American Journal of Theoretical and Applied Statistics
2024, Vol. 13, No. 6, pp. 213-226
https://doi.org/10.11648/j.ajtas.20241306.14

Review Article

Methods of Optimal Accelerated Life Test Plans: A Review

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Abstract

Accelerated life tests (ALT) have been used as a powerful tool to obtain time based information on the life span or performance characteristics over time of the items. Tests are performed under higher stressed levels instead of under normal use constraints. Obtained information as tests results are used to make predictions about life span over time at the real use. Accelerated testing under different stresses continuously helps in improving product reliability and in formulating warranty policies. This paper aims to provide insight into the methods of optimal acceleration life test designs. We first present a review of literature on optimum design of accelerated life tests in chronological order over the last six decades. Second, we present life time distributions with their mean lifetime or qth quantity and life stress relationship with their different factors level. We also present a flow chart outlining the process of accelerated life test planning. Further, we present the estimation methods commonly employed in the field of accelerated life testing, including least squares estimation, maximum likelihood estimation, graphical estimation, and Bayesian estimation. Finally, we provide an analytical discussion on accelerated life testing. This review aims to assist researchers, reliability engineers, and scientists in enhancing the design and planning of accelerated life tests.

Keywords

Accelerated Life Tests (ALT), Life-Stress Relationship, Lifetime Distribution Censoring, Estimation Methods

1. Introduction

In the pursuit of highly reliable products or systems, accelerated life tests (ALTs) have served as a crucial tool for assessing the longevity and performance of products within a compressed timeframe. By applying test units to higher stress levels, ALT accelerates failures, allowing practitioners or researchers to gather product failure characteristics within a short period. Optimal accelerated life testing strategies are pivotal in effectively utilizing resources while ensuring accurate estimations of product lifetimes and failure characteristics.

Many decades ago, accelerated-life tests began with constant stress. Since then, they have evolved to incorporate time varying stress loadings. Accelerated life testing enables the development of an effective test that addresses crucial issues, including design of stress, test duration, sample-allocation, budget-constraints, and frequency measurement. Overall, accelerated life tests fulfill diverse objectives. Some common objectives include, identifying design failures, manufacturing defects, burn-in time, quality control, and helps in evaluating other factors. It also helps in qualifying design, assessing production, helps in making manufacturing changes, revising validity by consistency check. Further, accelerating life tests helps in developing relationship between reliability and operating conditions, and drafting ser-

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Received: 1 November 2024; **Accepted:** 21 November 2024; **Published:** 9 December 2024

vice policies.

Many authors have been working to planning the practical methods in accelerated life tests in the past few decades and they provide valuable discussions on different scenarios of accelerated life test plans, data analysis and their application for reliability estimation [1, 2]. Meeker and Escobar [3] presented a review on basic statistical methods and some current issues to improve accelerated life test designs. Nelson [4, 5] presented a comprehensive list of research works published in the area of accelerated life test plans. Escobar and Meeker [6] presented a review on planning ALTs, statistical methods, and the importance of ALT. Limon et al. [7] presented a review of statistical techniques and optimal design for accelerated life test plans. Chen et al. [8] presented a review on the theory of optimal design of ALT by considering the engineering issues and provided the procedure for selecting suitable methods. Pinto-Santos et al. [9] presented a review on generalities of the degradation tests and their applications. Therefore, the purpose of this review is to bring together both established and emerging research on the design and planning of accelerated life tests.

In this paper, we present a comprehensive review and summarize the work on optimal accelerated life test designs available in the literature. Table 1 provides a list of key journals that have published over 75 percent of research papers on ALT plans. Figure 1 illustrates the flow chart of ALT plans. Further, we discuss various life-stress relationships, provide analytical insights and highlight key trends in ALT.

The remainder of the paper is structured as follows: Section 2 provides a review of related works on accelerated life tests. Section 3 presents lifetime distributions. Section 4 discusses several life-stress relationship models. The estimation methods and test plans are discussed in Sections 5 and 6, respectively. Section 7 offers a critical discussion, and finally, Section 8 provides a brief conclusion.

