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Research Article

Design of Reinforced Concrete Retaining Wall by Hybrid Teaching Learning Based Optimization

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Abstract

Reinforced concrete (RC) retaining walls are widely used in civil engineering applications, where economical and efficient designs are essential, given their extensive use and material demands. This study aims to optimize the weight and cost of RC cantilever retaining walls by developing a hybrid Teaching–Learning-Based Optimization (TLBO) algorithm with enhanced performance characteristics. The proposed method introduces a multi-population selection strategy that improves exploration of the design space in early iterations and promotes convergence in later stages. In addition, a pre-generated list of feasible reinforcement configurations is incorporated to eliminate repetitive constraint checks, thereby reducing computational effort. The optimization framework considers both geotechnical and structural constraints, including stability against sliding and overturning, bearing capacity, and compliance with ACI 318-19 design requirements. Two benchmark problems—retaining walls with and without a shear key—are analyzed to evaluate the effectiveness of the proposed hybrid TLBO. The results are compared with several established optimization techniques, including genetic algorithms, particle swarm optimization, grey wolf optimization, and other heuristic methods. The findings demonstrate that the hybrid TLBO algorithm provides more consistent, near-optimal solutions, as indicated by lower standard deviation values and improved convergence. The optimized designs achieve reduced cost and weight while satisfying all design constraints, with several critical constraints approaching their capacity limits, indicating optimal resource utilization. Furthermore, the proposed modifications reduce computational time by eliminating up to 20% of constraint evaluations. Overall, the study confirms that the hybrid TLBO approach is a robust and efficient tool for the optimal design of RC retaining walls, offering superior performance compared to conventional optimization methods.

Keywords

Reinforced Concrete Retaining Walls, Structural Optimization, Teaching-Learning-Based Optimization (TLBO), Hybrid Optimization Algorithm, Cost Minimization, Weight Minimization, Multi-Population Strategy, Mutation-Based Optimization

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1. Introduction

Reinforced concrete (RC) retaining walls are widely used in many civil engineering projects, including structural and geotechnical works. These structures are critical in highway construction, where bridges are used, and soils are retained. Due to their locations, residential buildings, recreational centers, water supply projects, and other civil engineering projects would require the construction of RC retaining walls. This universal application of RC cantilevered retaining walls encourages engineers to pursue low-cost, low-weight designs through optimization.

RC retaining walls have been the subject of numerous structural optimization studies over the past few decades. Camp & Akin [1] used big bang-big crunch optimization to minimize the cost and weight of retaining walls, and they compared their results with other evolutionary optimization designs. Kaveh & Farhoudi [2] investigated the optimum design of RC cantilevered walls using dolphin echolocation optimization. Temür & Bekdaş [3] applied teaching learning-based optimization (TLBO) to design RC cantilever retaining walls. They conducted a comparative study of the retaining wall's average and minimum weights compared to other heuristic optimization methods. Kaveh & Shakouri Mahmud Abadi [4] studied the optimum cost of retaining walls using a harmony search-based algorithm. Kayabekir et al. [5] studied the optimum design of RC retaining walls using a harmony search algorithm with three objective functions: cost minimization, CO_2 emission minimization, and multi-objective cost and CO_2 emission minimization. Kalemci et al. [6] used grey wolf optimization, a technique that mimics grey wolves' hierarchy and hunting methods, to minimize the weight of RC cantilevered retaining walls. They compared the results of the weight minimization with those from over 10 previous studies. Nandha Kumar and Suribabu [7] studied the weight reduction of the RC cantilevered retaining walls using a differential evolution algorithm. The optimum solution saved about 15% of the material weight. Gandomi et al. [8] investigated the efficiency of the accelerated particle swarm, firefly, and cuckoo search algorithms in minimizing the weight of RC cantilever retaining walls. They also conducted a parametric study investigating the effect of the base shear key, surcharge load, base soil friction angle, and backfill slope. Bekdaş et al. [9] used TLBO to determine a restricted-optimum design for RC retaining walls.

One aspect of the optimization process among all these researchers is consensus. When any algorithm selects a better population, it will likely converge on a better solution. None of these studies examines the effects of selecting a search population based on fitness at the early stages of the algorithm, which should improve the convergence of any optimization method.

Issa [10] used a distributed genetic algorithm to minimize the weight of steel portal frames. A mutation strategy was applied to encourage exploration of the design space, increasing mutation probabilities at the early stages of the algorithm and systematically decreasing mutation values as the generation increased. It was shown that this adaptation diversified the search population and consistently produced optimal solutions [11, 12].

Many researchers have shown that TLBO is robust and effective [3, 9]. Others, such as Camp & Farshchin [13] and Rao & Patel [14], have modified the TLBO algorithm to improve overall performance.

This study aims to develop and evaluate an enhanced hybrid TLBO framework for the optimal design of RC cantilever retaining walls, with the dual objective of minimizing structural weight and construction cost while satisfying all geotechnical and structural design constraints. The proposed approach introduces a mutation-based multi-population strategy to improve exploration of the design space and the reliability of convergence. In addition, a computationally efficient methodology is implemented by predefining feasible reinforcement configurations, thereby eliminating repetitive constraint checks associated with steel reinforcement limits and significantly reducing computational effort. The performance of the proposed hybrid TLBO algorithm is systematically assessed through benchmark problems and compared with established optimization techniques to demonstrate its accuracy, efficiency, and robustness in achieving near-optimal engineering designs.

2. Design Optimization Process

The design of RC retaining walls must satisfy both geotechnical and structural requirements. Feasible RC retaining wall designs must withstand overturning and sliding forces and provide minimum bearing displacement. Also, different elements of the RC retaining walls must provide adequate flexural and shear stresses.

2.1. Basis of Geotechnical Design

Figure 1 shows the geotechnical forces considered in the design of RC cantilevered retaining walls per unit meter length, where W_c is the weight of the retaining wall, W_b is the weight of backfill on the heel, W_t is the weight of soil on the toe surcharge load, q is the distributed surcharge load, Q is the resultant surcharge load, β is the backfill angle, P_A is the active earth force, P_t is the passive earth force on the front part of the toe, P_k is the passive earth force on the key, and P_B is the bearing stress force [13].

