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# On the Structural Properties of Translations of Neutrosophic Fuzzy Ideals in B-Algebras

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**Abstract:** In this article, we introduce and investigate the concepts of neutrosophic fuzzy ideals and their translations in B-algebras. We establish the fundamental properties of translations of neutrosophic fuzzy ideals and prove that the translation of a neutrosophic fuzzy ideal of a B-algebra is also a neutrosophic fuzzy ideal. Furthermore, we characterize neutrosophic fuzzy ideals through their level sets and show that a neutrosophic fuzzy set is a neutrosophic fuzzy ideal if and only if its corresponding level sets are also ideals. We also provide sufficient conditions under which the neutrosophic translation of a neutrosophic fuzzy ideal forms a neutrosophic fuzzy subalgebra. In addition, we introduce the concept of an extension of a neutrosophic fuzzy ideal and prove that the neutrosophic translation of a neutrosophic fuzzy ideal is a neutrosophic fuzzy ideal extension. An illustrative example is presented to demonstrate that not every neutrosophic fuzzy ideal extension is obtained as a translation of a neutrosophic fuzzy ideal. Moreover, we define the multiplication of neutrosophic fuzzy ideals and investigate its properties in B-algebras. We prove that the multiplication of neutrosophic fuzzy ideals is also a neutrosophic fuzzy ideal under appropriate conditions. The results establish meaningful relationships among neutrosophic fuzzy ideals, translations, level sets, extensions, and multiplication in B-algebras. These findings contribute to the development of neutrosophic fuzzy algebraic structures and provide a systematic framework for further investigations of ideals and their associated operations in B-algebras. The proposed characterizations also clarify structural behavior and broaden the existing theoretical framework for neutrosophic fuzzy algebraic systems.

**Keywords:** B-algebra, Translations, Multiplications, Neutrosophic Fuzzy Ideal, Neutrosophic Fuzzy Ideal Extension

## 1. Introduction

Zadeh,L.A.[3] developed the concept of fuzzy sets in 1965 as a mathematical method for handling uncertainty, expanding on the concept of standard crisp sets. In the subsequent decades, many researchers investigated different ways to expand on this idea. In 1986 [2], Atanassov,K.T proposed the concept of Intuitionistic Fuzzy Sets (IFS), where the truth value is determined by both membership and non-membership degrees, subject to the limitation that their sum does not exceed unity. Smarandache [11] developed the idea of Neutrosophic Sets (NS) in order to achieve a broader framework for addressing uncertainty. This structure generalizes fuzzy and intuitionistic fuzzy sets by adding an indeterminacy membership function ($I$) to the truth ($T$) and falsity ($F$) components. Neutrosophic logic has subsequently been utilised in a variety of fields, including game theory, operations research and information technology. By starting to study neutrosophic algebraic structures in 2006, Smarandache and Kandasamy [12] made the connection between neutrosophic logic and abstract algebra.

Algebraic structures play a crucial role in mathematics, with numerous applications in computer science, theoretical physics, and coding theory. Neggers and Kim [1] introduced the notion of B-algebras within this category, establishing significant connections with algebraic structures especially BCK/BCI-algebras. Imai and Isaki [7] started the investigation of BCK/BCI-algebras by expanding concepts from propositional calculus and set-theoretic difference. The subsequent studies by Walendziak [15] and Park and Kim [13] contributed to improving the structural properties and axiomatic foundations of B-algebras.

Fuzzy B-algebras were developed through the integration of the concept of fuzzy sets in B-algebras. Fuzzy (normal) B-algebras were initially presented by Jun et al. [8], and further research by Saeid focused on fuzzy topological B-algebras [16] and interval-valued fuzzy B-subalgebras [17]. In order to advance the general theory of these algebraic structures, ideals and subalgebras are key components. The concepts of fuzzy subalgebras and fuzzy closed ideals of B-algebras were proposed by Senapati et al. [18, 19] with respect to $t$-norms. These studies subsequently led to the development of Intuitionistic fuzzy B-algebras.

The introduction of the concept of translations is one of the significant advancements in fuzzy algebraic structures. In 2011, Jun [20] investigated fuzzy translations of fuzzy ideals in BCK/BCI-algebras. Subsequent investigations by Prasanna, Premkumar, and Mohideen, as well as Senapati [6, 21], focused on fuzzy and intuitionistic fuzzy translations in $B$-algebras. Although this research examined the variation of membership values in fuzzy and intuitionistic fuzzy systems, their development to the neutrosophic framework has not yet been investigated. To address this research gap, the present study introduces neutrosophic fuzzy translations of B-algebras and analyses their fundamental properties and characterisations.

In this paper, neutrosophic fuzzy translations, neutrosophic fuzzy extensions and neutrosophic fuzzy multiplications of neutrosophic fuzzy ideals (NFids) in B-algebras are introduced. Relations among neutrosophic fuzzy translations, neutrosophic fuzzy extensions and neutrosophic fuzzy multiplications of NFids in B-algebras are also investigated.

## 2. Preliminaries

In this section, we first present some elementary concepts and results that will be used in the subsequent sections.

*Definition* 2.1. [1] A non-empty set $M$ with a constant ' $0$ ' and a binary operation ' $\diamond$ ' is called a B-algebra if it satisfies the following axioms

$(\mathcal{BA}-1)$ $\mathfrak{q}_1 \diamond \mathfrak{q}_1 = 0$,

$(\mathcal{BA}-2)$ $\mathfrak{q}_1 \diamond 0 = \mathfrak{q}_1$,

$(\mathcal{BA}-3)$ $(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \diamond \mathfrak{q}_3 = \mathfrak{q}_1 \diamond (\mathfrak{q}_3 \diamond (0 \diamond \mathfrak{q}_2))$, for all $\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3 \in M$.

A slightly different set of axioms was independently introduced in [2].

*Definition* 2.2. [1] non-empty sub-set $A$ of a B-algebra $M$ is said to be a sub-algebra of $M$ if $\mathfrak{q}_1 \diamond \mathfrak{q}_2 \in A$ for any $\mathfrak{q}_1, \mathfrak{q}_2 \in A$.

*Definition* 2.3. [4] A non-empty sub-set $D$ of a B-algebra $M$ is said to be an ideal of $M$ if

$(D_1)$ $0 \in D$

$(D_2)$ $\mathfrak{q}_1 \diamond \mathfrak{q}_2 \in D$ for any $\mathfrak{q}_2 \in D \Rightarrow \mathfrak{q}_1 \in D$.

*Definition* 2.4. [3] Let $M$ be a B-algebra. Then a fuzzy set $\mathcal{B}$ in $M$ is defined as

$$\mathcal{B} = \{\langle \mathfrak{q}_1, \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1)\rangle \mid \mathfrak{q}_1 \in M\}$$

where $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1)$ is called the membership value of $\mathfrak{q}_1$ in $\mathcal{B}$ and $0 \leq \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) \leq 1$.

*Definition* 2.5. [5] Let $\mathcal{B}$ be a fuzzy set in $M$ and let $\gamma \in [0, 1 - \sup\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) | \mathfrak{q}_1 \in M\}]$. A mapping $(\mathcal{B}_{\mathcal{T}})^T_\gamma : M \to [0,1]$ is said to be a fuzzy-$\gamma$-translation of $\mathcal{B}$ if it fulfill

$$(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma$$

for all $\mathfrak{q}_1 \in M$.