Table 1. *List of Important Journals on Accelerated life test designs.*

S. No.	Name of journal	Publisher	No. of Articles	% of Articles
1.	IEEE Transactions on Reliability	IEEE	20	35
2.	Naval Research Logistic	John Wiley & Sons	9	16
3.	Technometrics	Taylor & Francis	7	12
4.	Quality & Reliability Engineering International	John Wiley & Sons	6	11
5.	Reliability Engineering & System Safety	Elsevier	4	7
6.	Journal of statistical computation and Simulation	Taylor & Francis	3	5
7.	Journal of Statistical Planning & Inference	Elsevier	3	5
8.	International Journal of Quality and Reliability Management	Emerald Insight	2	4
9.	Journal of Applied Statistics	Taylor & Francis	2	4
10.	Communications in Statistics-Theory & Methods	Taylor & Francis	1	2

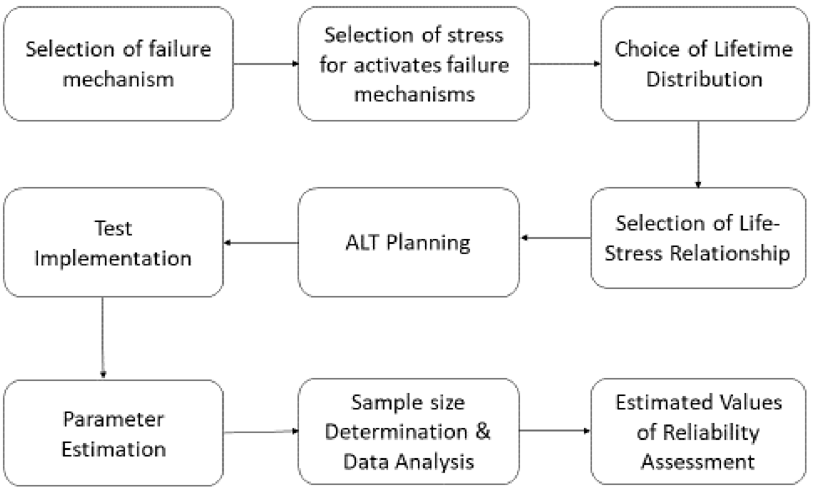

Figure 1. *Flow chart of Accelerated Life Test Plans.*

2. Review of Related Works

The history of accelerated life testing traces back to the early mid-20th century when engineers and researchers recognized the need to predict the reliability and life span of products and system more efficiently. Engineers and researchers were involved in development of resilient equipment, fighting warheads, aerospace, etc. during the 2nd World War. Post World War, applications of accelerated techniques expanded into commercial sectors such as automotive, electronics and consumer goods. Now, ALT emerged as essential tool to evaluate the long term performance of items and components used critically. In this section, we discuss the contribution of the researchers in last six decades.

Chernoff [10] provided an optimal accelerated life designs for effectively utilizing resources for estimating the parameters, such as, the number of failures, the mean lifetime of a device or system.

Meeker and Nelson [11] introduced a strategy to estimate the correlation between a stress and longevity of the product, with stress following either a smallest extreme-value-distribution or Weibull-distribution.

Kielpinski and Nelson [12] presented the life test plans for the same relationship for stress and the median of product life, where stress follow normal or lognormal distribution.

Nelson and Kielpinski [13] discussed a theory for optimal plans in accelerated life tests to estimate a basic relationship. They also recommend running more test units at low stress levels than at high stress levels, although acknowledging potential limitations in practical robustness.

The authors Nelson & Meeker [14] worked in collaboration with Meeker [15] on large sample Optimal ALT plans for transformed stress and product longevity, with stress follow Weibull or smallest extreme value distribution.

DeGroot and Goel [16] proposed a life testing method that integrates both ordinary and accelerated life-testing procedures. They explored two conditions under which an item can be tested and examined how stress influences the item's lifetime.

