American Journal of Mechanics and Applications
2026, Vol. 13, No. 3, pp. 35–40
https://doi.org/10.11648/j.ajma.20261303.11

Research Article

Heat Distribution Within a Raw Cotton Seed

Mamatov Alisher Zununovich[1,*], Pardaev Xanimkul Normamatovich[1], Bomurotov Sherbek Najmidinovich[2]

[1]Department of Textile Machinery Engineering, Tashkent Institute of Textile and Light Industry, Tashkent, Uzbekistan
[2]Department of Mathematics, Gulistan State University, Gulistan, Uzbekistan

Abstract

The drying and active ventilation of raw cotton are important technological processes that directly affect the quality, storage stability, and further processing of cotton. During drying, heat and moisture transfer within individual cotton seeds plays a significant role in determining the efficiency of the overall process. Therefore, mathematical modeling of heat distribution inside a cotton seed is essential for improving and optimizing drying conditions. The purpose of this study is to develop and validate a mathematical model describing the heat distribution process inside a raw cotton seed during active ventilation and drying. For this purpose, an individual cotton seed is represented as a spherical object, and the heat conduction process inside the seed is described by a system of differential equations considering moisture evaporation and convective heat transfer between the seed surface and the surrounding drying medium. The boundary value problem is solved using the method of separation of variables (Fourier method). The unknown coefficients of the mathematical model are determined by applying the least squares method to experimental data. The proposed mathematical model was evaluated by comparing theoretical calculations with experimental results obtained at drying-agent temperatures of 100°C and 150°C. The comparison demonstrated good agreement between the calculated and experimental temperature distributions, with a deviation of less than 5%. This confirms the adequacy and reliability of the developed model for describing the heat transfer process inside raw cotton seeds under active drying conditions. The results of the study provide a theoretical basis for determining rational drying parameters and optimizing technological operating modes of industrial installations used for the drying and storage of raw cotton. The developed approach can also be applied to further investigations of coupled heat and moisture transfer processes in cotton seeds.

Keywords

Raw Cotton, Seed, Heat Conduction, Mathematical Model, Active Ventilation, Drying Kinetics, Differential Equation, Method of Least Squares

1. Introduction

In the processes of primary processing and quality preservation of raw cotton, active ventilation and drying operations play a critical role. Non-uniform distribution of moisture and temperature within the cotton mass leads to self-heating of the raw material, which drastically degrades the quality of both the fiber and the seeds. Cotton seeds possess a complex capillary-porous structure enclosed by a fibrous covering. Consequently, determining the internal temperature field of the seed

*Correspondence: Mamatov Alisher Zununovich (maz54@mail.ru)

Received: 25 August 2026; **Accepted:** 4 September 2026; **Published:** 20 September 2026

during heat and mass transfer with the drying agent constitutes a highly relevant scientific and practical problem. Mathematical modeling serves as the most effective methodology for acquiring a profound understanding of the underlying physical laws of the process and for selecting optimal technological regimes.

2. Literature Review

The study of drying kinetics and the thermophysical properties of raw cotton has been the subject of numerous fundamental investigations conducted by domestic and foreign researchers. In particular, the core principles governing heat and moisture transport in porous media are detailed comprehensively in classical works on heat and mass transfer. However, analytical solutions that simultaneously account for the spherical geometry of the seed, the influence of the surrounding air-fiber mass, and the exponential nature of the moisture evaporation rate remain insufficiently developed. [1-7] The present study is aimed at addressing and fulfilling this specific scientific gap.

3. Research Methodology

Let us assume that the raw cotton seeds (conceived as spheres of radius R) are surrounded by an air-fiber mass whose temperature varies over time according to a specified law g(τ), Convective heat transfer occurs at the contact interface between the sphere and the surrounding mass in accordance with Newton's law of cooling. Taking moisture evaporation into account, the temperature field of the raw cotton seed can be determined by solving the following boundary value problem for the heat conduction equation: [8-12]:

$$c\rho\frac{\partial T}{\partial \tau}=\lambda\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right)+\varepsilon\rho r_{21}\frac{\partial U}{\partial \tau}, 0<r<R, \tau>0$$

$$\lambda\frac{\partial T}{\partial r}+\alpha\big(-T+g(\tau)\big)=0, r=R, \tau>0 \quad (1)$$

$$T(r,0)=T_H, \lim_{r\to 0}\left(r^2\frac{\partial T}{\partial r}\right)=0$$

where

Here, T and T_H denote the current and initial temperatures of the seed, respectively;