*Definition* 2.6. [11] A neutrosophic fuzzy set (briefly, NFS) $\mathcal{B}$ over $M$ is an object having the form

$$\mathcal{B} = \{\langle \mathfrak{q}_1, \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1)\rangle \mid \mathfrak{q}_1 \in M\}$$

where $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) : M \to [0,1]$, $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) : M \to [0,1]$ and $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) : M \to [0,1]$, with the condition $0 \leq \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) \leq 3$ for all $\mathfrak{q}_1 \in M$. The numbers $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1)$, $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1)$ and $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1)$ denote, respectively, the degree of membership, the degree of indeterminacy and the degree of non-membership of the element $\mathfrak{q}_1$ in the set $\mathcal{B}$. NFS simply denote by $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$

*Definition* 2.7. [9] Let $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ be a NFS of $M$ and let $\gamma \in [0, T]$. An object having the form $\mathcal{B}^T_\gamma = ((\mathcal{B}_{\mathcal{T}})^T_\gamma, (\mathcal{B}_{\mathcal{I}})^T_\gamma, (\mathcal{B}_{\mathcal{F}})^T_\gamma)$ is called a neutrosophic fuzzy $\gamma$-translation (briefly, NF-$\gamma$-T) of $\mathcal{B}$ if $(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma$, $(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma$ and $(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma$ for all $\mathfrak{q}_1 \in M$.

*Definition* 2.8. [9] Let $\mathcal{B}_1 = (\mathcal{B}_{1\mathcal{T}}, \mathcal{B}_{1\mathcal{I}}, \mathcal{B}_{1\mathcal{F}})$, $\mathcal{B}_2 = (\mathcal{B}_{2\mathcal{T}}, \mathcal{B}_{2\mathcal{I}}, \mathcal{B}_{2\mathcal{F}})$ be NFSs of $M$. If $\mathcal{B}_1 \leq \mathcal{B}_2$, that is, $\mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_1) \leq \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_1)$, $\mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_1) \leq \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_1)$ and $\mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_1) \geq \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_1)$ for all $\mathfrak{q}_1 \in M$. Then we say that $\mathcal{B}_2$ is a neutrosophic fuzzy extension (briefly, NFe) of $\mathcal{B}_1$.

*Definition* 2.9. [9] A NFS $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ in $M$ is said to be a neutrosophic fuzzy subalgebra (briefly, NFSA) of $M$ if it fulfill

$(NFSA_1)$ $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \geq \min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\}$

$(NFSA_2)$ $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \geq \min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\}$ and

$(NFSA_3)$ $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \leq \max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2)\}$, for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$.

*Definition* 2.10. [9] Let $\mathcal{B}$ be a NFS of $M$ and $\varsigma \in [0,1]$. An object having the form $\mathcal{B}^s_\varsigma = ((\mathcal{B}_{\mathcal{T}})^s_\varsigma, (\mathcal{B}_{\mathcal{I}})^s_\varsigma, (\mathcal{B}_{\mathcal{F}})^s_\varsigma)$ is called a neutrosophic fuzzy $\varsigma$-multiplication (briefly, NF-$\varsigma$-Mu) of $\mathcal{B}$ if $(\mathcal{B}_{\mathcal{T}})^s_\varsigma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1).\varsigma$, $(\mathcal{B}_{\mathcal{I}})^s_\varsigma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1).\varsigma$ and $(\mathcal{B}_{\mathcal{F}})^s_\varsigma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1).\varsigma$ for all $\mathfrak{q}_1 \in M$.

For any NFS $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ of $M$, a neutrosophic fuzzy 0-multiplication $\mathcal{B}^s_0 = ((\mathcal{B}_{\mathcal{T}})^s_0, (\mathcal{B}_{\mathcal{I}})^s_0, (\mathcal{B}_{\mathcal{F}})^s_0)$ of $\mathcal{B}$ is a NFSA of $M$.

# 3. Translations of Neutrosophic Fuzzy Ideals in B-algebras

In this section, translations of neutrosophic fuzzy ideals are defined with some results studied.

*Definition* 3.1. A NFS $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ in $M$ is said to be a neutrosophic fuzzy ideal (briefly, NFid) of $M$ if it fulfill

$(NFid_1)$ $\mathcal{B}_{\mathcal{T}}(0) \geq \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1)$, $\mathcal{B}_{\mathcal{I}}(0) \geq \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1)$, $\mathcal{B}_{\mathcal{F}}(0) \leq \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1)$,

$(NFid_2)$ $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) \geq \min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\}$

$(NFid_3)$ $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) \geq \min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\}$ and

$(NFid_4)$ $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) \leq \max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2)\}$, for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$.

*Theorem* 3.1. If $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ is a NFid of $M$, then the NF-$\gamma$-T $\mathcal{B}^T_\gamma = ((\mathcal{B}_{\mathcal{T}})^T_\gamma, (\mathcal{B}_{\mathcal{I}})^T_\gamma, (\mathcal{B}_{\mathcal{F}})^T_\gamma)$ of $\mathcal{B}$ is a NFid, for all $\gamma \in [0, T]$.

*Proof.* Let $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ be a NFids of $M$ and $\gamma \in [0, T]$. Then

$$\begin{aligned}
(\mathcal{B}_{\mathcal{T}})^T_\gamma(0) &= \mathcal{B}_{\mathcal{T}}(0) + \gamma \geq \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma = (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1),\\
(\mathcal{B}_{\mathcal{I}})^T_\gamma(0) &= \mathcal{B}_{\mathcal{I}}(0) + \gamma \geq \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma = (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1),\\
(\mathcal{B}_{\mathcal{F}})^T_\gamma(0) &= \mathcal{B}_{\mathcal{F}}(0) - \gamma \leq \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma = (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1),
\end{aligned}$$

for all $\mathfrak{q}_1 \in M$. Now,

$$\begin{aligned}
(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma\\
&\geq \min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\} + \gamma\\
&= \min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) + \gamma, \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) + \gamma\}\\
&= \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\}\\
(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma\\
&\geq \min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\} + \gamma\\
&= \min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) + \gamma, \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) + \gamma\}\\
&= \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\}\\
(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma\\
&\leq max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2)\} - \gamma\\
&= \max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) - \gamma, \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) - \gamma\}\\
&= \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\}
\end{aligned}$$

for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$. Therefore, the NF-$\gamma$-T of $\mathcal{B}$ is a NFid of $M$.

*Theorem* 3.2. Let $\mathcal{B}$ be a NFS of $M$ such that the NF-$\gamma$-T of $\mathcal{B}$ is a NFid of $M$ for some $\gamma \in [0, T]$. Then $\mathcal{B}$ is a NFid of $M$.

*Proof.* Suppose that $\mathcal{B}^T_\gamma$ is a NFid of $M$ for some $\gamma \in [0, T]$. Let $\mathfrak{q}_1, \mathfrak{q}_2 \in M$, we have

$$\begin{aligned}
\mathcal{B}_{\mathcal{T}}(0) + \gamma &= (\mathcal{B}_{\mathcal{T}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma\\
\mathcal{B}_{\mathcal{I}}(0) + \gamma &= (\mathcal{B}_{\mathcal{I}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma\\
\mathcal{B}_{\mathcal{F}}(0) - \gamma &= (\mathcal{B}_{\mathcal{F}})^T_\gamma(0) \leq (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma
\end{aligned}$$

$$\Rightarrow \mathcal{B}_{\mathcal{T}}(0) \geq \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1), \mathcal{B}_{\mathcal{I}}(0) \geq \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1)$$
$$\mathcal{B}_{\mathcal{F}}(0) \leq \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1)$$

Now, we have

$$\begin{aligned}
\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma &= (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1)\\
&\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) + \gamma, \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) + \gamma\}\\
&= \min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\} + \gamma\\
\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma &= (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1)\\
&\geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) + \gamma, \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) + \gamma\}\\
&= \min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\} + \gamma\\
\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma &= (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1)\\
&\leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) - \gamma, \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) - \gamma\}\\
&= \max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2)\} - \gamma
\end{aligned}$$

Therfore,

$$\begin{aligned}
\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) &\geq \min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\}\\
\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) &\geq \min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\}\\
\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) &\leq \max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2)\}
\end{aligned}$$

for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$. Therefore, $\mathcal{B}$ is a NFIs of $M$.

