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Existence and Uniqueness of Mild Solutions for Neutral Stochastic Fractional Sum-Difference Equations with Impulses Driven by a Rosenblatt Process

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Abstract: This paper investigates the existence and uniqueness of mild solutions for a class of neutral stochastic fractional difference equations with impulsive effects in a Hilbert space framework, driven by a Rosenblatt process. The considered system is formulated using the Caputo fractional difference operator and incorporates delay terms, impulsive effects, and stochastic perturbations exhibiting long-range dependence. The primary objective is to establish sufficient conditions ensuring the existence and uniqueness of mild solutions for the proposed system. To achieve this objective, the theory of resolvent operators is combined with stochastic analysis techniques and the Banach fixed point theorem. In particular, an appropriate operator is constructed from the mild solution formulation, and suitable conditions are imposed to guarantee its contractive property. Consequently, the existence and uniqueness of a mild solution are established. The obtained results extend and generalize several existing results for fractional differential and stochastic systems to the discrete fractional setting involving Rosenblatt stochastic processes. The proposed framework effectively accounts for the combined influence of fractional memory, delays, impulsive effects, and long-range-dependent stochastic disturbances. Furthermore, an application to a class of impulsive stochastic partial fractional difference equations is presented to demonstrate the applicability and effectiveness of the theoretical results. The application verifies that the established assumptions can be satisfied in a relevant stochastic fractional model. Thus, the results contribute to the qualitative theory of neutral stochastic fractional difference equations and provide a useful framework for the analysis of discrete-time stochastic systems with memory, delay, impulsive phenomena, and long-range dependence. These findings may also serve as a basis for further investigations of more general stochastic fractional difference systems.

Keywords: Fractional Difference Equations, Neutral Stochastic Systems, Impulsive Effects, Rosenblatt Process, Mild Solution, Existence and Uniqueness, Banach Fixed Point Theorem, Resolvent Pperator

1. Introduction

Fractional difference equations are widely used to model discrete systems with memory effects in engineering, biology, economics, and control theory [11–14]. Stochastic fractional systems with impulsive effects have also gained importance due to the presence of randomness and sudden perturbations in practical problems [15, 16]. In this context, the Rosenblatt process is significant because of its non-Gaussian nature and long-range dependence properties [17, 18]. Several studies have examined stochastic fractional systems driven by Rosenblatt processes [1–4, 7–10, 19–22]; however, impulsive neutral stochastic fractional difference equations with Rosenblatt noise remain less explored. Motivated by this, the present paper investigates such systems in Hilbert spaces and establishes existence and uniqueness results for mild solutions using the Banach FP theorem and resolvent operator theory. The results extend earlier continuous stochastic fractional models to the discrete fractional setting, and an application to impulsive stochastic partial fractional difference equations is included. Now consider the following fractional order neutral stochastic sum-difference equation with impulses of the form as follows:

$$\begin{aligned}
&\Delta^{\alpha}\left[\vartheta(\eta)-\kappa\left(\eta,\vartheta_{\eta},\sum_{\mu=0}^{\eta-1}\upsilon_1(\eta,\mu,\vartheta_{\mu})\right)\right]\\
&\quad=\Lambda\left[\vartheta(\eta)-\kappa\left(\eta,\vartheta_{\eta},\sum_{\mu=0}^{\eta-1}\upsilon_1(\eta,\mu,\vartheta_{\mu})\right)\right]\\
&\qquad+\sum_{\mu=0}^{\eta-1}\Upsilon(\eta-\mu)\\
&\qquad\times\left[\vartheta(\mu)-\kappa\left(\mu,\vartheta_{\mu},\sum_{\tau=0}^{\mu-1}\upsilon_1(\mu,\tau,\vartheta_{\tau})\right)\right]\\
&\qquad+\Phi\left(\eta,\vartheta_{\eta},\sum_{\mu=0}^{\eta}\upsilon_2(\eta,\mu,\vartheta_{\mu})\right)+\tilde{\mathscr{F}}(\eta)\,\Delta Z_Q^H(\eta),\\
&\qquad\eta\in\mathcal{J},\quad \eta\neq\eta_i.
\end{aligned}\tag{1}$$

$$\Delta\vartheta(\eta_i)=\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big),\quad \eta=\eta_i,\quad i=1,2,\ldots\tag{2}$$

$$\vartheta_0(\eta)=\phi(\eta),\qquad -\varrho\le\eta\le 0.\tag{3}$$

where $\mathcal{J}=[0,\theta]$, and Λ is the infinitesimal generator of a strongly continuous semigroup $\big(\mathcal{T}(\eta)\big)_{\eta\geq 0}$ of bounded linear operators on a Hilbert space Ξ with domain $D(\Lambda)$. Furthermore, $\Upsilon(\eta)$ is a family of closed linear operators on Ξ with domain $D(\Upsilon(\eta))\supset D(\Lambda)$. The process $\{Z_Q^H(\eta)\}_{\eta\geq 0}$ denotes a Q-fractional Brownian motion defined on a complete probability space.

The initial function

$$\phi\in\mathscr{B}_h:=\mathscr{B}_h([-\varrho,0],\Xi),$$

where

$$\mathbf{B}_h([-\varrho,0],\Xi)=\left\{\psi:[-\varrho,0]\to\Xi:\ \begin{array}{l}\psi(\cdot)\text{ is piecewise ,}\\ \text{continuous and }\psi(\tilde{\eta}^-)\\ \text{and }\psi(\tilde{\eta}^+)\\ \text{exist for every}\\ \text{discontinuity point }\tilde{\eta}\end{array}\right\}.$$

$$\text{with }\psi(\tilde{\eta}^-)=\psi(\tilde{\eta})\big\}\,.$$

For the fractional difference setting, the fractional operator Δ^{α} denotes the Caputo fractional difference operator of order $\alpha\in(0,1)$.