U is the volume-averaged moisture content of the seed;

C, ρ and λ represent the specific heat capacity, density, and thermal conductivity coefficient of the seed, respectively;

r_21 signifies the latent heat of vaporization;

ε is the phase transformation coefficient of liquid into vapor;

α is the heat transfer coefficient between the seed surface and the fibrous mass; and τ denotes the drying time [13-17]:

To determine the rate of moisture evaporation, a power-law relationship between the drying rate and moisture content can be utilized at m=1.

$$-\frac{dW}{d\tau}=k(W-W_P)^m, W|_{\tau=0}=W_H$$

$$\frac{\partial W}{\partial \tau}=-k(W_H-W_P)\cdot\exp(-k\tau) \quad (2)$$

where k – is the drying coefficient, and, W_H, and W_P represent the initial and equilibrium moisture content of the seed, respectively.

To evaluate problem (1), the method of separation of variables is applied. Introducing the dimensionless variable ξ = r/R we obtain:

$$\frac{\partial T}{\partial \tau}=\frac{\alpha}{R^2}\left(\frac{\partial^2 T}{\partial \xi^2}+\frac{2}{\xi}\ \frac{\partial T}{\partial \xi}\right)+f(\tau), 0<\xi<1$$

$$-\frac{\partial T}{\partial \xi}+H\cdot R\cdot(g(\tau)-T)=0, \xi=1 \quad (3)$$

$$T(\xi,0)=T_H, T(0,\tau)<\infty$$

where

$$\alpha=\frac{\lambda}{c\rho}, f(\tau)=\frac{0{,}01\varepsilon\ r_{21}\cdot\frac{dW}{d\tau}}{C}$$

The solution to this problem is sought in the following functional form: [13-17]:

$$T(\xi,\tau)=\sum_{n=1}^{\infty}\ T_n(\tau)\frac{\sin(\mu_n\ \xi)}{\xi}+g(\tau)$$

Utilizing the boundary condition yields:

$$T_n(\tau)\ \ (\mu_n cos\mu_n-sin\mu_n+H\ \ R\ \ sin\ \ \mu_n)=0$$

From which we derive the transcendental characteristic equation:

$$tg\mu\ \ =-\frac{\mu}{H\cdot R-1} \quad (4)$$

Consequently, μ_n - represents the roots of the characteristic equation. Substituting expression (4) into equation (3) results in:

$$\sum_{n=1}^{\infty}\left[T_n'(\tau)+a\cdot\frac{\mu_n^2}{R^2}T_n(\tau)\right]\cdot\frac{Sin(\mu_n\ \xi)}{\xi}=-g'(\tau)+f(\tau)$$

By invoking the orthogonality of the function $Sin(\mu_n\ \xi)$, we establish that:

$$T_n'(\tau)+\ \ a\frac{\mu_n^2}{R^2}\cdot T_n(\tau)=b_n\big(f(\tau)-g'(\tau)\big) \quad (5)$$

where

$$b_n=\frac{2(Sin\mu_n\ -\mu_n\ Cos\mu_n)}{\mu_n(\mu_n\ -\ Sin\mu_n\ Cos\mu_n)}$$

The general solution to the ordinary differential equation (5) takes the form:

$$T_n(\tau) = exp\left(-a\frac{\mu_n^2}{R^2} - \tau\right)\left[c + b_n \int_0^\tau f(\theta) exp\left(a\frac{\mu_n^2}{R^2}\ \theta\right) d\theta - b_n \int_0^\tau f(\theta) exp\left(a\frac{\mu_n^2}{R^2}\ \theta\right) d\theta\right]$$

Applying the initial condition, the integration constant is determined as:

$$c = -b_n\ g(0) + b_n T_H$$

Therefore, the analytical solution to problem (1) is expressed as:

$$T(\xi,\tau) = [T_H - g(0)]T_1(\xi,\tau) + \int_0^\tau [f(\theta) - g'(\theta)]d\theta * T_1(\xi,\tau-\theta)d\theta + g(\tau) \quad (6)$$

where

$$T_1(\xi,\tau) = \sum_{n=1}^{\infty} A_n \cdot \frac{Sin(\mu_n\ \xi)}{\mu_n \xi} \cdot exp(-\alpha_n \cdot \tau)$$

$$A_n = b_n \mu_n, \alpha_n = \frac{a\mu_n^2}{R^2}$$

Assuming the absence of evaporation $\varepsilon = 0$ and that the air-fiber mass maintains a constant boundary temperature $(\tau) = T_B$, the solution simplifies to the following expression:

$$T(\xi,\tau) = T_B + [T_H - T_B] \cdot T_1(\xi,\tau) \quad (7)$$

Incorporating the evaporation rate equation (2), the comprehensive formula for computing the temperature field of the raw cotton seed is derived as:

$$T(\xi,\tau) = [T_H - g(0)]T_1(\xi,\tau) + g(\tau) - \int_0^\tau g'(\theta) T_1(\xi,\tau-\theta)d\theta - T_2(\xi,\tau) \quad (8)$$

$$T_2(\xi,\tau) = k\ \ \varepsilon\ \ r\frac{W\ -\ W_1}{100\cdot\ C} \cdot \frac{A}{\alpha_n - k} \cdot \frac{Sin(\mu_n\ \ \xi)}{\mu_n\ \ \xi} - [exp(-k\tau) - exp(-\alpha_n\ \ \tau)] \quad (9)$$

In formula (8), the terms designated by $T_2(\xi,\tau)$ explicitly quantify the thermal energy expended on moisture evaporation during the drying process. If the seed undergoes heating without any concurrent moisture evaporation, then $T_2(\xi,\tau) = 0$.

Let us assume that the temperature of the air-fiber mass varies linearly over time:

$$g(\tau) = \beta_0\ + \beta_1\ \ \tau \quad (10)$$

Under this assumption, the heat distribution within the sphere is structured as follows:

$$T(\xi,\tau) = \ [T_H\ - \beta_0]T_1(\xi,\tau) + \beta_0\ + \beta_1\tau\ - \beta_1\ \ T_3(\xi,\tau) - \ T_2(\xi,\tau) \quad (11)$$

where

$$T_3(\xi,\tau) = \sum_{n=1}^{\infty} \frac{A_n}{a_n} \frac{sin(\mu_n\ \ \xi)}{\mu_n \xi}\ \ [1 - exp\ \ (-a_n\ \ \tau)]$$

If the constants β_0 and β_1 – are unknown, they can be determined via the method of least squares using the experimental values of temperature T_i and time τ_i at a fixed coordinate value: ξ

$$\beta_0 = \frac{c_3\ c_4\ -\ c_2\ c_5}{c_1\ c_4\ -\ c_2^2}, \beta_1 = \frac{c_1\ c_5\ -\ c_2\ c_3}{c_1\ c_4\ -\ c_2^2} \quad (12)$$

where

$$c_1 = \sum_{i=1}^{N} \left(1 - T_1(\xi,\tau_i)\right)^2$$

$$c_2 = \sum_{i=1}^{N} (1 - T_1(\xi,\tau_i))(\tau_i - T_3(\xi,\tau_i))$$

$$c_3 = \sum_{i=1}^{N} (1 - T_1(\xi,\tau_i))(T_i - T_H\ \ T_1(\xi,\tau_i) + T_2(\xi,\tau_i))$$

$$c_4 = \sum_{i=1}^{N} \left(\tau_i - T_3(\xi,\tau_i)\right)^2$$

$$c_5 = \sum_{i=1}^{N} (\tau_i - T_3(\xi,\tau_i))(T_i - T_H\ \ T_1(\xi,\tau_i) + T_2(\xi,\tau_i))$$

Here, N - denotes the total number of experimental data points.

Furthermore, if the temperature of the air-fiber mass varies according to a parabolic law:

$$g(\tau) = \beta_0 + \beta_1\tau\ \ + \beta_2\tau^2 \quad (13)$$

then the analytical solution to problem (1) takes the form:

$$T(\xi,\tau) = [T_H - \beta_0]T_1(\xi,\tau) + \beta_0 + \beta_1\tau - \beta_2\tau^2 - \beta_2 T_4(\xi,\tau) - \beta_1\ \ T_3(\xi,\tau) - T_2(\xi,\tau)$$

where

$$T_4(\xi,\tau) = -2 \quad \sum_{n=1}^{\infty} \frac{A_n}{a_n^2} \cdot \frac{sin(\mu_n \ \xi)}{\mu_n \xi} [1 - exp(-a_n \ \ \tau)] + 2\tau \quad \sum_{n=1}^{\infty} \frac{A_n}{a_n} \cdot \frac{sin(\mu_n \ \xi)}{\mu_n \xi}$$

When the constants β_0, β_1 and β_2, are unknown, they are uniquely determined from the minimization condition:

$$S = \sum_{i=1}^{N} [T_i - T(\xi, \tau_i)]^2 \rightarrow min$$

By setting the first partial derivatives of S with respect to the arguments β_0, β_1 and β_2, to zero, and solving the resulting system of three linear equations, the unknown parameters are obtained.