*Theorem* 3.3. Let the NF-$\gamma$-T of $\mathcal{B}$ be a NFid of $M$ for $\gamma \in [0, T]$. Then the following are valid:

(i). If $\mathfrak{q}_1 \diamond \mathfrak{q}_2 \leq \mathfrak{q}_3$, then

$$\left(\begin{array}{c}
(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_3)\right\}\\
(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_3)\right\}\\
(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) \leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_3)\right\}
\end{array}\right)$$

(ii). If $\mathfrak{q}_1 \leq \mathfrak{q}_2$, then $(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \geq (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)$, $(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \geq (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)$ and $(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) \leq (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)$.

*Proof.* (i) Suppose that $\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3 \in M$ are such that $\mathfrak{q}_1 \diamond \mathfrak{q}_2 \leq \mathfrak{q}_3$. Then $(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \diamond \mathfrak{q}_3 = 0$ and thus

$$\begin{aligned}
&(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \min\left\{\min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(0), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_3)\right\}, (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_3)\right\}\\
&(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \min\left\{\min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(0), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_3)\right\}, (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_3)\right\}\\
&(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) \leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \max\left\{\max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(0), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_3)\right\}, (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\}\\
&= \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_3)\right\}
\end{aligned}$$

(ii) Again take $\mathfrak{q}_1, \mathfrak{q}_2 \in M$ such that $\mathfrak{q}_1 \leq \mathfrak{q}_2$. Then $\mathfrak{q}_1 \diamond \mathfrak{q}_2 = 0$ and thus

$$\begin{aligned}
(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) &\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\} \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(0), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\} = (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \\
(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) &\geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\} \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(0), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\} = (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \\
(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) &\leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\} \\
&\leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(0), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\} = (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2).
\end{aligned}$$

Using induction on $n$ and by Theorem 3.2, we can easily prove that the following theorems.

*Theorem* 3.4. If the NF-$\gamma$-T of $\mathcal{B}$ be a NFid of $M$ for all $\gamma \in [0, T]$, then

$(\ldots((\mathfrak{q}_1 \diamond v_1) \diamond v_2) \diamond \ldots) \diamond v_n = 0$ for any $\mathfrak{q}_1, v_1, v_2, \ldots, v_n \in M, \Rightarrow$

$$\left(\begin{array}{c}
(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \\
\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(v_1), (\mathcal{B}_{\mathcal{T}})^T_\gamma(v_2), \ldots, (\mathcal{B}_{\mathcal{T}})^T_\gamma(v_n)\right\} \\
(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \\
\geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(v_1), (\mathcal{B}_{\mathcal{I}})^T_\gamma(v_2), \ldots, (\mathcal{B}_{\mathcal{I}})^T_\gamma(v_n)\right\} \\
(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) \\
\leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(v_1), (\mathcal{B}_{\mathcal{F}})^T_\gamma(v_2), \ldots, (\mathcal{B}_{\mathcal{F}})^T_\gamma(v_n)\right\}
\end{array}\right)$$

We now give a condition for the NF-$\gamma$-T of $\mathcal{B}$ which is a NFid of $M$ to be a NFSA of $M$.

*Theorem* 3.5. Suppose that NF-$\gamma$-T of $\mathcal{B}$ is a NFid of $M$ for all $\gamma \in [0, T]$. If $\mathfrak{q}_1 \diamond \mathfrak{q}_2 \leq \mathfrak{q}_1$ for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$, then $\mathcal{B}^T_\gamma$ is a NFSA of $M$.

*Proof.* Assume that $\mathcal{B}^T_\gamma$ of $\mathcal{B}$ is a NFid of $M$. Suppose that $\mathfrak{q}_1 \diamond \mathfrak{q}_2 \leq \mathfrak{q}_1$ for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$. Then by utilizing Theorem 3.2, we obtain

$$\begin{aligned}
(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2) &\geq (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\} \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\} \\
(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2) &\geq (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\} \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\right\} \\
(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2) &\leq (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) \\
&\leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\} \\
&\leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\right\}
\end{aligned}$$

Therefore, $\mathcal{B}^T_\gamma$ is a NFSA of $M$.

*Theorem* 3.6. If $\mathcal{B}$ is a NFS of $M$ such that the NF-$\gamma$-T of $\mathcal{B}$ is a NFid of $M$ for $\gamma \in [0, T]$, then the sets

$$\begin{aligned}
\mathfrak{T}_{\mathcal{B}_{\mathcal{T}}} &= \left\{\mathfrak{q}_1 \in M \mid \mathfrak{q}_1 \in M \text{ and } (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{T}})^T_\gamma(0)\right\} \\
\mathfrak{T}_{\mathcal{B}_{\mathcal{I}}} &= \left\{\mathfrak{q}_1 \in M \mid \mathfrak{q}_1 \in M \text{ and } (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{I}})^T_\gamma(0)\right\} \\
\mathfrak{T}_{\mathcal{B}_{\mathcal{F}}} &= \left\{\mathfrak{q}_1 \in M \mid \mathfrak{q}_1 \in M \text{ and } (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{F}})^T_\gamma(0)\right\}
\end{aligned}$$

are ideals of $M$.

*Proof.* Assume that $\mathcal{B}^T_\gamma = ((\mathcal{B}_{\mathcal{T}})^T_\gamma, (\mathcal{B}_{\mathcal{I}})^T_\gamma, (\mathcal{B}_{\mathcal{F}})^T_\gamma)$ be a NFid of $M$. Obviously, $0 \in \mathfrak{T}_{\mathcal{B}_{\mathcal{T}}}, \mathfrak{T}_{\mathcal{B}_{\mathcal{I}}}, \mathfrak{T}_{\mathcal{B}_{\mathcal{F}}}$. Let $\mathfrak{q}_1, \mathfrak{q}_2 \in M$ be such that $\mathfrak{q}_1 \diamond \mathfrak{q}_2 \in \mathfrak{T}_{\mathcal{B}_{\mathcal{T}}}$ and $\mathfrak{q}_2 \in \mathfrak{T}_{\mathcal{B}_{\mathcal{T}}}$. Then

$$(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2) = (\mathcal{B}_{\mathcal{T}})^T_\gamma(0) = (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2) \text{ and so}$$

$$\begin{aligned}
(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) &\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\right\} \\
&= (\mathcal{B}_{\mathcal{T}})^T_\gamma(0)
\end{aligned}$$

Since $\mathcal{B}^T_\gamma$ is a NFid of $M$, we conclude that $(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{T}})^T_\gamma(0) \Rightarrow \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma = \mathcal{B}_{\mathcal{T}}(0) + \gamma$ or, $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(0)$ so that $\mathfrak{q}_1 \in \mathfrak{T}_{\mathcal{B}_{\mathcal{T}}}$. Hence, $\mathfrak{T}_{\mathcal{B}_{\mathcal{T}}}$ is an ideal of $M$.
Again, Let $p, s \in M$ be such that $p \diamond s \in \mathfrak{T}_{\mathcal{B}_{\mathcal{I}}}$ and $s \in \mathfrak{T}_{\mathcal{B}_{\mathcal{I}}}$. Then

$$(\mathcal{B}_{\mathcal{I}})^T_\gamma(p \diamond s) = (\mathcal{B}_{\mathcal{I}})^T_\gamma(0) = (\mathcal{B}_{\mathcal{I}})^T_\gamma(s) \text{ and so}$$

$$(\mathcal{B}_{\mathcal{I}})^T_\gamma(p) \geq \min\left\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(p \diamond s), (\mathcal{B}_{\mathcal{I}})^T_\gamma(s)\right\} = (\mathcal{B}_{\mathcal{I}})^T_\gamma(0)$$

