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4D Gravitational Collapse Spherically Symmetric Spacetime in *f(R,T)* Theory with Cosmological Constant

Yasar Khan[1], Furqan Habib[2, *], Asaf Khan[1]

[1]Department of Mathematics and Statistic, University of Swat, Swat, Pakistan
[2]Department of Mathematics, University of Peshawar, Peshawar, Pakistan

Email address:
y.zada2130@gmail.com (Yasar Khan), furqanhabib125@gmail.com (Furqan Habib), asafkhan@uswat.edu.pk (Asaf Khan)
*Corresponding author

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Abstract: This study explores the behavior of an anisotropic fluid in a spherically symmetric spacetime by examining expanding and collapsing solutions to the Einstein Field Equations (EFEs) within the framework of $f(R,T)$ gravity. This modified theory of gravity extends General Relativity by allowing the gravitational action to depend on both the Ricci scalar R and the trace T of the energy-momentum tensor. The work incorporates a cosmological constant to assess its influence on the evolution of the fluid. A central aim of the study is to understand how the interaction between the Ricci scalar, the expansion scalar, and the trace of the energy-momentum tensor affects the dynamics of the system. Special attention is given to the anisotropic nature of the fluid, where radial and tangential pressures differ adding complexity to both expansion and collapse processes. The presence of a cosmological constant further modifies the pressure and density profiles, revealing how dark energy-like effects can shape the evolution of matter under gravity. The research identifies the existence of a single horizon in the system and uses a mass function to analyze the formation of trapped surfaces regions where outgoing light rays begin to converge, indicating gravitational collapse. Additionally, the relationship between the coupling constants Λ (cosmological constant) and λ (associated with the $f(R,T)$ theory) is explored for both collapsing and expanding scenarios. Graphical results highlight the influence of these parameters on pressure, mass, anisotropy, and energy density, offering valuable insights into modified gravity's role in astrophysical phenomena.

Keywords: Cosmological Constant, Trapped Surfaces, Collapsing Solution, Expanding Solution, Anisotropy

1. Introduction

The contraction of an astronomical object as a result of its natural gravitational force, which pushes objects toward the center of gravity, is known as gravitational collapse [1]. One of the main processes by which structures are formed in the universe. Following sufficient accretion, an initial, relatively smooth dispersion of matter over time, such as stars or black holes, may eventually collapse to a region of higher density a star's abrupt collapse near the end of its life cycle, which is an example of how matter tends to gravitate toward a single center of mass. Gravitational collapse is the process by which an object in space, like a star or gas cloud, collapses. It contracts because of its own incredibly strong gravity.

The Einstein Field Equations $(EFEs)$ are the foundation of the general theory of relativity, revealing the complex interaction of space, time, and matter. These equations help us comprehend the mechanics of gravity. The universe's evolution at present, the most prevalent intriguing problem in gravitation. In order to account for the universe's fast expansion, researchers have put proposed a number of modified gravity concepts. The cosmic constant, which controls acceleration and dark energy, is one method. The cosmological constant issue might be resolved by different models, like $f(R,T)$ theories [2, 3]. These models provide a foundation for interpreting cosmic acceleration and involve linear curvature-matter coupling $(R + \gamma T)$ and quadratic gravity $(R + \gamma R^2)$. Moreover, various hypotheses argue that the rapid expansion of the universe can be understood through the combination of dark energy and modified gravity. For example, a modified Einstein action model with the $1/R$ scalar has been laid out, proposing that the expansion of the universe is caused by cosmic scale distortion and curvature, predominantly by the repulsive gravity of dark energy [4].

General relativity, must be replaced with updated gravity theories to account for the universe's expansion. The f(R) theory, in which the Lagrangian is a generic function of the Ricci scalar, was initially introduced by Buchdahal in 1970. By setting R to a constant scalar value, general relativity is restored.The f(R) gravity theory naturally depicts the transition from slowdown to acceleration in a changing universe, providing a plausible foundation for comprehending cosmic expansion. Hawking and Penrose [1, 5, 6] were interested in singularities in spacetime and they went into great length about the formation and presence of singularities. They discussed their theories and noted that when a trapped surface is generated, the gravitational collapse of large objects causes singularities in space-time. Penrose proposed two model conjectures: weak and strong censorship conjectures. According to a weak form of the conjecture, a singularity may be concealed behind an event horizon and only be visible to a nearby observer. A singularity following the development of a gravitational process cannot be observed by a nearby or distant observer, according to the strong version of the conjecture.There was no empirical or mathematical proof to back up or disprove Penrose's theories at the time. Some authors [7–9] challenged Penrose's work by providing counterexamples to the Penrose conjectures. Gravitational lensing was later used by Virbhadra [10] to improve the articulation of the Penrose conjectures. For their dust model, Oppenheimer and Snyder [11] investigated gravitational collapse. Their research produced a black hole. Numerous researchers used the cosmological constant to examine gravitational collapse. Sharif and Ahmad [12] examined perfect fluid collapse with a non-zero cosmological constant using junction circumstances. It was later extended to five dimensions by the authors [13].

However, Dabnath et al. [14] investigated dust collapse with a cosmological constant in quasi-spherical geometry. Additionally, Sharif and Abbas [15, 16] assessed perfect fluid collapse in four and five dimensions with a cosmic constant and electromagnetic field. In Friedman's model with charge, Sharif and Abbas [17] also examined perfect fluid collapse and employed matching conditions. Yousaf and Sharif [18] investigated the collapse of charged ideal fluids. They concluded that the collapsing process is slowed down by electromagnetic fields. Guha and Banerji [19] used a charged anisotropic fluid to apply the Darmois junction condition for cylindrical collapse.Using the cosmic constant, Ahmad and Malik [20] examined the collapsing of anisotropic fluids. Khan et al. [21] went into additional detail about this work in a five-dimensional anisotropic fluid with a cosmological constant present. Ahmad et al. [22] also investigated heat flux and the collapse process of anisotropic fluid. Prisco et al. [23] investigated an anisotropic fluid in a cylindrical shear-free model.

Researchers are still particularly interested in studying collapsing items when there is an electromagnetic field present in the background. Sharif and Bhatti utilized their results for spherical collapse in the electromagnetic background [24]. According to their research, the electromagnetic field accelerates the process of stellar collapse by lowering the internal pressure of a star. Sharif and Abbas conducted research on the Friedmann model with electromagnetic charge and total fluid collapse [25]. Researchers have explored the collapse of charged cylinders with anisotropic fluids, employing various mathematical techniques. Notably, Guha and Banerji [19] utilized Darmois junction conditions to investigate this phenomenon, while Sharif and Fatima [26] focused on the charged cylindrical collapse of anisotropic fluid models, providing valuable insights into these complex astrophysical processes. Maurya and Gupta [27] employed general relativity to investigate the distribution of charged fluids to anisotropic fluids. Khan and associates have investigated the ultimate outcome of charged anisotropic fluid collapse [28].

The effects of an electromagnetic field on the expansion and collapse of an anisotropic gravitational source in a four-dimensional spacetime were recently studied by Abbas [29]. The collapsing scenario in general relativity and some modified theories pertaining to higher dimensional spacetimes were studied by many researchers. Shear-free relativistic models in higher dimensions with charge and heat flux were studied by Nyonyi and associates [30]. Recent studies have delved into the complexities of gravitational collapse in various contexts. For instance, Shah et al. [31] explored collapsing solutions in higher-dimensional spaces, while Khan et al. [32] built upon this work by incorporating cosmological constants and anisotropic fluids into their higher-dimensional models. Additionally, Sharif and Atiq [33] investigated charged collapse in the framework of $f(R)$ gravity, highlighting the need for continued research into the intricacies of gravitational collapse. We also find that this process is assisted by the cosmological constant in field models. In a comparable manner the fluid's develop and the presence of cosmological constant have an immense effect on the gravitational collapse summons. Researching spherical gravitational collapse with cosmological constant and an anisotropic fluid will therefore be beneficial. It will assist us in establishing out how the cosmological constant and the anisotropic fluid interact to affect the collapse phase.

In the $f(R,T)$ theory of gravity, the cosmological constant is essential for providing a more complex picture of the evolution of the universe. The concept offers a framework for investigating the changing character of dark energy and its consequences for cosmology by permitting Λ to fluctuate with R and T [34–36]. The impact of the cosmological constant on gravitational collapse has been investigated recently in a number of modified gravity theories, such as Gauss-Bonnet gravity [39], $f(R)$ theory [37], and $f(R,T)$ theory [38]. All of these studies have demonstrated that the cosmological constant can have a substantial effect on the collapse process, changing the way singularities emerge and how the gravitational field behaves.In addition, investigations have looked into how the cosmological constant affects gravitational collapse in the setting of modified gravity theories with higher-order curvature terms [41] and brane cosmology [40]. The aforementioned studies have shown that the fate of collapsing objects, such stars and black holes, can be significantly influenced by the cosmological constant. This field of study is developing quickly, according to an assessment of the literature on gravitational collapse in modified gravity theories with a cosmological constant [42]. Examining the cosmological constant's function in gravitational collapse is crucial as our knowledge of the cosmos and its fundamental rules of physics advances, especially in light of modified gravity theories.

