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Research Article

Complex Spacetime Geometry and the Geometric Origin of the Aharonov-Bohm Effect

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Abstract

The Aharonov-Bohm (AB) effect fundamentally demonstrates that electromagnetic potentials, rather than strictly classical fields, play a foundational, observable role in quantum mechanics. While standard quantum theory successfully accommodates this phenomenon via the axiomatic insertion of the minimal coupling principle, a purely geometric interpretation of the gauge potential has historically remained elusive. Building on a previously established complex spacetime framework—where conventional spacetime coordinates are extended into real and imaginary domains—this paper proposes a novel, localized geometric origin for the AB effect. We postulate that the imaginary spatial derivative of the complex wavefunction is physically equivalent to the electromagnetic gauge potential, thereby deriving the minimal coupling prescription directly from spacetime geometry rather than treating it as an independent axiom. Under this framework, the topological phase shift experienced by a charged particle in a field-free region arises naturally from its local evolution through the imaginary dimensions of complex spacetime. Furthermore, by treating the gauge potential as an affine connection or cross-metric term that links real and imaginary dimensions, we successfully redefine the complex metric tensor. This allows the classical electromagnetic field tensor to emerge naturally as the Riemann curvature of the complexified space. This approach provides a unified geometric foundation for both gauge potentials and quantum interference phenomena, resolving previous field-tensor metric limitations while offering a promising mathematical framework for future integrations into relativistic quantum mechanics and cosmic geometry.

Keywords

Aharonov-Bohm Effect, Complex Spacetime Geometry, Gauge Potential, Minimal Coupling Principle, Quantum Topology, Complex Momentum Operator, Electromagnetic Field Tensor, Affine Connection

1. Introduction

The intersection of quantum mechanics and electromagnetism has traditionally relied on the mathematical formalisms of gauge theories. In classical electrodynamics, the scalar potential (V) and vector potential (A) were long considered mere mathematical conveniences used to derive the physical electric and magnetic fields (E and B). However, the Aharonov-Bohm (AB) effect fundamentally challenged this classical paradigm [1]. By demonstrating that a charged particle's wavefunction accumulates an observable phase shift even when traversing a region where the electromagnetic field tensor is strictly zero ($F_{\mu\nu} = 0$), the AB effect proved that the gauge potential (A_μ) is a foundational property of physical reality, independent of the local fields [3].

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In a previous work, a complex spacetime geometry was introduced where conventional spacetime coordinates were extended into the complex domain [8, 9, 18] ($x = x_r + ix_i$, $t = t_r + it_i$). In that framework, applying the Cauchy-Riemann analyticity conditions to a complex wavefunction successfully generated dynamics analogous to classical Maxwell equations, suggesting a geometric origin for the electromagnetic field tensor. However, by tying the imaginary spacetime metric directly to the local field tensor $F_{\mu\nu}$, that initial framework was naturally constrained to classical field interactions and could not inherently account for topological phenomena like the AB effect, which occur in field-free regions.

This paper resolves that limitation by shifting the geometric coupling from the classical field to the quantum potential. We propose that the electromagnetic four-potential A_μ is not merely an external gauge field acting upon a particle, but is intrinsically linked to the imaginary dimensions of the complex spacetime itself. By mathematically correlating the imaginary spatial derivative with the gauge potential, we demonstrate that the Aharonov-Bohm phase shift is a direct geometric consequence of a quantum particle navigating the complexified fabric of spacetime. This model preserves the analyticity of the wavefunction while offering a purely geometric origin for electromagnetic potentials.

2. Redefining the Complex Momentum Operator

In standard quantum mechanics, the momentum of a particle is represented by the differential operator $\hat{p} = -i\hbar\nabla$. When a charged particle interacts with an electromagnetic field, standard theory accounts for this by modifying the momentum operator using the minimal coupling prescription [10, 11]:

$$\hat{p} \quad \rightarrow \quad \hat{p} - qA$$

where q is the charge of the particle and A is the electromagnetic vector potential. While mathematically effective, this substitution is traditionally treated as an axiomatic insertion rather than an emergent property [14] of the spacetime geometry itself.