Since $\mathcal{B}^T_\gamma$ is a NFid of $M$, we conclude that $(\mathcal{B}_{\mathcal{I}})^T_\gamma(p) = (\mathcal{B}_{\mathcal{I}})^T_\gamma(0) \Rightarrow \mathcal{B}_{\mathcal{I}}(p) + \gamma = \mathcal{B}_{\mathcal{I}}(0) + \gamma$ or, $\mathcal{B}_{\mathcal{I}}(p) = \mathcal{B}_{\mathcal{I}}(0)$ so that $p \in \mathfrak{T}_{\mathcal{B}_{\mathcal{I}}}$. Hence, $\mathfrak{T}_{\mathcal{B}_{\mathcal{I}}}$ is an ideal of $M$.

Now, again, let $v, w \in M$ be such that $v \diamond w \in \mathfrak{T}_{\mathcal{B}_{\mathcal{F}}}$ and $w \in \mathfrak{T}_{\mathcal{B}_{\mathcal{F}}}$. Then

$$(\mathcal{B}_{\mathcal{F}})^T_\gamma(v \diamond w) = (\mathcal{B}_{\mathcal{F}})^T_\gamma(0) = (\mathcal{B}_{\mathcal{F}})^T_\gamma(w) \text{ and so}$$

$$(\mathcal{B}_{\mathcal{F}})^T_\gamma(v) \leq \max\left\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(v \diamond w), (\mathcal{B}_{\mathcal{F}})^T_\gamma(w)\right\} = (\mathcal{B}_{\mathcal{F}})^T_\gamma(0).$$

Since $\mathcal{B}^T_\gamma$ is a NFid of $M$, we conclude that $(\mathcal{B}_{\mathcal{F}})^T_\gamma(v) = (\mathcal{B}_{\mathcal{F}})^T_\gamma(0). \Rightarrow \mathcal{B}_{\mathcal{F}}(v) + \gamma = \mathcal{B}_{\mathcal{F}}(0) + \gamma$ or, $\mathcal{B}_{\mathcal{F}}(v) = \mathcal{B}_{\mathcal{F}}(0)$ so that $v \in \mathfrak{T}_{\mathcal{B}_{\mathcal{F}}}$. Hence, $\mathfrak{T}_{\mathcal{B}_{\mathcal{F}}}$ is an ideal of $M$.

*Definition* 3.2. Let $\mathcal{B}_1 = (\mathcal{B}_{1\mathcal{T}}, \mathcal{B}_{1\mathcal{I}}, \mathcal{B}_{1\mathcal{F}})$, and $\mathcal{B}_2 = (\mathcal{B}_{2\mathcal{T}}, \mathcal{B}_{2\mathcal{I}}, \mathcal{B}_{2\mathcal{F}})$ be NFSs of $M$. Then $\mathcal{B}_2$ is called a neutrosophic fuzzy ideal extension (briefly, NFIe) of $\mathcal{B}$ if the following assertions are true:

(i). $\mathcal{B}_2$ is a NFe of $M$.

(ii). If $\mathcal{B}_1$ is a NFid of $M$, then $\mathcal{B}_2$ is a NFid of $M$.

From the definition of NF-$\gamma$-T, we get

$$\begin{aligned}
\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma, \\
(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma, \\
(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma.
\end{aligned}$$

Hence, we have the following theorem.

*Theorem* 3.7. Let $\mathcal{B}$ be a NFS of $M$ and $\gamma \in [0, T]$. Then the NF-$\gamma$-T of $\mathcal{B}$ is a NFIe of $M$.

A NFIe of a NFid $\mathcal{B}$ may not be represented as a NF-$\gamma$-T of $\mathcal{B}$, i.e., the converse of Theorem 3.7 is not true in general as seen in the following example.

*Example* 3.1. Consider $M = \{0, \mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4, \mathfrak{q}_5\}$ be a B-algebras as following Cayley table

***Table 1.*** *B-algebra.*

| $\diamond$ | $0$ | $\mathfrak{q}_1$ | $\mathfrak{q}_2$ | $\mathfrak{q}_3$ | $\mathfrak{q}_4$ | $\mathfrak{q}_5$ |
|---|---|---|---|---|---|---|
| $0$ | $0$ | $\mathfrak{q}_5$ | $\mathfrak{q}_4$ | $\mathfrak{q}_3$ | $\mathfrak{q}_2$ | $\mathfrak{q}_1$ |
| $\mathfrak{q}_1$ | $\mathfrak{q}_1$ | $0$ | $\mathfrak{q}_5$ | $\mathfrak{q}_4$ | $\mathfrak{q}_3$ | $\mathfrak{q}_2$ |
| $\mathfrak{q}_2$ | $\mathfrak{q}_2$ | $\mathfrak{q}_1$ | $0$ | $\mathfrak{q}_5$ | $\mathfrak{q}_4$ | $\mathfrak{q}_3$ |
| $\mathfrak{q}_3$ | $\mathfrak{q}_3$ | $\mathfrak{q}_2$ | $\mathfrak{q}_1$ | $0$ | $\mathfrak{q}_5$ | $\mathfrak{q}_4$ |
| $\mathfrak{q}_4$ | $\mathfrak{q}_4$ | $\mathfrak{q}_3$ | $\mathfrak{q}_2$ | $\mathfrak{q}_1$ | $0$ | $\mathfrak{q}_5$ |
| $\mathfrak{q}_5$ | $\mathfrak{q}_5$ | $\mathfrak{q}_4$ | $\mathfrak{q}_3$ | $\mathfrak{q}_2$ | $\mathfrak{q}_1$ | $0$ |

Let $\mathcal{B}_1 = (\mathcal{B}_{1\mathcal{T}}, \mathcal{B}_{1\mathcal{I}}, \mathcal{B}_{1\mathcal{F}})$ be a NFS of $M$ given by $\mathcal{B}_{1\mathcal{T}}(0) = 0.41$, $\mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_1) = 0.32$, $\mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_2) = \mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_3) = \mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_4) = \mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_5) = 0.31$, $\mathcal{B}_{1\mathcal{I}}(0) = 0.31$, $\mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_1) = 0.22$, $\mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_2) = \mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_3) = \mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_4) = \mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_5) = 0.21$ and $\mathcal{B}_{1\mathcal{F}}(0) = 0.15$, $\mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_1) = 0.18$, $\mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_2) = \mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_3) = \mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_4) = \mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_5) = 0.29$. Then $\mathcal{B}$ is a NFid of $M$.

Let $\mathcal{B}_2 = (\mathcal{B}_{2\mathcal{T}}, \mathcal{B}_{2\mathcal{I}}, \mathcal{B}_{2\mathcal{F}})$ be a NFS of $M$ is given as follows $\mathcal{B}_{2\mathcal{T}}(0) = 0.42$, $\mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_1) = 0.33$, $\mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_2) = \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_3) = \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_4) = \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_5) = 0.32$, $\mathcal{B}_{2\mathcal{I}}(0) = 0.32$, $\mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_1) = 0.23$, $\mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_2) = \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_3) = \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_4) = \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_5) = 0.22$ and $\mathcal{B}_{2\mathcal{F}}(0) = 0.12$, $\mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_1) = 0.15$, $\mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_2) = \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_3) = \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_4) = \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_5) = 0.26$. Then $\mathcal{B}_2$ is a NFIe of $\mathcal{B}_1$. But it is not NF-$\gamma$-T of $\mathcal{B}_1$ for all $\gamma \in [0, T]$.