Bamba et al. [44] evaluated collapse in $f(T)$ gravity, whereas Alavirad and Weller [43] studied gravitational collapse in f(R) gravity. The consequences of a cosmological constant on collapse of gravitation in f(R) gravity were studied by Capozziello et al. [45]. Also, Nojiri et al. [47] addressed collapse in Gauss-Bonnet gravity, and Harko et al. [46] analyzed collapse in $f(R,T)$ gravity. The area of modified gravity theories and their relevancy to cosmology and astrophysics are currently covered in a number of books in addition to these scholarly publications. Nojiri and Odintsov [49] offered a cohesive framework for comprehending cosmic history in modified gravity ideas, while Capozziello and Faraoni [48] offered an extensive review of gravitational concepts for cosmology and astrophysics.The subject of modified gravity concepts and gravitational collapse has also been the subject of numerous review papers. While Sotiriou [51] examined the consequences of modified gravity concepts for cosmology, De Felice and Tsujikawa [50] reviewed f(R) theories. A review of collapse processes in modified gravity models with a cosmological constant was presented by Harko et al. [52]. Alternative explanations for dark energy and dark matter include modified gravity theories $f(R,T)$ gravity, scalar-tensor theories and quintessence, phantom energy, and brane cosmology. Unified approaches combine modified gravity with quintessence or scalar fields. The study also aim to explore the new insights into early universe physics. It may also help in solving cosmological constant problem and will derive the impact on large-scale structure formation.

This paper examines an anisotropic fluid in a 4-dimensional gravitational collapsing and expanding solution with a cosmological constant in $f(R,T)$ gravity. This paper is divided into two sections. In the first section, we derived the spacetime field equations, in the second we derive collapsing and expanding solution in $f(R,T)$ theory og gravity and in the last there is some conclusion.

2. Field Equations

We use 4-dimensional spherically symmetric spacetime in the internal geometry given by

$$ds^2 = L^2(t,r)dt^2 - A^2(t,r)dr^2 - H^2(t,r)(d\theta^2 + \sin^2\theta d\phi^2). \tag{1}$$

The Ricci scalar (1) for the spacetime is defined by

$$\begin{aligned} R &= \frac{2L''}{LA^2} - \frac{2L'A'}{LA^3} - \frac{2\ddot{A}}{AL^2} - \frac{4\ddot{H}}{HL^2} + \frac{2\dot{L}\dot{A}}{AL^3} + \frac{4\dot{L}\dot{H}}{HL^3} \\ &+ \frac{4L'H'}{LHA^2} + \frac{4H''}{HA^2} - \frac{4\dot{A}\dot{H}}{AHL^2} - \frac{2A'H'}{HA^3} - \frac{2\dot{H}^2}{L^2H^2} + \frac{2H'^2}{A^2H^2} - \frac{2}{H^2} \end{aligned} \tag{2}$$

The field equations with the cosmic constant for the aforementioned action are,

$$\begin{aligned} G_{\Psi\eta} &= R_{\Psi\eta} - \frac{1}{2}Rg_{\Psi\eta} + \Lambda g_{\Psi\eta} = kT_{\Psi\eta} \\ G_{00} &= \frac{1}{L^2}\left(\frac{\dot{H}^2}{H^2} + 2\frac{\dot{A}\dot{H}}{AH}\right) - \frac{1}{A^2}\left(\frac{2H''}{H} + \frac{H'^2}{H^2} - \frac{2A'H'}{AH}\right) + \frac{1}{H^2} - \Lambda, \\ G_{11} &= \frac{1}{A^2}\left(\frac{H'^2}{H^2} + \frac{2L'H'}{LH}\right) - \frac{1}{L^2}\left(\frac{2\ddot{H}}{H} - \frac{2\dot{L}\dot{H}}{LH} + \frac{\dot{H}^2}{H^2}\right) - \frac{1}{H^2} + \Lambda, \end{aligned}$$

$$\begin{aligned} G_{22} &= \frac{1}{L^2}\left(-\frac{\ddot{A}}{A} - \frac{\ddot{H}}{H} + \frac{\dot{L}\dot{A}}{LA} - \frac{\dot{A}\dot{H}}{HA} + \frac{\dot{L}\dot{H}}{LH}\right) \\ &+ \frac{1}{A^2}\left(\frac{L''}{L} + \frac{H''}{H} - \frac{L'A'}{LA} + \frac{L'H'}{LH} - \frac{A'H'}{AH}\right) + \Lambda, \\ G_{33} &= \sin^2\theta G_{22} \\ G_{01} &= \frac{\dot{H}'}{H} - \frac{L'\dot{H}}{LH} - \frac{\dot{A}H'}{HA} = 0 \end{aligned} \tag{3}$$

The $f(R,T)$ gravity action with cosmic canstant and anisotropic contribution is described mathematically as

$$S=\int d^4x\sqrt{-g}(f(R,T)+\mathcal{L}_m) \qquad (4)$$

where g is the determinant of metric g_{ab}, and the lagrangian matter is $\mathcal{L}_m$. In this instance, we choose $\mathcal{L}_m=\rho$. The field equation in modified theory is

$$G_{\Psi\eta} = \frac{1}{f_R}[(f_T+1)T^{(m)}_{\Psi\eta}-\rho g_{\Psi\eta}f_T+\frac{f-Rf_R}{2}g_{\Psi\eta}+(\nabla_\Psi\nabla_\eta-g_{\Psi\eta}\Box)f_R], \qquad (5)$$

In our current investigation, the *f(R,T)* model can be selected in the way outlined listed below:

$$f(R,T)=f_R+f_T,\ \text{Here we take}\ f_R=R,\ \text{and}\ f_T=2\lambda T \qquad (6)$$

where R is the Ricci scalar, T is the trace of the stress energy tensor, and λ is a positive constant. For annisotropic fluids with a cosmological constant present, the energy momentum is defined by

$$T^{(m)}_{\Psi\eta}=(\rho+P_\perp)V_\Psi V_\eta-P_\perp g_{\Psi\eta}+(P_r-P_\perp)U_\Psi U_\eta, \qquad (7)$$

Additionally, V^Ψ and U^Ψ are vectors that meet the following relations:

$$V^\Psi=L^{-1}\Psi^\Psi_0, V^\Psi V_\Psi=1, U^\Psi=A^{-1}\Psi^\Psi_1, U^\Psi U_\Psi=-1, \qquad (8)$$

For interior metric Eq.(1), the factors L^Ψ, L_Ψ, A^Ψ and A_Ψ are given by

$$\begin{aligned} V^\Psi &= [\frac{1}{L},0,0,0], V_\Psi=[-L,0,0,0,], \\ U^\Psi &= [0,\frac{1}{A},0,0], U_\Psi=[0,A,0,0]. \end{aligned} \qquad (9)$$

The expansion scalar Θ for the spherically symetric spacetime (1) is specified by

$$\Theta=\frac{1}{L}\left(\frac{\dot{A}}{A}+2\frac{\dot{H}}{H}\right), \qquad (10)$$

Here dot stand for derivative with regard to t. The dimensionless metric of anisotropy is defined as [53]

$$\Delta a=\frac{P_r-P_\perp}{P_r}. \qquad (11)$$

The Eq.(5)) for the spacetime (1) is defined by

$$\begin{aligned}
G_{00} &= \frac{L^2}{f_R}\left[\rho+\frac{f-Rf_R}{2}+\frac{f''_R}{A^2}-\dot{f_R}\left(\frac{\dot{A}}{A}-2\frac{\dot{H}}{H}\right)\frac{1}{L^2}-\frac{f'_R}{A^2}\left(\frac{A'}{A}-\frac{2H'}{H}\right)\right] \\
G_{01} &= \frac{\dot{f_R}}{f_R}\left[1-\frac{L'\dot{f_R}}{\dot{f_R}L}-\frac{\dot{A}f'_R}{A\dot{f_R}}\right], \\
G_{11} &= \frac{A^2}{f_R}\left[P_r+(\rho+P_\perp)f_T-\frac{f-f_R}{2}-\frac{\dot{f_R}}{HLA^2}\left(H\dot{L}-2L\dot{H}\right)-\frac{f'_R}{HLA_2}\left(HL'+2LH'\right)\right]
\end{aligned}$$

$$\begin{aligned}
G_{22} &= \frac{H^2}{f_R}\left[P_\perp+(\rho+P_\perp)f_T+\frac{Rf_R-f}{2}+\frac{\ddot{f_R}}{L^2}-\frac{f''_R}{A^2}\right. \\
&- \left.\frac{\dot{f_R}}{L^2}\left(\frac{\dot{L}}{L}-\frac{\dot{A}}{A}-\frac{\dot{H}}{H}\right)-\frac{f'_R}{A^2}\left(\frac{L'}{L}-\frac{A'}{A}-\frac{H'}{H}\right)\right],
\end{aligned} \qquad (12)$$