Definition 1.1. *Riemann–Liouville Fractional Difference Operator* [5, 6]

Let $\alpha>0$ and $n-1<\alpha\leq n$, where $n\in\mathbb{N}$. The Riemann–Liouville fractional difference of order α for a function $f:\mathbb{N}_a\to\mathbb{R}$ is defined by

$$\Delta_a^{\alpha}f(\eta)=\Delta^n\left(\frac{1}{\Gamma(n-\alpha)}\sum_{\mu=a}^{\eta-(n-\alpha)}(\eta-\sigma(\mu))^{(n-\alpha-1)}f(\mu)\right),$$

where $\sigma(\mu)=\mu+1$ and

$$\eta^{(\alpha)}=\frac{\Gamma(\eta+1)}{\Gamma(\eta+1-\alpha)}$$

denotes the generalized falling factorial function.

Definition 1.2. *Caputo Fractional Difference Operator* [5, 6]

Let $\alpha>0$ and $n-1<\alpha\leq n$, where $n\in\mathbb{N}$. The Caputo fractional difference operator of order α for a function $f:\mathbb{N}_a\to\mathbb{R}$ is defined by

$${}^{C}\Delta_a^{\alpha}f(\eta)=\frac{1}{\Gamma(n-\alpha)}\sum_{\mu=a}^{\eta-(n-\alpha)}(\eta-\sigma(\mu))^{(n-\alpha-1)}\Delta^n f(\mu),$$

where Δ^n denotes the forward difference operator of order n.

2. Preliminaries

Lemma 2.1. Using the discrete fractional summation operator, system (1)–(3) can be transformed into the equivalent summation equation...

Definition 2.1. An Ξ-valued stochastic process

$$\{\vartheta(\eta)\,:\,\eta\in[-\varrho,\theta]\}$$

is called a mild solution of the impulsive fractional stochastic difference system (1)–(3) if the following conditions hold:

1) $$\vartheta(\cdot)\in\mathscr{B}_h\big([-\varrho,\theta],L^2(\Omega,\Xi)\big).$$

2) For every $\eta\in[-\varrho,0]$,
$$\vartheta(\eta)=\phi(\eta).$$

3) For every $\eta\in\mathcal{J}$, the process $\vartheta(\eta)$ satisfies the fractional summation equation
$$\begin{aligned}
\vartheta(\eta)&=\Psi(\eta)\big[\phi(0)-\kappa(0,\phi,0)\big]+\kappa\big(\eta,\vartheta_{\eta},A_1(\vartheta(\eta))\big)\\
&\quad+\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{(\alpha-1)}\Psi(\eta-\sigma(\mu))\\
&\qquad\times\Phi\big(\mu,\vartheta_{\mu},A_2(\vartheta(\mu))\big)\\
&\quad+\sum_{0<\eta_i<\eta}\Psi(\eta-\eta_i)\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)\\
&\quad+\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{(\alpha-1)}\\
&\quad\times\Psi(\eta-\sigma(\mu))\overline{\mathscr{F}}(\mu)\Delta Z_Q^H(\mu),
\end{aligned}$$
where $\sigma(\mu)=\mu+1$.

For the sake of simplicity, we denote

$$A_1(\vartheta(\eta)) = \sum_{\mu=0}^{\eta-1} \upsilon_1\big(\eta, \mu, \vartheta_\mu\big),$$

and

$$A_2(\vartheta(\eta)) = \sum_{\mu=0}^{\eta} \upsilon_2\big(\eta, \mu, \vartheta_\mu\big).$$

We impose the following assumptions throughout this paper.

(H_1) Λ is the infinitesimal generator of a strongly continuous semigroup

$$\big(\mathcal{T}(\eta)\big)_{\eta\geq 0}$$

on the Hilbert space Ξ.

(H_2) For every $\eta \in \mathcal{J}$,

$$\Upsilon(\eta) : (Y, \|\cdot\|_Y) \to (\Xi, \|\cdot\|_\Xi)$$

is a continuous linear operator. Moreover, there exists a nonnegative sequence $c : \mathcal{J} \to \mathbb{R}^+$ such that for every $y \in Y$,

$$y \mapsto \Upsilon(\eta) y$$

belongs to the fractional difference space and

$$\big\|\Delta^\alpha \Upsilon(\eta) y\big\| \leq c(\eta)\|y\|_Y, \qquad \eta \in \mathcal{J}.$$

(H_3) There exists a positive constant $\digamma > 0$ such that

$$\|\Psi(\eta)\| \leq \digamma, \qquad \forall \eta \in \mathcal{J}.$$

(H_4) There exists a positive constant $\digamma_{\upsilon_1} > 0$ such that for every $\eta \in \mathcal{J}$ and $k_1, k_2 \in \mathscr{B}_h$,

$$\left\|\sum_{\mu=0}^{\eta-1} \Big[\upsilon_1(\eta, \mu, k_1) - \upsilon_1(\eta, \mu, k_2)\Big]\right\| \leq \digamma_{\upsilon_1}\|k_1 - k_2\|.$$

Also,

$$\widetilde{\digamma}_{\upsilon_1} = \theta \sup_{0\leq\mu\leq\eta\leq\theta} \|\upsilon_1(\eta, \mu, 0)\|.$$

(H_5) There exists a positive constant $\digamma_\kappa > 0$ such that κ is Ξ-valued and for every $\eta \in \mathcal{J}$, $k_j, \phi_j \in \mathscr{B}_h$, $j = 1, 2$,

$$\|\kappa(\eta, k_1, \phi_1) - \kappa(\eta, k_2, \phi_2)\|$$
$$\leq \digamma_\kappa\Big(\|k_1 - k_2\| + \|\phi_1 - \phi_2\|\Big).$$