If we assign $\beta_0 = T_H$, the constants β_1 and β_2 are defined via formula (11) by replacing β_0 with β_2 and introducing the alternative constant structures:

$$c_1 = \sum_{i=1}^{N} \left(\tau_i^2 - T_4(\xi, \tau_i)\right)^2$$

$$c_2 = \sum_{i=1}^{N} (\tau_i^2 - T_4(\xi, \tau_i))(\tau_i - T_3(\xi, \tau_i))$$

$$c_3 = \sum_{i=1}^{N} (\tau_i^2 - T_4(\xi, \tau_i))(T_i - T_H + T_2(\xi, \tau_i))$$

$$c_5 = \sum_{i=1}^{N} (\tau_i - T_3(\xi, \tau_i))(T_i - T_H + T_2(\xi, \tau_i))$$

4. Analysis and Results

To verify the validity and accuracy of the proposed mathematical model, the calculation of the thermal field of the seed was carried out using the experimental case of raw cotton drying within an elementary layer.

The experimental parameters were established as follows: initial seed temperature $T_H = 15°C$, initial moisture content $W_H = 19\%$. The dynamics of the seed heating temperature were investigated under drying agent temperatures of $T_B = 100°C$ and $T_B = 150°C$ at the specific spatial nodes $\xi = 0.2$ and $\xi = 0.5$.

A comprehensive comparison between the experimental and numerically calculated data is presented in Table 1.

To verify the validity of the proposed mathematical model (1) and its solution, the calculation of the seed temperature field is considered using the example of raw cotton drying in an elementary layer.

Table 1: Evaluation of experimental and calculated seed heating temperatures at:

$$T_B = 100°C, 150°C, T_H = 15°C, W_H = 19\%$$

Table 1. *Comprehensive comparison between the experimental and numerically.*

Drying Time (sec.)	Seed heating temperature, °C					
	at $T_B = 100°C$			at $T_B = 150°C$		
	exp.	calc. $\xi = 0,2$	calc. $\xi = 0,5$	exp.	calc. $\xi = 0,2$	calc. $\xi = 0,5$
0	15	14,67	15,01	15	14,54	15,01
15	21	20,03	21,17	23	24,05	25,77
30	26	26,47	27,44	34	35,23	36,68
45	31	31,48	32,32	46	44,24	45,49
60	36	35,55	36,30	52	51,82	52,93
90	42	42,04	42,67	64	64,43	65,36

The comparative analysis demonstrates a high level of agreement between the calculated values and experimental metrics. The results conclusively validate the capacity of the proposed model and its derived analytical formulations to accurately predict the heating temperature of cotton seeds under varying thermal environments.

5. Conclusion and Recommendations

A mathematical model tracking the heat distribution within a raw cotton seed has been successfully developed and implemented, incorporating moisture evaporation kinetics and variable thermal boundary conditions from the ambient environment. The high efficacy of integrating the method of least squares to identify unknown parameters of thermal exposure from sparse experimental data points was verified.

Based on the analytical solutions derived, it is highly recommended that cotton ginning enterprises adjust the operational thermal regimes of drying and ventilation installations to ensure that the internal temperature of the seeds remains within safe margins (not exceeding 42-45 ℃). This operational constraint guarantees the strict preservation of both the seed germination properties and the technological quality parameters of the raw material.

Author Contributions

Mamatov Alisher Zununovich: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Resources, Software, Validation, Writing – original draft, Writing – review & editing

Pardaev Xanimkul Normamatovich: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Methodology, Resources, Software, Validation, Writing – original draft, Writing – review & editing

Bomurotov Sherbek Najmidinovich: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Methodology, Resources, Software, Validation, Writing – original draft, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Parpiev, A. P. (1977). Research on ways to intensify the process of drying raw cotton [Candidate dissertation, Technical Sciences]. Tashkent.

[2] Mamatov, A. Z., & Parpiev, A. P. (1993). An approximate method for calculating the drying process of raw cotton. Izvestiya Vuzov. Tekhnologiya Tekstil'noi Promyshlennosti, (1), 99–101.

[3] Mamatov, A. Z., & Parpiev, A. P. (1992). On the generalized mass transfer equation for calculating the drying kinetics of raw cotton. Reports of the Academy of Sciences of the Republic of Uzbekistan (DAN RUz), (6–7), 21–23.