these equations implies that

$$
\begin{aligned}
&(1+\lambda)\rho-\lambda P_r-2\lambda P_\perp=\frac{1}{L^2}\left(\frac{\dot{H}^2}{H^2}+2\frac{\dot{A}\dot{H}}{AH}\right)-\frac{1}{A^2}\times\left(\frac{2H''}{H}+\frac{H'^2}{H^2}-\frac{2A'H'}{AH}\right)+\frac{1}{H^2}-\Lambda,\\
&\lambda\rho+(1+3\lambda)P_r+2\lambda P_\perp=\frac{1}{A^2}\left(\frac{H'^2}{H^2}+\frac{2L'H'}{LH}\right)-\frac{1}{L^2}\times\left(\frac{2\ddot{H}}{H}-\frac{2\dot{L}\dot{H}}{LH}+\frac{\dot{H}^2}{H^2}\right)-\frac{1}{H^2}+\Lambda,\\
&\lambda\rho+\lambda P_r+(1+4\lambda)P\perp=\frac{1}{L^2}\left(-\frac{\ddot{A}}{A}-\frac{\ddot{H}}{H}+\frac{\dot{L}\dot{A}}{LA}\right.\\
&\left.-\frac{\dot{A}\dot{H}}{HA}+\frac{\dot{L}\dot{H}}{LH}\right)+\frac{1}{A^2}\left(\frac{L''}{L}+\frac{H''}{H}-\frac{L'A'}{LA}+\frac{L'H'}{LH}-\frac{A'H'}{AH}\right)+\Lambda,\\
&G_{01}=\frac{\dot{H}'}{H}-\frac{L'\dot{H}}{LH}-\frac{\dot{A}H'}{HA}=0\\
&G_{33}=G_{22}=\lambda\rho+\lambda P_r+(1+4\lambda)P\perp,
\end{aligned}
\tag{13}
$$

Where Λ is cosmological constant. For the Misner-Sharp mass in 4-dimensions is defined as [54–56]

$$
m(t,r)=\frac{H}{2}\left([1+\frac{\dot{H}^2}{L^2}-\frac{H'^2}{A^2}]-\frac{\Lambda H^2}{3}\right),
\tag{14}
$$

For the $G01$ element in the field equations, we have found an auxiliary solution.

$$
L=\frac{\dot{H}}{H^\Psi},\quad A=H^\Psi,
\tag{15}
$$

where Ψ is arbitrary constant. Now using Eq.(15) in Eq.(10), the expansion scalar becomes

$$
\Theta=(2+\Psi)H^{(\Psi-1)}.
\tag{16}
$$

now use Eq.(16) into field eqyation we get

$$
\begin{aligned}
&(1+\lambda)\rho-\lambda P_r-2\lambda P_\perp=(2\Psi+1)H^{2\Psi-2}+\frac{1}{H^2}-\frac{1}{H^{2\Psi}}(\frac{2H''}{H}+(1-2\Psi)(\frac{H'}{H})^2)-\Lambda,\\
&\lambda\rho+(1+3\lambda)P_r+2\lambda P_\perp=-(2\Psi+1)H^{2\Psi-2}-\frac{1}{H^2}+\frac{1}{H^{2\Psi}}((1-2\Psi)(\frac{H'}{H})^2+\frac{2\dot{H}'H'}{\dot{H}H})+\Lambda,\\
&\lambda\rho+\lambda P_r+(1+4\lambda)P\perp=\frac{1}{H^{2\Psi}}(\frac{(1-\Psi)H''}{H}+\frac{\dot{H}''}{\dot{H}}+(2\Psi^2-\Psi)(\frac{H'}{H})^2-(2\Psi^2+\Psi)H^{2\Psi-2}\\
&+\frac{(1-3\Psi)\dot{H}'H'}{\dot{H}H})+\Lambda,
\end{aligned}
\tag{17}
$$

solving these equation we get

$$
\begin{aligned}
\rho &= -\frac{1}{(2\lambda+1)(4\lambda+1)}[\Lambda-(2\Psi+1)H^{2\Psi-2}-\frac{1}{H^2}\\
&- 2\lambda(\Lambda-(2\Psi^2+\Psi)H^{2\Psi-2}+H^{-2\Psi}(\frac{(1-\Psi)H''}{H}+\frac{\dot{H}''}{\dot{H}}+\frac{(1-3\Psi)\dot{H}'H'}{H\dot{H}}+\frac{(2\Psi^2-\Psi)(H')^2}{H^2}))\\
&- 5\lambda(-\Lambda+(2\Psi+1)H^{2\Psi-2}+\frac{1}{H^2}+H^{-2\Psi}(-(\frac{2H''}{H}+\frac{(1-2\Psi)(H')^2}{H^2})))-\lambda(\Lambda-(2\Psi+1)H^{2\Psi-2}-\frac{1}{H^2}\\
&+ H^{-2\Psi}(\frac{2\dot{H}'H'}{H\dot{H}}+\frac{(1-2\Psi)(H')^2}{H^2}))+H^{-2\Psi}(\frac{2H''}{H}+\frac{(1-2\Psi)(H')^2}{H^2})]
\end{aligned}
\tag{18}
$$

$$\begin{aligned}
p_r \;=& -\frac{1}{(2\lambda+1)(4\lambda+1)}[-\Lambda+(2\Psi+1)H^{2\Psi-2}+\frac{1}{H^2} \\
+& 2\lambda(\Lambda-(2\Psi^2+\Psi)H^{2\Psi-2}+H^{-2\Psi}(\frac{(1-\Psi)H''}{H}+\frac{\dot{H}''}{\dot{H}}+\frac{(1-3\Psi)\dot{H}'H'}{H\dot{H}}+\frac{(2\Psi^2-\Psi)(H')^2}{H^2})) \\
+& \lambda(-\Lambda+(2\Psi+1)H^{2\Psi-2}+\frac{1}{H^2}+H^{-2\Psi}-(\frac{2H''}{H}+\frac{(1-2\Psi)(H')^2}{H^2})))-3\lambda(\Lambda \\
-& (2\Psi+1)H^{2\Psi-2}-\frac{1}{H^2}+H^{-2\Psi}(\frac{2\dot{H}'H'}{H\dot{H}}+\frac{(1-2\Psi)(H')^2}{H^2}))+H^{-2\Psi}(-(\frac{2\dot{H}'H'}{H\dot{H}} \\
+& \frac{(1-2\Psi)(H')^2}{H^2}))]
\end{aligned} \tag{19}$$

$$\begin{aligned}
p_t \;=& -\frac{1}{(2\lambda+1)(4\lambda+1)}[-\Lambda+(2\Psi^2+\Psi)H^{2\Psi-2} \\
-& 2\lambda(\Lambda-(2\Psi^2+\Psi)H^{2\Psi-2}+H^{-2\Psi}(\frac{(1-\Psi)H''}{H}+\frac{\dot{H}''}{\dot{H}}+\frac{(1-3\Psi)\dot{H}'H'}{H\dot{H}}+\frac{(2\Psi^2-\Psi)(H')^2}{H^2})) \\
+& H^{-2\Psi}(-(\frac{(1-\Psi)H''}{H}+\frac{\dot{H}''}{\dot{H}}+\frac{(1-3\Psi)\dot{H}'H'}{H\dot{H}}+\frac{(2\Psi^2-\Psi)(H')^2}{H^2}))+\lambda(-\Lambda+(2\Psi+1)H^{2\Psi-2}+\frac{1}{H^2} \\
+& H^{-2\Psi}(-(\frac{2H''}{H}+\frac{(1-2\Psi)(H')^2}{H^2})))+\lambda(\Lambda-(2\Psi+1)H^{2\Psi-2}-\frac{1}{H^2}+H^{-2\Psi}(\frac{2\dot{H}'H'}{H\dot{H}} \\
+& \frac{(1-2\Psi)(H')^2}{H^2}))]
\end{aligned} \tag{20}$$

The mass function now becomes Eq. (15) in Eq. (14)

$$\frac{2m}{H}+\frac{\Lambda H^2}{3}-1=H^{2\Psi}-\frac{H^{'2}}{H^{2\Psi}}. \tag{21}$$

It is clear from the equation above that trapped surfaces exist for $H'=H^{2\Psi}$. As a result, the trapped surface condition is $H'=H^{2\Psi}$. Using $H'=H^{2\Psi}$, Eq.(21) now becomes

$$\Lambda H^3-3H+6m=0. \tag{22}$$

For $\Lambda=0$, we clearly have the Schwarzschild horizon, or $H=2m$, whereas for $m=0$, we have a de-Sitter horizon, or $H=\left(\frac{3}{\Lambda}\right)^{\frac{1}{2}}$.

