



Research Article

Wave-Particle Duality and Electron Kinetic Energy in Quantum Physics and Nanoelectronics

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Abstract

Wave-particle duality is widely regarded as one of the conceptual pillars of modern quantum theory, providing the framework through which the behavior of microscopic matter can be consistently interpreted. According to this principle, microscopic entities such as electrons, photons, atoms and even complex molecules exhibit both wave-like and particle-like properties depending on the experimental configuration used to observe them. This duality is quantitatively formalized through the De Broglie relation, which associates a wavelength with any particle possessing momentum. The present work provides a comprehensive theoretical analysis of the physical and mathematical foundations of wave-particle duality, focusing particularly on the De Broglie wavelength and the kinetic energy of the electron in both non-relativistic and relativistic regimes. The connection between the De Broglie hypothesis and Schrödinger's equation is discussed in detail, highlighting the role of the kinetic energy operator and the interpretation of matter waves in terms of quantum wave packets. Experimental evidence supporting the wave nature of matter, including electron diffraction experiments and interferometry techniques, is also reviewed. Beyond their conceptual importance, these principles underpin several modern technologies, including electron microscopy, nanoscale semiconductor devices and quantum metrology. The analysis demonstrates that the De Broglie relation provides a unifying framework connecting momentum, wavelength and kinetic energy, thereby forming a fundamental bridge between classical mechanics, quantum theory and modern technological applications.

Keywords

Wave-particle Duality, De Broglie Wavelength, Electron Kinetic Energy, Quantum Mechanics, Electron Diffraction, Matter Waves

1. Introduction

The emergence of quantum mechanics during the early twentieth century fundamentally transformed the understanding of physical reality at microscopic scales. One of the most

profound conceptual developments of this scientific revolution was the recognition that particles and waves are not mutually exclusive categories but rather complementary aspects

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of quantum systems. This idea, known as wave-particle duality, challenged the classical distinction between localized particles and extended wave phenomena that had dominated physics since the nineteenth century.

Classical mechanics describes matter as consisting of particles with well-defined positions and trajectories, governed by deterministic equations of motion such as Newton's laws.

In contrast, classical wave theory describes phenomena such as sound or electromagnetic radiation as continuous oscillations propagating through space. Prior to the development of quantum theory, these two descriptions were considered fundamentally distinct. The classical description of optical interference is developed in [4].

The first signs that this distinction might be incomplete emerged from studies of electromagnetic radiation. Planck's explanation of black-body radiation in 1900 introduced the concept of quantized energy exchange, suggesting that radiation interacts with matter in discrete packets called quanta [21]. Einstein extended this idea in his explanation of the photoelectric effect, proposing that light itself behaves as a collection of localized particles known as photons [11].

These discoveries established the particle-like properties of radiation. The next major step occurred when Louis de Broglie proposed that matter might also exhibit wave-like behavior. In his doctoral thesis, de Broglie suggested that any particle with momentum should be associated with a wave of wavelength given by

$$\lambda = \frac{h}{p} \quad (1)$$

where h is Planck's constant and p is the particle momentum [9]. This simple but revolutionary idea unified the description of matter and radiation by extending the concept of wave behavior to material particles.

Shortly after this theoretical prediction, electron diffraction experiments by Davisson and Germer [8] and independently by G. P. Thomson [27] provided direct evidence of electron wave behavior. Electrons scattered by crystalline structures produced diffraction patterns consistent with the de Broglie wavelength.

Wave-particle duality is now fully incorporated within the framework of quantum mechanics. In this theory, particles are described by quantum states represented mathematically by wave functions whose evolution is governed by Schrödinger's equation [25]. However, measurements produce discrete outcomes, reflecting the particle-like aspect of quantum systems. This dual description is formalized in Bohr's principle of complementarity, which states that wave and particle descriptions are mutually exclusive but jointly necessary for a complete understanding of quantum phenomena [3].