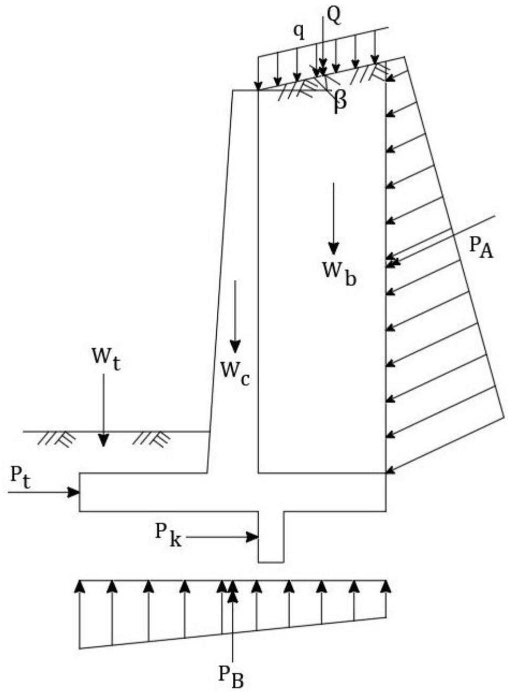

Figure 1. The forces acting on RC cantilevered retaining walls.

The active and passive earth pressure coefficients, K_a and K_p, respectively, are calculated using Rankine theory as

$$K_a = cos\beta \frac{cos\beta - \sqrt{cos^2\beta - cos^2\phi}}{cos\beta + \sqrt{cos^2\beta - cos^2\phi}} \tag{1}$$

The coefficient of the passive earth pressure is:

$$K_p = cos\beta \frac{cos\beta + \sqrt{cos^2\beta - cos^2\phi}}{cos\beta - \sqrt{cos^2\beta - cos^2\phi}} \tag{2}$$

where ϕ is the angle of the internal friction in the soil.

The forces shown in Figure 1 produce an overturning moment; if the retaining wall cannot resist them, it will fail by overturning. The factor of safety for overturning SF_O:

$$SF_O = \frac{\sum M_R}{\sum M_A} \tag{3}$$

where ΣM_R is the sum of the moments of resisting forces, and ΣM_A is the sum of moments of applied forces around the front face of the toe. To avoid an overturning hazard, the safety factor SF_O must be greater than the predetermined value, typically between 1.5 and 2.5.

The factor of safety for the sliding forces, SF_S, is

$$SF_S = \frac{\sum F_R}{\sum F_A} \tag{4}$$

where ΣF_R is the sum of the horizontal resisting forces, and ΣF_A is the sum of the horizontal applied forces.

The applied force is the summation of the horizontal components of the active earth pressure:

$$\sum F_A = P_A cos\beta \tag{5}$$

The resisting forces are calculated as

$$\sum F_R = \sum N * \tan\left(\frac{2}{3} * \varphi_{base}\right) + \frac{2}{3} * B * c_{base} + \sum P_P \tag{6}$$

where ϕ_{base} is the internal friction angle of the base soil, c_{base} is the cohesion of the base soil, ΣN is the sum of the weight of the retaining wall, surcharge load, and the vertical components of the active forces

$$\sum N = W_C + W_b + Q + P_A sin\beta \tag{7}$$

The ΣP_p is the passive force expressed as

$$\sum P = P_t + P_k \tag{8}$$

with

$$P_t = \frac{1}{2}\gamma_{fill} D^2 K_p + 2cD^2\sqrt{K_p} \tag{9}$$

$$P_k = \frac{1}{2}\gamma_{base} h_k^2 K_p + 2c_{base} h_k^2\sqrt{K_p} \tag{10}$$

where D is the total depth of the retained soil causing passive earth pressure, γ_{fill} is the unit weight of the retained soil, c is the cohesion of the retained soil, h_k is the depth of the soil in front of the base shear key, and γ_{base} is the unit weight of the base soil.

Since the foundation of the wall is considered a shallow foundation, the factor of safety for bearing capacity, SF_B, is expressed as:

$$SF_B = \frac{q_u}{q_{max}} \tag{11}$$

where q_u and q_{max} are the ultimate bearing capacity and the maximum applied bearing stress, respectively. Terzaghi's bearing capacity theory is used to compute the ultimate bearing capacity as

$$q_u = c_{base} N_c \gamma_{fill} D N_q + \frac{1}{2}\gamma_{base} N_g (B - 2e) \tag{12}$$

where N_c, N_q, and N_g are Terzaghi's coefficients, B is the width of the slab, γ_{fill} is the unit weight of the retained soil, and e is the eccentricity given as

$$e = \frac{\sum M_R - \sum M_O}{\sum V} \tag{13}$$

where ΣM_O is the sum of the moments of the applied force around the middle of the slab, ΣM_R is the sum of the moments of resisting forces, and ΣV is the sum of the vertical forces acting on the foundation.

The minimum and maximum values of the soil bearing capacity of the shallow foundation could be calculated from:

$$q_{min.max} = \frac{\sum V}{B}\left(1 \mp \frac{6e}{B}\right) \quad (14)$$

2.2. Basis of Structural Design

The critical sections, including the stem, toe, and heel, are checked for compliance with applied forces and moments during the structural analysis and design of RC retaining walls. These sections must meet the strength capacity defined by ACI 318-19 [15].

The nominal flexural strength M_n of the cross-section shall be greater than the ultimate flexural strength M_u.

$$M_n \geq M_u \quad (15)$$

The nominal flexural strength of the RC retaining wall is calculated as

$$M_n = 0.9A_s f_y\left(d - \frac{a}{2}\right) \quad (16)$$

where A_s is the area of the reinforcing steel, f_y is the yield strength of the steel reinforcement, d is the effective depth of the cross-section, and a is the depth of equivalent rectangular stress block given as

$$a = \frac{A_s f y}{0.85 f_c b} \quad (17)$$

where f_c is the concrete compressive strength and b is the width of the cross-section.

The nominal shear strength V_n should be greater than the ultimate applied shear force V_u

$$V_n \geq V_u \quad (18)$$

where V_u is

$$V_n = 0.75\left(0.17\sqrt{f_c}bd\right) \quad (19)$$

2.3. Loads

When investigating the external stability of RC retaining walls, i.e., bearing capacity of the foundation, sliding, and overturning, in the geotechnical basis of design, service loads are used to compute the earth pressure with a load factor value of 1.0. On the other hand, the loads used in the structural design of the RC retaining walls should be consistent with ACI 318-19 [15], which requires increasing service loads by specified factors.

If resistance against the soil pressure, S (weight of soil), is included in combination with the dead load, D (weight of concrete), and live load, L (surcharge load), the factor is computed from the following combination:

$$F_L = 1.2D + 1.6L + 1.6S \quad (20)$$

For the design of the toe where D or L reduces the effect of soil pressure, the factor load used in the design is computed from the following combination:

$$F_L = 0.9D + 1.6S \quad (21)$$

For any combination of dead load, live load, and soil pressure, the required strength shall not be less than:

$$F_L = 1.2D + 1.6L \quad (22)$$

For the design of the arm and heel, Eq. (20) usually governs, whereas for the toe, Eq. (21) governs.