Using the perturbation approach, we will find an approximate solution of Eq. (22) up to first order in m and Λ

$$H_{ch}=(H)_0+m(H)_1+\cdots \tag{23}$$

be a solution that is specified up to order m in Eq. (21). Once Eq.(21) is imitated in Eq.(22), a comparison of coefficients of powers of m provides us with:

$$(H)_0=\left(\frac{3}{\Lambda}\right)^{\frac{1}{2}}. \tag{24}$$

$$(H)_1=-1. \tag{25}$$

After entering $(H)_0$ and $(H)_1$ into Eq. (23), we obtain

$$H_{ch}=\left(\frac{3}{\Lambda}\right)^{\frac{1}{2}}-m\cdots \tag{26}$$

The radius of the cosmic horizon is denoted by H_{ch}. Regarding Λ, we assume that

$$H_{bh}=(H)_0+\Lambda(H)_1+\cdots \tag{27}$$

serve as the reaction to Eq. (21). After replacing Eq. (21) in Eq. (27), we may compare the coefficients of powers of Λ and obtain the following conclusion:

$$(H)_0=2m. \tag{28}$$

$$(H)_1=\frac{1}{3}(2m)^3. \tag{29}$$

Now putting $(H)_0$ and $(H)_1$ in Eq.(27), we get

$$H_{bh}=2m+\frac{\Lambda}{3}(2m)^3\cdots \tag{30}$$

where the black hole horizon's radius is H_{bh}.We now obtain $H_{bh}\rightarrow 2m$ (Schwarzschild horizon) and $H_{ch}\rightarrow\infty$ (Cosmological horizon does not exist) when $\Lambda\rightarrow 0$. Additionally, for $m\rightarrow 0$, $H_{bh}\rightarrow 0$, and $H_{ch}\rightarrow\left(\frac{3}{\Lambda}\right)^{\frac{1}{2}}$ (de-Sitter horizon) [57]. Given that $H'=H^{2\Psi}$ is the criterion for a trapped surface, gravitational collapse results in either two trapping surfaces or a single trapped surface.
The trapping condition's integral suggests:

$$H^{1-2\Psi}_{trap}=(1-2\Psi)r+B(t), \tag{31}$$

where a function created during the integration is denoted by $B(t)$. Equations (15) and (31) provide the source variables in the explicit form shown below.

$$\begin{aligned}
\rho &= \frac{1}{8\lambda^2+6\lambda+1}(b-2\Psi r+r)^{\frac{2}{2\Psi-1}}((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{-2\Psi}(4\Psi^2\lambda((b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}}-((b\\
&- 2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi})-2\lambda\Lambda(b-2\Psi r+r)^{\frac{2}{1-2\Psi}}\times((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}+4\lambda(((b-2\Psi r\\
&+ r)^{\frac{1}{1-2\Psi}})^{2\Psi}+((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi}-(b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}})-2\Psi(3\lambda+1)((b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}}-((b\\
&- 2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi})+\Lambda(-(b-2\Psi r+r)^{\frac{2}{1-2\Psi}})((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}+((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}\\
&+ ((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi}-(b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}}),\\
P_r &= \frac{1}{8\lambda^2+6\lambda+1}(b-2\Psi r+r)^{\frac{2}{2\Psi-1}}((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{-2\Psi}(-4\Psi^2\lambda((b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}}-((b\\
&- 2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi})+2\lambda(\Lambda(b-2\Psi r+r)^{\frac{2}{1-2\Psi}}\times((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}-2((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}\\
&- 2((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi}+2(b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}})+2\Psi(3\lambda+1)((b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}}-((b-2\Psi r\\
&+ r)^{\frac{1}{1-2\Psi}})^{4\Psi})+\Lambda(b-2\Psi r+r)^{\frac{2}{1-2\Psi}}((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}-((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}-((b-2\Psi r\\
&+ r)^{\frac{1}{1-2\Psi}})^{4\Psi}+(b-2\Psi r+r)^{\frac{4\Psi}{1-2\Psi}})
\end{aligned}$$

$$\begin{aligned}
P_\perp &= \frac{1}{4\lambda+1}((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{-2\Psi}(\Psi^2(\frac{2}{(b-2\Psi r+r)^2}-2(b-2\Psi r+r)^{\frac{2}{2\Psi-1}}((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi})\\
&+ \Lambda((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{2\Psi}+\Psi(\frac{1}{(b-2\Psi r+r)^2}-(b-2\Psi r+r)^{\frac{2}{2\Psi-1}}((b-2\Psi r+r)^{\frac{1}{1-2\Psi}})^{4\Psi})) \qquad (32)
\end{aligned}$$

3. Generating Solutions

We clarify solutions for various values of Ψ. In collapsing solution, $\Psi=-\frac{5}{2}$, and in expanding solution, $\Psi=\frac{3}{2}$.

3.1. Collapse Solution $\Psi=-\frac{5}{2}$

Since the rate of collapse for a collapsing system must be negative, Ψ must be smaller than -2. Consequently, we maintain that $\Psi=-\frac{5}{2}$. The trapping requirement $H'=H^{2\Psi}$ becomes $H'=H^{-5}$ for $\Psi=-\frac{5}{2}$, which further provides

$$H_{trap}=(6r+b(t))^{\frac{1}{6}}. \qquad (33)$$

Here an arbitary function of integration is $b(t)=b$. The pressures and density equations for $\Psi=-\frac{5}{2}$ becomes

$$\begin{aligned}
\rho &= \frac{4\lambda}{(8\lambda^2+6\lambda+1)\sqrt[3]{b+6r}}+\frac{1}{(8\lambda^2+6\lambda+1)\sqrt[3]{b+6r}}\frac{2\lambda\Lambda}{8\lambda^2+6\lambda+1}-\frac{\Lambda}{8\lambda^2+6\lambda+1}\\
P_r &= -\frac{4\lambda}{(8\lambda^2+6\lambda+1)\sqrt[3]{b+6r}}-\frac{1}{(8\lambda^2+6\lambda+1)\sqrt[3]{b+6r}}+\frac{2\lambda\Lambda}{8\lambda^2+6\lambda+1}+\frac{\Lambda}{8\lambda^2+6\lambda+1}\\
P_t &= -\frac{1}{(2\lambda+1)(4\lambda+1)}(\lambda(\frac{1}{\sqrt[3]{b+6r}}-\Lambda)+\lambda(\Lambda-\frac{1}{\sqrt[3]{b+6r}})-2\lambda\Lambda-\Lambda) \qquad (34)
\end{aligned}$$

The anisotropy become

$$\Delta a = 1+\frac{2\lambda\Lambda}{\lambda(\frac{1}{Z}-\Lambda)-3\lambda(\Lambda-\frac{1}{Z})+\frac{1}{Z}+2\lambda\Lambda-\Lambda}+\frac{\Lambda}{\lambda(\frac{1}{Z}-\Lambda)-3\lambda(\Lambda-\frac{1}{Z})+\frac{1}{Z}+2\lambda\Lambda-\Lambda} \qquad (35)$$

where $Z=\sqrt[3]{b+6r}$ and the misner sharp mass we get

$$m(t,r)=\frac{1}{2}\sqrt[6]{b+6r}(1-\frac{1}{3}\Lambda\sqrt[3]{b+6r}). \qquad (36)$$

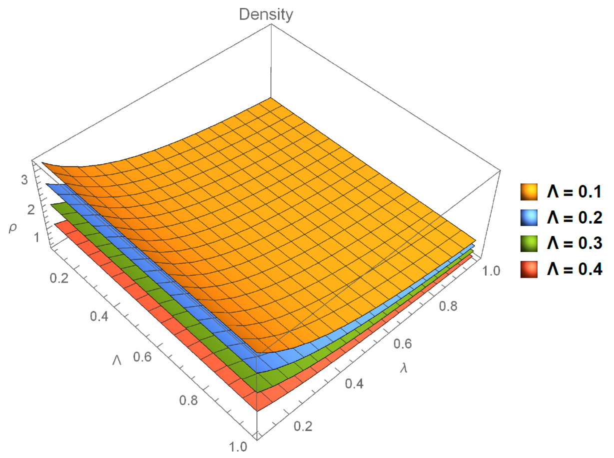

Figure 1. For varying values of $b(t) = 5$*, the variation of* 'ρ' *with regards to* 'Λ', 'r' *and* 'λ'.

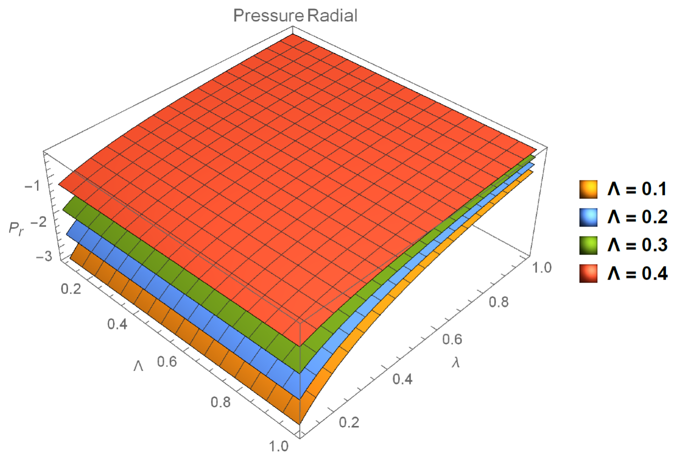

Figure 2. For varying values of $b(t) = 5$*, the variation of* P_r *with regards to* 'Λ', 'r' *and* 'λ'.

