase 1b:

For $n \geq 4$, the vertices in the sets $\{\beta_1, \beta_{2i-1}: \ 2 \leq i \leq n\}$ and $\{\beta_1, \beta_{2i}: \ 2 \leq i \leq n\}$ induce subgraph isomorphic to K_n. Also, K_n is the clique in $<N \ [\beta_1] \ >$, which implies that,

$$rr(<N \ [\beta_1] \ >) \geq n \tag{4}$$

Define ξ': $N[u_1] \to \{1,2,3,\ldots\}$ by $\xi'(\beta_1) = 1$; $\xi'(\beta_{2i-1}) = i$, $1 \leq i \leq n$; $\xi'(\beta_{2i}) = i$, $2 \leq i \leq n$. The induced subgraph $<N[\beta_1]>$ of $K_{11} - T$ is illustrated in Figure 5.

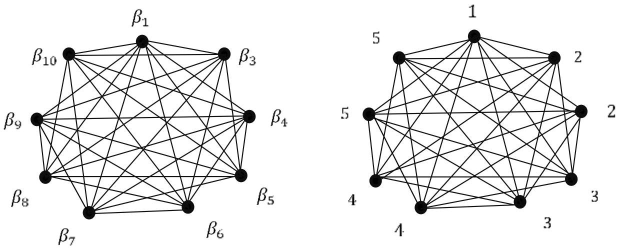


Figure 5. *a* $<N \ [\beta_1] \ >$ *in* $K_{11} - T$, *b Labeling* ξ' *of* $<N \ [\beta_1] \ >$ *in* $K_{11} - T$.

By Lemma, we confirm that ξ' is a radial radio labeling for$<N \ [\beta_1] \ >$, as the non-adjacent vertices get the distinct positive integers as labels. Thus $span \ (\xi') = n$ and so

$$rr(<N \ [\beta_1] \ >) \leq n \tag{5}$$

By inequalities (4) and (5), we have $rr(<N[\beta_1]>) = n$.

Case 2: The vertices which are not adjacent to β_1, that is, β_2 and β_{2n+1}.

We observe the following:

1) $<N \ [\beta_2] \ > \cong < N \ [\beta_{2n+1}] \ >$ this implies that, $rr(<N[\beta_2]>) = rr(<N \ [\beta_{2n+1}] \ >)$.
2) $N[\beta_2] = \{\beta_i: \ 2 \leq i \leq 2n+1\}$.
3) $rad(<N[\beta_2]>) = 1$.

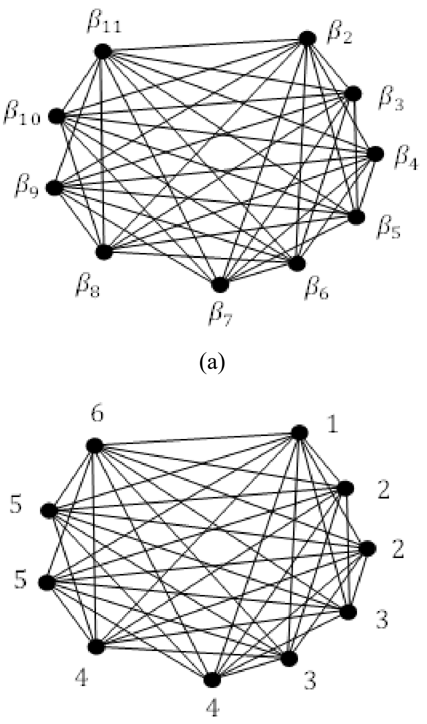
Assume the induced subgraph induced by β_2. Consider the sets of vertices:

i) $\{\beta_2, \beta_{2n+1}, \beta_{2i-1}: \ 2 \leq i \leq n\}$
ii) $\{\beta_2, \beta_{2n+1}, \beta_{2i}: \ 2 \leq i \leq n\}$

Both the vertex sets (i) and (ii) induce subgraphs isomorphic to K_{n+1}, which is a clique in $N[\beta_2]$ and hence by Theorem A,

$$rr(<N[\beta_2]>) \geq n+1 \tag{6}$$

Define ς: $N[\beta_2] \to \{1,2,3,\ldots\}$ such that $\varsigma(\beta_2) = 1$; $\varsigma(\beta_{2i-1}) = i$, $2 \leq i \leq n+1$; $\varsigma(\beta_{2i}) = i$, $2 \leq i \leq n$. The illustration of the function ς is shown in Figure 6.