Dead loads, such as concrete weight, should be multiplied by 0.9 for the toe slab and by 1.2 for the heel slab and stem.

Each critical flexural section should satisfy the minimum and maximum reinforcement ratios and minimum and maximum reinforcement spacing as specified by ACI 318-19 [15].

The minimum area of flexural reinforcement, A_{smin}, shall not be less than the following.

$$A_{Smin} = \frac{\sqrt{fc}}{4fy}bd \geq \frac{1.4}{fc}bd \quad (23)$$

The minimum reinforcement ratio ρ_{min} can be computed as

$$\rho_{min} = \frac{A_{Smin}}{bd} \quad (24)$$

The limit for the maximum steel ratio ρ_{max} is the balanced steel ratio, which produces balanced strain conditions under flexure:

$$\rho_b = \left(\frac{0.85\beta_1 f_c}{f_y}\right)\left(\frac{600}{600+f_y}\right) \quad (25)$$

The development length, l_d, should comply with ACI 318-19 [15] requirements to ensure adequate bond between concrete and steel reinforcement. For a deformed steel bar diameter d_b of 19mm or less, l_d should be:

$$l_d = \left(\frac{f_y\psi_t\psi_e}{2.1\lambda\sqrt{f_c}}\right)db \quad (26)$$

where ψ_t is the modification factor for casting location, ψ_e is the modification factor for development length, and λ It is a factor based on the reinforcement coating, usually set to 1.0.

For steel reinforcing bar diameters greater than 19mm, l_d should be

$$l_d = \left(\frac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f_c}}\right)db \quad (27)$$

Provided that l_d is not less than 300mm.

The development length for hooks l_{dh} is calculated as

$$l_{dh} = \left(\frac{0.24f_y}{\sqrt{fc}}\right)db \quad (28)$$

and shall not be smaller than #8 steel reinforcing bar diameters or 150mm.

2.4. Optimum Design

In many RC retaining walls, the objective function may be formulated to minimize the weight of concrete and steel reinforcement, or to minimize the total cost, including concrete, steel reinforcement, and formwork.

The objective function for weight F_W is

$$F_W = W_C + W_{st} \tag{29}$$

where W_C is the weight of concrete, and W_{st} is the weight of steel reinforcement

The objective function for cost F_C is

$$F_C = C_{st}W_{st} + C_C V_C + C_F A_F \tag{30}$$

where C_{st} is the cost of steel reinforcement per weight, C_C is the cost of concrete per volume, V_C is the volume of concrete, C_F is the cost of formwork per area, and A_F is the area of formwork.

2.5. Design Variables

Two groups of design variables are used in RC retaining walls: those representing the wall geometry, including cross-sections, and those for reinforcement. As shown in Figure 2, twelve variables are commonly used for the optimum design of RC retaining walls.

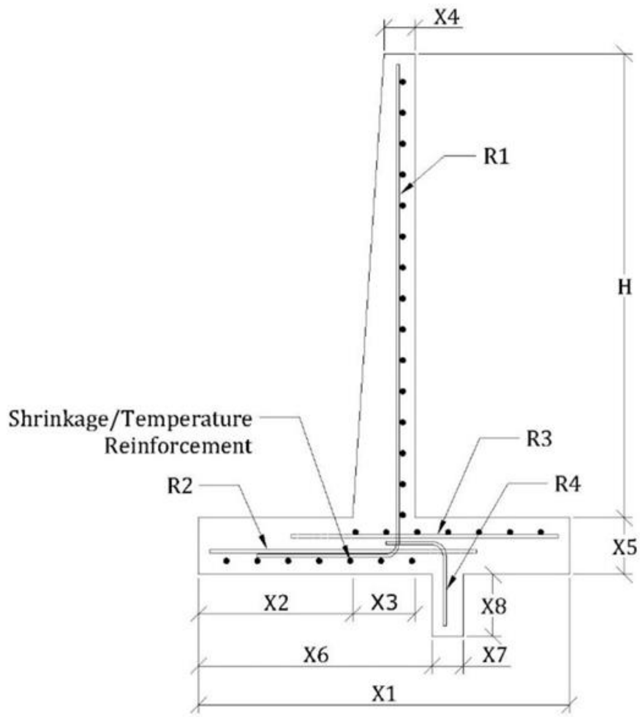

Figure 2. *Design variables of RC cantilevered retaining wall.*

Variables that express the sections of the wall include X1: width of the base slab, X2: width of the slab, X3: stem thickness at the bottom of the wall, X4: stem thickness at the top of the wall, X5: base slab thickness, X6: distance from the front of the toe slab to the front of the shear key, X7: width of the base shear key, X8: height of the base shear key.

The reinforcement design variables for the retaining wall are R1: the vertical steel area in the stem per unit length of the wall, R2: the horizontal steel area of the toe slab, R3: the horizontal steel area of the heel slab, and R4: the vertical steel area of the shear key per unit length of the wall.

In this study, both geometric design and reinforcement variables are discrete variables.

3. Hybrid TLBO

In many learning environments, teachers are essential to transferring knowledge to students, helping them learn at a higher level. There is a correlation between a teacher's knowledge and competence and students' learning outcomes. The more qualified the teacher, the higher the mean of learning outcome. In addition, learners gain knowledge from their peers through interaction, communication, information sharing, research, extracurricular activities, and tutorial work, all of which significantly support the learning process.

In this model, students learn from teachers and peers in a classroom with varying levels of learning. Rao *et al.* [16] first conceptualize the teaching-learning process of teachers and students in classrooms into an optimization algorithm. The teaching-learning analogy guides the optimization process to identify the best learner as a teacher, who can transfer information to the student body and bring the classroom knowledge mean closer to the teacher's level. Then, students are randomly paired and interact to improve the less knowledgeable student's level. Rao *et al.* [16] organized this process into a cycle of teacher learning followed by student learning. In the TLBO algorithm, after a predetermined number of learning cycles, the best teacher represents the best solution to an optimization problem.

There have been efforts to modify the original TLBO algorithm to improve its performance and generate better solutions. This study makes two significant modifications to the TLBO algorithm to improve convergence and reduce computational time.