By extending spacetime into the complex plane, this potential emerges naturally. As established in previous work, extending the space coordinate into a complex variable $x = x_r + ix_i$ requires redefining spatial derivatives. In three dimensions, the gradient operator in complex spacetime becomes:

$$\nabla_c = \nabla_r + i \ \nabla_i$$

where ∇_r represents differentiation with respect to real spatial coordinates, and ∇_i represents differentiation with respect to imaginary spatial coordinates.

If we apply the standard quantum momentum operator to this complexified space, we obtain a generalized complex momentum operator $\hat{P}$:

$$\hat{P} = -i\hbar(\nabla_r + \mathrm{i}\nabla_i)$$

Distributing the terms yields:

$$\hat{P} = -i\hbar\nabla_r + \hbar\nabla_i$$

We immediately recognize the first term, $-i\hbar\nabla_r$, as the standard momentum operator operating in real, observable space. Therefore, the generalized momentum equation can be rewritten as:

$$\hat{P} = \hat{p} + \hbar\nabla_i$$

By comparing this geometrically derived operator with the standard minimal coupling prescription ($\hat{p} - qA$), a profound physical equivalence is revealed. We can perfectly map the gauge potential to the imaginary spatial gradient by asserting:

$$\hbar\nabla_i = -qA$$

or, rearranging for the imaginary derivative:

$$\nabla_i = -\frac{q}{\hbar}A$$

This mathematical identity fundamentally redefines the nature of the vector potential. Under this framework, A is no longer an external gauge field arbitrarily acting upon a particle. Instead, the gauge potential is the physical manifestation of the particle's spatial evolution through the imaginary dimensions of complex spacetime.

When a particle traverses a region with a non-zero vector potential, it is essentially moving through a region of spacetime possessing an imaginary gradient. Therefore, the minimal coupling principle is not merely a rule for adding electromagnetic interactions; it is a geometric necessity for preserving the analyticity of the wavefunction in complex spacetime.

3. Geometric Derivation of the Aharonov-Bohm Phase

To demonstrate the validity of the complex spacetime framework, we apply it to the Aharonov-Bohm (AB) effect, a phenomenon that cannot be explained by classical field geometry alone.

3.1. The Aharonov-Bohm Experimental Setup

Consider the standard AB thought experiment (experimentally verified by Chambers in 1960 [2]): an electron beam is split into two paths, Path 1 and Path 2, which travel around opposite sides of a perfectly shielded, infinitely long solenoid before recombining on a screen to form an interference pattern.

Inside the solenoid, there is a constant magnetic field ($B \neq 0$). Outside the solenoid, the magnetic field is strictly zero ($B = 0$), meaning the classical electromagnetic field tensor $F_{\mu\nu}$ vanishes entirely in the region where the electrons travel. Classically, the electrons experience zero Lorentz force. However, the vector potential outside the solenoid is non-zero ($A \neq 0$).

In standard quantum mechanics, the presence of this vector potential alters the phase of the electron's wavefunction, shifting the interference pattern on the screen. We will now derive this exact phase shift purely from the geometry of complex spacetime.

3.2. Phase Accumulation in Complex Spacetime

The evolution of a quantum wavefunction's phase ϕ as it propagates along a trajectory is determined by the action integral of its momentum. Using the generalized complex momentum operator $\hat{P}$ derived in Section 2, the total phase accumulated along a spatial path [4] Γ is:

$$\phi = \frac{1}{\hbar} \int_{\Gamma} \hat{P} . dx_r$$

Substitute the complex momentum operator $\hat{P} = \hat{p} + \hbar\nabla_i$:

$$\phi = \frac{1}{\hbar} \int_{\Gamma} (\hat{p} + \hbar\nabla_i) . dx_r$$

This integral splits into two distinct components:

$$\phi = \frac{1}{\hbar} \int_{\Gamma} \hat{p}. dx_r + \int_{\Gamma} \nabla_i . dx_r$$

The first term, $\frac{1}{\hbar} \int_{\Gamma} \hat{p}. dx_r$, represents the standard dynamical phase accumulated by a particle moving through real space in the absence of any fields.