Escobar and Meeker [17] formulated a new method for ALT planning for models where the logarithm of time to failure followed normal distribution as stress function.

Yin & Sheng [18] presented the lifetime of an item under an accelerated life test with progressive stress. They assumed that the constant stress follows either an exponential or Weibull distribution. Additionally, they explained the unique case where the acceleration equation fits the inverse-power law and the stress is time proportional.

Yum et al. [19] discussed the optimal plan for exponentially distributed product lifespan under periodic inspection with Type I censoring data. They also conducted a sensitivity analysis to study the variation in the estimated mean concerning the uncertainty involved in use and –stress conditions.

Seo and Yum [20] demonstrated the advantages of practical ALT plans over statistically optimal ALT plans. Practical plans assumed periodic-inspection and Type I censoring. While they adopt the same design criteria, however, involved three in place of two over-stress levels and featuring easily calculable inspection schemes. They also discussed efficiency for practical plan and statistically optimal one.

Bai and Chung [21] designed an optimal accelerated life test with two constant-stress levels, high and low, where failed test items are replaced with good ones.

Barton [22] introduced a change in the optimum ALT plans described by Nelson and other authors, and discussed how to minimize the accelerated test stress.

Bai and Chung [23] introduced optimal designs based on partially accelerated life tests (PALTs) for which test can be changed during change in acceleration and use condition.

Bai et al. [24] proposed an optimal simple ramp-test for the Weibull distribution under Type I censoring considering different linearly increasing stresses.

Menzefricke [25] presented on the idea of sample size planning for accelerated life tests with type II censoring. Author determined the optimal accelerated stress levels at which observations are to be taken for cost constrain.

Chaloner et al. [26] discussed experimental designs for accelerated life tests with lifetimes following lognormal or Weibull distributions. They focused on quantiles of lifetime distribution at specified stress levels, assuming the log-lifetime decreases with stress under Type I censoring.

Bai et al. [27] developed optimal designs for PALTs, wherein items undergo initial testing under normal conditions for a specified period before transitioning to accelerated conditions.

Yang [28] presented an optimal design featuring various censoring in planning multiple level accelerated life tests. Further, the author compared with existing lower-level constant accelerated life test plans.

Meeter and Meeker [29] examined an optimal test plan for non-constant scale parameters aimed at minimizing the asymptotic variance of the maximum likelihood estimator for a specified quantile at the design stress.

Ahmad at al. [30] introduced a Rayleigh based optimal ALT plans under periodic inspection, including inspection and two stress levels. Islam and Ahmad [31] also developed optimal accelerated test plans where lifetime distribution followed Weibull distribution.

Yang and Jin [32] investigated the optimal compromise among three-level constant stress accelerated life test plans for Weibull distributions with varying censoring times. They also compared these plans to existing three-level test plans.

Escobar and Meeker [33] presented their understanding on the requirement of considering more than one acceleration factor in planning. They described approaches and standards for planning two-factors testing experiments for models without interaction between the factors.

Park and Yum [34] developed an optimal design for accelerated life tests involving two stresses, considering the possibility of interaction between the stresses.

Ahmad and Islam [35] presented optimal ALT designs for Burr Type XII distribution, considering periodic-inspection and Type-I censoring, with the assumption of stress-independent shape parameters and a log-linear scale parameter. They also highlighted the similarities between these designs and those used for exponential or Weibull distributions.

Bai et al. [36] investigated ramp-tests for Weibull distributions, incorporating constraints on test stress and time, based on the inverse power law and cumulative exposure model. They proposed optimal test plans aimed at minimizing the asymptotic variance of the maximum likelihood estimator for a specified log(life) or quantile at constant stress.

Tang et al. [37] discussed optimal ALT plans for lifetime distributions with failure free life, assuming a two-parameter exponential distribution and an inverse power law model for the stress-life relationship. They developed optimal ALT plans for constant stress under both failure and time censoring.

Tang et al. [38] discussed two alternative approaches for planning constant stress accelerated life tests (CSALTs) with three stress levels considering a Weibull lifetime distribution and optimized the stress levels and sample allocation.