3.1. Modifying TLBO

Like many other heuristic search techniques, a TLBO algorithm begins with a randomly selected initial population. The diversity and overall fitness of the initial population are essential for the algorithm to converge rapidly to a good solution.

Issa [17] modified a distributed genetic algorithm by applying higher mutation probabilities in the first few generations to encourage exploration of the design space, then lowering mutation rates later to promote convergence. In this study, this generational mutation probability strategy is integrated into a TLBO algorithm to generate multiple populations over cycles rather than selecting a single population at the beginning. The probability of selecting a new population P_S^{Gc} is given as

$$P_S^{Gc} = P_S^{max} - \frac{e^{-\frac{1}{20}} - e^{-\frac{G_C}{20}}}{e^{-\frac{1}{20}} - e^{-\frac{N_G}{20}}}\left(P_S^{max} - P_S^{min}\right) \quad (31)$$

where P_S^{max} is the predetermined maximum probability value, P_S^{min} is the predetermined minimum probability value, G_C is the current generation, and N_G is the number of predetermined generations. Figure 3 shows the variation of P_S^{Gc} for 200 generations with a maximum probability value of 80% and a minimum of 0.05%.

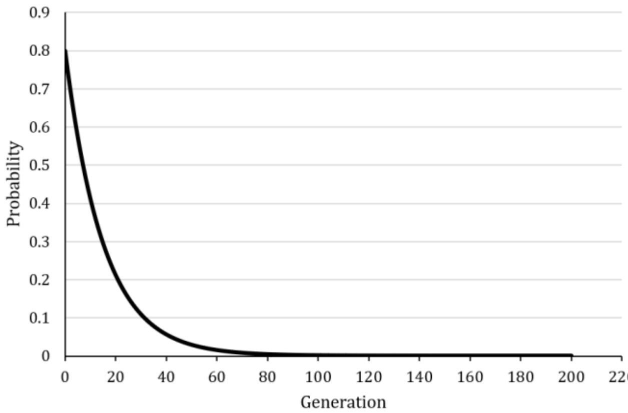

Figure 3. *Selection probability of generating a new population.*

As indicated in Figure 3, the probability value of selecting a new population P_S^{Gc} varies with the number of generations. The probability is high during the initial generations, allowing frequent selection of new populations, exploration of a larger portion of the design space, and diversification of solutions. As the number of generations increases, P_S^{Gc} decreases, allowing TLBO to converge to the optimum solution. For each generation, P_S^{Gc} is computed using Eq. (31) and compared to a random number [0, 1]. Suppose the random number is less than or equal to the probability value of the current generation. In that case, a new population is selected for that generation, and the algorithm proceeds to the teacher phase. Also, an elitist strategy is used to preserve the best solutions from the current population in the next generation.

3.2. Modifying Fitness Computation

A design's feasibility is generally assessed during optimization by checking a series of geotechnical and structural constraints. Violations of these constraints are usually reflected in a penalty applied to the objective function. Some constraints are associated with minimum and maximum amounts of reinforcing steel and rebar spacing, and checking these constraints for each increases the computational time. This study developed a list of feasible steel reinforcement configurations for various thicknesses and lengths of structural elements to ensure compliance with these constraints. In addition to Eqs. (23) & (24), the following constraints were considered to generate a list of feasible reinforcement configurations following ACI 318-19 [15]:

$$\rho_{max} \leq 0.85 \frac{f_c}{f_y} \frac{3}{7} \beta_1 \quad (32)$$

where β_1 is the ratio of the depth of the rectangular stress block to the depth of the neutral axis, determined as

$$\beta_1 = 0.85 \; for \; f_c \leq 28 \; MPa \quad (33)$$

$$\beta_1 = 0.85 - 0.5 \frac{f_c - 28}{7} \geq 0.65 \; for \; f_c > 28 \; MPa \quad (34)$$

The reinforcement ratio ρ is defined as

$$\rho = \frac{A_S}{bd} \quad (35)$$

The minimum and maximum reinforcement spacing, S_{min} and S_{max} are

$$S_{min} = \max\left(d_b, \frac{4}{3} d_A, 25mm\right) \quad (36)$$

$$S_{max} = \min(t, 300mm) \quad (37)$$

where d_b is the diameter of a steel bar, d_A is the maximum diameter of coarse aggregate, and t is the thickness of the structural element.

Table 1 shows a sample list of feasible reinforcement configurations for various thicknesses. Each number in the list represents the area of steel reinforcement, considering the number of steel bars used. For instance, 497 mm^2 indicated as the first number under 200 mm thickness is equivalent to 7-ϕ10mm, and 568 mm^2 under 220 mm is equivalent to 2-ϕ20mm.

Table 1. *List of feasible reinforcement configurations.*

Thickness, *t* (mm)	Feasible Reinforcement Configurations, mm^2
200	497, 568, 639, 645, 710, 770, 774, 781, 852, 903, 923, 924, 994, 1000, 1032, 1065, 1078, 1136, 1161, 1200, 1207, 1232, 1278, 1290, 1349, 1386, 1400, 1419, 1420, 1491, 1540, 1548, 1562, 1600, 1633, 1677, 1694, 1704, 1800,

Thickness, *t* (mm)	Feasible Reinforcement Configurations, mm^2
	1806, 1848, 1935
210	497, 568, 639, 645, 710, 770, 774, 781, 852, 903, 923, 924, 994, 1000, 1032, 1065, 1078, 1136, 1161, 1200, 1207, 1232, 1278, 1290, 1349, 1386, 1400, 1419, 1420, 1491, 1540, 1548, 1562, 1600, 1633, 1677, 1694, 1704, 1800, 1806, 1848, 1935, 1988, 2000, 2002, 2064, 2156
220	568, 639, 645, 710, 770, 774, 781, 852, 903, 923, 924, 994, 1000, 1032, 1065, 1078, 1136, 1161, 1200, 1207, 1232, 1278, 1290, 1349, 1386, 1400, 1419, 1420, 1491, 1540, 1548, 1562, 1600, 1633, 1677, 1694, 1704, 1800, 1806, 1848, 1935, 1988, 2000, 2002, 2064, 2156, 2193, 2200, 2272, 2310, 2322
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600	903, 923, 924, 994, 1000, 1032, 1065, 1078, 1136, 1161, 1200, 1207, 1232, 1278, 1290, 1349, 1386, 1400, 1419, 1420, 1491, 1540, 1548, 1562, 1600, 1633, 1677, 1694, 1704, 1800, 1806, 1848, 1935, 1988, 2000, 2002, 2036, 2064, 2156, 2193, 2200, 2272, 2310, 2322, 2400, 2451, 2464, 2545, 2556, 2580, 2600, 2618, 2709, 2772, 2800, 2838, 2840, 2926, 3000, 3018, 3054, 3080, 3096, 3124, 3200, 3225, 3234, 3276, 3400, 3408, 3483, 3563, 3600, 3692, 3800, 3870, 3976, 4000

The reinforcement areas are assigned to the structural elements from the feasible reinforcement configuration list before computing the fitness function, ensuring compliance with the minimum and maximum reinforcement area and spacing requirements. The remaining geotechnical and structural constraints are presented in the following section.