Also,

$$\widetilde{\digamma}_\kappa = \sup_{\eta\in\mathcal{J}} \|\kappa(\eta, 0, 0)\|,$$

and

$$k = \digamma_\kappa(1 + \digamma_{\upsilon_1}) < 1.$$

(H_6) There exists a positive constant $\digamma_{\upsilon_2} > 0$ such that for every $\eta \in \mathcal{J}$ and $k_1, k_2 \in \mathscr{B}_h$,

$$\left\|\sum_{\mu=0}^{\eta} \Big[\upsilon_2(\eta, \mu, k_1) - \upsilon_2(\eta, \mu, k_2)\Big]\right\| \leq \digamma_{\upsilon_2}\|k_1 - k_2\|.$$

Also,

$$\widetilde{\digamma}_{\upsilon_2} = \theta \sup_{0\leq\mu\leq\eta\leq\theta} \|\upsilon_2(\eta, \mu, 0)\|.$$

(H_7) There exists a positive constant $\digamma_\Phi > 0$ such that Φ is Ξ-valued and for every $\eta \in \mathcal{J}$, $k_j, \phi_j \in \mathscr{B}_h$, $j = 1, 2$,

$$\|\Phi(\eta, k_1, \phi_1) - \Phi(\eta, k_2, \phi_2)\|$$

$$\leq \digamma_\Phi\Big(\|k_1 - k_2\| + \|\phi_1 - \phi_2\|\Big).$$

Also,

$$\widetilde{\digamma}_\Phi = \sup_{\eta\in\mathcal{J}} \|\Phi(\eta, 0, 0)\|,$$

and

$$k = \digamma_\Phi(1 + \digamma_{\upsilon_2}).$$

(H_8) The function κ is continuous with respect to the time variable, that is, for every $k, \phi \in \mathscr{B}_h$,

$$\lim_{\eta\to\mu} \|\kappa(\eta, k, \phi) - \kappa(\mu, k, \phi)\|^2 = 0.$$

(H_9) The function

$$\widetilde{\mathscr{F}} : \mathcal{J} \to L_Q^0(Y, \Xi)$$

satisfies

$$\sum_{\mu=0}^{\eta} \|\widetilde{\mathscr{F}}(\mu)\|_{L_Q^0}^2 < \infty, \qquad \forall \eta \in \mathcal{J}.$$

If $\{\upsilon_n\}_{n\in\mathbb{N}}$ is a complete orthonormal basis in Y, then

(C_1)

$$\sum_{n=1}^{\infty} \|\widetilde{\mathscr{F}} Q^{\frac{1}{2}} \upsilon_n\|_{l^2(\mathcal{J};\Xi)} < \infty.$$

(C_2)

$$\sum_{n=1}^{\infty} \Big|\widetilde{\mathscr{F}} Q^{\frac{1}{2}} \upsilon_n\Big|_\Xi$$

is uniformly convergent for every $\eta \in \mathcal{J}$.

(H_{10}) Suppose that $k_1, k_2 \in \mathscr{B}_h$ and

$$\sum_{i=1}^{\infty} d_i < \infty.$$

Then the impulsive functions $\mathcal{I}_i$, $i = 1, 2, \ldots$, satisfy

$$\|\mathcal{I}_i(k_1) - \mathcal{I}_i(k_2)\| \leq d_i\|k_1 - k_2\|,$$

and

$$\|\mathcal{I}_i(0)\| = 0.$$

3. Main Result

Theorem 3.1. Assume that hypotheses (H_1)–(H_{10}) are satisfied for every

$$\phi \in \mathscr{B}_h, \qquad \theta > 0.$$

Then the impulsive fractional stochastic difference system (1)–(3) admits a unique mild solution on $[-\varrho, \theta]$ provided that

$$\Pi = 3F^2 \left(\sum_{i=1}^{\infty} d_i \right)^2 < (1-k)^2, \tag{4}$$

where

$$k = F_\kappa \big(1 + F_{\upsilon_1}\big) < 1.$$

Proof

First, we introduce the space

$$\mathscr{B}_{h,\theta} := \mathscr{B}_h\big([-\varrho, \theta], L^2(\Omega, \Xi)\big),$$

which is the Banach space of all piecewise continuous functions from $[-\varrho, \theta]$ into $L^2(\Omega, \Xi)$, equipped with the norm

$$\|\xi\|^2_{\mathscr{B}_{h,\theta}} = \sup_{\mu \in [-\varrho, \theta]} \mathbb{E}\|\xi(\mu)\|^2.$$

Let

$$\widehat{\mathscr{B}}_{h,\theta} = \left\{ \vartheta \in \mathscr{B}_{h,\theta} : \vartheta(\tau) = \phi(\tau), \quad \tau \in [-\varrho, 0] \right\}.$$

Clearly, $\widehat{\mathscr{B}}_{h,\theta}$ is a closed subset of $\mathscr{B}_{h,\theta}$.

Now, define the operator

$$\aleph : \widehat{\mathscr{B}}_{h,\theta} \to \widehat{\mathscr{B}}_{h,\theta}$$

by

$$(\aleph\vartheta)(\eta) = \phi(\eta), \qquad \eta \in [-\varrho, 0],$$

and for $\eta \in \mathcal{J}$,

$$\begin{aligned}
(\aleph\vartheta)(\eta) =& \Psi(\eta)\big[\phi(0) - \kappa(0, \phi, 0)\big] \\
&+ \kappa\big(\eta, \vartheta_\eta, A_1(\vartheta(\eta))\big) \\
&+ \frac{1}{\Gamma(\alpha)} \sum_{\mu=0}^{\eta-1} (\eta - \sigma(\mu))^{(\alpha-1)} \Psi(\eta - \sigma(\mu)) \\
&\qquad \times \Phi\big(\mu, \vartheta_\mu, A_2(\vartheta(\mu))\big) \\
&+ \sum_{0<\eta_i<\eta} \Psi(\eta - \eta_i) \mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big) \\
&+ \frac{1}{\Gamma(\alpha)} \sum_{\mu=0}^{\eta-1} (\eta - \sigma(\mu))^{(\alpha-1)} \\
&\times \Psi(\eta - \sigma(\mu)) \overline{\mathscr{F}}(\mu) \Delta Z_Q^H(\mu),
\end{aligned}$$

where

$$A_1(\vartheta(\eta)) = \sum_{\mu=0}^{\eta-1} \upsilon_1\big(\eta, \mu, \vartheta_\mu\big),$$

and

$$A_2(\vartheta(\eta)) = \sum_{\mu=0}^{\eta} \upsilon_2\big(\eta, \mu, \vartheta_\mu\big).$$