Clearly, the intersection of NFIe of a NFid $\mathcal{B}$ of $M$ is a NFIe of $\mathcal{B}$. But the union of NFIes of a NFid $\mathcal{B}$ of $M$ is not a NFIe of $\mathcal{B}$ as seen in the following example.

*Example* 3.2. Consider $M = \{0, \mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be a B-algebras with the following Cayley table:

***Table 2.*** *B-algebra.*

| $\diamond$ | $0$ | $\mathfrak{q}_1$ | $\mathfrak{q}_2$ | $\mathfrak{q}_3$ |
|---|---|---|---|---|
| $0$ | $0$ | $\mathfrak{q}_1$ | $\mathfrak{q}_2$ | $\mathfrak{q}_3$ |
| $\mathfrak{q}_1$ | $\mathfrak{q}_1$ | $0$ | $\mathfrak{q}_3$ | $\mathfrak{q}_2$ |
| $\mathfrak{q}_2$ | $\mathfrak{q}_2$ | $\mathfrak{q}_3$ | $0$ | $\mathfrak{q}_1$ |
| $\mathfrak{q}_3$ | $\mathfrak{q}_3$ | $\mathfrak{q}_2$ | $\mathfrak{q}_1$ | $0$ |

Let $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ be a NFS of $M$ given by $\mathcal{B}_{\mathcal{T}}(0) = 0.32$, $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_3) = 0.22$, $\mathcal{B}_{\mathcal{I}}(0) = 0.23$, $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_3) = 0.21$ and $\mathcal{B}_{\mathcal{F}}(0) = 0.18$, $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_3) = 0.2$. Then $\mathcal{B}$ is a NFid of $M$.

Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be two NFSs of $M$ is given by $\mathcal{B}_{1\mathcal{T}}(0) = \mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_2) = 0.41$, $\mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_1) = \mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_3) = 0.38$, $\mathcal{B}_{1\mathcal{I}}(0) = \mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_2) = 0.32$, $\mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_1) = \mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_3) = 0.21$, $\mathcal{B}_{1\mathcal{F}}(0) = \mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_2) = 0.3$, $\mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_1) = \mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_3) = 0.31$. $\mathcal{B}_{2\mathcal{T}}(0) = \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_1) = 0.42$, $\mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_2) = \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_3) = 0.35$, $\mathcal{B}_{2\mathcal{I}}(0) = \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_1) = 0.22$, $\mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_2) = \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_3) = 0.18$ and $\mathcal{B}_{2\mathcal{F}}(0) = \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_1) = 0.1$, $\mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_2) = \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_3) = 0.15$.

Then $\mathcal{B}_1$ and $\mathcal{B}_2$ are NFIes of $\mathcal{B}$. Obviously, the union $\mathcal{B}_1 \cup \mathcal{B}_2$ is a NFe of $M$, but it is not a NFIe of $\mathcal{B}$ since

$$\begin{aligned}
\mathcal{B}_{1\mathcal{T}} \cup \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_3) &= max\{\mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_3), \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_3)\} \\
&= max\{0.38, 0.35\} = 0.38, \\
\mathcal{B}_{1\mathcal{I}} \cup \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_3) &= max\{\mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_3), \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_3)\} \\
&= max\{0.21, 0.18\} = 0.21, \\
\mathcal{B}_{1\mathcal{F}} \cup \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_3) &= min\{\mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_3), \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_3)\} \\
&= min\{0.31, 0.15\} = 0.15.
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{1\mathcal{T}} \cup \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_3 \diamond \mathfrak{q}_1) &= \mathcal{B}_{1\mathcal{T}} \cup \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_2) \\
&= max\{\mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_2), \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_2)\} \\
&= max\{0.41, 0.35\} = 0.41, \\
\mathcal{B}_{1\mathcal{I}} \cup \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_3 \diamond \mathfrak{q}_1) &= \mathcal{B}_{1\mathcal{I}} \cup \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_2) \\
&= max\{\mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_2), \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_2)\} \\
&= max\{0.32, 0.18\} = 0.32, \\
\mathcal{B}_{1\mathcal{F}} \cup \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_3 \diamond \mathfrak{q}_2) &= \mathcal{B}_{1\mathcal{F}} \cup \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_1) \\
&= min\{\mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_1), \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_1)\} \\
&= min\{0.31, 0.1\} = 0.1.
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{1\mathcal{T}} \cup \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_1) &= max\{\mathcal{B}_{1\mathcal{T}}(\mathfrak{q}_1), \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_1)\} \\
&= max\{0.38, 0.42\} = 0.42, \\
\mathcal{B}_{1\mathcal{I}} \cup \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_1) &= max\{\mathcal{B}_{1\mathcal{I}}(\mathfrak{q}_1), \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_1)\} \\
&= max\{0.21, 0.22\} = 0.22, \\
\mathcal{B}_{1\mathcal{F}} \cup \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_2) &= min\{\mathcal{B}_{1\mathcal{F}}(\mathfrak{q}_2), \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_2)\} \\
&= min\{0.3, 0.15\} = 0.15.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathcal{B}_{1\mathcal{T}} \cup \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_3) \\
&\ngeq min\{\mathcal{B}_{1\mathcal{T}} \cup \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_3 \diamond \mathfrak{q}_1), \mathcal{B}_{1\mathcal{T}} \cup \mathcal{B}_{2\mathcal{T}}(\mathfrak{q}_1)\}, \\
&\mathcal{B}_{1\mathcal{I}} \cup \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_3) \\
&\ngeq min\{\mathcal{B}_{1\mathcal{I}} \cup \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_3 \diamond \mathfrak{q}_1), \mathcal{B}_{1\mathcal{I}} \cup \mathcal{B}_{2\mathcal{I}}(\mathfrak{q}_1)\}, \\
&\mathcal{B}_{1\mathcal{F}} \cup \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_3) \\
&\nleq max\{\mathcal{B}_{1\mathcal{F}} \cup \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_3 \diamond \mathfrak{q}_2), \mathcal{B}_{1\mathcal{F}} \cup \mathcal{B}_{2\mathcal{F}}(\mathfrak{q}_2)\}.
\end{aligned}$$

If $\mathcal{B}$ is a NFid of $M$, then it is clear that $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{i})$, $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{j})$ and $\mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{k})$ are ideals of $M$ for all $\mathfrak{i} \in Im(\mathcal{B}_{\mathcal{T}})$, $\mathfrak{j} \in Im(\mathcal{B}_{\mathcal{I}})$ and $\mathfrak{k} \in Im(\mathcal{B}_{\mathcal{F}})$ with $\mathfrak{i} \geq \gamma$. But, if we do not give a condition that $\mathcal{B}$ is a NFid of $M$, then $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{i})$, $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{j})$ and $\mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{k})$ are not ideals of $M$ as seen in the following example.