Elsayed and Jiao [39] presented an optimal design for accelerated lifetime test plans using the proportional hazard model, focusing on determining appropriate stress levels and sample sizes.

Zhao and Elsayed [40] developed an optimum ALT plan considering a proportional mean residual life (PMRL) regression model. The proposed plan does not depend on life stress relationship.

Yang and Tse [41] discussed accelerated life test plans considering exponentially distributed lifetimes for progressive Type I interval censoring. They focused on minimizing the asymptotic variance for various combinations of total inspections and removal probabilities.

Pascual and Montepiedra [42] developed formulas for the asymptotic distribution of maximum likelihood estimators of model parameters in accelerated life tests when the model is misspecified. Their analysis focused on two widely used models such as the lognormal and Weibull Arrhenius-type models.

Ahmad at al. [43] generalized the previous research works on the accelerated life test designs. They proposed optimal plan of accelerated life tests for exponentiated-Weibull lifetime distribution under periodic inspection.

Pascual [44] introduced a methodology for planning accelerated life tests in scenarios involving two or more failure modes, or competing risks, driven by a single accelerating factor. He assumed that each failure mode is associated with a latent failure time, and the product lifetime is determined by the minimum of these times. The failure times were modeled using an S-independent Weibull distribution with known shape parameter.

Pascual [45] investigated the planning of accelerated life tests in the context of competing risks. The time to failure associated with each specific risk was modeled using a Weibull distribution, with the failure times considered to be S-independent.

Ahmad et al. [46] developed optimal accelerated life test designs for periodic inspection and Type I censoring, assuming failure lifetimes follow Burr Type III distributions with stress-dependent mean lifetimes and stress-independent shape parameters. They also discussed procedures for planning accelerated life tests and selecting sample sizes.

Tse et al. [47] presented optimal ALT design with progressive Type I interval censoring and random removals for Weibull-distributed lifetimes, aiming to minimize the asymptotic variance of the qth quantile.

Elsayed and Zhang [48] proposed optimal multiple-stress accelerated life test plans for the proportional odds (PO) model. They determined the best combinations of stress levels and the allocation of test units to each combination in order to minimize the variance in reliability predictions for the product over a specified period of time.

Ismail at al. [49] presented partially accelerated life tests for type II censoring with stress followed Weibull distribution and, presented a statistical method to reduce asymptotic variance of model parameters.

Liao [50] presented an improved accelerated life testing with optimal test plans. He introduced an approach for designing ALT plans that align with a mandatory periodic replacement schedule and account for a discounted penalty.

Meeker et al. [51] proposed a ALT model that uses life-stress data and field data to estimate failure time distribution for a component or product working in the same use condition.

Ahmad [52] proposed accelerated life test designs for the general exponential (GE) distribution using a log-linear model with periodic inspection and Type I censoring. The author derived the asymptotic variance of the maximum likelihood estimator for the log mean life or the qth quantile at the design stress, using this variance as the optimality criterion. The inspection times were assumed to be equally spaced.

Pascual [53] introduced methods for planning accelerated life tests in the presence of competing risks. He provided expressions for calculating the Fisher information matrix when the risks are independently distributed according to a lognormal distribution.

Liao and Elsayed [54] considered ALT planning with Type-I censoring for a log-location-scale distribution. Further, they presented equivalence of different ALT plans involving various stress loadings.

Hong et al. [55] presented an ALT plan for log location scale distribution considering time varying stress and censoring. They also showed its compatibility with ramp-stress test plan where they found new plan has a smaller variance.

Zhu and Elsayed [56] developed test plans for various stress applications, aiming to achieve reliability predictions with the same statistical precision as constant stress. They demonstrated that equivalent test plans exist that can reduce test duration, minimize costs, and maintain the same level of accuracy in reliability predictions.

Ahmad at al. [57] worked with Burr Type X distribution to design optimal accelerated life test plans considering periodic inspection and Type-I censoring. Authors also presented procedures for an ALT planning, and strategy for deciding sample size.