[4] Mamatov, A. Z., Parpiev, A. P., & Usmankulov, A. (2006). Solution to a problem of mass and heat transfer in a raw cotton seed. Journal of Textile Problems, (4).

[5] Mamatov, A., Parpiev, A., & Shorakhmedova, M. (2021). Mathematical model for calculating the temperature field of a direct-flow drying drum. Journal of Physics: Conference Series, 2131(5), 052067.

[6] Mamatov, A. Z., Usmankulov, A. K., Abbazov, I. Z., Norboyev, U. A., & Mukhametshina, E. T. (2021). Determination of temperature of components of cotton-raw material in a drum dryer with a constant. IOP Conference Series: Earth and Environmental Science, 939(1), 012052.

[7] Mamatov, A. Z., Pardaev, Kh. N., Mardonov, J. Sh., & Plekhanov, A. F. (2021). Determining of the heat-moisture state of raw cotton in a drum dryer. Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Tekhnologiya Tekstil'noi Promyshlennosti, 391(1), 46–49.

[8] Lykov, A. V. (1978). Heat and mass transfer. Energiya.

[9] Ladyzhenskaya, O. A., Solonnikov, V. A., & Ural'tseva, N. N. (1967). Linear and quasilinear equations of parabolic type. Nauka.

[10] Ladyzhenskaya, O. A., & Ural'tseva, N. N. (1973). Linear and quasilinear equations of elliptic type. Nauka.

[11] Mikhlin, S. G. (1966). Numerical realization of variational methods. Nauka.

[12] Mamatov, A., Bakhramov, S., & Narmamatov, A. (2021). An approximate solution by the Galerkin method of a quasilinear equation with a boundary condition containing the time derivative of the unknown function. AIP Conference Proceedings, 2365(1), 020036.

[13] Wheeler, M. F. (1973). A priori error estimates for Galerkin approximations to parabolic partial differential equations. SIAM Journal on Numerical Analysis, 10(4), 723–759.

[14] Douglas, J., Jr., & Dupont, T. (1973). Galerkin methods for parabolic equations with nonlinear boundary conditions. Numerische Mathematik, 20(3), 213–237.

[15] Dendy, J. E., Jr. (1977). Galerkin methods for some highly nonlinear problems. SIAM Journal on Numerical Analysis, 14(2), 327–347.

[16] Mitchell, A. R. (1979). A Galerkin method for nonlinear parabolic equations with nonlinear boundary conditions. SIAM Journal on Numerical Analysis, 16(2), 254–299.

[17] Mamatov, A. Z., Raxmanov, J. T., & Sulaymanova, N. O. (2023). On the stability of the Galerkin method for solving the problem of determining the warm-moisture state of raw cotton. International Bulletin of Engineering and Technology, 3(6), 101–107.

Biography

Mamatov Alisher Zununovich is a highly respected Doctor of Technical Sciences and a professor, renowned for his extensive contributions to the field of machine details. He has authored numerous articles and books, which are considered fundamental references in the industry. His expertise and profound knowledge have earned him recognition both locally and internationally.

Pardaev Xanimkul Normamatovich is a distinguished Doctor of Technical Sciences and professor at the Tashkent Institute of Textile and Light Industry, recognized for his impactful research on raw cotton drying and thermal processing systems. He has published extensive scholarly work across high-impact international journals, helping shape modern approaches to industrial heat-moisture dynamics. His innovative engineering solutions and dedication to academic excellence have made him an influential figure in the textile sector.

Bomurotov Sherbek Najmidinovich is an accomplished researcher and academic specializing in agricultural engineering and textile materials processing. His work focuses on optimizing mechanical operations, enhancing equipment efficiency, and applying advanced mathematical models to industrial drying processes. Through his scholarly publications and innovative technical solutions, he continues to advance research standards and contribute valuable insights to the modern textile and processing industries.

Research Field

Mamatov Alisher Zununovich: Mathematical Modeling of Heat & Mass Transfer, Raw Cotton Primary Processing & Drying Dynamics, Numerical Methods (Galerkin Method, Differential Equations), and Textile Machinery Optimization.

Pardaev Xanimkul Normamatovich: Raw Cotton Primary Processing, Heat and Mass Transfer Dynamics, Mathematical Modeling of Thermal Processes, and Textile Industry Equipment Optimization.

Bomurotov Sherbek Najmidinovich: Agricultural Machinery & Equipment, Raw Cotton Primary Processing, Heat-Mass Transfer Dynamics, and Mathematical Modeling of Drying Processes.