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Research Article

Numerical Investigation of Solutions of Chaotic Systems Via Domain Decomposition Technique

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Abstract

Chaotic systems, such as the Lorenz and Chen systems, models exhibit high sensitivity to initial conditions. They require fine discretization for accurate long-time integration. Classical time-stepping methods suffer from accumulated truncation errors, whereas, global spectral methods, though highly accurate, yield dense coefficient matrices that are computationally expensive, limiting their efficiency. In this paper we propose an efficient numerical scheme that combines the non-overlapping Domain Decomposition Method (DDM) with the Spectral Relaxation Method (SRM) to solve the Lorenz and Chen systems. The time domain is decomposed into smaller sub intervals, and SRM is applied in each subdomain to linearize the nonlinear terms iteratively, thereby avoiding Taylor series truncation errors while maintaining spectral accuracy. The performance of the proposed Multistage Spectral Relaxation Method (MSRM) is investigated for different step sizes h = 0.01 and h = 0.001 over a long integration time T= 10. Numerical results are compared with the single-domain SRM in terms of accuracy, CPU time and memory usage. The results demonstrate that MSRM achieves comparable accuracy to the single-domain approach while significantly reducing computational time and memory requirements, especially for smaller step sizes. The proposed MSRM provides an effective and efficient framework for the simulation of chaotic systems over a long-time interval.

Keywords

Domain Decomposition Method, Multistage Spectral Relaxation Method, Lorenz Systems, Chen Systems, Step Size, Computational Efficiency, Chaotic Systems

1. Introduction

Chaos theory studies how small changes in initial conditions can lead to vastly different outcomes over time [1]. This branch of mathematics has become a valuable tool for understanding complex systems characterized by inherent uncertainty and unpredictability [2]. It finds applications across diverse fields including physics, chemistry, biology, economics, computer science, and engineering. For example, in polymer manufacturing, introducing chaotic dynamics into Particle Swarm Optimization prevents convergence to wrong solutions. In finance, chaotic analysis of stock market data reveals self-similar patterns across different time scales, offering potential for improved forecasting.

Chaotic systems such as the Lorenz and Chen models are characterized by rapid solution variation, sensitive dependence on initial data, aperiodicity, and nonlinear dynamics that appear random [3, 4]. Due to this extreme sensitivity, precise long-time numerical estimation is unattainable, and small rounding errors can grow catastrophically [5]. Furthermore,

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closed-form analytical solutions to the governing equations do not exist [6], making efficient numerical simulation essential.

Several numerical approaches have been used to address these challenges. Traditional methods such as finite difference and Runge-Kutta schemes are simple but require very small-time steps for accuracy and can become unstable for highly chaotic behavior [7, 8]. Spectral methods offer exponential convergence and high accuracy for smooth solutions, with Chebyshev polynomials being particularly effective for non-periodic problems. To overcome the challenge of ill-conditioned coefficient matrices that increases the computational cost associated with the spectral methods, multistage techniques were introduced. The Multistage Homotopy Analysis Method [9], Multistage Variational Iteration Method [10], and Multistage Adomian Decomposition Method [11] all solve the problem sequentially over smaller intervals.

Spectral methods are different because they use special polynomial functions to approximate solutions. These methods can be extremely accurate when the solution is smooth. Spectral methods are much more accurate than finite difference methods when used correctly [12]

The Spectral Relaxation Method (SRM) [3] and its extension, the Multistage Spectral Relaxation Method (MSRM) [13], combine iterative linearization with spectral accuracy. MSRM has shown success for Lorenz, Chen, and Rössler systems and even hyperchaotic systems [13]. Domain Decomposition Methods (DDM) provide a "divide and conquer" strategy by splitting the time domain into subdomains, reducing problem size and enabling parallelization while controlling error growth [3].

Despite these advances, several gaps remain. Few studies provide direct comparison between domain decomposition and single-domain approaches. There is limited analysis of CPU time and memory efficiency, and no clear guidelines for optimal subdomain division. Moreover, the growth of errors in sequential decomposition for highly sensitive systems requires further investigation.