The second term, $\int_{\Gamma} \nabla_i . dx_r$, is the geometric phase shift introduced by the complexification of spacetime [4]. It represents the integration of the imaginary spatial gradient along the particle's real spatial trajectory.

3.3. The Topological Loop Integral

To find the observable interference shift on the screen, we calculate the phase difference $\Delta\Phi$ between the electron traveling Path 1 and the electron traveling Path 2. This difference is mathematically equivalent to taking the closed loop integral ($\oint$) around the central solenoid:

$$\Delta\phi = \oint (\frac{1}{\hbar}\hat{p} + \nabla_i). dx_r$$

Because the classical field is zero outside the solenoid ($F_{\mu\nu} = 0$), there are no classical forces acting on the particle, meaning the real momentum $\hat{p}$ is conservative. The closed loop integral of the conservative real momentum vanishes (or contributes only to the baseline interference pattern), leaving only the phase shift generated by the imaginary geometry:

$$\Delta\phi = \oint \nabla_i . dx_r$$

By applying the geometric equivalence established in Section 2.

($\nabla_i = -\frac{q}{\hbar} A$), this purely geometric integral perfectly reproduces the renowned Aharonov-Bohm phase shift [1, 10]:

$$\Delta\phi = -\frac{q}{\hbar} \oint A . dx_r$$

3.4. Physical Implications

This derivation provides a profound reinterpretation of the AB effect. The phase shift is not the result of an invisible, non-local interaction with the shielded magnetic field, nor is it a mere mathematical artifact of gauge theory.

Instead, the Aharonov-Bohm effect is a direct, local consequence of the complex topology of spacetime. While the classical field ($F_{\mu\nu}$) represents the curvature of *real* spacetime, the gauge potential (A) represents the gradient of *imaginary* spacetime. As the electron wave travels through the real spatial dimensions, it continuously samples the underlying imaginary spatial gradient (∇_i), integrating it into an observable geometric phase shift [16].

4. Redefining the Complex Metric Tensor

To fully integrate this framework into a cosmological or relativistic context, we must address how the geometry of complex spacetime relates to the metric tensor. In a previous formulation, it was proposed that the complex spacetime metric could be written as $g^{c}_{\mu\nu} = g_{\mu\nu} + ih_{\mu\nu}$, where the imaginary tensor $h_{\mu\nu}$ was directly proportional to the classical electromagnetic field tensor $F_{\mu\nu}$.

While mathematically consistent with classical Maxwell equations in curved spacetime, the Aharonov-Bohm effect exposes a limitation in that specific mapping.

4.1. The Limitation of the Field Tensor Metric

If we define the imaginary curvature strictly by the field tensor ($h_{\mu\nu} \propto F_{\mu\nu}$), the imaginary geometry completely flattens out in any region where the electric and magnetic fields are zero. In the Aharonov-Bohm experiment, the electron travels entirely through a field-free vacuum ($F_{\mu\nu} = 0$). Under the previous metric definition, the imaginary metric

$h_{\mu\nu}$ would also be zero in this region, which would mathematically predict no phase shift—contradicting experimental reality.

Since the topological phase shift relies on the gauge potential (A_μ), our complex metric tensor must fundamentally couple to A_μ, not just its classical derivative $F_{\mu\nu}$.

4.2. Gauge Potentials as Complex Affine Connections

In general relativity, the gravitational potential is not a force field, but is embedded directly in the metric tensor $g_{\mu\nu}$, with gravity emerging from the spacetime connection [12] (Christoffel symbols). We propose a parallel geometric treatment for the electromagnetic potential within complex spacetime.

Rather than the imaginary metric being the field tensor itself, the gauge potential A_μ functions as the *affine connection* (or a mixed real-imaginary metric cross-term) that links the real and imaginary dimensions of spacetime [13]. Drawing inspiration from Kaluza-Klein theory—which successfully mapped [6, 7] A_μ to the off-diagonal components of a 5D metric—we can map A_μ to the geometric interaction between real and imaginary coordinates [6, 7, 17].