4. Constraints

Constraints define the search space for the optimization problems. RC retaining walls have two sets of constraints: geotechnical and structural constraints.

Geotechnical Constraints

The stability factor of safety against overturning (FS_O), sliding (FS_S), and bearing capacity failure (FS_B) must be adequate.

$$g(1) = \frac{M_O(FS_O)}{M_{RO}} - 1 \le 0 \tag{38}$$

$$g(2) = \frac{F_S(FS_S)}{F_{RS}} - 1 \le 0 \tag{39}$$

$$g(3) = \frac{q_B(FS_B)}{q_{RB}} - 1 \le 0 \tag{40}$$

where, M_O is the overturning moment, M_{RO} is the resisting moment against overturning, F_S is the applied sliding force, F_{RS} is the resisting force against sliding, q_B is the applied soil pressure on the footing, and q_{RB} is the allowable soil pressure.

Structural Constraints

In addition to the reinforcement configurations, the moment and shear capacity, and the development length requirement must be satisfied.

The constraints for moment capacity are

$$g(4-7) = \frac{M_d}{M_n} - 1 \le 0 \tag{41}$$

where M_d is the applied moment to the arm, toe, heel, and key.

For shear strength, the constraints are

$$g(8-11) = \frac{V_d}{V_n} - 1 \le 0 \tag{42}$$

where V_d is the applied shear to the arm, toe, heel, and key,

The development length constraints are

$$g(12) = \frac{l_{db_{arm}}}{X5-cc} - 1 \le 0 \tag{43}$$

$$g(13) = \frac{l_{dh_{arm}}}{X5-cc} - 1 \le 0 \tag{44}$$

$$g(14) = \frac{l_{db_{toe}}}{X1-X2-cc} - 1 \le 0 \tag{45}$$

$$g(15) = \frac{12 l_{dh_{toe}}}{X5-cc} - 1 \le 0 \tag{46}$$

$$g(16) = \frac{l_{db_{heel}}}{X2+X3-cc} - 1 \le 0 \tag{47}$$

$$g(17) = \frac{12 l_{dh_{heel}}}{X5-cc} - 1 \le 0 \tag{48}$$

$$g(18) = \frac{l_{db_{key}}}{X5-cc} - 1 \le 0 \tag{49}$$

$$g(19) = \frac{l_{dh_{key}}}{X5-cc} - 1 \le 0 \tag{50}$$

where l_{db} and l_{dh} are the development lengths of the main reinforcement bars and hooks of the arm, toe, heel, and key.

To avoid negative stress values,

$$g(20) = q_B \ge 0 \tag{51}$$

where q_B is the soil pressure under the base.

A penalty is applied for any violation of a constraint that would increase the value of the fitness function through a penalty coefficient C, as

$$F_C = (C_{st}W_{st} + C_CV_C)(1+C) \quad (52)$$

The penalty coefficient C is computed using two categories:

$$\begin{cases} C = \sum_i^n 10g(i)\ if\ < g(i) \le 1 \\ C = \sum_i^n 50g(i)\ if\ g(i) > 1 \end{cases} \quad (53)$$

Figure 4 shows a flowchart of the hybrid TLBO algorithm, where r(0, 1) denotes a probability between 0 and 1, r(1, 2) denotes a probability of selecting either 1 or 2, and mean(i) denotes the average fitness across the population.

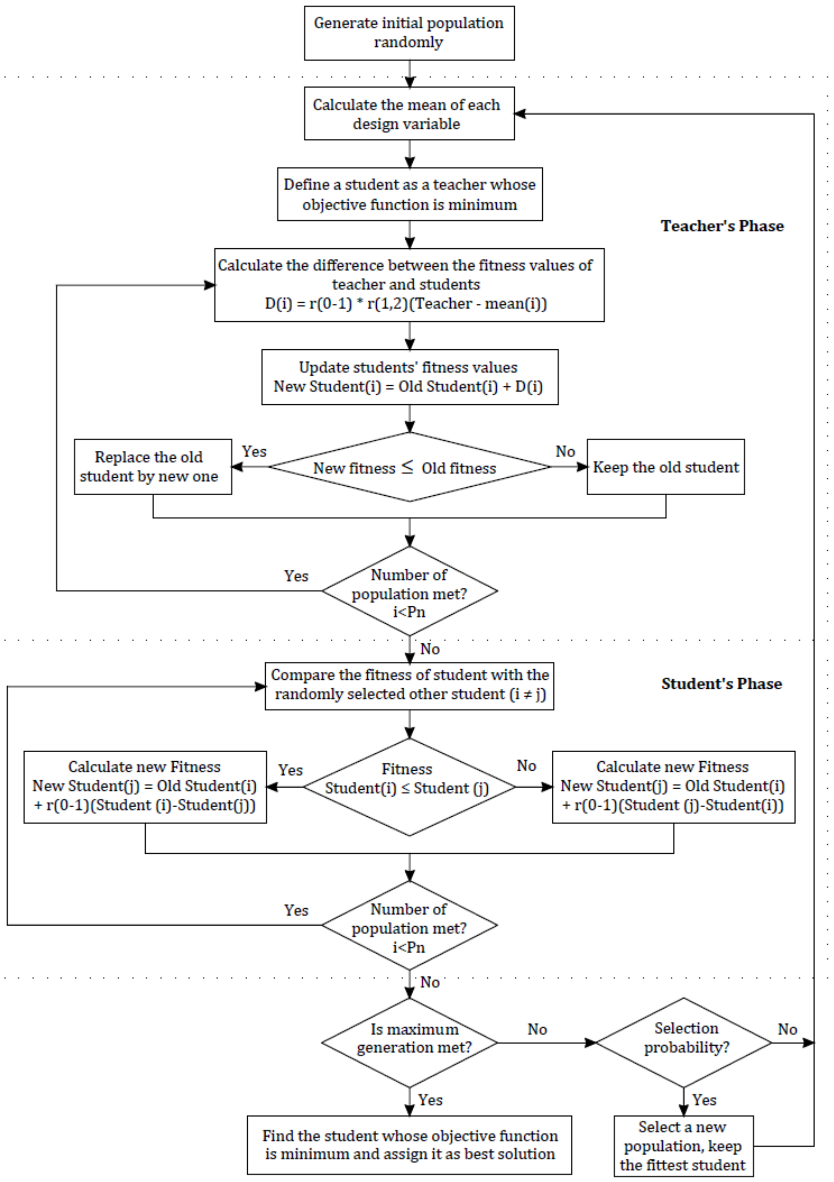

Figure 4. The flowchart of hybrid TLBO.