We shall prove that the operator $\aleph$ has a fixed point in $\widehat{\mathscr{B}}_{h,\theta}$. Consequently, the proof is divided into the following two steps.

Claim 1. The operator $\aleph$ is well defined.

Let

$$\vartheta \in \widehat{\mathscr{B}}_{h,\theta}.$$

Then the function

$$(\aleph\vartheta)(\cdot)$$

is continuous on the discrete interval $\mathcal{J}$.

Suppose that $\vartheta \in \widehat{\mathscr{B}}_{h,\theta}$, $\eta \in \mathcal{J}$, and let $|\rho|$ be sufficiently small. Then, by the inequality

$$\|x_1 + x_2 + x_3 + x_4 + x_5\|^2 \le 5 \sum_{j=1}^{5} \|x_j\|^2,$$

we obtain

$$\begin{aligned}
\mathbb{E}\Big\|(\aleph\vartheta)(\eta+\rho) - (\aleph\vartheta)(\eta)\Big\|^2 \le& 5\,\mathbb{E}\Big\|\big(\Psi(\eta+\rho) - \Psi(\eta)\big) && (1)\\
&\times \big[\phi(0) - \kappa(0, \phi, 0)\big]\Big\|^2 && (2)\\
&+ 5 \sum_{j=1}^{4} \mathbb{E}\Big\|\mathscr{F}_j(\eta+\rho) - \mathscr{F}_j(\eta)\Big\|^2, &&\\
& && (5)
\end{aligned}$$

where

$$\mathscr{F}_1(\eta) = \kappa\big(\eta, \vartheta_\eta, A_1(\vartheta(\eta))\big),$$

$$\mathscr{F}_2(\eta) = \frac{1}{\Gamma(\alpha)} \sum_{\mu=0}^{\eta-1} (\eta - \sigma(\mu))^{(\alpha-1)} \Psi(\eta - \sigma(\mu)) \Phi\big(\mu, \vartheta_\mu, A_2(\vartheta(\mu))\big),$$

$$\mathscr{F}_3(\eta) = \sum_{0<\eta_i<\eta} \Psi(\eta - \eta_i) \mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big),$$

and

$$\mathscr{F}_4(\eta) = \frac{1}{\Gamma(\alpha)} \sum_{\mu=0}^{\eta-1} (\eta - \sigma(\mu))^{(\alpha-1)} \Psi(\eta - \sigma(\mu)) \overline{\mathscr{F}}(\mu) \Delta Z_Q^H(\mu).$$

Using hypothesis (H_3), we obtain

$$\begin{aligned}
&\mathbb{E}\Big\|\big(\Psi(\eta+\rho) - \Psi(\eta)\big)\big[\phi(0) - \kappa(0, \phi, 0)\big]\Big\|^2 \\
&\quad \le 2F^2\, \mathbb{E}\|\phi(0) - \kappa(0, \phi, 0)\|^2.
\end{aligned}$$

Since $\Psi(\eta)$ is strongly continuous, it follows from the dominated convergence theorem that

$$\lim_{\rho \to 0} \mathbb{E}\Big\|\big(\Psi(\eta+\rho) - \Psi(\eta)\big)\big[\phi(0) - \kappa(0, \phi, 0)\big]\Big\|^2 = 0.$$

From assumption (H_8), we have

$$\mathbb{E}\|\mathscr{F}_1(\eta+\rho) - \mathscr{F}_1(\eta)\|^2 \to 0, \qquad |\rho| \to 0.$$

Next, using assumptions (H_3), (H_4), (H_6), and (H_7), we obtain

$$\begin{aligned}
&\mathbb{E}\|\mathscr{F}_2(\eta+\rho)-\mathscr{F}_2(\eta)\|^2\\
&\leq 2\,\mathbb{E}\Bigg\|\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}\Big[\Psi(\eta+\rho-\sigma(\mu))-\Psi(\eta-\sigma(\mu))\Big]\\
&\times(\eta-\sigma(\mu))^{(\alpha-1)}\Phi\big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\big)\Bigg\|^2\\
&+2\,\mathbb{E}\Bigg\|\frac{1}{\Gamma(\alpha)}\sum_{\mu=\eta}^{\eta+\rho-1}\Psi(\eta+\rho-\sigma(\mu))\\
&\times(\eta+\rho-\sigma(\mu))^{(\alpha-1)}\Phi\big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\big)\Bigg\|^2.
\end{aligned}\tag{6}$$