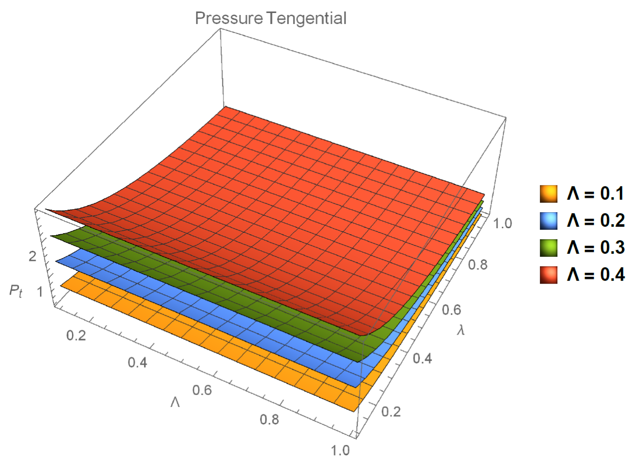

Figure 3. For varying values of $b(t) = 5$*, the variation of* P_t *with regards to* 'Λ', 'r' *and* 'λ'.

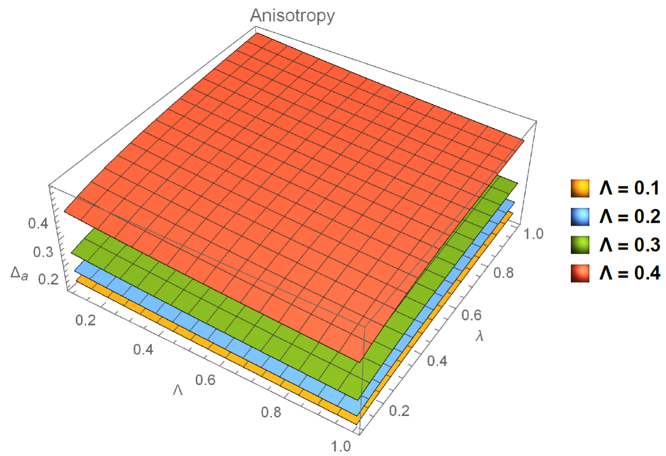

Figure 4. For varying values of $b(t) = 5$*, the variation of* 'Δ_a' *with regards to* 'Λ', 'r' *and* 'λ'.

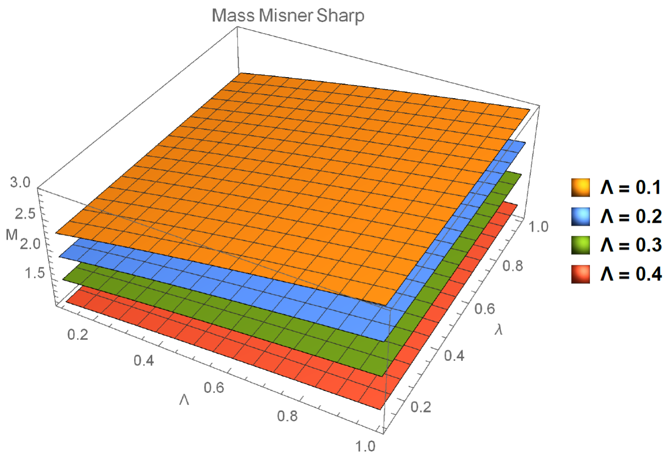

Figure 5. For varying values of $b(t) = 5$*, the variation of* $M_{t,r}$ *with regards to* 'Λ', 'r' *and* 'λ'.

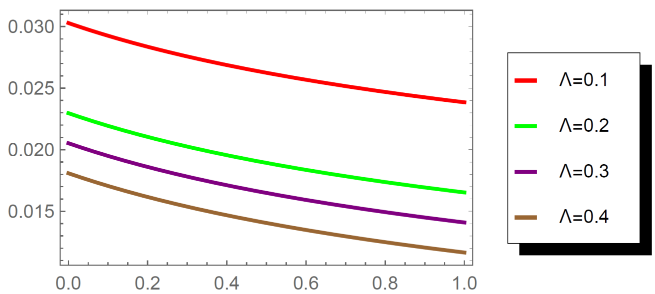

Figure 6. For varying of $\lambda = 4$ *and* $b(t) = 5$*, the variation of* ρ *with regards* 'r' *and* 'Λ'

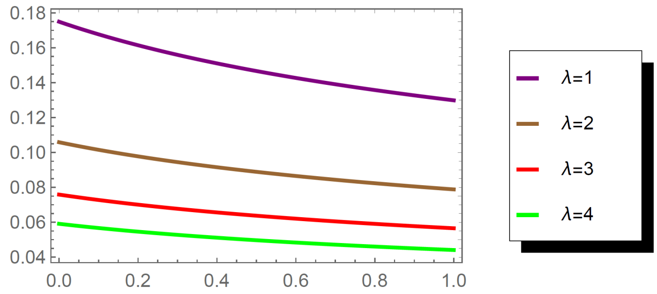

Figure 7. For varying of $\Lambda = 0.4$ and $b(t) = 5$, the variation of ρ with regards 'r' and 'λ'

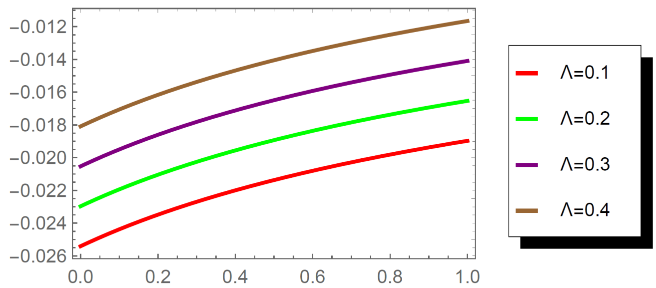

Figure 8. For varying of $\lambda = 4$ and $b(t) = 5$, the variation of P_r with regards 'r' and 'Λ'

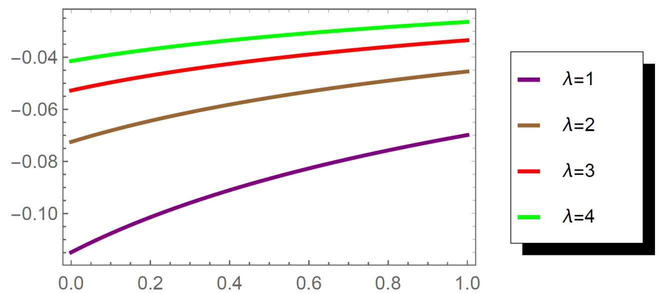

Figure 9. For varying of $\Lambda = 0.4$ and $b(t) = 5$, the variation of P_r with regards 'r' and 'λ'

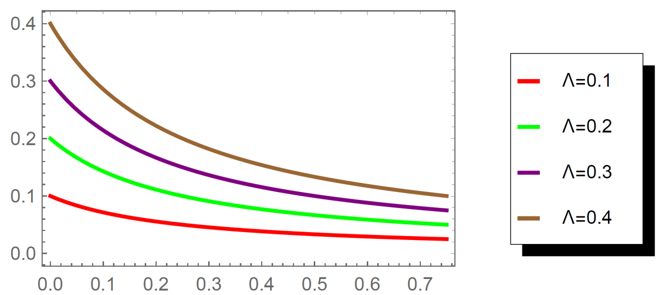

Figure 10. For varying of $\lambda = 4$ and $b(t) = 5$, the variation of P_t with regards 'r' and 'Λ'

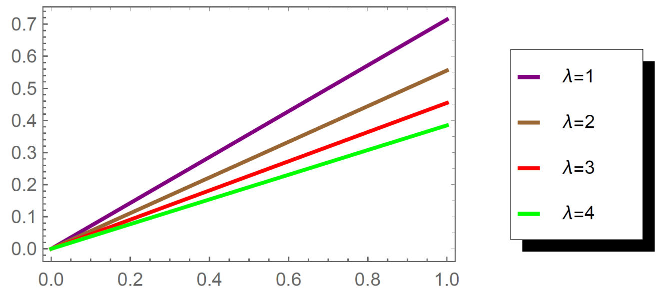

Figure 11. For varying of $\Lambda = 0.4$ and $b(t) = 5$, the variation of P_t with regards 'r' and 'λ'

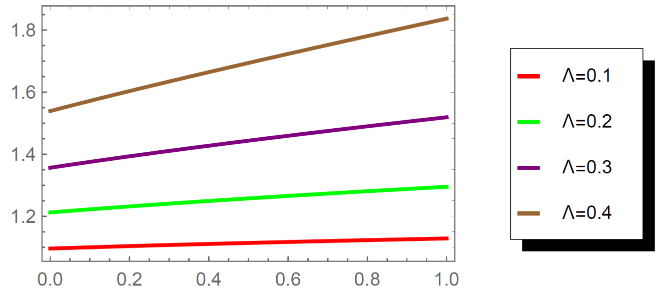

Figure 12. For varying of $\lambda = 4$ and $b(t) = 5$, the variation of Δ_a with regards 'r' and 'Λ'

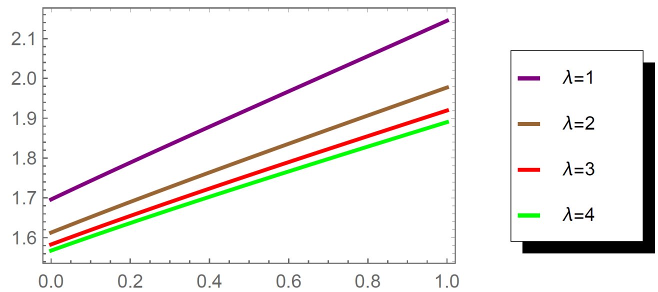

Figure 13. For varying of $\Lambda = 0.4$ and $b(t) = 5$, the variation of Δ_a with regards 'r' and 'λ'.