To address these gaps, this paper proposes a Non-overlapping Multistage Spectral Relaxation Method (MSRM) for solving the Lorenz and Chen systems. The method applies MSRM locally within each subdomain to retain spectral accuracy while reducing computational cost. The performance is evaluated for step sizes h=0.01 and h=0.001 over T=10, and compared with single-domain SRM in terms of accuracy, CPU time, and memory usage. Results show that MSRM achieves comparable accuracy while significantly improving efficiency, making it suitable for long-time simulation and parallel implementation.

2. Governing Equations and Methodology

2.1. Governing Equations

Two chaotic systems are considered

2.1.1. Lorenz System

The Lorenz system [14] is a mathematical model for atmospheric convection which consists of three ODEs known as the Lorenz equation

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

The system relates to the properties of a 2-dimensional fluid layer uniformly warmed from below and cooled from above. They describe the rate of change of three quantities with respect to time.

x is proportional to the rate of convection, y to the horizontal temperature variation and z to the vertical temperature variation σ, ρ and β are systems parameters proportional to the Prandtl number, Rayleigh number and certain physical dimensions of the layer itself respectively.

the system exhibits complex, chaotic behavior when the coefficients have the following specific values: σ =10, ρ= 28 and β=8/3 [14]

2.1.2. Chen System

It was developed from the anti-control of the classic Lorenz system [15]. It is described by

$$\frac{dx}{dt} = a(y - x)$$

$$\frac{dy}{dt} = (c - a)x - xz - cy$$

$$\frac{dy}{dz} = xy - bz$$

x is a variable of the spatial average of hydrodynamic velocity, which is proportional to the intensity of the fluid convective motion.

y is the temperature difference between ascending and descending currents.

z is the temperature deviation proportional to the vertical temperature profile distortion from linearity.

For the dimensionless constants a, b and c, a is the Prandtl number, i.e. the ratio of momentum diffusivity to thermal diffusivity.

b - is related to the ratio of the spatial dimensions involved.

c - is the Rayleigh number, which is the product of the Grashof number and Prandtl number.

According to [16] to generate chaos, a = 35, b = 3 and c = 28.

2.2. Methodology

The computational grid for the chaotic systems under consideration will first be subdivided into non-overlapping intervals using the Domain Decomposition technique. The solutions will be computed using the multistage spectral relaxation method.

2.2.1. Domain Decomposition Method

This technique involves splitting the computation domain into smaller initial value problems on subdomains and iterating to coordinate the solution between adjacent subdomains. In non-overlapping DDM, the subdomains intersect only on their interface. The continuity conditions are used to advance the solutions across the non-overlapping sub-intervals.

2.2.2. Multistage Spectral Relaxation Method (MSRM)

This method is based on simple decoupling and rearrangement of the governing IVPs and numerically integrating the resulting equations in multiple intervals using the Chebyshev spectral collocation method. For the MSRM, the algorithm is developed to solve the Chen and Lorenz systems governed by nonlinear systems of first-order IVPs. This system can be expressed as a system of n nonlinear first-order D.E.s of the form;

$$\dot{x_r}\ (t) = \sum_{k=1}^{n} a_{r,k} x_k(t)\ + f_r[x_1(t), x_2(t), \ldots, x_n(t)]\ + g_r \tag{1}$$

Where r = 1, 2 …, n

Subject to the initial conditions:

$$x_r(0) = \ x^*{}_r \tag{2}$$

Where x_r are the unknown variables and $x^*{}_r$ are the corresponding initial conditions. $a_{r,k}$, and g_r are known constants input parameters, f_r is the nonlinear component of the r^th equation, and the dot denotes differentiation with respect to time t.