Understanding the relationship between particle momentum, wavelength and kinetic energy is essential not only for the conceptual foundations of quantum mechanics but also for many modern technological applications. Electron diffraction

techniques, electron microscopy and nanoscale semiconductor devices all rely on the wave properties of electrons. Consequently, the De Broglie relation and the associated expressions for electron kinetic energy play a central role in both fundamental physics and applied technologies.

The present article examines these concepts in detail. First, the theoretical foundations of wave-particle duality and the De Broglie hypothesis are analyzed. The connection between the De Broglie relation and Schrödinger's wave mechanics is then discussed, highlighting the role of the kinetic energy operator. Finally, the implications of these ideas for experimental physics and modern technological applications are explored.

2. Theoretical Background

The de Broglie wavelength of a particle with momentum p is given by Equation (2). Here m denotes the rest mass in the free-particle relations; non-relativistic expressions require speeds much smaller than c [5, 24].

$$\lambda = \frac{h}{p} \quad (2)$$

For a particle moving with velocity v , the classical momentum is

$$p = mv \quad (3)$$

Substituting Equation (3) into Equation (2) gives

$$\lambda = \frac{h}{mv} \quad (4)$$

For relativistic particles the momentum becomes

$$p = \gamma mv \quad (5)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (6)$$

Thus, the relativistic De Broglie wavelength becomes

$$\lambda = \frac{h}{\gamma mv} \quad (7)$$

These expressions show that particles with higher momentum have shorter wavelengths.

3. Relation with Schrödinger Wave Mechanics

For a non-relativistic particle in a scalar potential $V(r)$, Schrödinger's equation is [6, 14].

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \psi \quad (8)$$

The kinetic energy operator is therefore

$$\hat{T} = -\frac{\hbar^2}{2m} \nabla^2 \quad (9)$$

For a free-particle plane wave, up to an overall normalization factor,

$$\psi(\mathbf{r}, t) = e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \quad (10)$$

the operator yields

$$\hat{T}\psi = \frac{\hbar^2 k^2}{2m} \psi \quad (11)$$

Using the relations

$$p = \hbar k \quad (12)$$

and

$$E = \hbar \omega \quad (13)$$

we obtain [10, 26]:

$$T = \frac{p^2}{2m} \quad (14)$$

4. Methodology

The present work adopts a theoretical and analytical methodology based on the established framework of quantum mechanics and solid-state physics. The analysis integrates mathematical derivations, conceptual interpretations and comparisons with experimental observations reported in the scientific literature.

The methodology consists of three main components:

4.1. Theoretical Derivation

The De Broglie relation establishes a quantitative connection between classical momentum and the wave description of matter, thereby linking classical mechanics with the formal structure of quantum theory. These equations were analyzed in both non-relativistic and relativistic regimes.

4.2. Literature Analysis

Experimental results from electron diffraction, interferometry and electron microscopy were examined to validate theoretical predictions. These experiments provide empirical confirmation of the wave behavior of electrons.

4.3. Technological Interpretation

The theoretical concepts were applied to modern technological systems, particularly semiconductor nanostructures and nanoscale electronic devices. The implications of electron wavelength and quantum confinement were examined in relation to modern nanoelectronics.

This methodology allows a unified analysis connecting theoretical quantum physics with experimental evidence and practical technological applications.

5. Wave-Particle Duality

Wave-particle duality refers to the observation that microscopic entities exhibit properties traditionally associated with both waves and particles. Rather than implying that particles are literally half wave and half particle, the concept reflects the fact that different experimental arrangements reveal different aspects of the same underlying quantum system [7].

For example, when electrons are detected by localized detectors such as photographic plates or semiconductor sensors, they produce discrete impacts characteristic of particle behavior. However, when electrons propagate through systems that allow multiple spatial paths—such as double-slit interferometers or crystalline lattices, they produce interference patterns characteristic of wave phenomena [13].