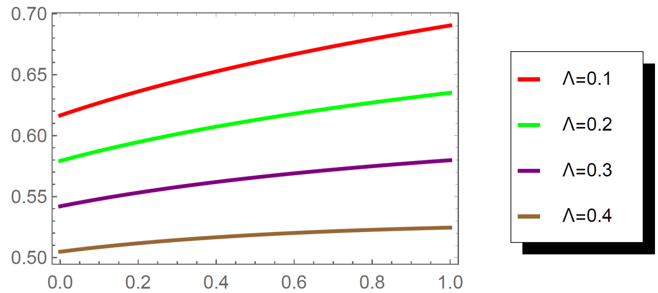

Figure 14. For varying of $\lambda = 4$ and $b(t) = 5$, the variation of $M_{t,r}$ with regards 'r' and 'Λ'.

This study examines the graphical behavior of various physical quantities during gravitational collapse, including energy density ρ, radial $P_R$and tangential P_t pressures, anisotropy Δ_a, and mass function$m_{r,t}$. The results show that ρ decreases with increasing ($'\Lambda'$) cosmological constant (Figure 1), while remaining a positive function of $'r'$ and $'\lambda'$. In contrast, $'P_r'$ (Figure 2) remains negative regardless of the values of $'r'$, $'\lambda'$, and $'\Lambda'$, but increases rapidly with higher $'\Lambda'$ values. $'P_t'$ (Figure 3) exhibits similar behavior to ρ, increasing with higher $'\Lambda'$ values. Δ_a (Figure 4) is a positive function of $'r'$, λ, and $'\Lambda'$, increasing rapidly with higher $'\Lambda'$ values. Finally, the $m_{r,t}$ (Figure 5) is a positive function of $'r'$ and $'\Lambda'$, increasing rapidly with higher $'\Lambda'$ values before stabilizing. Also these quantities shown in 2d with respect to Λ and λ as shown in the (Figure 6) to (Figure 14).

Unraveling the intricate relationships between density, pressure, and anisotropy is essential for comprehending the universe's evolution. As the universe expands, matter and energy density decrease, resulting in a diluted cosmos [58, 59]. The emergence of density fluctuations gives rise to structures, which in turn shape the universe's large-scale structure. The cosmos can continue to expand as pressure, a measurement of a fluid's resistance to expansion, falls. Pressure gradients affect the rate of expansion and, depending on the equation of state, can either accelerate or decelerate the expansion. Additionally, as the cosmos expands, anisotropy decreases and the universe becomes more isotropic. An image of the early universe's anisotropies is provided by the cosmic microwave background, which sheds light on the universe's beginnings. The observed behaviours point to a substantial dark energy component that is accelerating the expansion of the cosmos. Ultimately, understanding the interplay between pressure and density is crucial for deciphering the universe's fate and expansion history

3.2. Expansion with $\Psi = \frac{3}{2}$

When the expansion scalar gains positive values, we have an expanding solution, therefore Eq. (11) implies that if $\Psi > -2$, then $\Theta > 0$,. For ease of use, we assume that $\Psi = \frac{3}{2}$

$$H = (r^2 + r_1^2) + k(t), \tag{37}$$

where $r_1 > 0$ and $k(t) = k$ are integration functions. Using $E(t,r) = 1 + k(t)(r^2 + r_1^2)$ and $H = \frac{E}{(r^2+r_1^2)}$, we can obtain

$$\begin{aligned}
\rho &= \frac{1}{E^5(2\lambda+1)(4\lambda+1)(r^2+r_1^2)}(4E^6(\lambda+1) - E^5(2\lambda+1)\Lambda(r^2+r_1^2) + E^3(4\lambda+1)(r^2+r_1^2)^3 \\
&- 2E(3r^2 - r_1^2)(r^2+r_1^2)^2(\lambda + 2(5\lambda+1)r^2 + 2(5\lambda+1)r_1^2) + 8(7\lambda+1)r^2(r^2+r_1^2)^2) \\
P_r &= \frac{1}{E^5(2\lambda+1)(4\lambda+1)(r^2+r_1^2)}(-4E^6(\lambda+1) + E^5(2\lambda+1)\Lambda(r^2+r_1^2) - E^3(4\lambda+1)(r^2+r_1^2)^3 \\
&+ 2E\lambda(r^2+r_1^2)^2(6r^4 + r^2(4r_1^2+3) - r_1^2(2r_1^2+1)) - 8(7\lambda+1)r^2(r^2+r_1^2)^2) \\
P_t &= \frac{1}{4\lambda+1}(\frac{12r^2(r^2+r_1^2)}{E^5} + \frac{(3r^2-r_1^2)(r^2+r_1^2)(\lambda(4r^2+4r_1^2-2)-1)}{E^4(2\lambda+1)} - \frac{6E}{r^2+r_1^2} + \Lambda)
\end{aligned} \tag{38}$$

Anisotropy in gravitational collapse and expansion can have a big effect on how the system behaves. Increased density and pressure gradients, elongated structures, and faster collapse can result from positive anisotropy, when the pressure or $'\rho'$ is higher in one direction [60]. The implications of anisotropy on both collapse and expansion have been intensively explored. The collapse phase may slow down in instances where pressure or density fluctuates in various directions, culminating in flattened structures and diminished pressure and density contrasts. Development on anisotropic cosmos models has demonstrated that anisotropy may profoundly impact the cosmic evolution [61–64]. Moreover, anisotropy can influence the polarization and amplitude of gravitational waves, and it can also influence the formation of black holes and neutron stars. Then anisotropic fluid is

$$\begin{aligned}
\Delta a &= (-6E^6(2\lambda+1) + E^5(2\lambda+1)\Lambda G + E(3r^2 - r_1^2) \times G^2(\lambda(4r^2+4r_1^2-2)-1) + 12(2\lambda+1)r^2G^2)G \\
&\div (4E^6(\lambda+1) - E^5(2\lambda+1)\Lambda + E^3(4\lambda+1)G^3G - 2E\lambda(3r^2-r_1^2)G^2(2r^2+2r_1^2+1) + 8(7\lambda G \\
&+ 1)r^2G^2) + 1
\end{aligned} \tag{39}$$

where $G = (r^2 + r_1^2)$ and the misner sharp mass is

$$m = \frac{E(\frac{E^3}{(r^2-r_1^2)^3} - \frac{4r^2}{E^3(r^2-r_1^2)} - \frac{E^2\Lambda}{3(r^2-r_1^2)^2} + 1)}{2(r^2-r_1^2)} \tag{40}$$

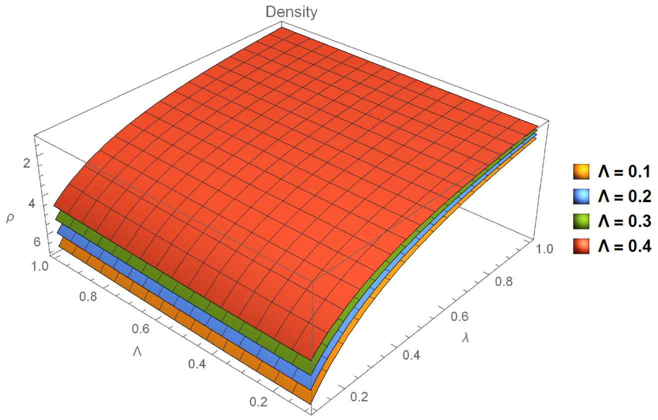

Figure 15. For varying of value of $k(t) = 5$, the variation of 'ρ' with regards to 'Λ', 'r' and $1\lambda'$.