*Example* 3.3. Consider $M = \{0, \mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be a B-algebras as defined in the Example 3.2 and $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ be a NFS of $M$ given by $\mathcal{B}_{\mathcal{T}}(0) = 0.42$, $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) = 0.4$, $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_3) = 0.41$, $\mathcal{B}_{\mathcal{I}}(0) = 0.32$, $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) = 0.03$, $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_3) = 0.31$ and $\mathcal{B}_{\mathcal{F}}(0) = 0.2$, $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) = 0.5$, $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_3) = 0.25$. Since,

$$\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) = 0.4 \ngeq 0.41 = min\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\},$$

$$\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) = 0.03 \ngeq 0.31 = min\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\}$$

$$\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) = 0.5 \nleq 0.25 = max\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_3), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_3)\}$$

Hence, $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ is not a NFid of $M$. For $\gamma = 0.12$, $\mathfrak{i} = 0.38$, $\mathfrak{j} = 0.3$ and $\mathfrak{k} = 0.15$, we get $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{i}) = \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{j}) = \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{q}_1) = \{0, \mathfrak{q}_2, \mathfrak{q}_3\}$, which are not ideals of $M$, since $\mathfrak{q}_2 \diamond \mathfrak{q}_3 = \mathfrak{q}_1 \notin \{0, \mathfrak{q}_2, \mathfrak{q}_3\}$.

*Theorem* 3.8. Let $\mathcal{B}$ be a NFS of $M$ and $\gamma \in [0, T]$. Then the NF-$\gamma$-T of $\mathcal{B}$ is a NFid of $M \Leftrightarrow \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$, $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$ and $\mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$ are ideals of $M$ for $\mathfrak{m}_1 \in Im(\mathcal{B}_{\mathcal{T}})$, $\mathfrak{m}_2 \in Im(\mathcal{B}_{\mathcal{I}})$ and $\mathfrak{m}_3 \in Im(\mathcal{B}_{\mathcal{F}})$ with $\mathfrak{m}_1, \mathfrak{m}_2, \mathfrak{m}_3 \geq \gamma$.

*Proof.* Let $\mathcal{B}^T_\gamma = ((\mathcal{B}_{\mathcal{T}})^T_\gamma, (\mathcal{B}_{\mathcal{I}})^T_\gamma, (\mathcal{B}_{\mathcal{F}})^T_\gamma)$ be a NFid of $M$. Let $\mathfrak{m}_1 \in Im(\mathcal{B}_{\mathcal{T}})$, $\mathfrak{m}_2 \in Im(\mathcal{B}_{\mathcal{I}})$ and $\mathfrak{m}_3 \in Im(\mathcal{B}_{\mathcal{F}})$ with $\mathfrak{m}_1, \mathfrak{m}_2, \mathfrak{m}_3 \geq \gamma$. Since, $(\mathcal{B}_{\mathcal{T}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1)$ for all $\mathfrak{q}_1 \in M$, we have

$$\mathcal{B}_{\mathcal{T}}(0) + \gamma = (\mathcal{B}_{\mathcal{T}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma \geq \mathfrak{m}_1$$

for $\mathfrak{q}_1 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$. Thus $0 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$. Let $\mathfrak{q}_1, \mathfrak{q}_2 \in M$ such that $\mathfrak{q}_1 \diamond \mathfrak{q}_2, \mathfrak{q}_2 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$. Then $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \geq \mathfrak{m}_1 - \gamma$ and $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) \geq \mathfrak{m}_1 - \gamma$, that is, $(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) + \gamma \geq \mathfrak{m}_1$ and $(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) + \gamma \geq \mathfrak{m}_1$. Since, $\mathcal{B}^T_\gamma$ is a NFid of $M$, hence, we have $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) + \gamma = (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \geq \min\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\} \geq \mathfrak{m}_1$, i.e., $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) \geq \mathfrak{m}_1 - \gamma$ so that $\mathfrak{q}_1 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$. Hence, $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$ is an ideal of $M$.

Again, since $(\mathcal{B}_{\mathcal{I}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1)$ for all $\mathfrak{q}_1 \in M$, we have

$$\mathcal{B}_{\mathcal{I}}(0) + \gamma = (\mathcal{B}_{\mathcal{I}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma \geq \mathfrak{m}_2$$

for $\mathfrak{q}_1 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$. Thus $0 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$. Let $\mathfrak{q}_1, \mathfrak{q}_2 \in M$ such that $\mathfrak{q}_1 \diamond \mathfrak{q}_2, \mathfrak{q}_2 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$. Then $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \geq \mathfrak{m}_2 - \gamma$ and $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) \geq \mathfrak{m}_2 - \gamma$, that is, $(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) + \gamma \geq \mathfrak{m}_2$ and $(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) + \gamma \geq \mathfrak{m}_2$. Since, $\mathcal{B}^T_\gamma$ is a NFid of $M$, hence, we have $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) + \gamma = (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \geq \min\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\} \geq \mathfrak{m}_2$, i.e., $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) \geq \mathfrak{m}_2 - \gamma$ so that $\mathfrak{q}_1 \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$. Hence, $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$ is an ideal of $M$.

And, again since $(\mathcal{B}_{\mathcal{F}})^T_\gamma(0) \leq (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1)$ for all $\mathfrak{q}_1 \in M$, we have

$$\mathcal{B}_{\mathcal{F}}(0) - \gamma = (\mathcal{B}_{\mathcal{F}})^T_\gamma(0) \leq (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma \leq \mathfrak{m}_3$$

for $\mathfrak{q}_1 \in \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$. Thus $0 \in \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$. Let $\mathfrak{q}_1, \mathfrak{q}_2 \in M$ such that $\mathfrak{q}_1 \diamond \mathfrak{q}_2, \mathfrak{q}_2 \in \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$. Then $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \leq \mathfrak{m}_3 + \gamma$ and $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) \leq \mathfrak{m}_3 + \gamma$, that is, $(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) - \gamma \leq \mathfrak{m}_3$ and $(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) - \gamma \leq \mathfrak{m}_3$. Since, $\mathcal{B}^T_\gamma$ is a NFid of $M$, hence, we have $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) - \gamma = (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) \leq \max\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\} \leq \mathfrak{m}_3$, i.e., $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) \leq \mathfrak{m}_3 + \gamma$ so that $\mathfrak{q}_1 \in \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$. Hence, $\mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$ is an ideal of $M$.

Conversely, suppose that $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$, $\mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$ and $\mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$ are ideals of $M$ for $\mathfrak{m}_1 \in Im(\mathcal{B}_{\mathcal{T}})$, $\mathfrak{m}_2 \in Im(\mathcal{B}_{\mathcal{I}})$ and $\mathfrak{m}_3 \in Im(\mathcal{B}_{\mathcal{F}})$ with $\mathfrak{m}_1, \mathfrak{m}_2, \mathfrak{m}_3 \geq \gamma$. If there exists $e \in M$ such that $(\mathcal{B}_{\mathcal{T}})^T_\gamma(0) < \mu \leq (\mathcal{B}_{\mathcal{T}})^T_\gamma(e)$, then $\mathcal{B}_{\mathcal{T}}(e) \geq \mu - \gamma$ but $\mathcal{B}_{\mathcal{T}}(0) < \mu - \gamma$. This shows that $e \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$ and $0 \notin \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$. This is a contradiction, and $(\mathcal{B}_{\mathcal{T}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1)$ for all $\mathfrak{q}_1 \in M$.

Again, if there exists $g \in M$ such that $(\mathcal{B}_{\mathcal{I}})^T_\gamma(0) < \vartheta \leq (\mathcal{B}_{\mathcal{I}})^T_\gamma(g)$, then $\mathcal{B}_{\mathcal{I}}(g) \geq \vartheta - \gamma$ but $\mathcal{B}_{\mathcal{I}}(0) < \vartheta - \gamma$. This shows that $g \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}};\ \mathfrak{m}_2)$ and $0 \notin \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$. This is a contradiction, and $(\mathcal{B}_{\mathcal{I}})^T_\gamma(0) \geq (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1)$ for all $\mathfrak{q}_1 \in M$.