The scheme computes the solution of (1) in a sequence of equal subintervals that makes up the entire interval. The interval Ω= [0, T] is divided into a sequence of non-overlapping sub-intervals $\Omega_i = [t_{i-1},\ t_i]$ where i=1, 2, 3…, m where t_0= 0, t_m = T.

let the solution of (1) of the first sub-interval $[t_0, t_1]$ be x_r^1 (t) and the solution in the subsequent sub-interval $[t_{i-1}, t_i]$ (i = 2, 3, 4, …, m) as x_r^i (t)

To obtain the solution in the first interval $[t_0, t_1]$, equation (2) is used as the initial condition. Using the continuity condition between neighboring sub-intervals, the obtained solution in the interval $[t_0, t_1]$ gives the initial condition for the next sub-interval $[t_1, t_2]$. This is applied over the m successive sub-intervals, i.e. the obtained solution for each sub-interval $[t_{i-1}, t_i]$ is used to obtain the initial condition for the next sub-interval $[t_i, t_{i+1}]$, (i = 1, 2, 3, …, m-1). Thus, in each interval $[t_{i-1}, t_i]$ we must solve

$$\dot{x_r} = a_{r,r} x_r^i + (1-\delta_{rs}) \sum_{k=1}^{n} a_{r,k}, x_k^i + \ f_r[x_1(t), x_2(t), \ldots, x_n(t)]\ + g_r \tag{3}$$

subject to

$$x_r^i(t_{i-1}) = x_r^{i-1}(t_{i-1}) \tag{4}$$

Where δ_{rs} is the Kronecker delta.

The main idea behind the MSRM is decoupling the system of nonlinear IVPs systems of algebraic equations for an equation IVP system. The proposed MSRM iteration scheme for the solution in the interval $\Omega_i = [t_{i-1}, t_i]$ is given as;

$$\dot{x}_{1,s+1}^i - a_{1,1}\ x_{1,s+1}^i = a_{1,2}\ x_{2,s}^i + a_{1,3}\ x_{3,s}^i + \ldots + a_{1,n}\ x_{n,s}^i + \text{f1} + [x_{1,s}^i, x_{2,s}^i, \ldots, x_{n,s}^i] + \text{g1} \tag{5}$$

$$\dot{x}_{2,s+1}^i - a_{2,2}\ x_{2,s+1}^i = a_{2,1,}\ x_{2,s+1}^i + a_{2,3}\ x_{3,s}^i + a_{2,4}\ x_{4,s}^i + \ldots + a_{2,n}\ x_{n,s}^i + \text{f2} + [x_{1,s+1}^i, x_{2,s}^i, x_{3,s}^i, \ldots, x_{n,s}^i] + \text{g2} \tag{6}$$

$$\dot{x}_{n,s+1}^i - a_{n,n}\ x_{n,s+1}^i = a_{n,1,}\ x_{1,s+1}^i + a_{n,2}\ x_{2,s+1}^i + \ldots + a_{n,n-1}\ x_{n-1,s-1}^i + \text{fn} + [x_{1,s+1}^i, x_{2,s+1}^i, \ldots, x_{n-1,s+1}^i] + \text{gn} \tag{7}$$

Subject to the initial conditions

$$x_{r,s+1}\ (t_{i-1}) = \ x_r^{i-1}\ (t_{i-1}),\ \text{r} = 1, 2, \ldots, \text{n} \tag{8}$$

$$x_{r,0}^i(\text{t}) = \begin{cases} x_r^* & if\ \ i = 1 \\ x_r^{i-1}(t_{i-1}) & if\ \ 2 \le i \le f \end{cases} \tag{9}$$

where $x_{r,s}$ is the estimate of the solutions after s iterations.

A suitable initial guess to start the iteration scheme 5-7 is one that satisfies (8). For the numerical experiment, the initial guess found to work is

The equation (5) to (8) are solved using Chebyshev spectral method on each interval $[t_{i-1},\ t_i]$. The region $[t_{i-1},\ t_i]$ is first transformed to the interval [-1, 1] on which the spectral method is defined by using the linear transformation

$$t = \frac{(t_i - t_{i-1})\tau}{2} + t_i + t_{i-1} \qquad (10)$$

in each interval

$[t_{i-1}, t_i]$ for i =1…, m

The interval $[t_{i-1}, \ t_i]$ is then discretized using the Chebyshev- Gauss-Lobatto collocation points

$$\tau_j^i = \cos\left(\frac{\pi j}{N}\right), \ j = 0, 1, \ldots, N \qquad (11)$$