By estimating the terms on the right-hand side of (6), we obtain

$$\begin{aligned}
&\Big\|\big(\Psi(\eta+\rho-\sigma(\mu))-\Psi(\eta-\sigma(\mu))\big)\\
&\times\Phi\big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\big)\Big\|^2\frac{1}{\zeta^*}\\
&\leq 2F^2\Big[F_\Phi(1+F_{\upsilon_2})\|\vartheta_\mu\|^2+F_\Phi\widetilde{F}_{\upsilon_2}+\widetilde{F}_\Phi\Big]\frac{1}{\zeta^*},
\end{aligned}$$

and

$$\begin{aligned}
&\Big\|\Psi(\eta+\rho-\sigma(\mu))\Phi\big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\big)\Big\|^2\frac{1}{\zeta^*}\\
&\leq F^2\Big[F_\Phi(1+F_{\upsilon_2})\|\vartheta_\mu\|^2+F_\Phi\widetilde{F}_{\upsilon_2}+\widetilde{F}_\Phi\Big]\frac{1}{\zeta^*},
\end{aligned}$$

where

$$\zeta=\Gamma(\alpha)(\eta-\sigma(\mu))^{(1-\alpha)},$$

and

$$\|\zeta\|=\zeta^*.$$

Using the dominated convergence theorem together with the above inequalities and the norm continuity of $\Psi(\eta)$, we conclude that

$$\mathbb{E}\|\mathscr{F}_2(\eta+\rho)-\mathscr{F}_2(\eta)\|^2\to 0,\qquad |\rho|\to 0.$$

Next, using assumptions (H_3) and (H_{10}), we obtain

$$\begin{aligned}
&\mathbb{E}\|\mathscr{F}_3(\eta+\rho)-\mathscr{F}_3(\eta)\|^2\\
&\leq 2\,\mathbb{E}\Bigg\|\sum_{0<\eta_i<\eta}\Big[\Psi(\eta+\rho-\eta_i)-\Psi(\eta-\eta_i)\Big]\\
&\times\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)\Bigg\|^2\\
&+2\,\mathbb{E}\Bigg\|\sum_{\eta<\eta_i<\eta+\rho}\Psi(\eta+\rho-\eta_i)\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)\Bigg\|^2.
\end{aligned}$$

Evaluating the terms on the right-hand side, we obtain

$$\begin{aligned}
&\Big\|\big(\Psi(\eta+\rho-\eta_i)-\Psi(\eta-\eta_i)\big)\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)\Big\|^2\\
&\leq\|\Psi(\eta+\rho-\eta_i)-\Psi(\eta-\eta_i)\|^2\Big\|\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)\Big\|^2\\
&\leq 2F^2d_i^2\|\vartheta(\overline{\eta_i})\|^2,
\end{aligned}$$

and

$$\begin{aligned}
&\Big\|\Psi(\eta+\rho-\eta_i)\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)\Big\|^2\\
&\leq F^2d_i^2\|\vartheta(\overline{\eta_i})\|^2.
\end{aligned}$$

Hence,

$$\mathbb{E}\|\mathscr{F}_3(\eta+\rho)-\mathscr{F}_3(\eta)\|^2\to 0,\qquad |\rho|\to 0.$$

Now,

$$\begin{aligned}
&\mathbb{E}\|\mathscr{F}_4(\eta+\rho)-\mathscr{F}_4(\eta)\|^2\\
&\leq 2\mathbb{E}\Bigg\|\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}\Big[\Psi(\eta+\rho-\sigma(\mu))-\Psi(\eta-\sigma(\mu))\Big]\\
&\times(\eta-\sigma(\mu))^{(\alpha-1)}\overline{\mathscr{F}}(\mu)\Delta Z_Q^H(\mu)\Bigg\|^2\\
&+2\mathbb{E}\Bigg\|\frac{1}{\Gamma(\alpha)}\sum_{\mu=\eta}^{\eta+\rho-1}\Psi(\eta+\rho-\sigma(\mu))\\
&\times(\eta+\rho-\sigma(\mu))^{(\alpha-1)}\overline{\mathscr{F}}(\mu)\Delta Z_Q^H(\mu)\Bigg\|^2\\
&=:\mathcal{N}_1+\mathcal{N}_2.
\end{aligned}$$

Using Lemma 1 in [8] together with assumption (H_3), we get

$$\begin{aligned}
\mathcal{N}_1&\leq 2c(H)\eta^{2H-1}\sum_{\mu=0}^{\eta-1}\\
&\times\Big\|\big[\Psi(\eta+\rho-\sigma(\mu))-\Psi(\eta-\sigma(\mu))\big]\overline{\mathscr{F}}(\mu)\Big\|^2_{L_Q^0}\frac{1}{\zeta^*}.
\end{aligned}$$

Furthermore,

$$\mathcal{N}_1\leq 2c(H)\eta^{2H-1}F^2\sum_{\mu=0}^{\eta-1}\|\overline{\mathscr{F}}(\mu)\|^2_{L_Q^0}\frac{1}{\zeta^*}\to 0,\ |\rho|\to 0.$$

Indeed, for every fixed μ,

$$\begin{aligned}
&\Big\|\big[\Psi(\eta+\rho-\sigma(\mu))-\Psi(\eta-\sigma(\mu))\big]\overline{\mathscr{F}}(\mu)\Big\|^2_{L_Q^0}\\
&\leq 2F^2\|\overline{\mathscr{F}}(\mu)\|^2_{L_Q^0}.
\end{aligned}$$

Similarly,

$$\mathcal{N}_2\leq 2c(H)\eta^{2H-1}F^2\sum_{\mu=\eta}^{\eta+\rho-1}\|\overline{\mathscr{F}}(\mu)\|^2_{L_Q^0}\frac{1}{\zeta^*}\to 0,\ |\rho|\to 0.$$

Furthermore, we have

$$\lim_{\rho\to 0}\mathbb{E}\|\mathscr{F}_4(\eta+\rho)-\mathscr{F}_4(\eta)\|^2=0.$$

Consequently,

$$\lim_{\rho\to 0}\mathbb{E}\Big\|(\aleph\vartheta)(\eta+\rho)-(\aleph\vartheta)(\eta)\Big\|^2=0.$$

Hence, the mapping

$$\eta\longmapsto(\aleph\vartheta)(\eta)$$

is continuous on the discrete interval $\mathcal{J}$.

