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Research Article

Characterization of Rectangular Waveguides Loaded E-Plane Dielectrics Using the Newton-Raphson Method

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Abstract

Homogeneous metallic waveguides have long been used to carry high powers. They are often filled with inhomogeneous, isotropic dielectrics to reduce their size and cut-off frequencies. To characterize these inhomogeneous rectangular waveguides made of homogeneous and isotropic media, the Newton-Raphson method is used in this article. Frequency of cutoff, attenuation, and power flow distribution are all properties of the EM wave that are highly dependent on the physical structure and composition within the guide. This article presents the characterization of an inhomogeneous and isotropic rectangular guide. The analysis of this type of guide is based on the Borgnis potential method for determining the components of the electric field E and the magnetic field H, to obtain the guide's dispersion equations. The modes that were found to exist in these waveguides are hybrid, meaning that they have both axial E- and H-fields. Numerical resolution of these equations using the Newton-Raphson method obtains the guide's propagation constants. A MATLAB program is used to plot these dispersion curves. The propagation constant increases as a function of frequency, and the d/a ratio influences the dispersion curves. Increasing the relative permittivity of the dielectric leads to an increase in the ratio of the propagation constant in the z direction to the wave number.

Keywords

Characteristic, Inhomogeneous Rectangular Guide, E-Plane, Newton-Raphson Method

1. Introduction

Microwave equipment and its applications play a very important role in our daily lives. For this reason, the use of sophisticated systems has their use in various telecommunications systems [1, 2].

Rectangular waveguides are one of the most widely used transmission lines. The main application for this type of guide was the transmission of microwave signals. There are still some critical applications, such as couplers, detectors, isolators, attenuators and slotted lines, available on the market in a wide variety of different bands from 1 to 220 GHz. Nowadays, modern devices use flat transmission lines such as strip-lines or microstrips rather than waveguides. This also helps to miniaturize devices. However, waveguides still have important applications, in high-power systems, millimeter-wave

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applications, satellite systems and so on.

In order to reduce their size and cut-off frequencies, these guides are often filled with inhomogeneous, isotropic dielectrics. In the general case, the propagation media in each guide may be different (inhomogeneous guides). Classical tools for propagation in these media can be used to establish analytically the expressions of the dispersion equations [3-6].

In 1950, Harlington proposed a modal connection method based on the use of LSE and LSM modes, thus obtaining dispersion equations [7]. These dispersion equations are solved for small values of the propagation constants in each medium by approximating the circular functions. In 1988, TAO et al. used the Transverse Operator Method (MOT) to define a transfer matrix linking the electromagnetic fields at the interfaces of each layer, and thus, by chaining these matrices, obtain a global transfer matrix for the guide [8, 9]. To improve these results, a rigorous characterization of a rectangular guide is proposed, taking into account all the values of the propagation constants in each medium.

Yu-Bo Tian (2004) [10] is developed An effective new algorithm combining Genetic Algorithm (GA) with Parameter Tracking Scheme (PTS) and Dynamic Searching

Area (DSA) technique. They used the algorithm, the propagating constants of waveguide symmetrically or asymmetrically loaded with dissipative magnetic slab are solved successfully and tightly linked to certain designated modes. The results also reveal some interesting phenomena, such as for some high-order modes and certain thickness of loaded slab, the negative phase velocity may occur.

Felipe L et al [11] presented an efficient root-finder method for the cutoff wave-number resolution in a rectangular partially filled waveguide based upon the Cauchy integral method.

This method is valid for lossy dielectric and magnetic materials, has over others is that no initial seed is necessary for the localization of zeros, and also that no roots are lost inside the region under study, which guarantees that all modes are taken into account when the full-wave problem is solved.

Coşkun DENIZ (2017) [12] proposed a fast computational algorithm design based on the Newton-Raphson (N-R) numerical method to determine the first n zeros of these special functions. He showed that the determination of the zeros of the first two types of Bessel functions and their derivatives by fast, reliable and accurate calculations is essential for the determination of the required transverse (TE) and transverse electrical functions. Determining the zeros of the first two types of Bessel functions and their derivatives using fast, reliable calculations is essential for determining the transverse electrical (TE) and transverse magnetic (TM) modes required for circular waveguides. Their proposal was to scan the given function in the selected domain according to the chosen number of iteration steps (or the number of divisions of the domain) and find their zeros by the N-R method at each step.