Quantum mechanics reconciles these observations by describing particles through a wave function $\psi(\mathbf{r}, t)$, whose squared magnitude represents the probability density of finding the particle at a given position and time. The wave function evolves deterministically according to Schrödinger's equation, but individual measurement outcomes appear as discrete events. A spatially localized particle is represented by a wave packet with a spread of momenta, consistent with the uncertainty principle [15, 19, 28].

This framework removes the apparent contradiction between wave and particle descriptions: the wave function describes the probability amplitudes of possible outcomes, while measurements produce localized particle-like results.

5.1. De Broglie Hypothesis

The De Broglie relation connects the momentum of a particle with its associated wavelength:

$$\lambda = \frac{h}{p} \quad (15)$$

For a particle of mass m and velocity v , the momentum in classical mechanics is

$$p = mv \quad (16)$$

Substituting into Equation (15) gives

$$\lambda = \frac{h}{mv} \quad (17)$$

For relativistic particles the momentum becomes

$$p = \gamma mv \quad (18)$$

with

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (19)$$

Thus, the relativistic De Broglie wavelength is

$$\lambda = \frac{h}{\gamma mv} \quad (20)$$

These relations show that particles with larger momentum possess shorter wavelengths.

5.2. Connection with the Schrödinger Equation

For a non-relativistic particle in a scalar potential V , Schrödinger's equation reads [23].

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi \quad (21)$$

The first term inside the Hamiltonian represents the kinetic energy operator

$$\hat{T} = -\frac{\hbar^2}{2m} \nabla^2 \quad (22)$$

For a free-particle plane wave, up to an overall normalization factor,

$$\psi(\mathbf{r}, t) = e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \quad (23)$$

we obtain

$$\hat{T}\psi = \frac{\hbar^2 k^2}{2m} \psi \quad (24)$$

Using the relations

$$p = \hbar k \quad (25)$$

and

$$E = \hbar \omega \quad (26)$$

we recover the classical kinetic energy expression

$$T = \frac{p^2}{2m} \quad (27)$$

6. Kinetic Energy of the Electron

6.1. Non-relativistic Regime

The kinetic energy of an electron moving at non-relativistic velocities is

$$T = \frac{p^2}{2m} \quad (28)$$

Using the De Broglie relation

$$p = \frac{h}{\lambda} \quad (29)$$

we obtain

$$T = \frac{h^2}{2m\lambda^2} \quad (30)$$

This relation is fundamental in electron diffraction experiments.

6.2. Relativistic Regime

For high-energy electrons the relativistic energy relation must be used:

$$E^2 = (pc)^2 + (mc^2)^2 \quad (31)$$

The kinetic energy becomes

$$T = E - mc^2 \quad (32)$$

or

$$T = mc^2(\gamma - 1) \quad (33)$$

These expressions are relevant in transmission electron microscopes operating at hundreds of kilovolts [22].

7. Experimental Evidence

Electron diffraction experiments provide direct confirmation of the De Broglie hypothesis. In the Davisson-Germer experiment, electrons scattered from a nickel crystal produced diffraction maxima consistent with Bragg's law.

$$2d \sin \theta = n\lambda \quad (34)$$

where d is the lattice spacing [8].

Subsequent matter-wave experiments extended these observations beyond electrons; interference with C_{60} molecules provides a particularly clear demonstration [1].

8. Discussion

Wave-Particle Duality in Semiconductor Quantum Physics and Electronic Transport.

Wave-particle duality is not merely a conceptual principle of quantum mechanics but also a fundamental element in the physical description of charge transport in modern semiconductor systems. As electronic devices shrink to nanometer dimensions, the De Broglie wavelength of electrons becomes comparable to the characteristic dimensions of the device. Under these conditions, classical models of electron transport—based on particle trajectories and scattering events—must be replaced by quantum mechanical descriptions based on wave propagation and probability amplitudes [7].