5. Effectiveness of Hybrid TLBO

In this study, the hybrid TLBO is compared with other optimization methods to assess the effectiveness of the proposed modifications. Two common benchmark examples, with and without a shear key, are used by many researchers and employed in this study to evaluate the effectiveness of Hybrid-TLBO.

Example 1

Example 1 is the design of an RC cantilevered retaining wall without a shear key. The objective function considers only the weight of the wall's concrete and steel per unit length. Table 2 lists the geotechnical and cost parameters. Table 3 lists the upper and lower limits on geometric design variables; the design considers a 10mm increment for all dimensions.

Table 2. RC Cantilevered Retaining Walls Parameters for Example 1.

Parameters	Symbol	Value	Unit
Arm height	H	3	m
Depth of soil in front of the wall	D	0.5	m
Surcharge load	Q	20	kPa
Cohesion of the base soil	c_{base}	125	kPa
Internal friction angle of base soil	φ_{base}	0	°
Internal friction angle of retaining soil	φ	36	°
Compressive strength of concrete	f_c	21	MPa
Yield strength of reinforcing steel	f_y	400	MPa
Concrete cover	cc	70	mm
Percentage of shrinkage and temperature reinforcement	ρ_{st}	0.002	-
Backfill slope	B	10	°
Unit weight of retained soil	γ_{fill}	17.5	kN/m^3
Unit weight of base soil	γ_{base}	18.5	kN/m^3
Unit weight of concrete	γ_C	23.5	kN/m^3
Unit weight of steel	G_s	78.5	kN/m^3
Factor of safety for overturning stability	FS_O	1.5	-
Factor of safety for sliding	FS_S	1.5	-
Factor of safety for soil bearing capacity	FS_B	1.5	-
Cost of steel	C_S	0.4	USD/kg
Cost of concrete	C_C	40	USD/m^3

Table 3. Limits of geometric design variables.

Design Variable	Unit	Lower Limit	Upper Limit
X1	mm	1,309	2,333.3
X2	mm	436.3	777.7
X3	mm	200	333.3
X4	mm	200	333.3
X5	mm	272.2	333.3

In this example, the maximum and minimum selection probabilities are 80% and 0.05%, respectively, with $N_G = 30$. The population size is 120, and the algorithm runs for 200 generations. Table 4 compares the design developed by the hybrid TLBO algorithm with those of other optimization methods. The comparisons are made with grey wolf optimization (GWO) by Kalemci [6], search group algorithm (SGA) and backtracking search algorithm (BSA) by Lopez [18], big bang-big crunch (BB-BC) optimization by Camp and Akin [1], interior search algorithm (ISA) by Gandomi *et al.* [8], differential evolution (DE), genetic algorithm (GA), biogeography based optimization algorithm (BBO) and evolutionary strategy (ES) by Gandomi *et al.* [12], and accelerated particle swarm optimization (APSO) and particle swarm optimization (PSO) algorithm by Gandomi *et al.* [11]. Figure 5 compares the results using the hybrid TLBO algorithm and other methods for low-cost designs.

Table 4. Low-weight design for RC wall without shear key.

Search Technique	X1 mm	X2 mm	X3 mm	X4 mm	X5 mm	R1	R2	R3	Cost
Hybrid TLBO Min Cost	1,740	660	270	200	270	15-ϕ10mm	10-ϕ10mm	10-ϕ10mm	$70.92
Hybrid TLBO Min Weight	1,740	700	210	200	270	14-ϕ12mm	10-ϕ10mm	10-ϕ10mm	$74.47
GWO	1,800	670	210	200	280	28-ϕ10mm	3-ϕ18mm	3-ϕ18mm	$83.00
SGA	1,710	650	200	200	270	27-ϕ10mm	12-ϕ10mm	10-ϕ10mm	$76.84
BSA	1,710	640	200	200	270	27-ϕ10mm	9-ϕ10mm	10-ϕ10mm	$74.50
BB-BC	1,740	650	200	200	270	27-ϕ10mm	10-ϕ10mm	10-ϕ10mm	$70.96
ISA	1,840	750	290	200	270	6-ϕ16mm	3-ϕ18mm	3-ϕ18mm	$73.06
DE	1,870	620	290	200	270	6-ϕ16mm	4-ϕ16mm	5-ϕ14mm	$75.49
GA	1,910	580	270	200	280	17-ϕ10mm	8-ϕ12mm	3-ϕ18mm	$77.63
BBO	1,840	740	270	200	270	16-ϕ10mm	9-ϕ10mm	9-ϕ10mm	$73.08
ES	1,840	690	320	220	280	9-ϕ12mm	6-ϕ14mm	7-ϕ14mm	$78.07
APSO	1,840	570	270	200	270	17-ϕ10mm	13-ϕ10mm	10-ϕ10mm	$73.06
PSO	1,840	740	290	200	270	15-ϕ10mm	9-ϕ10mm	9-ϕ10mm	$73.06

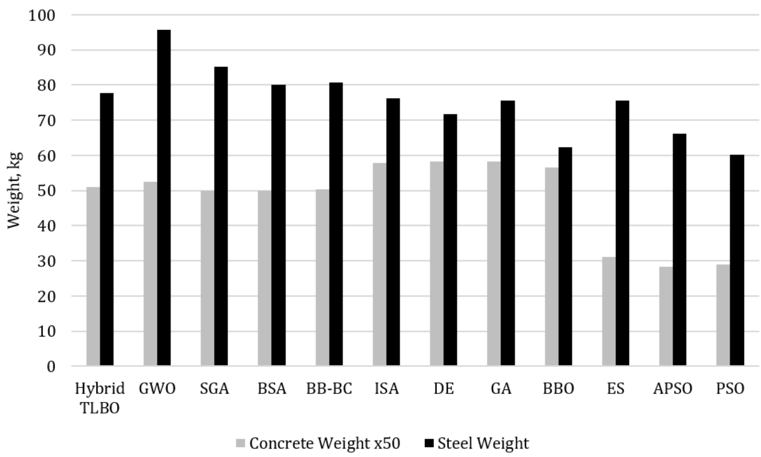

Figure 5. Comparisons of design weight between the Hybrid TLBO and other search techniques for retaining walls without a shear key.