In this study, the analysis of inhomogeneous and isotropic rectangular waveguides is developed, although they have been the subject of numerous studies.

The evolution of computational media (computers, etc.) has led to better development of numerical methods, making it possible to analyze more complex microwave structures [2-9, 13]. One such method is the Newton Raphson method. This method provides an efficient algorithm capable of numerically finding an approximate zero (root) of the dispersion equation with a good degree of precision.

The aim of this work is to apply Newton Raphson's method to the study of an inhomogeneous E-plane waveguide as an aid to solving the dispersion equation.

2. Theory Study

A waveguide formed by two different dielectrics is a discontinuous medium. Medium 1 can be air with relative permittivity $\varepsilon_r = 1$ and medium 2 a dielectric with any relative permittivity other than unity. This is illustrated in figure 1.

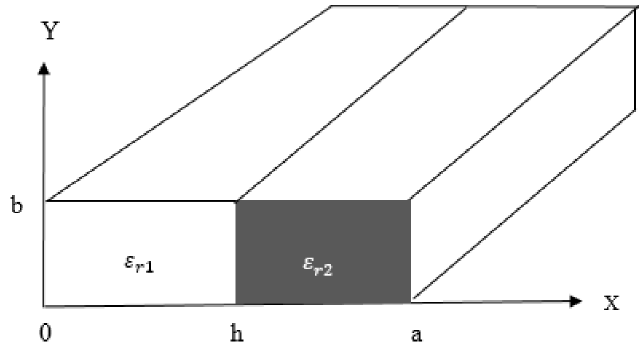

Figure 1. *Dielectric-filled inhomogeneous guide.*

Since the LSE mode is transverse to x, it can be characterized with Hx. Similarly, the LSM mode can be characterized with Ex. The field has to have $e^{jk_z z}$ dependence everywhere inside the waveguide due to the phase matching condition. Consequently, the equations satisfied by Hx and Ex are:

For LSM modes, the axial component of the magnetic field is zero ($H_x = 0$) and $E_x \neq 0$.

$$\vec{E}\begin{cases} E_x \\ E_y \\ E_z \end{cases} \text{and } \vec{H}\begin{cases} H_x = 0 \\ H_y \\ H_z \end{cases}$$

The generating function E_x obeys the Helmholtz equation below:

$$\nabla^2 E_x + \omega^2 \varepsilon \mu_o E_{ix} = 0 \rightarrow (\nabla^2 + k_i^2 - k_z^2) E_{ix} = 0 \quad (1)$$

LSM modes

$$\nabla^2 H_{ix} + \omega^2 \varepsilon \mu_o H_{ix} = 0 \rightarrow (\nabla^2 + k_i^2 - k_z^2) H_{ix} = 0 \quad (2)$$

LSE modes

$$E_x(x,y)=\begin{cases}E_{x1}=[A_1\cos(k_{x1}x)+B_1\sin(k_{x1}x)]\sin\left(\frac{n\pi}{b}y\right) & 0\le x\le x_1\\ E_{x2}=[A_2\cos(k_{x2}x)+B_2\sin(k_{x2}x)]\sin\left(\frac{n\pi}{b}y\right) & x_1\le x\le x_2\end{cases} \quad (3)$$

where subscript i denotes the region i

Expression of $E_y$and E_z of the field

$$\begin{cases}E_{y1,2}=\frac{1}{k_0^2-k_x^2}\frac{\partial}{\partial y}\left(\frac{\partial E_{x1,2}}{\partial x}\right)\\ E_{z1,2}=\frac{1}{k_0^2-k_x^2}\frac{\partial}{\partial z}\left(\frac{\partial E_{x1,2}}{\partial x}\right)\end{cases} \quad (4)$$