(a)

(b)

Figure 6. *a:* $<N \ [\beta_2] \ >$ *in* $K_{11} - T$; *b: Labeling* ς *of* $<N \ [\beta_2] \ >$ *in* $K_{11} - T$.

Since $d(\beta_{2i-1}, \beta_{2i}) = 2$, $1 \leq i \leq n$; $d(\beta_2, \beta_i) = 1$, $3 \leq i \leq n+1$ and all adjacent vertices receive distinct positive integers as labels, by Lemma, we conclude that, ς is a radial radio labeling of $<N \ [\beta_2] \ >$. Hence $span(\varsigma) = n+1$ and so

$$rr(<N[\beta_2]>) \leq n+1 \tag{7}$$

From inequalities, (6) and (7), we conclude that, $rr(<N[\beta_2]>) = n+1$. Also, $rr(<N[\beta_2]>) = rr(<N[\beta_{2n+1}]>) = n+1$.

Case 3: The vertices which are adjacent to β_1. That is, the vertices belong to $\{\beta_i: \ 3 \leq i \leq 2n\}$.

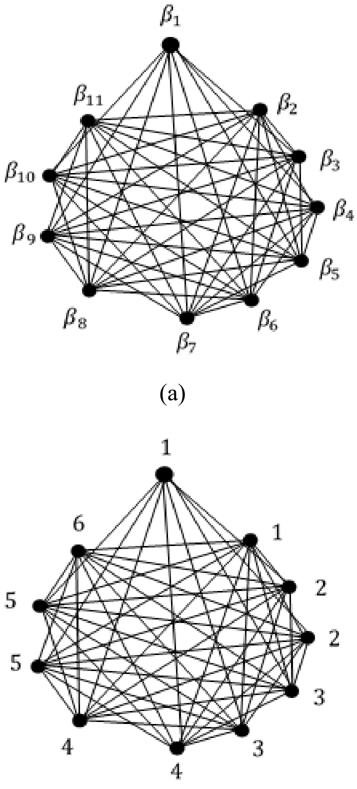
Note that: $<N[\beta_i]> \cong <N[\beta_j]>$ and $rr(<N[\beta_i]>) \cong rr(<N[\beta_j]>)$, for every $3 \leq i \neq j \leq 2n$. Here, $rad(<N[\beta_i]>) = 1$. Without loss of generality, assume that, $i = 3$. It is observed that, the set of vertices in (i) and (ii) induce subgraphs which are isomorphic to K_{n+1}, which is a clique in $<N[\beta_3]>$:

i) $\{\beta_1, \beta_2, \beta_3, \beta_{2i-1}: \ 3 \leq i \leq n+1\}$
ii) $\{\beta_1, \beta_2, \beta_3, \beta_{2n+1}, \beta_{2i}: \ 3 \leq i \leq n\}$

By Theorem A,

$$rr(<N[\beta_3]>) \geq n+1 \tag{8}$$

Now, Define φ: $N[\beta_3] \to \{1,2,3,\ldots\}$ such that, $\varphi(\beta_{2i-1}) = \varphi(\beta_{2i}) = i$, $1 \leq i \leq n$; $\varphi(\beta_{2n+1}) = n+1$. Figure 7 shows the optimal radial radio labeling of the induced subgraph.



(a)

(b)

Figure 7. *a:* $< N\ [\beta_3]\ >$ *in* $K_{11} - T$*; b: Labeling* φ *of* $< N\ [\beta_3]\ >$ *in* $K_{11} - T$.

By Lemma, we can prove that, φ is a radial radio labeling of $< N[\beta_3] >$ and $span(\varphi) = n + 1$ and hence

$$rr(< N[\beta_3] >) \leq n + 1 \qquad (9)$$

Inequalities (8) and (9), proves that $rr(< N[\beta_3] >) = n + 1$. Hence $r(< N[\beta_i] >) = n + 1,\ 3 \leq i \leq 2n$.

Cases 1, 2 and 3 concludes that the radial radio sequence of $K_{2n+1} - T$ is $(\underbrace{n+1, n+1, \dots, n+1}_{n\ times}, n)$. This completes the proof.

4. Conclusion

In this paper, we determined the radial radio sequences of the connected induced subgraphs of the complete graph K_n, which serves as our primary graph of interest. These results may be extended in future work to other specific families of graphs. It is important to note that the resulting graph must remain connected after the deletion of a perfect matching or an edge covering in order to define the radial radio sequence.