Yang and Pan [58] introduced ALT plans which utilizes readout data and discussed a new approach to design ALT test plans considering generalized linear model (GLM) for censored data.

Zhu and Elsayed [59] proposed a method for designing accelerated life test plans involving multiple stresses. They formulated multi-stress test plans based on various objectives and practical constraints. To efficiently determine the test plan parameters, they developed a simulated annealing algorithm.

Liu and Tang [60] introduced an ALT which simultaneously optimize stress levels, sample allocation under scheduled inspections where lifetime followed Weibull and Lognormal distributions.

Ding and Tse [61] explored the design of accelerated life test plans with random removals under progressive Type II interval censoring, where the product's lifetime is modeled using a Weibull distribution. They assumed that the number of random removals at each inspection follows a binomial distribution.

Haghighi [62] proposed an optimal ALT considering the stress level follows extended exponential distribution.

Balakrishnan and Ling [63] proposed accelerated life tests for single use device testing stress followed weibull distribution.

Xu et al. [64] proposed an analysis of ALT considering constant stress and Type II censored samples depending on fuzzy theory.

Xu et al. [65] introduced a framework for planning accelerated life tests based on maximizing the expected Shannon information between the prior and posterior density functions. This approach helps in optimizing the test design by improving the knowledge gained about the parameters of interest through the test data.

Gao et al. [66] addressed the optimal accelerated life test design for Type I censoring under constant stress. They transformed the challenge of determining an optimal design for an ALT with a univariate or multivariate nonlinear stress lifetime relationship into one that involves an optimal plan for an ALT with a univariate linear stress lifetime relationship.

Huang and Wu [67] investigated the optimization problem of sample size allocation for competing risks data obtained from progressive Type II censoring in a constant stress accelerated life test with multiple-levels. Their study aimed to enhance the efficiency of sample size distribution among different test conditions.

Subramanian at al. [68] discussed their work on both, the planning (design) stage as well as the inference (analysis) stage of ALT and discussed the importance of deciding number of factors, censoring, levels of each factor, sample size, etc.

Dey and Nassar [69] presented a comparative evaluation of MLE and eight other parameters estimation methods of the exponential Lindley distribution.

Ma et al. [70] proposed a hybrid testing plan under several experimental design constraints for the first time to meet the estimation requirement which were difficult to obtain using only ALT or accelerated degradation test.

Ayasse and Seo [71] discussed the computational difficulties and complexity of traditional methods in developing optimal ALT designs. They also introduced a practical method for finding optimal experimental designs for accelerated life testing using run tests and statistical modeling.

Kumar et al. [72] examined nine methods for estimating accelerated life test parameters for the generalized inverse distribution under constant stress conditions, including the maximum likelihood method. They evaluated the performance of these methods by analyzing mean relative estimates and mean square errors across small, medium, and large sample sizes.

Wu et al. [73] focused on finding the interval estimation of an ALT plan with constant stress following a two-parameter exponential distribution to test failure data under Type-II censoring. They further investigated the performance of the proposed confidence interval using Monte Carlo simulation.

Smit et al. [74] introduced a novel procedure for a Bayesian accelerated life testing model that employs the Weibull distribution for lifetime failure data and the generalized Eyring model as the time transformation function.

Lv et al. [75] developed a model that accounts for varying mechanisms and random effects resulting from time and budget constraints, using a specific percentile in place of the scale parameter. They estimated the model parameters employing the Bayesian method.

Smith et al. [76] presented a failure model that relates failures or life to stress using a binomial distribution. They developed a binomial representation called the acceleration factor (AF), which is the ratio between the binomial distributions of use and accelerated life.

Nassar et al. [77] discussed the estimation of unknown parameters and the reliability function. They advocated for the use of likelihood estimation, maximum product of spacing, and Bayesian methods when the lifespan of units followed the Lindley model under constant stress ALT.

3. Lifetime Distribution

In this section, we present different distributions used in accelerated life testing, along with their probability density functions (pdf) and mean lifetimes and is presented in Table 2.