The magnetic field components are continuous

$$\begin{cases}H_{y1,2}=\frac{\omega k_z}{v^2\mu}\frac{\omega k_z E_{x1,2}}{k_{01,2}^2-k_x^2}\\ H_{z1,2}=-\frac{j\omega}{v^2\mu(k_{01,2}^2-k_x^2)}\left(\frac{\partial E_{x1,2}}{\partial y}\right)\end{cases} \quad (5)$$

where the cutoff constants $k_{x1,2}$along the (Ox) axis are given by:

$$k_{x1,2}=\sqrt{k_{01,2}^2\varepsilon_{r1,2}-(\frac{n\pi}{b})^2-k_z^2}$$

After finding the components of the electric and magnetic fields in the two media according to the LSM modes, the dispersion equation for this mode is determined. This equation is given by the following formula:

$$\varepsilon_{r2}k_{x1}tank_{x1}h=\varepsilon_{r1}k_{x2}tank_{x2}(h-a) \quad (6)$$

Numerical Resolution of the Dispersion Equation Using the Newton-Raphson Method

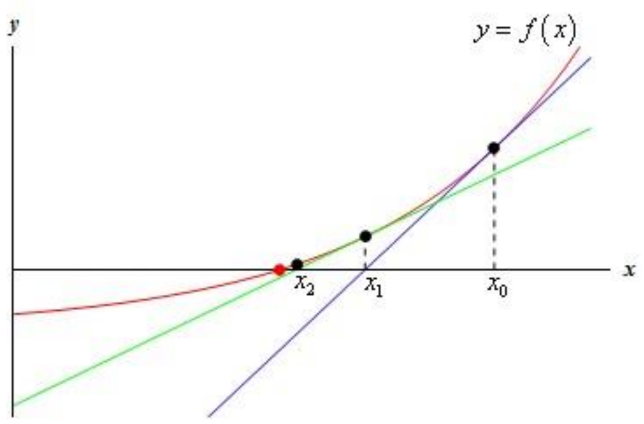

Figure 2*. Principle of Newton's method.*

The principle of this method is illustrated in figure 2. A first estimate x_0 is chosen, the second estimate is x_1 determined by the intersection of the tangent line of the function f(x) at the point $(x_1; f(x_1))$ and the straight line y = 0. The third estimate x_2 is determined by the intersection of the tangent line of the function f(x) at the point $(x_2; f(x_2))$ and the straight line y=0, and so on.

The Newton-Raphson method is an algorithm for numerically finding a precise approximation to a zero (or root) of a function of one real variable. In other words, it's a method for solving an equation of the form f(x) = 0. It involves successively finding the best approximations to the roots of the real function [7, 14-16].

For LSM modes, the dispersion equation is written as follows:

$$\varepsilon_{r2}k_{x1}\tan(k_{x1}h)=\varepsilon_{r1}k_{x2}\tan k_{x2}(h-a)$$

The function f(x) is defined as:

$$f(x)=\varepsilon_{r2}k_{x1}\tan(k_{x1}h)-\varepsilon_{r1}k_{x2}\tan k_{x2}(h-a)$$

Posing f(x) = 0:

$$\varepsilon_{r2}k_{x1}\tan(k_{x1}h)-\varepsilon_{r1}k_{x2}\tan k_{x2}(h-a)=0 \quad (7)$$

It therefore boils down to solving equation (6) using the Newton-Raphson method. looking for the points x1, x2, x3...xp until convergence such that:

$$f(x)=f(x_o)+(x-x_o)f'(x_o) \quad (8)$$

The equation (4) allows us to write:

$$x_1=x_o-\frac{f(x_o)}{f'(x_o)}$$

By successive iterations, we construct a sequence $(x_p)_{p\in N}$ such that:

$$x_{p+1}=x_p-\frac{f(x_p)}{f'(x_p)} \quad (9)$$

where $f'(x)$ is the derivative of $f(x)$.