Therefore,

$$\aleph:\widehat{\mathscr{B}}_{h,\theta}\to\widehat{\mathscr{B}}_{h,\theta}$$

is well defined.

Claim 2. The operator $\aleph$ is a contraction mapping.

Let $\vartheta, y\in\widehat{\mathscr{B}}_{h,\theta}$. Then, for every $\eta\in\mathcal{J}$, we have

$$\begin{aligned}
&\|(\aleph\vartheta)(\eta)-(\aleph y)(\eta)\|^2\\
&\le\frac{1}{k}\Big\|\kappa\big(\eta,\vartheta_\eta,A_1(\vartheta(\eta))\big)-\kappa\big(\eta,y_\eta,A_1(y(\eta))\big)\Big\|^2\\
&+\frac{3}{1-k}\Bigg[\Bigg\|\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{(\alpha-1)}\Psi(\eta-\sigma(\mu))\\
&\times\Big(\Phi\big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\big)-\Phi\big(\mu,y_\mu,A_2(y(\mu))\big)\Big)\Bigg\|^2\\
&+\Bigg\|\sum_{0<\eta_i<\eta}\Psi(\eta-\eta_i)\Big(\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)-\mathcal{I}_i\big(y(\overline{\eta_i})\big)\Big)\Bigg\|^2\Bigg].
\end{aligned}$$

Consequently,

$$\begin{aligned}
&\|(\aleph\vartheta)(\eta)-(\aleph y)(\eta)\|^2\\
&\le\frac{1}{k}\Big\|\kappa\big(\eta,\vartheta_\eta,A_1(\vartheta(\eta))\big)-\kappa\big(\eta,y_\eta,A_1(y(\eta))\big)\Big\|^2\\
&+\frac{3}{1-k}\Bigg\|\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{(\alpha-1)}\Psi(\eta-\sigma(\mu))\\
&\times\Big(\Phi\big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\big)-\Phi\big(\mu,y_\mu,A_2(y(\mu))\big)\Big)\Bigg\|^2\\
&+\frac{3}{1-k}\Bigg\|\sum_{0<\eta_i<\eta}\Psi(\eta-\eta_i)\Big(\mathcal{I}_i\big(\vartheta(\overline{\eta_i})\big)-\mathcal{I}_i\big(y(\overline{\eta_i})\big)\Big)\Bigg\|^2.
\end{aligned}$$

Using Hölder's inequality together with the Lipschitz conditions of κ, Φ, and $\mathcal{I}_i$, $i=1,2,\dots$, we obtain

$$\begin{aligned}
&\mathbb{E}\|(\aleph\vartheta)(\eta)-(\aleph y)(\eta)\|^2\\
&\le\mathcal{K}\,\mathbb{E}\|\vartheta_\eta-y_\eta\|^2\\
&\quad+\frac{3}{1-k}\eta F^2\widetilde{F}^{\,2}\sum_{\mu=0}^{\eta-1}\mathbb{E}\|\vartheta_\mu-y_\mu\|^2\frac{1}{\zeta^*},
\end{aligned}$$

where $\mathcal{K}=\frac{F_\kappa(1+F_{v_1})}{1-k}$.

Hence, $\begin{aligned}&\sup_{\mu\in[-\varrho,\eta]}\mathbb{E}\|(\aleph\vartheta)(\mu)-(\aleph y)(\mu)\|^2\\&\le\omega(\eta)\sup_{\mu\in[-\varrho,\eta]}\mathbb{E}\|\vartheta(\mu)-y(\mu)\|^2,\end{aligned}$ where

$$\omega(\eta)=\mathcal{K}+\frac{3F^2\widetilde{F}^{\,2}}{1-k}\eta^2+\frac{3F^2}{1-k}\left(\sum_{i=1}^{\infty}d_i\right)^2.$$

By condition (4), we have

$$\omega(0)=\mathcal{K}+\frac{3F^2}{1-k}\left(\sum_{i=1}^{\infty}d_i\right)^2<1.$$

Therefore, there exists a sufficiently small constant $\theta_1>0$ such that

$$0<\theta_1\le\theta\qquad\text{and}\qquad 0<\omega(\theta_1)<1.$$

Hence, $\aleph$ is a contraction mapping on $\widehat{\mathscr{B}}_{h,\theta}$. Hence, $\mathcal{N}$ is a contraction mapping on $B_{h,\theta}$.

Since $B_{h,\theta}$ is a complete Banach space and $\mathcal{N}$ maps $B_{h,\theta}$ into itself, the Banach Fixed Point Theorem guarantees the existence of a fixed point $\vartheta\in B_{h,\theta}$ such that

$$\mathcal{N}\vartheta=\vartheta.$$

This fixed point is the mild solution of system (1)–(3). Moreover, since $\mathcal{N}$ is a contraction mapping, the fixed point is unique.

Consequently, by the Banach Fixed Point Theorem, the impulsive fractional stochastic difference system (1)–(3) admits a unique mild solution on $[-\varrho,\theta]$. Consequently, by Banach's fixed point theorem, the impulsive fractional stochastic difference system (1)–(3) admits a unique mild solution on $[-\varrho,\theta]$.

4. Mean Square Exponential Stability

4.1. Mean Square Exponential Stability

In this section, we investigate the mean square exponential stability of the mild solution for the impulsive neutral stochastic fractional difference system driven by a Rosenblatt process.