8.1. Quantum Transport Regimes in Semiconductors

Electron transport is characterized by the device length, the mean free path and the phase-coherence length. Diffusive or ballistic propagation and phase coherence describe distinct, potentially overlapping properties [7, 12, 16]:

1) Diffusive transport

In conventional macroscopic devices, electron motion is dominated by scattering processes involving phonons, impurities and defects. In this regime, transport is well described by classical drift-diffusion equations derived from the Boltzmann transport equation [2].

2) Ballistic transport

When the device length is much shorter than the mean free path, electrons propagate with little momentum-relaxing scattering. Ballistic transport alone does not guarantee phase coherence; the phase-coherence length must also exceed the relevant propagation distance [7, 12].

For one spin-degenerate channel in the coherent, elastic limit, the Landauer formalism gives

$$I = \frac{2q}{h} \int T(E) [f_S(E) - f_D(E)] dE \quad (35)$$

where q is the positive elementary charge, $T(E)$ is the channel transmission probability, and $f_S(E)$ and $f_D(E)$ are the source and drain Fermi-Dirac distributions. For multiple channels, $T(E)$ is replaced by the sum of their transmission probabilities [7, 18].

3) Quantum coherent transport

When the propagation distance is shorter than the phase-coherence length, interference effects can survive in either ballistic or diffusive systems. Coherent superpositions of quantum states then produce effects such as resonant tunneling and quantum interference [12, 16].

8.2. Quantum Confinement and Energy Quantization

When electrons are confined in semiconductor structures

with dimensions comparable to their De Broglie wavelength, their allowed energy states become quantized. This effect is known as quantum confinement and is central to the physics of nanostructures such as:

- 1) quantum wells
- 2) quantum wires
- 3) quantum dots

The electron wave function must satisfy the confining boundary conditions. For an ideal one-dimensional infinite square well of width L and constant carrier mass m , the energy levels are [14].

$$E_n = \frac{n^2 \hbar^2}{8mL^2} \quad (36)$$

where n is an integer quantum number.

This quantization arises directly from the wave nature of electrons and has profound consequences for semiconductor device behavior.

8.3. Effective Mass and Band Structure Effects

In crystalline semiconductors, electrons move within a periodic potential created by the atomic lattice. As a result, their behavior is described by the band structure $E(k)$ of the material [17, 30].

Near an isotropic band minimum, a parabolic approximation gives

$$E(k) = E_c + \frac{\hbar^2 k^2}{2m^*} \quad (37)$$

where m^* is the electron effective mass, E_c is the band-edge energy and k is measured from the band minimum [20].

In one dimension, the inverse effective mass is the band curvature divided by the squared reduced Planck constant:

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k^2} \quad (38)$$

This parameter determines how the electron responds to external forces within the semiconductor lattice.

In anisotropic crystals this scalar relation generalizes to an effective-mass tensor. The quantity $\hbar k$ is crystal momentum and should be distinguished from the free-electron mechanical momentum. Effective mass captures the effect of the periodic lattice potential on electron dynamics [17, 30].

8.4. Quantum Tunneling in Semiconductor Devices

One of the most important consequences of wave-particle duality in semiconductor physics is quantum tunneling. According to classical mechanics, a particle with energy E cannot cross a potential barrier higher than its energy. However, quantum mechanics predicts a finite probability of transmission even when

$$E < V_0$$

For an opaque rectangular barrier of height V_0 and width L , the leading exponential dependence of the transmission probability is [14].

$$T \approx e^{-2\kappa L} \quad (39)$$

where

$$\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} \quad (40)$$

Here the approximation retains the exponential attenuation and omits the interface prefactor; it requires κL to be much greater than one. Quantum tunneling plays a critical role in devices including:

- 1) tunnel diodes
- 2) flash memory cells
- 3) resonant tunneling diodes
- 4) advanced nanoscale MOSFETs

In modern transistor technologies, tunneling currents can become a limiting factor for device scaling, contributing to leakage currents in ultra-thin gate oxides.

8.5. Quantum Effects in Modern Nanoelectronics

At nanometer length scales, quantum confinement and tunneling can strongly affect device operation. Their importance depends on carrier energy, effective mass, barrier geometry and coherence, rather than on a universal 10 nm threshold [7, 20].