3. Results and Discussions

To plot the dispersion curves of the fundamental mode with respect to the LSM mode dispersion equation, a homogeneous guide of dimensions a and b composed of two isotropic dielectric media of relative permittivities εr1 and εr2 is considered.

Dispersion curves for a homogeneous, isotropic guide.

The rectangular guide is homogeneous and empty, with εr2 = εr1= 1. Considering the fundamental mode.

mode (n = 1 and m = 0), the dispersion curve expressing the propagation constant is obtained as a function of frequency for this guide, shown in figure 3.

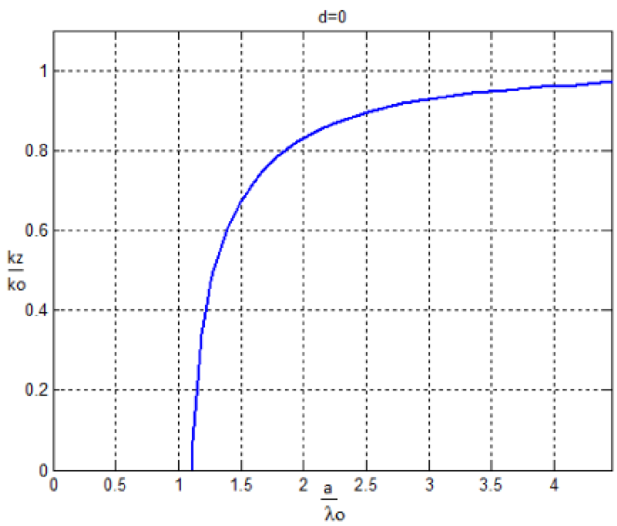

Figure 3. *Propagation constant variation curve with εr2 = εr1= 1.*

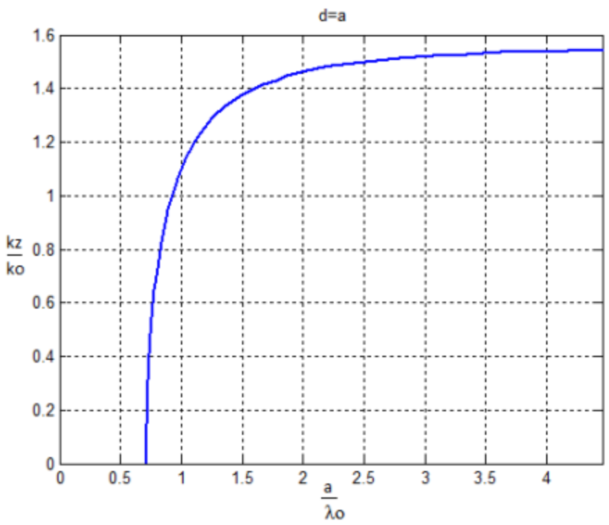

Figure 4. *Propagation constant variation curve with εr2 = εr1= 2.45.*

Figure 4 shows the variation of the propagation constant as a function of the ratio a/λ_o for a rectangular guide is homogeneous and loaded with a dielectric with εr1 = 2.45. Considering the fundamental mode (n = 1 and m = 0). The curve grows exponentially and becomes constant from a certain value of a/λ_o.

Dispersion curves for an inhomogeneous and isotropic waveguide.

Figure 5 shows the different dispersion curves for different d values. It can be seen that the propagation constant increases as a function of frequency. The higher the d/a ratio, the higher the propagation constant as a function of frequency. Let us note that the higher the d/a ratio, the faster the dispersion curves tend towards their asymptotic value.