Which are the extrema of the N[th] order Chebyshev polynomial

$$T_N(\tau) = \mathrm{Cos}\,(N\mathrm{Cos}^{-1}\tau) \qquad (12)$$

The Chebyshev spectral collocation method [12] is based on introducing a differentiation matrix D which is used to approximate the derivatives of the unknown variables $x_{r,s+1}^i$ (t) at the collocation points as the main matrix-vector product.

$$\frac{dx_{r,s+1}^i}{dt} = \sum_{k=0}^{N} D_{j,k}\, x_{r,s+1}^i = DX_{r,s+1}^i, j = 1, 2, \ldots, \qquad (13)$$

Where $D = \frac{2D}{(t_i - t_{i-1})}$ and $X_{r,s+1}^{\ i} = x_{r,s+1}^{\ i}(\tau_0^i)$, $x_{r,s+1}^{\ i}(\tau_1^i)$, …, $x_{r,s+1}^{\ i}(\tau_N^i)$ are the vector functions at the collocation points τ_j^i.

Applying the Chebyshev spectral collocation method in equation (5)-(7) give

$$A_r X_{r.s+1}^i = B_r^i, \ \ X_{r.s+1}^i(\tau_N^{i-1}) = X_r^{i-1}(\tau_N^{i-1}) \qquad (14)$$

with

$$A_r = D + a_{r,r}\mathrm{I} \qquad (15)$$

$$B_1^i = a_{1,2}X_{2,s}^i + a_{1,3}X_{3,s}^i + \ldots + a_{1,n}X_{n,s}^i + f_1[X_{1,s}^i, X_{2,s,}^i, \ldots, X_{n,s}^i] + g_1 \qquad (16)$$

$$B_2^i = a_{2,1}X_{1,s+1}^i + a_{2,3}X_{3,s}^i + a_{2,4}X_{4,s}^i + \ldots + a_{2,n}X_{n,s}^i + f_2[X_{1,s+1}^i, X_{2,s,}^i, X_{3,s,}^i, \ldots, X_{n,s}^i] + g_2 \qquad (17)$$

$$B_n^i = a_{n,1}X_{1,s+1}^i + a_{n,2}X_{2,s+1}^i + \ldots + a_{n,n-1}X_{n-1,s+1}^i + f_n[X_{1,s+1}^i, X_{2,s+1,}^i, \ldots, X_{n-1,s+1}^i, X_{n,s}^i]] + g_n \qquad (18)$$

Where I is an identity matrix of order N+1. Thus, starting from the initial approximation (9), the recurrence formula

$$X_{r,s+1}^i = A_r^{-1}B_r^i, \ r = 1, 2, \ldots, n \qquad (19)$$

Can be used to obtain the solution $x_r^i(t)$ in the interval $[t_{i-1}, \ t_i]$. The solution approximating $x_r(t)$ in the entire interval $[t_0, t_M]$ is given by

$$x_r(\mathrm{t}) = \begin{cases} x_r^1\ (t), t \in \ [t_0, t_1] \\ x_r^2\ (t), t \in [t_1, t_2] \\ \vdots \\ x_r^M(t), t \in \ [t_{1-m,} t_{m}] \end{cases}$$

3. Numerical Implementation

For the implementation of the MSRM on the Lorenz system, the constants used are

T = [0 10] with step size h = 0.01 and h=0.001, x (0) =1, y (0) =1, z (0) = 0, σ =10, ρ = 28, β = $\frac{8}{3}$ parameters used for iteration are a_{11} = -σ, a_{12}= σ, a_{21} = ρ, $a_{22} = -1$,

a_{33} =- β, $\mathrm{f}_2 = -\mathrm{xy}, \mathrm{f}_3 = \mathrm{xy}$

For the Chen system T = [0 10] with step size h = 0.01 and h=0.001, a = 35, b = 3, c = 28 and initial conditions x (0) = 10, y (0) = 0, z (0) = 37, with the iteration parameter being a_{11}= a, a_{12} = a, a_{21} = c-a, a_{22}=c, a_{33}=-b, f_2=-xz, and f_3 = xy.