Using weight as the objective function in minimization leads to a higher cost. Since the objective function treats the weight of concrete and steel similarly, the higher the steel weight, the more costly the design. On the other hand, if cost minimization is the objective, a higher concrete weight is preferred, as it reduces the design cost. Hybrid TLBO generates a design with material costs of 74.47 USD ($40/concrete volume and $0.4/steel weight) for weight minimization. In contrast, the material cost for cost minimization is 70.92 USD, slightly lower than the best cost-minimizing BB-BC result ($70.96).

In this example, the significant difference between hybrid TLBO and other optimization methods was that three strength constraints approached 100% of their capacity, indicating an optimal or near-optimal solution. Table 5 lists those constraints above 90% of the maximum limit for various optimization techniques. As shown in Table 5, several optimization methods violated constraints below the 5% acceptable range in practice. The best designs, including those generated by the hybrid TLBO algorithm, maximized the sliding, bearing, and arm moment capacities. A few designs reached near capacity for the heel moment.

Table 5. *Controlling constraints and the ratio applied to the capacity value.*

Search Technique	Sliding	Bearing	Arm Moment	Heel Moment	SD of Minimum Cost	SD of Minimum Weight
Hybrid TLBO	99.70%	98.82%	99.13%	89.06%	0.1269	0.3068
GWO	94.92%				-	-
SGA	99.12%	Violation	97.39%	95.62%	-	-
BSA	98.68%	Violation	97.39%	97.85%	-	-
BB-BC	97.50%	Violation	97.39%	Violation	0.57	5.89
ISA	98.71%				0.2761	10.4974
DE	98.71%				1.67	33.83
GA		92.01%			1.6	33.99
BBO	97.77%		95.70%	98.54%	0.827	30.45
ES	95.85%				1.308	34.23
APSO	90.92%	96.70%			2.279	12.918
PSO	98.27%			93.85%	0.136	0.345

This study also examined the effectiveness of selection probability on the algorithm's performance. Table 6 lists the weight and cost optimization results with and without population selection probability. The multi-population strategy generated lower mean and standard deviation (SD) values for weight and cost optimization. Comparing the hybrid TLBO SD values with those of other optimization methods in Table 5 shows improvements of 6.7% over the best PSO cost-minimization result and 11.1% over the PSO weight-minimization result. Furthermore, selecting the reinforcement area from the list of reinforcement configurations has eliminated 4 of 18 constraint checks in Example 1, saving 22% in constraint value computation in each generation of hybrid TLBO.

Table 6 shows the results of changes in the traditional TLBO.

Table 6. *Results of variables' effect on Hybrid TLBO.*

Objective	Variables	Mean	Median	Fitness Value	Standard Deviation
Weight Minimization	Multi-Population Selections	2,627.5	2,627	2,627	0.3068
	Single Population Selection	2,627.8	2,627	2,627	0.6052
Cost Minimization	Multi-Population Selections	\$71.00	\$70.92	\$70.92	0.1269
	Single Population Selection	\$71.05	\$70.92	\$70.92	0.8355

Example 2 is the design of an RC cantilevered retaining wall with a shear key. The objective function considers only the weight of the wall's concrete and steel per unit length. Table 7 lists the geotechnical and cost parameters. Table 8 lists the upper and lower limits on geometric design variables; the design considers a 10mm increment in all dimensions. In this example, the maximum and minimum selection probabilities are 80% and 0.05%, respectively, with $N_G = 30$. The population size is 200, and the algorithm is run for 200 generations.

Table 7. RC Cantilevered Retaining Walls Parameters for Example 2.

Parameters	Symbol	Value	Unit
Arm height	H	4.5	m
Depth of soil in front of the wall	D	0.3	m
Surcharge load	Q	30	kPa
Cohesion of the base soil	c_{base}	0	kPa
Internal friction angle of base soil	φ_{base}	34	°
Internal friction angle of retaining soil	φ	28	°
Compressive strength of concrete	f_c	21	MPa
Yield strength of reinforcing steel	f_y	400	MPa
Concrete cover	cc	70	mm
Percentage of shrinkage and temperature reinforcement	ρ_{st}	0.002	-
Backfill slope	β	0	°
Unit weight of retained soil	γ_{fill}	18.5	kN/m^3
Unit weight of base soil	γ_{base}	17	kN/m^3
Unit weight of concrete	γ_C	23.5	kN/m^3
Unit weight of steel	G_s	78.5	kN/m^3
Factor of safety for overturning stability	FS_O	1.5	-
Factor of safety for sliding	FS_S	1.5	-
Factor of safety for soil bearing capacity	FS_B	1.5	-
Cost of steel	C_S	0.4	USD/kg
Cost of concrete	C_C	40	USD/m^3

Table 8. Limits of geometric design variables.

Design Variable	Unit	Lower Limit	Upper Limit
X1	mm	1,960	5,500
X2	mm	650	1,160
X3	mm	250	500
X4	mm	250	500
X5	mm	400	500
X6	mm	1,960	5,500
X7	mm	200	500
X8	mm	200	500

Table 9 compares the results of Hybrid TLBO with those of other optimization methods. The comparisons are made with grey wolf optimization (GWO) by Kalemci *et al.* [6], the search group algorithm (SGA) and the backtracking search algorithm (BSA) by Lopez [18], and big bang-big crunch (BB-BC) optimization by Camp and Akin [1].

Table 9. *Low-weight design for RC wall with a shear key.*

Search Technique	X1 mm	X2 mm	X3 mm	X4 mm	X5 mm	X6 mm	X7 mm	X8 mm	R1	R2	R3	R4	Cost
Hybrid TLBO Min Cost	3,290	1,150	490	250	490	1,960	200	490	17-ϕ14mm	21-ϕ10mm	16-ϕ12mm	7-ϕ10mm	$206.75
Hybrid TLBO Min Weight	3,280	1,150	480	250	490	1,960	200	490	5-ϕ25mm	21-ϕ10mm	16-ϕ12mm	7-ϕ10mm	$212.96
GWO	3,210	660	390	250	470	2,490	200	490	19-ϕ16mm	20-ϕ10mm	21-ϕ12mm	4-ϕ12mm	$234.33
SGA	3,450	650	410	250	420	2,320	200	500	22-ϕ14mm	20-ϕ10mm	24-ϕ12mm	27-ϕ10mm	$233.81
BSA	3,460	650	410	250	420	2,720	200	500	30-ϕ12mm	16-ϕ10mm	25-ϕ12mm	6-ϕ10mm	$244.21
BB-BC	3,760	680	410	250	400	3,220	200	490	22-ϕ14mm	18-ϕ10mm	20-ϕ14mm	6-ϕ10mm	$233.81

Table 10 shows the results of changes in the traditional TLBO.