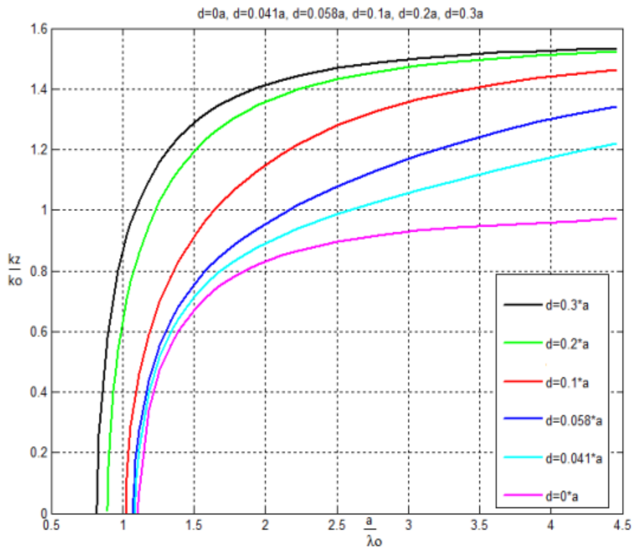

Figure 5. *Propagation constant versus frequency curve for different values of d.*

Figure 6 shows the dispersion curve for large values of d, the results agree with those reported in the literature [15] and show an improvement in the accuracy of the results. The higher the d/a ratio, the higher the propagation constant as a function of frequency. The higher the d/a ratio, the faster the dispersion curves tend towards their asymptotic value.

Figure 7 shows the evolution of ${k_z}/{k_0}$ as a function of frequency, increasing the relative permittivity of the dielectric leads to an increase in the ratio ${k_z}/{k_0}$.

The constant k_z is always greater than the wave number k_0 over the interval of $1 \leq \frac{k_z}{k_o} \leq \sqrt{\varepsilon_r}$. Consequently, the phase velocity is lower than that of light. This is why these waves are called slow waves. They cannot radiate. For high values of a/λ the ratio of $\frac{k_z}{k_o}$ remains constant.

For a given frequency, the guide with a higher permittivity dielectric gives a higher propagation constant value.

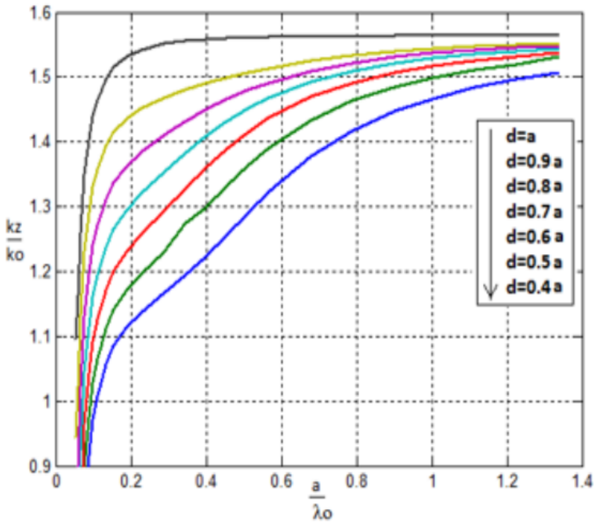

Figure 6. *Dispersion curve for some d/a values above 0.4.*

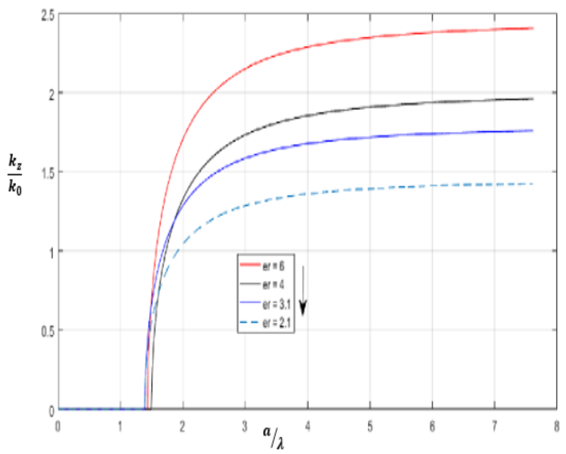

Figure 7. *Effect of permittivity on dispersion curves.*

4. Conclusion

In this study, a rigorous characterization of an inhomogeneous and isotropic rectangular waveguide was carried out. Solving the dispersion equation for the fundamental mode of the LSM using the Newton-Raphson method improved the accuracy of the results.

Abbreviations

LSE: Longitudinal-Section Electric
LSM: Longitudinal-Section Magnetic

Conflicts of Interest

The authors declare no conflicts of interest.

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