Abbreviations

K_n	Complete Graph on n Vertices
$rad(G)$	Radius of Graph G
$rr(G)$	Radial Radio Number of Graph G
$d(u, v)$	Distance Between Vertices u and v
$N\ [u]$	Closed Neighbourhood of Vertex u
$rr(\langle N\ [u]\ \rangle)$	Radial Radio Number of the Induced Subgraph by Closed Neighbourhood of u

Acknowledgments

The author sincerely thanks the Management of Ayya Nadar Janaki Ammal College (Autonomous), Sivakasi, Tamil Nadu, India, for their constant support and encouragement. The authors also gratefully acknowledge the Postgraduate Department of Data Science, Computer Science Lab and the Science Instrumentation Centre of Ayya Nadar Janaki Ammal College for providing the necessary facilities and support to carry out this research work.

Author Contributions

Vimalajenifer Selvaraj: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing

Conflicts of Interest

The author declares that she has no conflicts of interest.

References

[1] Selvam Avadayappan, M. Bhuvaneshwari, S. Vimalajenifer, The Radial Radio Number and the Clique Number of a Graph, International Journal of Engineering and Advanced Technology (IJEAT), Volume 9, Issue 1S4, pp. 1002-1006, December 2019.

[2] Selvam Avadayappan, M. Bhuvaneshwari, S. Vimalajenifer, A Note on Radial Radio Number of a Graph, International Journal of Applied and Advanced Scientific Research, pp. 62–68, 2017.

[3] Selvam Avadayappan, M. Bhuvaneshwari, S. Vimalajenifer, Radial Radio Sequence of a Graph, International Journal of Mathematical Combinatorics, Special Issue, pp. 1–8, 2018.

[4] Selvam Avadayappan, M. Bhuvaneshwari and S. Vimalajenifer, rr-sequence product graphs, Advances in Mathematics: Scientific Journal, no. 3, pp. 927-936, 2020. https://doi.org/10.37418/amsj.9.3.18

[5] R. Balakrishnan and K. Ranganathan, A Textbook of Graph Theory, Second Edition, Springer, New York, 2012. https://doi.org/10.1007/978-1-4614-4529-6

[6] G. Chartrand, D. Erwin, F. Harary and P. Zhang, Radio labeling of graphs, Bulletin of the Institute of Combinatorics and its Applications, vol. 33, pp. 77-85, 2001.

[7] W. K. Hale, Frequency Assignment: Theory and Applications, Proceedings of the IEEE, vol. 68, no. 12, pp. 1497-1514, 1980. https://doi.org/10.1109/PROC.1980.11899

[8] M. K. A. Kaabar, et al., Radio and Radial Radio Numbers of Certain Sunflower Extended Graphs, Journal of Mathematics, vol. 2022, Article ID 9229409, 2022. https://doi.org/10.1155/2022/9229409

[9] B. Mari, Further results on the radio number for some construction of the path, complete, and complete bipartite graphs, Heliyon, vol. 10, no. 15, e35678, 2024. https://doi.org/10.1016/j.heliyon.2024.e35678

[10] K. M. Paramasivam, K. Yenoke, B. S. K. Muralidharan, Radial radio number of chess board graph and king's graph, TELKOMNIKA Telecommunication Computing Electronics and Control, vol. 20, no. 1, pp. 189–196, 2022. https://doi.org/10.12928/TELKOMNIKA.v20i1.19493

[11] R. Ponraj, S. Sathish Narayanan, Radio Mean Number of Certain Graphs, International Journal of Mathematical Combinatorics, vol. 4, pp. 45–53, 2021.

[12] A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966), Gordon and Breach, N. Y. and Dunod Paris, pp. 349-355, 1967.

[13] S. Vimalajenifer, Radial Radio Sequence of Path-Deleted, Cycle-Deleted and Star-deleted subgraphs of the complete Graph. (Submitted).

[14] S. Vimalajenifer, Selvam Avadayappan, M. Bhuvaneshwari, Radial Radio Numbers of Subgraphs from Structured Deletions. (Communicated).

[15] K. Yenoke, et al, Radial Radio Number of Hexagonal and Its Derived Networks, International Journal of Mathematics and Mathematical Sciences, vol. 2021, Article ID 5101021, 2021. https://doi.org/10.1155/2021/5101021