Table 2. Life time Distributions for ALT.

Name of Distribution	Probability Density Function (pdf)	Mean Lifetime or qth quantile
Normal	$f\ (t) = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{1}{2}\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]}, -\infty < t < \infty,$	$\tau_q = \mu + z_q\sigma$
Log-normal	$f(t) = \left\{\frac{0.4343}{(2\pi)^{\frac{1}{2}}t\sigma}\right\}.\ e^{\left\{\frac{-(log(t)-\mu)^2}{(2\sigma^2)}\right\}}, t > 0$	$log(\tau_q) = \mu + z_q\sigma$
Weibull	$f(t) = ((\beta\alpha^{\beta})t^{\beta-1}e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, t > 0$	$\tau_q = \alpha[-\ln(1-q)]^{1/\beta}$
Rayleigh	$f(t) = \frac{t}{\sigma^2} e^{\left(-\frac{t^2}{2\sigma^2}\right), t, \sigma > 0}$	$ln(\tau_q) = \frac{1}{2}ln2 + \mu_0 + \frac{1}{2}\ln\{-\ln(1-q)\}$
EW	$f(t) = \sigma\frac{\delta}{\theta}\left(\frac{t}{\theta}\right)^{\delta-1}(1 - e^{-\left(\frac{t}{\theta}\right)^{\delta}})^{\sigma-1}e^{-(\frac{t}{\theta})^{\delta}}$	$ln(\tau_q) = \beta_0 + \beta_1 s_0 + \frac{1}{\delta} + ln[-\ln\left(1 - q^{\frac{1}{\sigma}}\right)]$
Generalized Exp.	$f(t) = \left(\frac{\sigma}{\theta}\right)e^{-\frac{t}{\theta}}\left(1 - e^{-\frac{t}{\theta}}\right)^{\sigma-1}, t \geq 0$	$\ln(\tau_q) = \beta_0 + \beta_1 s_0 + \ln[-\ln\left(1 - q^{\frac{1}{\sigma}}\right)]$
Burr Type III	$f(t) = m\left(\frac{\delta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\delta m-1}\left(1 + \left(\frac{t}{\theta}\right)^{\delta}\right)^{-(m+1)}, t \geq 0, m, \delta, \theta > 0$	$\ln(\tau_q) = \beta_0 + \beta_1 s_0 + (\frac{1}{\delta})ln(\frac{q^{\frac{1}{m}}}{1-q^{\frac{1}{m}}})$
Burr Type X	$f(t) = 2(\sigma/\theta)(t/\theta)e^{-\left(\frac{t}{\theta}\right)^2}, t \geq 0, \sigma > 0, \delta > 0$	$ln(\tau_q) = \beta_0 + \beta_1 s_0 + \frac{1}{2}ln[-\ln\left(1 - q^{\frac{1}{\sigma}}\right)]$
Burr Type XII	$f(t) = m\left(\frac{\delta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\delta-1}\left(1 + \left(\frac{t}{\theta}\right)^{\delta}\right)^{-(m+1)}, t \geq 0, m, \delta, \theta > 0$	$\ln(\tau_q) = \beta_0 + \beta_1 s_0 + (\frac{1}{\delta})\ln\{\frac{1-(1-q)^{\frac{1}{m}}}{(1-q)^{\frac{1}{m}}}\}$
Extreme value	$f(t) = 1 - e^{-e^{\frac{(t-\mu)}{\sigma}}, -\infty<t<\infty}$	$\tau_q = \mu + u_q\delta$

4. Life-Stress Relationship Models

The life stress relationship is a fundamental concept in reliability engineering and product testing, describing how different stress factors, such as temperature, pressure, or load, influence the lifespan and performance of materials and products. Understanding this relationship is crucial for predicting product durability, ensuring safety, and optimizing maintenance schedules. By studying how stress accelerates aging or failure, engineers can develop more robust designs, improve quality control, and reduce costs associated with warranty claims and unexpected failures. This relationship is often modeled using empirical data from accelerated life testing, where products under high test stress levels to induce failures more quickly, providing valuable insights into their long-term behavior under normal use conditions.