And again, if there exists $o \in M$ such that $(\mathcal{B}_{\mathcal{F}})^T_\gamma(0) > \varepsilon \geq (\mathcal{B}_{\mathcal{F}})^T_\gamma(o)$, then $\mathcal{B}_{\mathcal{F}}(o) \leq \varepsilon + \gamma$ but $\mathcal{B}_{\mathcal{F}}(0) > \varepsilon + \gamma$. This shows that $o \in \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$ and $0 \notin \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$. This is a contradiction, and $(\mathcal{B}_{\mathcal{F}})^T_\gamma(0) \leq (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1)$ for all $\mathfrak{q}_1 \in M$.

Let $a, b \in M$ be such that $(\mathcal{B}_{\mathcal{T}})^T_\gamma(a) < \delta \leq \min\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(a \diamond b), (\mathcal{B}_{\mathcal{T}})^T_\gamma(b)\}$. Then $\mathcal{B}_{\mathcal{T}}(a \diamond b) \geq \delta - \gamma$ and $\mathcal{B}_{\mathcal{T}}(b) \geq \delta - \gamma$, but $\mathcal{B}_{\mathcal{T}}(a) < \delta - \gamma$. This shows that $(a \diamond b) \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$ and $b \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$, but $a \notin \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{T}}; \mathfrak{m}_1)$. This is a contradiction, hence, $(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1) \geq \min\{(\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^T_\gamma(\mathfrak{q}_2)\}$ for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$.

Again, suppose that there exists $c, d \in M$ such that $(\mathcal{B}_{\mathcal{I}})^T_\gamma(c) < \omega \leq \min\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(c \diamond d), (\mathcal{B}_{\mathcal{I}})^T_\gamma(d)\}$, then $\mathcal{B}_{\mathcal{I}}(c \diamond d) \geq \omega - \gamma$ and $\mathcal{B}_{\mathcal{I}}(d) \geq \omega - \gamma$, but $\mathcal{B}_{\mathcal{I}}(c) < \omega - \gamma$. This shows that $c \diamond d \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$ and $d \in \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$, but $c \notin \mathfrak{U}_\gamma(\mathcal{B}_{\mathcal{I}}; \mathfrak{m}_2)$. This is a contradiction, hence, $(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1) \geq \min\{(\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^T_\gamma(\mathfrak{q}_2)\}$ for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$.

And again, suppose that there exists $r, u \in M$ such that $(\mathcal{B}_{\mathcal{F}})^T_\gamma(r) > \tau \geq \max\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(r \diamond u), (\mathcal{B}_{\mathcal{F}})^T_\gamma(u)\}$, then $\mathcal{B}_{\mathcal{F}}(r \diamond u) \leq \tau + \gamma$ and $\mathcal{B}_{\mathcal{F}}(u) \leq \tau + \gamma$, but $\mathcal{B}_{\mathcal{F}}(r) > \tau + \gamma$. This shows that $r \diamond u \in \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$ and $u \in \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$, but $r \notin \mathfrak{L}_\gamma(\mathcal{B}_{\mathcal{F}}; \mathfrak{m}_3)$. This is a contradiction, hence, $(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1) \leq \max\{(\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^T_\gamma(\mathfrak{q}_2)\}$ for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$.

Consequently, $\mathcal{B}^T_\gamma = ((\mathcal{B}_{\mathcal{T}})^T_\gamma, (\mathcal{B}_{\mathcal{I}})^T_\gamma, (\mathcal{B}_{\mathcal{F}})^T_\gamma)$ is a NFid of $M$.

*Theorem* 3.9. Let $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ be a NFid of $M$ and let $\gamma, \delta \in [0, T]$. If $\gamma \geq \delta$, then the NF-$\gamma$-T $\mathcal{B}^T_\gamma = ((\mathcal{B}_{\mathcal{T}})^T_\gamma, (\mathcal{B}_{\mathcal{I}})^T_\gamma, (\mathcal{B}_{\mathcal{F}})^T_\gamma)$ of $\mathcal{B}$ is a NFIe of the neutrosophic fuzzy $\delta$-translation $H^T_\delta = ((\mathcal{B}_{\mathcal{T}})^T_\delta, (\mathcal{B}_{\mathcal{I}})^T_\delta, (\mathcal{B}_{\mathcal{F}})^T_\delta)$ of $\mathcal{B}$.

*Proof.* Theorem proof is straightforward.

For every NFid $\mathcal{B}$ of $M$ and $\delta \in [0, T]$, the neutrosophic fuzzy $\delta$-translation $H^T_\delta$ of $\mathcal{B}$ is a NFid of $M$. If Let $I = (\Gamma_I, \Lambda_I, \Sigma_I)$ is a NFIe of $H^T_\delta$, then there exists $\gamma \in [0, T]$ such that $\gamma \geq \delta$ and $I \geq \mathcal{B}^T_\gamma$, meaning $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) \geq (\mathcal{B}_{\mathcal{T}})^T_\gamma$, $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) \geq (\mathcal{B}_{\mathcal{I}})^T_\gamma$ and $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) \leq (\mathcal{B}_{\mathcal{F}})^T_\gamma$ for all $\mathfrak{q}_1 \in M$. Hence, we have the following theorem.

*Theorem* 3.10. Let $\mathcal{B}$ be a NFid of $M$ and let $\delta \in [0, T]$. For every NFIe $I = (\Gamma_I, \Lambda_I, \Sigma_I)$ of the neutrosophic fuzzy $\delta$-translation $H^T_\delta$ of $\mathcal{B}$, there exists $\gamma \in [0, T]$ such that $\gamma \geq \delta$ and $I$ is a NFIe of the NF-$\gamma$-T of $\mathcal{B}$.

Let us illustrate Theorem 3.10 using the following example.

*Example* 3.4. Consider $M = \{0, \mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4, \mathfrak{q}_5\}$ be a B-algebras in Example 3.9 and $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ be a NFS of $M$ given by $\mathcal{B}_{\mathcal{T}}(0) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_4) = 0.41$, $\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_3) = \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_5) = 0.38$, $\mathcal{B}_{\mathcal{I}}(0) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) =$

$\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_4) = 0.35$, $\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_3) = \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_5) = 0.31$ and $\mathcal{B}_{\mathcal{F}}(0) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_4) = 0.03$, $\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_3) = \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_5) = 0.04$. Then $\mathcal{B}$ is a NFid of $M$ and $T = 0.03$. If we take $\delta = 0.01$, then the neutrosophic fuzzy $\delta$-translation $H^T_\delta = ((\mathcal{B}_{\mathcal{T}})^T_\delta, (\mathcal{B}_{\mathcal{I}})^T_\delta, (\mathcal{B}_{\mathcal{F}})^T_\delta)$ of $\mathcal{B}$ is given as follows:
$(\mathcal{B}_{\mathcal{T}})^T_\delta(0) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_2) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_4) = 0.42$,
$(\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_3) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_5) = 0.39$,
$(\mathcal{B}_{\mathcal{I}})^T_\delta(0) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_2) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_4) = 0.36$,
$(\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_3) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_5) = 0.32$ and
$(\mathcal{B}_{\mathcal{F}})^T_\delta(0) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_2) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_4) = 0.02$,
$(\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_3) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_5) = 0.03$.
Let $I = (\Gamma_I, \Lambda_I, \Sigma_I)$ be a NFS of $M$ is given as follows: $\Gamma_I(0) = \Gamma_I(\mathfrak{q}_2) = \Gamma_I(\mathfrak{q}_4) = 0.45$, $\Gamma_I(\mathfrak{q}_1) = \Gamma_I(\mathfrak{q}_3) = \Gamma_I(\mathfrak{q}_5) = 0.32$, $\Lambda_I(0) = \Lambda_I(\mathfrak{q}_2) = \Lambda_I(\mathfrak{q}_4) = 0.35$, $\Lambda_I(\mathfrak{q}_1) = \Lambda_I(\mathfrak{q}_3) = \Lambda_I(\mathfrak{q}_5) = 0.30$ and $\Sigma_I(0) = \Sigma_I(\mathfrak{q}_2) = \Sigma_I(\mathfrak{q}_4) = 0$, $\Sigma_I(\mathfrak{q}_1) = \Sigma_I(\mathfrak{q}_3) = \Sigma_I(\mathfrak{q}_5) = 0.01$. Then $I$ is a NFIe of $\mathcal{B}$. But it is not NF-$\gamma$-T of $\mathcal{B}$ for all $\gamma \in [0, T]$.