The generalized line element ds^2 for a particle moving through complex spacetime can be updated to include cross-terms coupling the real spatial differentials $dx_r^\mu$with the imaginary spatial differentials dx_i^ν:

$$ds^2 = \ g_{\mu\nu}dx_r^\mu dx_r^\nu - g_{\mu\nu}dx_i^\mu dx_i^\nu + 2ikA_\mu \ \ dx_r^\mu dx_i^\nu$$

where k is a coupling constant proportional to the particle's charge-to-mass ratio.

4.3. Recovering the Classical Field as Curvature

By embedding A_μ at the fundamental level of the metric's connection, we elegantly recover both quantum and classical regimes:

1. The Quantum Regime: In a field-free region (like the exterior of the AB solenoid), the overall classical curvature is zero, but the non-zero A_μ terms in the metric actively twist the phase of the complex wavefunction as it propagates through$d\ \ x_r^\mu$, producing the topological phase shift.

2. The Classical Regime: The classical electromagnetic field tensor $F_{\mu\nu}$ is no longer a fundamental metric component. Instead, it emerges naturally as the *Riemann curvature* of this complexified space. Just as spacetime curvature $R_{\mu\nu\rho\sigma}$ is calculated from the derivatives of the gravitational metric, the electromagnetic field $F_{\mu\nu} = \ \partial_\mu A_\nu - \ \partial_\nu A_\mu$ is simply the observable geometric curvature generated by [3, 15] the spatial derivatives of the underlying complex gauge connection.

Consequently, classical electromagnetic fields are merely the localized, macroscopic curvatures of complex spacetime, while the gauge potential is the fundamental, global geometric fabric experienced by the quantum wavefunction.

5. Conclusion

The Aharonov-Bohm effect has long stood as a profound indicator that classical electromagnetic fields are insufficient to fully describe the quantum world. While standard quantum mechanics accommodates this reality through the minimal coupling principle and gauge theory [14, 15], these mathematical tools have historically lacked a clear, underlying physical geometry.

In this paper, we have demonstrated that the geometric origin of the Aharonov-Bohm effect can be naturally explained by extending spacetime into the complex plane. By redefining the complex momentum operator, we established a strict physical equivalence between the electromagnetic gauge potential (A_μ) and the spatial gradient of imaginary spacetime (∇_i).

This complex geometric framework yields several key theoretical advancements:

1. A Local Origin for the AB Effect: The topological phase shift observed in field-free regions is no longer a mysterious, non-local interaction. Instead, it is the direct, local result of a quantum wavefunction integrating the imaginary geometry of spacetime as it propagates along its real spatial trajectory.

2. Potentials as Fundamental Geometry: The vector potential is elevated from a mathematical gauge choice to a fundamental structural component of the universe—specifically, the affine connection or cross-metric term linking real and imaginary spacetime dimensions.

3. Fields as Macroscopic Curvature: Classical electromagnetic fields ($F_{\mu\nu}$) are re-contextualized not as the primary physical reality, but as the emergent macroscopic curvature (Riemann-like tensor) of this underlying complex gauge connection.

By shifting the geometric focus from the classical field tensor to the quantum vector potential, this model successfully unifies the macroscopic dynamics of electromagnetism with the topological phenomena of quantum mechanics.

6. Future Research Directions

This complexified spacetime model opens several promising avenues for future research. Subsequent work should explore the integration of this complex metric tensor into the Dirac equation to account for intrinsic spin [5]. Additionally, extending this framework into general relativity may provide novel insights into quantum gravity, where both gravitational and electromagnetic interactions emerge from the unified curvature of real and imaginary spacetime dimensions. Finally, identifying testable experimental predictions—such as modified interference patterns in extreme topological geometries—will be crucial for validating the physical reality of imaginary spacetime dimensions.

Abbreviations

AB	Aharonov-Bohm

Author Contributions

Bhushan Poojary: Conceptualization, Formal Analysis, Methodology, Writing – original draft

Conflicts of Interest

The author declares no conflicts of interest.

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