Definition 4.1. The mild solution $\vartheta(\eta)$ of system (1)–(3) is said to be mean square exponentially stable if there exist positive constants M and λ such that

$$\mathcal{E}\|\vartheta(\eta)\|^2\le Me^{-\lambda\eta}\mathcal{E}\|\varphi\|^2_{Bh},\quad\eta\ge 0.$$

Theorem 4.1. Assume that hypotheses $(H1)$–$(H10)$ are satisfied. Further suppose that there exist positive constants $\delta_1,\delta_2,\delta_3$ such that

$$\|\Psi(\eta)\|\le\delta_1 e^{-\delta_2\eta},\quad\eta\ge 0,$$

and

$$3\delta_1^2\left[\frac{\widetilde{\Phi}^2(1+\widetilde{v}_2)^2}{\Gamma^2(\alpha)}+\left(\sum_{i=1}^{\infty}d_i\right)^2\right]<\delta_3<1.$$

Then the mild solution of system (1)–(3) is mean square exponentially stable.

Proof Let $\vartheta(\eta)$ be the mild solution of system (1)–(3). From the definition of mild solution, we have

$$\begin{aligned}\vartheta(\eta) &= \Psi(\eta)\Big[\varphi(0)-\kappa(0,\varphi,0)\Big]+\kappa\Big(\eta,\vartheta_\eta,A_1(\vartheta(\eta))\Big)\\ &+\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}\\ &\times\Psi(\eta-\sigma(\mu))\Phi\Big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\Big)\\ &+\sum_{0<\eta_i<\eta}\Psi(\eta-\eta_i)I_i(\vartheta(\eta_i))\\ &+\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}\\ &\times\Psi(\eta-\sigma(\mu))F(\mu)\Delta Z_Q^H(\mu).\end{aligned}$$

Using the inequality

$$\Big\|\sum_{j=1}^{5}x_j\Big\|^2\le 5\sum_{j=1}^{5}\|x_j\|^2,$$

we obtain

$$\begin{aligned}\mathcal{E}\|\vartheta(\eta)\|^2 \le& 5\mathcal{E}\Big\|\Psi(\eta)[\varphi(0)-\kappa(0,\varphi,0)]\Big\|^2\\ &+5\mathcal{E}\Big\|\kappa(\eta,\vartheta_\eta,A_1(\vartheta(\eta)))\Big\|^2\\ &+\frac{5}{\Gamma^2(\alpha)}\mathcal{E}\Big\|\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}\Psi(\eta-\sigma(\mu))\\ &\times\Phi(\mu,\vartheta_\mu,A_2(\vartheta(\mu)))\Big\|^2\\ &+5\mathcal{E}\Big\|\sum_{0<\eta_i<\eta}\Psi(\eta-\eta_i)I_i(\vartheta(\eta_i))\Big\|^2\\ &+\frac{5}{\Gamma^2(\alpha)}\mathcal{E}\Big\|\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}\\ &\times\Psi(\eta-\sigma(\mu))F(\mu)\Delta Z_Q^H(\mu)\Big\|^2.\end{aligned}$$

Using assumptions $(H3)$–$(H10)$ together with the Lipschitz continuity conditions, we derive

$$\mathcal{E}\|\vartheta(\eta)\|^2\le C_1e^{-2\delta_2\eta}\mathcal{E}\|\varphi\|_{Bh}^2+C_2\sup_{0\le\mu\le\eta}\mathcal{E}\|\vartheta(\mu)\|^2,$$

where

$$C_2=3\delta_1^2\left[\frac{\widetilde{\Phi}^2(1+\widetilde{\upsilon}_2)^2}{\Gamma^2(\alpha)}+\left(\sum_{i=1}^{\infty}d_i\right)^2\right].$$

Since $C_2<1$, it follows that

$$\sup_{0\le\mu\le\eta}\mathcal{E}\|\vartheta(\mu)\|^2\le\frac{C_1}{1-C_2}e^{-2\delta_2\eta}\mathcal{E}\|\varphi\|_{Bh}^2.$$

Hence,

$$\mathcal{E}\|\vartheta(\eta)\|^2\le Me^{-\lambda\eta}\mathcal{E}\|\varphi\|_{Bh}^2,$$

where

$$M=\frac{C_1}{1-C_2},\qquad \lambda=2\delta_2.$$

Therefore, the mild solution of system (1)–(3) is mean square exponentially stable.

4.2. Discrete Grönwall Inequality

The following result is required in the stability analysis.

Lemma 4.1. Let $\{u(\eta)\}_{\eta\in\mathbb{N}_0}$ be a nonnegative sequence satisfying

$$u(\eta)\le a+b\sum_{\mu=0}^{\eta-1}u(\mu),\quad \eta\ge 0,$$

where $a,b>0$ are constants. Then

$$u(\eta)\le a(1+b)^\eta,\quad \eta\ge 0.$$

Proof The conclusion follows by repeated application of the recursive inequality.

Theorem 4.2. Assume that conditions $(H1)$–$(H10)$ hold. Suppose further that there exist constants $M>0$ and $\lambda>0$ such that

$$\|\Psi(\eta)\|\le Me^{-\lambda\eta},\quad \eta\ge 0.$$

If

$$\Theta=\frac{4M^2\widetilde{\Phi}^2(1+\widetilde{\upsilon}_2)^2}{\Gamma^2(\alpha)}+4M^2\left(\sum_{i=1}^{\infty}d_i\right)^2<1,$$

then the mild solution of system (1)–(3) is mean square exponentially stable.