4.1. Arrhenius Model

This law is effective in modeling the effect of temperature on degradation processes affecting products. It is specifically applicable in predicting the longevity of products under varying temperature conditions. The Arrhenius model is given below,

$$AF(S_0, S_1) = \exp[\frac{Ea}{k_B}(\frac{1}{S_o} - \frac{1}{S_1})], \tag{1}$$

where S_0, and S_1 are temperature at base and accelerated level respectively, E_a *is* activation energy, K_B is Boltzmann constant

4.2. Eyring Model

It extends the concepts of the Arrhenius law by combining temperature with other stress factors. The Eyring model is given by:

$$AF(S_0, S_1) = \frac{S_1}{S_0}\exp[\frac{Ea}{k_B}(\frac{1}{S_0} - \frac{1}{S_1})], \quad (2)$$

where S_0 and S_1 are respective temperature at the base and accelerated level, E_a is activation energy, k_B is Boltzmann constant.

4.3. Inverse Power Model

The Inverse Power Law is a model use to describe how the lifespan of a product diminishes with the increase in stress. This law is widely used in failure analysis. The Inverse Power Law model is expressed as:

$$AF(S_0, S_1) = (\frac{S_1}{S_0})^{\alpha} \quad (3)$$

where S_0 and S_1 are stress value of base level and accelerate level, α is acceleration parameter.

4.4. Multifactor Models

There are many situations in which operational conditions involve multiple stress factors. Miner's rule drives remaining life as a function of various stress levels [1]. The model is given as

$$U = \sum_{i=1}^{k}\frac{c_i}{C_i} \quad (4)$$

where U is the unused lifetime of the product, c_i is the number of the cycles under i^{th} stress level at a point of time, and C_i is the average number of cycles to failure at that stress level.

The common form of the model is given by exponential relationship,

$$\lambda(t) = \lambda_0(t)e^{\sum_{i=1}^{k}\beta_i x_i} \quad (5)$$

where $\lambda_0(t)$is the base level failure rate, x_i is the i^{th} predictor variable, and β_i is the regression coefficient.

5. Estimation Methods

There are mainly four estimation methods in use in the field of accelerated life testing, namely, least square estimation, maximum likelihood estimation, graphical estimation and Bayesian estimation.

5.1. Least Squares Methods

The least squares method is an established procedure in regression analysis to determine the best-fitting line or model to a set of observed data points. Assuming that the random samples of n units are run on test until all units meet failures where there are J test stress levels $x_j, j = 1,2,3, \ldots J$. Here, total number of stress units are $n = n_1 + n_2 + \cdots + n_j$. The least square method in ALT helps in obtaining important parameters.

The model for the ith failure time at stress $x_j$is

$$Y_{ij} = \mu(x_j) + e_{ij}, \quad (6)$$

The least square estimation of mean life can be obtained by minimizing the following expression

$$L = \sum_{i=1}^{n_j}\sum_{j=1}^{J}(y_{ij} - \mu(x_j))^2 \quad (7)$$

for $i = 1, 2, \ldots, n_j$ and $j = 1, 2, \ldots, J$. The error term, e_{ij}, follows extreme value distribution a mean, 0 and an unknown scaling parameter, δ. The mean life is given as linear life-stress relationship

$$\mu(x_j) = \beta_0 + \beta_i x_j. \quad (8)$$

5.2. Maximum Likelihood Methods

Maximum likelihood method in accelerated life testing is employed to estimate the parameters of life distribution models under different stress conditions. To obtain estimates using maximum likelihood estimation, it is assumed that units censored at any particular time and those that continue beyond that time are from the same life distribution. However, this assumption does not hold true if units are removed from service intact when they appear to be near failure.