Then $I$ is clearly a NFid of $M$, a NFIe of the neutrosophic fuzzy $\delta$-translation $H^T_\delta$ of $\mathcal{B}$. But $I$ is not a NF-$\gamma$-T of $\mathcal{B}$ for all $\gamma \in [0, T]$. If we take $\gamma = 0.02$, then $\gamma = 0.02 > 0.01 = \delta$ and the NF-$\gamma$-T $\mathcal{B}^T_\gamma = ((\mathcal{B}_{\mathcal{T}})^T_\gamma, (\mathcal{B}_{\mathcal{I}})^T_\gamma, (\mathcal{B}_{\mathcal{F}})^T_\gamma)$ of $\mathcal{B}$ is given as follows:
$(\mathcal{B}_{\mathcal{T}})^T_\delta(0) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_2) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_4) = 0.43$,
$(\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_3) = (\mathcal{B}_{\mathcal{T}})^T_\delta(\mathfrak{q}_5) = 0.4$,
$(\mathcal{B}_{\mathcal{I}})^T_\delta(0) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_2) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_4) = 0.37$,
$(\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_3) = (\mathcal{B}_{\mathcal{I}})^T_\delta(\mathfrak{q}_5) = 0.33$ and
$(\mathcal{B}_{\mathcal{F}})^T_\delta(0) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_2) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_4) = 0.01$,
$(\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_1) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_3) = (\mathcal{B}_{\mathcal{F}})^T_\delta(\mathfrak{q}_5) = 0.02$.

Note that $I(\mathfrak{q}_1) \geq \mathcal{B}^T_\gamma(\mathfrak{q}_1)$ i.e., $\Gamma_I(\mathfrak{q}_1) \geq (\mathcal{B}_{\mathcal{T}})^T_\gamma$, $\Lambda_I(\mathfrak{q}_1) \geq (\mathcal{B}_{\mathcal{I}})^T_\gamma$ and $\Sigma_I(\mathfrak{q}_1) \leq (\mathcal{B}_{\mathcal{F}})^T_\gamma$ for all $\mathfrak{q}_1 \in M$, and thus, $I$ is a NFIe of NF-$\gamma$-T of $\mathcal{B}$.

For any NFS $\mathcal{B} = (\mathcal{B}_{\mathcal{T}}, \mathcal{B}_{\mathcal{I}}, \mathcal{B}_{\mathcal{F}})$ of $M$, a neutrosophic fuzzy 0-multiplication $\mathcal{B}^s_0 = ((\mathcal{B}_{\mathcal{T}})^s_0, (\mathcal{B}_{\mathcal{I}})^s_0, (\mathcal{B}_{\mathcal{F}})^s_0)$ of $\mathcal{B}$ is a NFid of $M$.

*Theorem* 3.11. If $\mathcal{B}$ is a NFid of $M$, then the neutrosophic fuzzy $\varsigma$-multiplication of $M$ is a NFid of $M$ for all $\varsigma \in [0, 1]$.

*Proof.* Theorem proof is straightforward.

*Theorem* 3.12. If $\mathcal{B}$ is any NFS of $M$, then the following assertions are equivalents:

(i). $\mathcal{B}$ is a NFid of $M$.

(ii). For all $\varsigma \in [0, 1]$, $\mathcal{B}^s_\varsigma$ is a NFid of $M$.

*Proof.* Necessity follows from Theorem 3.11. For sufficient part let $\varsigma \in [0, 1]$ be such that $\mathcal{B}^s_\varsigma = ((\mathcal{B}_{\mathcal{T}})^s_\varsigma, (\mathcal{B}_{\mathcal{I}})^s_\varsigma, (\mathcal{B}_{\mathcal{F}})^s_\varsigma)$ is a NFid of $M$. Then for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$, we have

$$
\begin{aligned}
(\mathcal{B}_{\mathcal{T}})^s_\varsigma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) \cdot \varsigma \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{T}})^s_\varsigma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{T}})^s_\varsigma(\mathfrak{q}_2)\right\} \\
&= \min\left\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \cdot \varsigma, \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2) \cdot \varsigma\right\} \\
&= \min\left\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\right\} \cdot \varsigma, \\
(\mathcal{B}_{\mathcal{I}})^s_\varsigma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) \cdot \varsigma \\
&\geq \min\left\{(\mathcal{B}_{\mathcal{I}})^s_\varsigma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{I}})^s_\varsigma(\mathfrak{q}_2)\right\} \\
&= \min\left\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \cdot \varsigma, \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2) \cdot \varsigma\right\} \\
&= \min\left\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\right\} \cdot \varsigma. \\
(\mathcal{B}_{\mathcal{F}})^s_\varsigma(\mathfrak{q}_1) &= \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) \cdot \varsigma \\
&\leq \max\left\{(\mathcal{B}_{\mathcal{F}})^s_\varsigma(\mathfrak{q}_1 \diamond \mathfrak{q}_2), (\mathcal{B}_{\mathcal{F}})^s_\varsigma(\mathfrak{q}_2)\right\} \\
&= \max\left\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2) \cdot \varsigma, \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2) \cdot \varsigma\right\} \\
&= \max\left\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2)\right\} \cdot \varsigma.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1) &\geq \min\left\{\mathcal{B}_{\mathcal{T}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{T}}(\mathfrak{q}_2)\right\}, \\
\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1) &\geq \min\left\{\mathcal{B}_{\mathcal{I}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{I}}(\mathfrak{q}_2)\right\}, \\
\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1) &\leq \max\left\{\mathcal{B}_{\mathcal{F}}(\mathfrak{q}_1 \diamond \mathfrak{q}_2), \mathcal{B}_{\mathcal{F}}(\mathfrak{q}_2)\right\},
\end{aligned}
$$

for all $\mathfrak{q}_1, \mathfrak{q}_2 \in M$, since $\varsigma \neq 0$. Therefore, $\mathcal{B}$ is a NFid of $M$.

# 4. Conclusion

In this article, neutrosophic fuzzy translation of NFids in B-algebras are introduced with some of their useful properties investigated. The relationships between NFTs and NFes of NFids have constructed. It is our hope that this work would become another foundations for further study of the theory of B-algebras.

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# Abbreviations

| | |
|---|---|
| NFS | Neutrosophic Fuzzy Set |
| NFid | Neutrosophic Fuzzy Ideal |
| NF-$\gamma$-T | Neutrosophic Fuzzy $\gamma$-translation |
| NF-$\varsigma$-Mu | Neutrosophic Fuzzy $\varsigma$-multiplication |
| NFe | Neutrosophic Fuzzy Extension |
| NFIe | Neutrosophic Fuzzy Ideal Extension |

# Author Contributions

**Bavanari Satyanarayana:** Conceptualization, Supervision

**Kamidi Madhavi:** Investigation, Methodology
**Shake Baji:** Writing – review & editing
**Dasari Ramesh:** Writing – review & editing

## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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