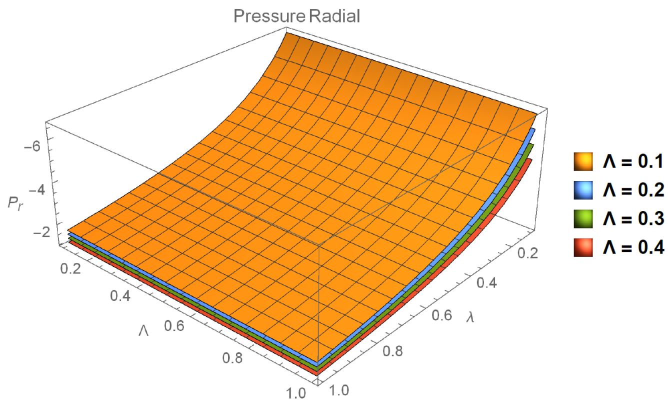

Figure 16. For varying of value of $k(t) = 5$, the variation of 'P_r' with regards to 'Λ', 'r' and 'λ'.

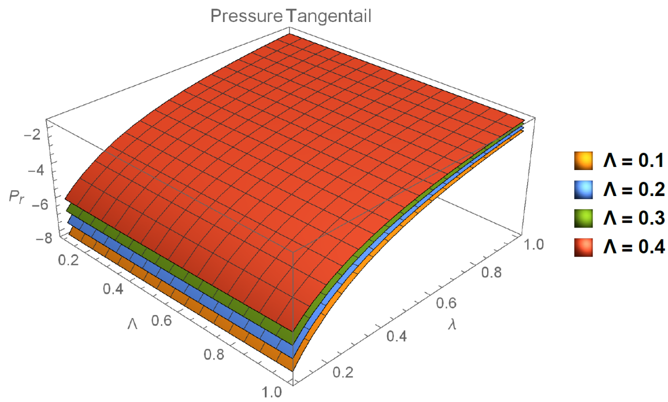

Figure 17. For varying of value of $k(t) = 5$, the variation of 'P_t' with regards to 'Λ', 'r' and 'λ'.

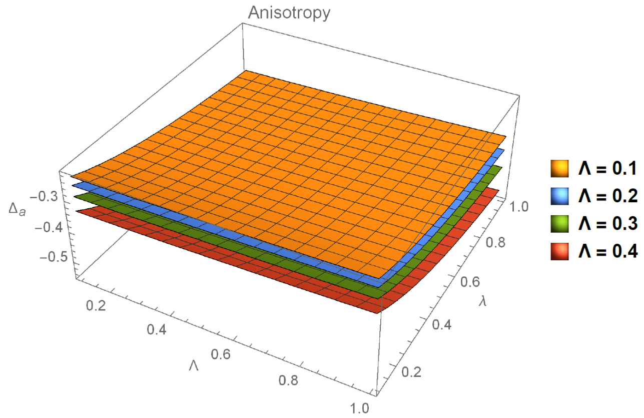

Figure 18. For varying of value of $k(t) = 5$, the variation of 'Δ_a' with regards to 'Λ', 'r' and 'λ'.

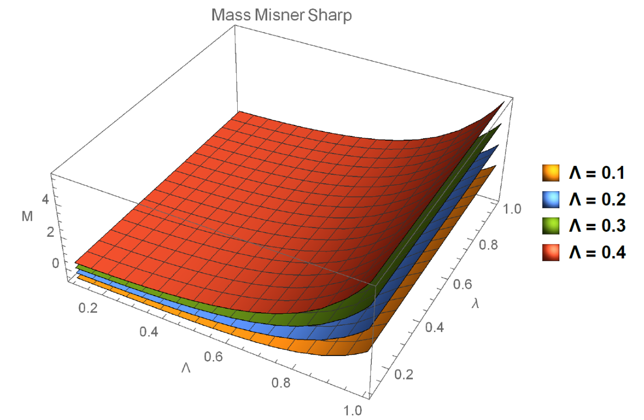

Figure 19. For varying of value of $k(t) = 5$, the variation of '$M_{t,r}'$ with regards to 'Λ' and 'r'.

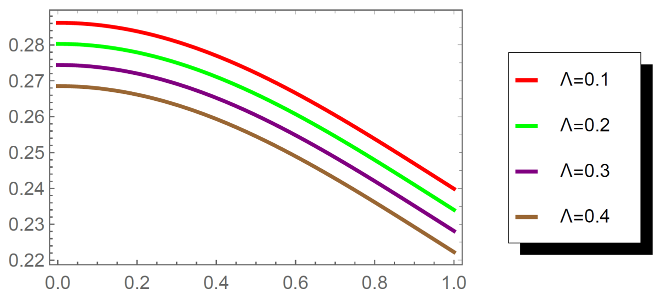

Figure 20. For varying of value of $\lambda = 4$ and $k(t) = 5$, the variation of ρ with regards to 'Λ' and 'r'

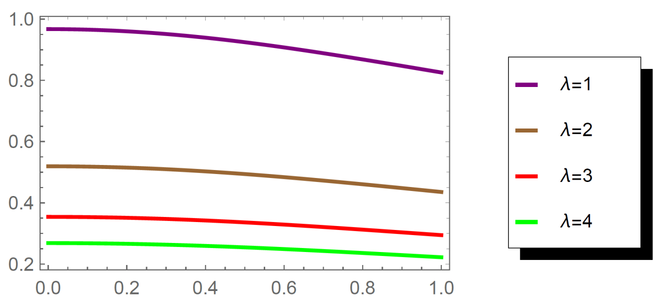

Figure 21. For varying of value of $\Lambda = 0.4$ and $k(t) = 5$, the variation of ρ with regards to λ and 'r'.

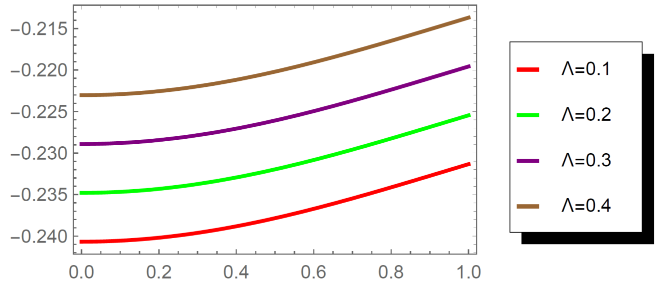

Figure 22. *For varying of value of* $\lambda = 4$ *and* $k(t) = 5$*, the variation of* P_r *with regards to* r *and* Λ*.*

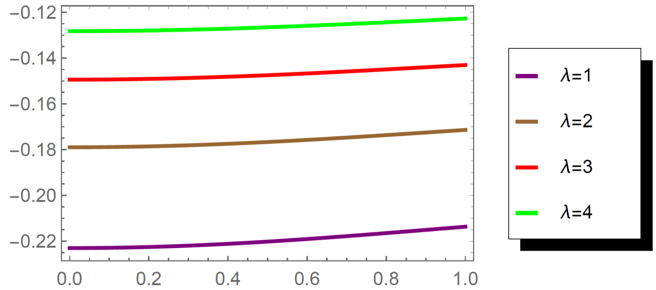

Figure 23. *For varying of value of* $\Lambda = 0.4$ *and* $k(t) = 5$*, the variation of* P_r *with regards to* r *and* λ*.*

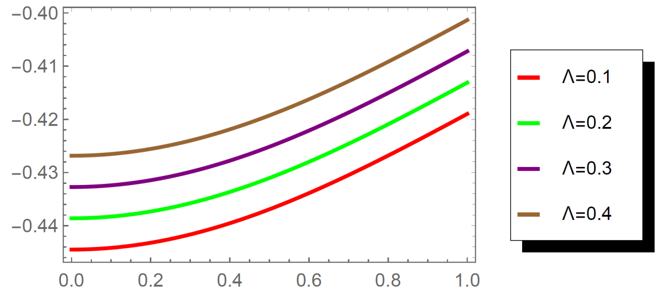

Figure 24. *For varying of value of* $\lambda = 4$ *and* $k(t) = 5$*, the variation of* P_t *with regards to* r *and* Λ*.*

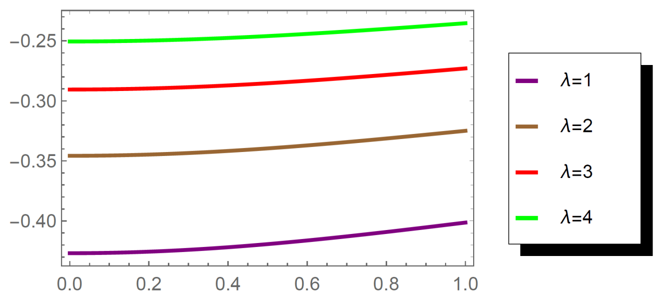

Figure 25. *For varying of value of* $\Lambda = 0.4$ *and* $k(t) = 5$*, the variation of* P_t *with regards to* r *and* λ*.*