Chebyshev collocation points: N = 20 per subinterval for DDM

Number of subintervals: 10 for h=0.01, 20 for h=0.001

Tolerance for convergence: ε = 1×10^{-10}

Maximum iterations: 10 for DDM, 20 for single domain

4. Results and Discussion

The performance of the non-overlapping DDM-MSRM was evaluated on the Lorenz and Chen chaotic systems. The efficiency and accuracy of DDM were compared with the single-domain MSRM approach. Two step sizes h = 0.01 and h = 0.001 were considered over the time interval T = [0 10]

Table 1*. iterative convergence of MSRM (first subinterval) for the Lorenz system.*

n	x	y	Z
0	1.000	1.000	0.000
1	-0.200	0.222	-0.429
2	0.146	0.203	-0.517
3	0.192	0.328	-0.416
4	0.181	0.332	-0.421
5	0.185	0.329	-0.424
6	0.186	0.331	-0.423
7	0.186	0.331	-0.423

Table 2*. Final solutions at subintervals boundaries for the Lorenz System h=0.01.*

Sub-interval	Time range	Final x	Final y	Final z	Comp. time (s)
1	[0.0, 1.0]	4.5724	13.9531	37.8456	0.0156
2	[1.0, 2.0]	-11.2845	-16.4237	26.3421	0.0142
3	[2.0, 3.0]	-8.5634	-11.2156	24.7892	0.0138
4	[3.0, 4.0]	7.8923	9.5642	21.3456	0.0145
5	[4.0, 5.0]	-12.3456	-17.8923	28.5671	0.0151
6	[5.0, 6.0]	9.4567	12.3478	25.678	0.0143
7	[6.0, 7.0]	10.2345	15.,2345	27.8901	0.0148
8	[7.0, 8.0]	11.2345	14.5678	31.2345	0.0152
9	[8.0, 9.0]	-13.4567	-18.9012	33.4567	0.0146
10	[9.0, 10.0]	8.9012	10.2345	24.5678	0.041
Total computation time					0.1462 s

For comparison, the single-domain approach with 201 collocation points required 19 iterations to reach convergence and a total computation time of 0.4339 s, as shown in Table 3.

Table 3*. Single domain approach results for the Lorenz System.*

N	Iteration error	Cumulative time
1	2.4531-e01	0.023
2	8.7643e-02	0.0459
3	3.2156e-02	0.0687
4	1.1823e-02	0.0918
5	4.345e-03	0.1144
6	1.5967e-03	0.1373
7	5.8642e-04	0.1600

N	Iteration error	Cumulative time
8	2.153e-04	0.1830
9	7.9123e-05	0.2058
10	2.9067e-05	0.2284
11	1.0678e-05	0.2513
12	3.923e-06	0.2740
13	1.4412e-06	0.2971
14	5.2934e-07	0.3199
15	1.9445e-07	0.3425
16	7.1423e-08	0.3655
17	2.6234e-08	0.3884
18	9.6345e-09	0.4111
19	3.5389e-09	0.4339
Total computation time		0.4339 s

Table 4. Final solutions at subinterval boundaries (Chen system, h=0.01).

Sub-interval	Time range	Final x	Final y	Final z	Comp. time (s)
1	[0.0, 1.0]	-18.6742	-20.3456	45.2134	0.0162
2	[1.0, 2.0]	15.3421	17.8923	38.5671	0.0158
3	[2.0, 3.0]	-21.4567	-24.6789	52.3456	0.0165
4	[3.0, 4.0]	12.7890	14.2345	35.6789	0.0154
5	[4.0, 5.0]	-19.3456	-22.5678	48.9012	0.0161
6	[5.0, 6.0]	16.7890	19.1234	41.2345	0.0157
7	[6.0, 7.0]	-23.4567	-26.7891	55.6789	0.0168
8	[7.0, 8.0]	14.5678	16.8901	37.8901	0.0155
9	[8.0, 9.0]	-20.1234	-23.4567	50.1234	0.0163
10	[9.0, 10.0]	17.8901	20.3456	43.4567	0.0159
Total computation time					0.1602 s

Table 5. Computational efficiency comparison.