Proof Let $\vartheta(\eta)$ denote the mild solution of system (1)–(3). By the definition of mild solution, we have

$$\begin{aligned}\vartheta(\eta) &= \Psi(\eta)\Big[\varphi(0)-\kappa(0,\varphi,0)\Big]\\ &+\kappa\Big(\eta,\vartheta_\eta,A_1(\vartheta(\eta))\Big)\\ &+\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}\Psi(\eta-\sigma(\mu))\\ &\qquad\times\Phi\Big(\mu,\vartheta_\mu,A_2(\vartheta(\mu))\Big)\\ &+\sum_{0<\eta_i<\eta}\Psi(\eta-\eta_i)I_i(\vartheta(\eta_i))\\ &+\frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}\Psi(\eta-\sigma(\mu))F(\mu)\Delta Z_Q^H(\mu).\end{aligned}$$

Applying the inequality

$$\left\|\sum_{j=1}^{5}x_j\right\|^2\le 5\sum_{j=1}^{5}\|x_j\|^2,$$

and taking mathematical expectation, we obtain

$$\begin{aligned}\mathcal{E}\|\vartheta(\eta)\|^2 \leq & C_1 e^{-2\lambda\eta}\mathcal{E}\|\varphi\|^2_{Bh} \\ & + C_2 \sup_{0\leq\mu\leq\eta} \mathcal{E}\|\vartheta(\mu)\|^2 \\ & + \frac{4M^2}{\Gamma^2(\alpha)}\mathcal{E}\left\|\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}F(\mu)\Delta Z_Q^H(\mu)\right\|^2,\end{aligned}$$

where

$$C_2 = \frac{4M^2\widetilde{\Phi}^2(1+\widetilde{\upsilon}_2)^2}{\Gamma^2(\alpha)} + 4M^2\left(\sum_{i=1}^{\infty} d_i\right)^2.$$

Using the stochastic estimate corresponding to the Rosenblatt process yields

$$\begin{aligned}&\mathcal{E}\left\|\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{\alpha-1}F(\mu)\Delta Z_Q^H(\mu)\right\|^2 \\ \leq & c(H)\sum_{\mu=0}^{\eta-1}(\eta-\sigma(\mu))^{2\alpha-2}\|F(\mu)\|^2_{L_Q^0},\end{aligned}$$

where $c(H) > 0$ depends only on the Hurst parameter H.

Consequently, there exists a constant $C_3 > 0$ such that

$$\mathcal{E}\|\vartheta(\eta)\|^2 \leq C_3 e^{-2\lambda\eta} + \Theta\sum_{\mu=0}^{\eta-1}\mathcal{E}\|\vartheta(\mu)\|^2.$$

Applying the discrete Grönwall inequality, we derive

$$\mathcal{E}\|\vartheta(\eta)\|^2 \leq C_3 e^{-2\lambda\eta}(1+\Theta)^\eta.$$

Since $\Theta < 1$, there exists $\delta > 0$ such that

$$(1+\Theta)^\eta \leq e^{\delta\eta}.$$

Therefore,

$$\mathcal{E}\|\vartheta(\eta)\|^2 \leq C_3 e^{-(2\lambda-\delta)\eta}.$$

Hence, the mild solution of system (1)–(3) is mean square exponentially stable.

5. Numerical Example and Simulation

In this section, we present a numerical example to illustrate the applicability and effectiveness of the obtained theoretical results for the impulsive neutral stochastic fractional difference system driven by a Rosenblatt process.

Consider the following stochastic fractional difference equation:

$$\begin{aligned}\Delta^\alpha\left[\vartheta(\eta)-\frac{1}{8}\vartheta(\eta-1)\right] =& -\lambda\left[\vartheta(\eta)-\frac{1}{8}\vartheta(\eta-1)\right] \\ & + \frac{1}{10}\sum_{\mu=0}^{\eta-1}e^{-0.2(\eta-\mu)}\vartheta(\mu) \\ & + \frac{1}{12}\sin(\vartheta(\eta)) + 0.05\Delta Z_Q^H(\eta), \\ & \eta\neq\eta_i,\end{aligned} \tag{7}$$

subject to the impulsive condition

$$\Delta\vartheta(\eta_i) = \frac{1}{15}\vartheta(\eta_i), \quad i = 1,2,\ldots, \tag{8}$$

and the initial condition

$$\vartheta(\eta) = 0.5, \quad -1\leq\eta\leq 0. \tag{9}$$

For the numerical simulation, we choose

$$\alpha = 0.8, \qquad \lambda = 0.6, \qquad H = 0.75.$$

The discrete fractional numerical approximation corresponding to system (7)–(9) is given by

$$\begin{aligned}\vartheta_{\eta+1} =& \vartheta_\eta + \frac{1}{\Gamma(\alpha)}\sum_{\mu=0}^{\eta}(\eta-\mu+1)^{\alpha-1}\Bigg[-\lambda\Big(\vartheta_\mu - \frac{1}{8}\vartheta_{\mu-1}\Big) \\ & + \frac{1}{10}\sum_{j=0}^{\mu-1}e^{-0.2(\mu-j)}\vartheta_j + \frac{1}{12}\sin(\vartheta_\mu)\Bigg] \\ & + 0.05\Delta Z_Q^H(\eta).\end{aligned} \tag{10}$$

The impulsive perturbation is imposed at discrete times

$$\eta_1 = 5, \qquad \eta_2 = 10, \qquad \eta_3 = 15.$$

Using MATLAB simulation, the approximate trajectories of the solution are obtained over the interval

$$0\leq\eta\leq 20.$$

The numerical results indicate that the solution trajectories remain bounded and converge asymptotically toward the equilibrium state despite the presence of stochastic perturbations and impulsive effects. Furthermore, the simulation confirms the mean square exponential stability established in the theoretical analysis.

The graphical behavior demonstrates that the combined influence of the fractional memory term and impulsive perturbations does not destroy the stability of the system. Hence, the proposed theoretical results are validated numerically.

6. Conclusion

The present work investigated neutral stochastic fractional difference systems with impulsive perturbations under the influence of a Rosenblatt process. Existence and uniqueness results for mild solutions were obtained through fixed point arguments and resolvent operator methods. Moreover, sufficient conditions guaranteeing mean square exponential stability were derived. A representative numerical example was included to demonstrate the applicability of the theoretical results. These findings enrich the current literature on discrete fractional stochastic systems and provide a useful framework for the analysis of models involving memory effects and long-range dependence.

Author Contributions

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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