Table 10. *Results of variables' effect on Hybrid-TLBO for retaining wall with a shear key.*

Objective	Variables	Mean	Median	Fitness Value	SD
Weight Minimization	Multi-Population Selections	7,875.5	7,844.2	7,844.2	0.9318
	Single Population Selection	7,987.4	7,844.2	7,844.2	1.8356
Cost Minimization	Multi-Population Selections	$213.83	$206.75	$206.75	3.4261
	Single Population Selection	$216.20	$206.75	$206.75	4.5800

As shown in Table 11, the controlling constraints, those exceeding 90% of capacity, for the design of the RC retaining wall with a shear key are bearing, arm moment, shear, and heel moment. A few optimization methods violated constraints; however, their values are below the 5% acceptable range in practice. The hybrid TLBO developed better designs when minimizing the cost. Three strength constraints reached nearly 100% capacity, indicating that the design is optimal or near-optimal.

The design obtained for cost minimization uses less steel and more concrete than weight minimization, and a higher amount of concrete since the steel price per unit weight is higher than that of concrete. In addition, the variation of solutions obtained from hybrid TLBO is lower than that of other optimization methods, as indicated by the lower SD values. There is an 84.2% improvement in the SD values, indicating that the design results are very close across runs. Furthermore, using a feasible list of reinforcement configurations eliminated 4 of 20 constraint checks in Example 2, saving 20% in constraint-value computation in each generation of the hybrid TLBO.

Figure 6 compares the results of hybrid TLBO and other optimization methods for low-weight designs.

Table 11. *Controlling constraints and the ratio applied to the capacity value with a shear key.*

Search Technique	Bearing	Arm Moment	Arm Shear	Heel Moment	Heel Shear	SD
Hybrid TLBO Min Weight	90.92%	99.8%		91.4%	99.65%	3.4261
Hybrid TLBO Min Cost	99.97%	98.69%		99.46%	92.39%	0.9318
GWO	98.34%	91.55%	94.15%	Violation	Violation	-
SGA		94.8%		99.98%	Violation	-
BSA			90.2%	99.43%	93.03%	-
BB-BC		93.76%		99.51%	93.08%	5.89

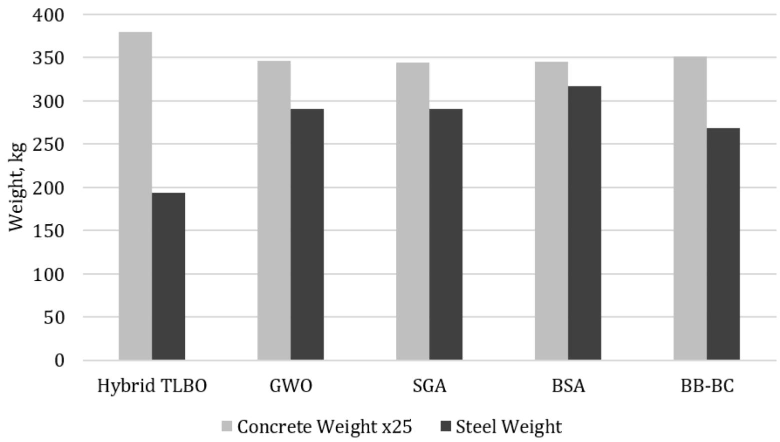

Figure 6. *Comparisons of design weight between the Hybrid TLBO and other search techniques for retaining wall with a shear key.*

6. Conclusion

This study presented a modified TLBO approach for the optimal design of RC retaining walls, demonstrating both improved computational efficiency and solution quality. The proposed hybrid method integrates a multi-selection population strategy and a predefined feasible reinforcement configuration list, addressing two key limitations in conventional optimization approaches: premature convergence and excessive computational cost associated with constraint checking.

The results confirm that the hybrid TLBO significantly enhances the exploration of the design space while maintaining robust convergence toward near-optimal solutions. Compared to established optimization techniques, the method consistently achieved lower standard deviation values, indicating improved stability and reliability of the solutions. Moreover, several governing constraints approached their capacity limits, which is a strong indicator of optimal or near-optimal structural performance. The introduction of the feasible reinforcement configuration list reduced the number of constraint evaluations by approximately 20%, thereby improving computational efficiency without compromising design accuracy.

From a practical design perspective, the findings reinforce that cost-based optimization yields more economical solutions than weight minimization, primarily because of the relative cost differences between steel and concrete. The hybrid TLBO method, therefore, provides a balanced and efficient framework for achieving cost-effective, structurally sound retaining wall designs.

Overall, the proposed approach offers a reliable and efficient tool for structural optimization of RC retaining walls and has strong potential for broader application in other civil engineering optimization problems. Future research can extend this work by incorporating multi-objective optimization (e.g., cost, sustainability, and CO_2 emissions) and by applying the methodology to more complex geotechnical conditions, dynamic loading scenarios, and real-world design case studies.

Abbreviations

RC	Reinforced Concrete
TLBO	Teaching-Learning-Based Optimization
Hybrid TLBO	Modified Teaching-Learning-Based Optimization
GWO	Grey Wolf Optimization
SGA	Search Group Algorithm
BSA	Backtracking Search Algorithm

BB-BC	Big Bang–Big Crunch Optimization
ISA	Interior Search Algorithm
DE	Differential Evolution
GA	Genetic Algorithm
BBO	Biogeography-Based Optimization
ES	Evolutionary Strategy
APSO	Accelerated Particle Swarm Optimization
PSO	Particle Swarm Optimization
CO_2	Carbon Dioxide
ACI	American Concrete Institute
SF_S	Safety Factor for Sliding
SF_B	Safety Factor for Bearing
FS_O	Factor of Safety for Overturning
FS_S	Factor of Safety for Sliding

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Honar Issa: Formal Analysis, Methodology, Software, Validation, Visualization, Writing – original draft

Charles Camp: Conceptualization, Methodology, Supervision, Validation, Writing – review & editing

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper. The research presented in this study was conducted independently, and no financial, personal, or institutional relationships influenced the design, analysis, or reporting of the results.

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