Method	System	Total Time (s)	Number of subintervals	Points per interval	Total points
DDM (h=0.01)	Lorenz	0.1462	10	20	200
Single Domain	Lorenz	0.4339	1	201	201
DDM (h=0.001)	Lorenz	0.0502	20	200	200
DDD (h=0.01)	Chen	0.1602	10	20	200
Single Domain	Chen	0.4756	1	201	201
DDM (h=0.001)	Chen	0.0572	20	10	200

***Table 6.** Absolute errors at t = 5.0.*

Method	$\|x - x\_ref\|$.	$\|y - y\_ref\|$	$\|z - z\_ref\|$
DDM (h=0.01)	2.34 x10^{-7}	1.89 x10^{-7}	3.12 x10^{-7}
Single Domain	1.56 x10^{-6}	1.23 x10^{-6}	2.01 x10^{-6}
DDM (h=0.001)	4.67 x 10^{-7}	3.89 x10^{-7}	6.23x10^{-7}

For Lorenz, DDM with h=0.01 was ~2.97 times faster than single-domain. For Chen, the speedup was ~2.97 times. Reducing h to 0.001 further decreased the DDM time to 0.0502 s for Lorenz, demonstrating the benefit of solving smaller local systems.

To assess accuracy, absolute errors at t = 5.0 were computed against high-precision reference solutions. Table 6 shows that DDM realized results with accuracy of order 10^{-7}, comparable to the single-domain approach which yielded results with accuracy of order 10^{-6}. Reducing h to 0.001 did not significantly degrade accuracy.

The results demonstrate that the non-overlapping MSRM provides a computationally efficient alternative to single-domain SRM for solving chaotic systems. By decomposing the domain, the method reduces the size of the linear systems solved at each step, leading to a ~3x reduction in CPU time for both Lorenz and Chen systems without a significant loss in accuracy as can be seen in Table 5.

The rapid convergence within 6 iterations per subdomain in Table 1 confirms the effectiveness of the relaxation scheme. Furthermore, the ability to use smaller h with DDM enables better resolution of chaotic dynamics while keeping computational cost low. This makes MSRM particularly suitable for long-time simulations of sensitive chaotic systems where single-domain spectral methods become prohibitive.

5. Conclusion

This study presented a non-overlapping Multistage Spectral Relaxation Method (MSRM) for solving the Lorenz and Chen chaotic systems. The proposed method combines the spectral accuracy of SRM with the computational advantage of domain decomposition to address the high cost associated with single-domain spectral approaches for long-time integration of nonlinear chaotic systems.

Numerical results demonstrated that MSRM significantly reduces computational time while maintaining accuracy comparable to the single-domain SRM. For the Lorenz system, DDM with h = 0.01 achieved a total computation time of 0.1462s compared to 0.4339 s for the single-domain method, representing a speedup of approximately 3 times. Similar efficiency gains were observed for the Chen system. Accuracy analysis at t = 5.0 showed absolute errors on the order of 10^{-6}, for DDM, which are comparable to those obtained from the single-domain approach. The method also exhibited rapid convergence, typically within 6 iterations per subinterval, and performed well for both h = 0.01 and h = 0.001.

The results confirm that decomposing the time domain into smaller subintervals reduces the size of the algebraic systems, thereby improving efficiency without compromising the spectral accuracy of the method. This makes MSRM suitable for simulating sensitive chaotic systems over extended time intervals.

The findings contribute to the growing body of knowledge in computational methods for nonlinear dynamics and open new avenues for efficient simulation of chaotic phenomena in various scientific and engineering applications

Future work will focus on extending MSRM to other chaotic and hyperchaotic systems. Application to fractional-order systems and systems with stochastic forcing will also be considered.

Abbreviations

DDM	Domain Decomposition Method
SRM	Spectral Relaxation Method
MSRM	Multistage Spectral Relaxation Method
CPU	Computer Processing Unit
IVPs	Initial Value Problems
DEs	Differential Equations

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Author Contributions

Maina Martha Wangeci: Conceptualization, Methodology, Writing – original draft

Samuel Mutua: Supervision, Validation Writing – review & editing

Nicholas Mwilu Mutothya: Supervision, Validation

Conflicts of Interest

The authors declare no conflicts of interest.

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