When those effects are significant, classical drift-diffusion models alone become insufficient. Instead, quantum transport models such as:

- 1) the Non-Equilibrium Green's Function (NEGF) method
 - 2) the Landauer-Büttiker formalism
- are required to accurately describe electron dynamics [7].

These models treat electrons as quantum waves interacting with potential landscapes defined by the device geometry and electrostatic fields.

8.6. Role of Wave Coherence in Electronic Devices

The phase coherence of electron wave functions can lead to interference effects in mesoscopic systems. When electrons propagate through multiple paths, their wave functions may interfere constructively or destructively depending on the phase difference.

Such effects are observed in phenomena such as:

- 1) Aharonov-Bohm interference
- 2) weak localization

- 3) universal conductance fluctuations

These phenomena illustrate how the wave nature of electrons influences electronic transport at the nanoscale [16].

8.7. Implications for Future Semiconductor Technologies

The continued scaling of semiconductor devices has pushed modern electronics into regimes where quantum mechanical effects cannot be ignored. As transistor dimensions approach the scale of a few nanometers, wave-particle duality becomes a central design consideration rather than a purely theoretical concept.

Emerging device architectures that rely explicitly on quantum effects include:

- 1) tunnel field-effect transistors (TFETs)
- 2) quantum dot devices
- 3) single-electron transistors
- 4) topological electronic systems

In these technologies, the wave properties of electrons are exploited to control transport mechanisms that differ fundamentally from classical conduction.

The de Broglie wavelength therefore helps assess quantum confinement, while scattering, coherence and electrostatic length scales are also needed to determine the appropriate transport model [7, 12].

9. Implications for Emerging Technologies

The principles of wave-particle duality and the De Broglie relation are central to several emerging technologies.

9.1. Electron Microscopy

The spatial resolution of electron microscopes depends on the wavelength of accelerated electrons. Because electron wavelengths can be thousands of times smaller than visible light wavelengths, electron microscopes can achieve atomic-scale resolution [29]. Wavelength alone does not determine practical resolution; electron optics, aberrations, stability and specimen conditions also matter [22].

9.2. Nanoelectronic Devices

In nanoscale semiconductor devices, quantum effects such as tunneling and quantum confinement strongly influence device behavior. When these effects dominate, device models must include quantum confinement and transport [7, 20].

9.3. Quantum Technologies

Matter-wave interferometry and electron phase manipulation play an important role in quantum sensing and precision metrology. Interference experiments involving electrons and

atoms provide extremely sensitive measurements of electromagnetic fields and gravitational effects.

10. Conclusion

Wave-particle duality and the De Broglie relation form the conceptual and mathematical foundation for understanding the quantum behavior of matter. The relation between momentum and wavelength provides a unified description of interference phenomena and particle dynamics, bridging classical mechanics and quantum theory.

The kinetic energy of the electron can be naturally expressed in terms of the De Broglie wavelength, illustrating how wave properties influence measurable physical quantities. These principles underpin a wide range of experimental techniques and technological applications, from electron diffraction to nanoscale electronic devices.

By integrating wave mechanics, operator formalism and experimental evidence, quantum theory provides a coherent framework for describing microscopic physical systems. Mastery of these concepts is therefore essential for both fundamental research and the development of advanced technologies in physics and engineering.

Abbreviations

NEGF	Non-Equilibrium Green's Function
TEM	Transmission Electron Microscopy
MOSFET	Metal-Oxide-Semiconductor Field-Effect Transistor
TFET	Tunnel Field-Effect Transistor

Author Contributions

Baldo Alberto Luigi Dalporto: Conceptualization, Formal Analysis, Investigation, Methodology, Writing – original draft, Writing – review & editing

Sabine Mary: Data curation, Formal Analysis, Project administration, Validation, Writing – review & editing

Santiago Gallur: Methodology, Supervision, Validation, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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