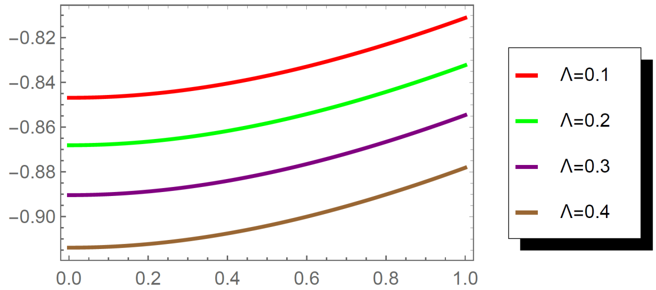

Figure 26. *For varying of value of* $\lambda = 4$ *and* $k(t) = 5$*, the variation of* Δ_a *with regards to* r *and* Λ*.*

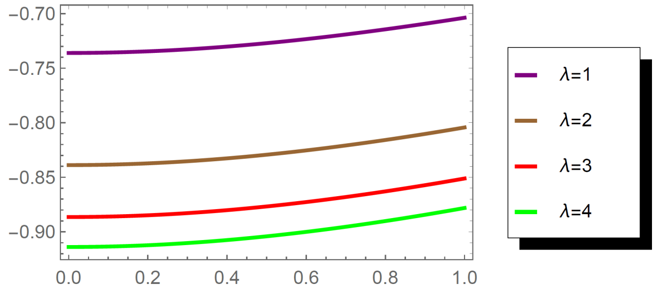

Figure 27. *For varying of value of* $\Lambda = 0.4$ *and* $k(t) = 5$*, the variation of*Δ_a *with regards to* r *and* λ*.*

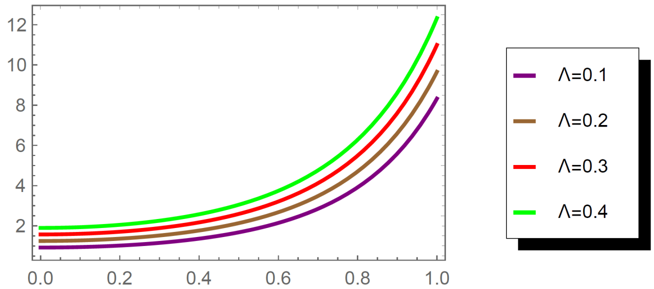

Figure 28. *For varying of value of* $\lambda = 4$ *and* $k(t) = 5$ *the variation of* $M_{t,r}$ *with regards to* r *and* Λ*.*

The energy density ($'\rho'$), radial pressure ($'P_r'$), tangential pressure ($'P_r'$), and anisotropy $'\Delta_a'$ throughout gravitational expansion are all graphically addressed in the present study. The results show that $'\rho'$ is a positive function of $'r'$, $'\lambda'$, and $'\Lambda'$ (Figure 15), increasing as the $'\Lambda'$ increases. In contrast, $'P_r'$ (Figure 16)and $'P_t'$ pressures (Figure 17) remain negative regardless of the values of $'r'$, $'\lambda'$, and $'\Lambda'$. However, $'P_r'$ decreases monotonically with increasing $'\Lambda'$, while $'P_t'$ pressure increases. The $'\Delta_a'$ (Figure 18) of the system is found to be a negative function of 'r', $'\lambda'$, and $'\Lambda'$, decreasing as the $'\Lambda'$ increases. Finally, the $m_{r,t}$ mass (Figure 19)is a positive function of $'r'$, lambda, and $'\Lambda'$, increasing as these values increase. Also we shown these quantities in 2d graph wrt to λ and Λ in Figure 20 to Figure 28.

4. Conclusion

Based on the fundamental work of Abbas and Ahmed [65], this study investigates the gravitational collapse and expansion of anisotropic fluids within the context of $f(R,T)$ gravity. Notably, they did not take into account the existence of a $'\Lambda'$ when studying the collapse and expansion of anisotropic fluids in the context of $f(R,T)$ gravity. By including the effects of the $'\Lambda'$ on the collapse and expansion of anisotropic fluids, the new study builds on this work. The thermal energy released during gravitational collapse is examined and a novel solution for the confined surface of the inner matter distribution is obtained. This study adds to the current body of knowledge in $f(R,T)$ gravity, which has attracted a lot of interest lately because of its potential to explain intricate astrophysical events [66–68].

A $4x4$ matrix referred as the stress-energy tensor offers important insights into the momentum flux, pressure, and energy density of an entire system. The statement $\rho + p = \mu$, where μ corresponds to energy density, describes the interaction between pressure (p) as well as density (ρ) in general relativity. Pressure and density are related by the equation of state, which is frequently referred to as $p = w\rho$, where w is the equation of state parameter. Interpreting the behavior of cosmic objects and the universe's evolution depends heavily on this characteristic. Different kinds of matter and energy are represented by different values of w. As an illustration, non-relativistic matter, like galaxy clusters and stars, is indicated by $w = 0$, although relativistic stuff, like photons and neutrinos, is characterized by $w = 1/3$. Stiff matter can be expressed by a value of $w = 1$, which denotes exceedingly high pressure. On the other hand, $w < -1/3$ shows negative pressure, which is linked to dark energy and propels the universe's rapid expansion. Determining the behavior of pressure and density is crucial in modified gravity models. A fundamentally realistic model necessitates an expanding universe with positive density and pressure that decrease over cosmic time.Dark energy-related negative pressure has the ability to speed up this expansion. Additionally, we look at the potential for singularities, which can happen when the pressure-to-density ratio is zero or less than $-1/3$. The presence or absence of singularities depends critically on the pressure to density ratio. [69–80] suggests that a naked singularity might exist if this ratio is less than $-1/3$. Due to favorable attractive effects, the expansion may be slowed down in some situations when the density and pressure stay positive.

A thorough examination of $'\rho'$, $'P_r'$, $'P_t'$, $m_{r,t}$, and $'\Delta_a'$ with respect to collapsing and expanding solutions has been carried out in the framework of $f(R,T)$ gravity. Although it lowers as $'\Lambda'$ increases, the $'\rho'$ (Figure 1) is found to be positively linked with $'r'$, λ, and $'\Lambda'$. In contrast, regardless of the values of $'r'$, λ, and $'\Lambda'$, the $'P_r'$ (Figure 2) stays negative. Interestingly, as $'\Lambda'$ increases, the $'P_r'$ falls, and its negative value implies a link to dark energy. Additionally, the ratio of $'P_r'$ to $'\rho'$ is smaller than $1/3$, suggesting that a naked singularity may exist.On the other hand, the tangential pressure (Figure 3) increases with the $'\Lambda'$ and behaves equivalent to $'\rho'$. Additionally, $'r'$ and $'\Lambda'$ are discovered to have a positive correlation with anisotropy (Figure 4), which causes a faster collapse in the direction of positive anisotropy. Lastly, it is discovered that the mass function (Figure 5) is an exponential function of $'r'$ and $'\Lambda'$, growing quickly with larger $'\Lambda'$ values before stabilizing.

Interesting patterns may be seen when examining the graphical behavior of $'\rho'$, $'P_r'$, $'P_t'$, $m_{r,t}$, and $'\Delta_a'$ during gravitational expansion. It is shown that $'\rho'$ (Figure 15) is exactly proportional to $'r'$, λ, and $'\Lambda'$, with $'\rho'$ increasing noticeably as $'\Lambda'$ increases. The values of $'r'$, λ, and $'\Lambda'$, on the other hand, always stay negative for $'P_r'$ (Figure 16) and $'P_t'$ (Figure 17). While $'P_t'$ shows an inverse connection, growing with rising values of the $'\Lambda'$, $'r'$, and λ, $'P_r'$ steadily declines as the $'\Lambda'$ grows. In addition, the system's $'\Delta_a'$ (Figure 18) shows a negative correlation with $'r'$, λ, and $'\Lambda'$, suggesting that these variables interact intricately during gravitational expansion. Additionally, we observed that the system's $'\Delta_a'$ drops as the value of Λ increases. Because of the reduced pressure or $'\rho'$, the collapse may proceed more slowly in the direction of negative $'\Delta_a'$. It may be more difficult to detect and investigate the collapse if the $'\Delta_a'$ is negative since it can decrease the emission of gravitational waves.The stability and eventual fate of the collapsing system, including the creation of a neutron star or a black hole, can be influenced by $'\Delta_a'$. A positive function of r, λ, and Λ is the misner sharp mass. As we increase the value of r and Λ and r, as illustrated in Figure 19, the misner sharp mass increases as well. In summary, the existence of anisotropy in gravitational collapse and expansion can have a substantial effect on the system's behavior, affecting the universe's evolution, structure building, and gravitational wave emission.

ORCID

0009-0007-2642-1881 (Yasar Khan)
0000-0002-0745-5476 (Furqan Habib)
0000-0002-5915-7032 (Asaf Khan)

Author Contributions

Yasar Khan: Played a role in research article, generating new problem ideas, devising problem-solving methods, and reviewing literature

Furqan Habib: Applied general techniques, contributed to solving the main problem and Responsible for writing, reviewing, editing, and submitting the initial draft

Asaf Khan: Provided the core problem idea and contributed to draft review

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There is no competing interest by the authors to declare.

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