Suppose y_{ij} be the dependent variable of lifetime distribution have the following cdf,

$$F(y; \theta_1, \theta_2, \ldots, \theta_k),$$

where $\theta_1, \theta_2, \ldots, \theta_k$ are the k parameters of the lifetime distribution and the pdf is given as

$$f(y; \theta_1, \theta_2, \ldots, \theta_k) = dF(y; \theta_1, \theta_2, \ldots, \theta_k)/dy.$$

Hence, the likelihood function of y_i is given as

$$L_i(\theta_i) = f(y; \theta_{1i}, \theta_{2i}, \ldots, \theta_{ki}), i = 1, \ldots, n.$$

If the ith item censored on right at y_i, then the likelihood function is

$$L_i(\theta_i) = 1 - F(y; \theta_{1i}, \theta_{2i}, \ldots, \theta_{ki}), i = 1, \ldots, n.$$

If the lifetime distribution of an items are independent, then the likelihood function, L is given as

$$L = L_1 \times L_2 \times \ldots \times L_n. \quad (9)$$

For periodic inspection, the likelihood function of the set of observations $\{x_{ij}\}_{j=1}^{K(i)+1}$ with Type I censoring cut off at

$K(i)$ follows multinomial distribution with n_j and probability $\{P_{ij}\}_{j=1}^{K(i)+1}$ at stress level s_i is given by

$$L = \prod_{i=1}^{2} L_i = \prod_{i=1}^{2} n_i! \left(\prod_{j=1}^{K(i)+1} x_{ij}!\right)^{-1} \left(\prod_{j=1}^{K(i)+1} P_{ij}{}^{x_{ij}}!\right) \quad (10)$$

5.3. Graphical Estimation Methods

In accelerated life testing, graphical estimation methods are generally use to analyze data and project a product's life expectancy under standard operating condition. The Arrhenius plot is a common graphical estimation method used in ALT. These methods are valuable for predicting product reliability and durability under real-world conditions based on accelerated testing data. They provide insights into how products will perform over time, aiding manufacturers in making informed decisions about design improvements and warranty policies.

5.4. Bayesian Estimation

Bayesian estimation often involves challenges related to eliciting prior information and formulating it into an appropriate prior distribution. The prior distribution $P(\theta)$ is subsequently integrated with the data to derive the posterior distribution $P(\frac{\theta}{D})$ representing the parameter values. The posterior distribution provides Bayesian estimates and probability limits for both the true parameter values and their corresponding functions. However, based on the data, some practitioners may revise the prior distribution repeatedly until a satisfactory posterior distribution is achieved.

6. Test Plans

A test-plan for Accelerated Life Testing is a comprehensive document that outlines the strategy, methodology, and procedures for conducting accelerated life tests on products to estimate their reliability and lifespan under normal usage conditions. Some common test plans for ALT are given below and the characterization of experimental region is presented in Table 3.

Optimum Plans: An optimal plan is the plan which provides the most precise estimates of product life at the design stress. Assuming the model and data are valid, it produces the most precise estimation of the mean log-life. They typically involve two stress levels. However, optimal plans have some drawbacks.

Traditional Plans: In traditional plan stress level spacing and number of specimens are equal. This approach ensures a balanced distribution of samples across different stress conditions, providing a comprehensive analysis of the product's reliability and lifespan under various stress scenarios. However, these plans are the least recommended.

Good Plans: This design allows for a detailed understanding of the product's performance across a range of conditions while optimizing resource use. Its' robustness under multiple scenarios helps in obtaining accurate estimates. Here specimens' distribution under different stress level is not equal.

Table 3. *Characterization of one, two and multi-factor ALT test plans.*

	One-factor ALT	**Two-factor ALT**	**Multi-factor ALT**
Factors	X	$\mathrm{x} = (x_1, x_2)^T$	$\mathrm{x} = (x_1, x_2, .., x_k)^T$
Design Conditions	x_d	$x_d = (x_{1d}, x_{2d})^T$	$x_d = (x_{1d}, x_{2d,} .. x_{kd})^T$
Maximum factor levels	$x \le x_h$	$x_1 \le x_{1h}, x_2 \le x_{2h}$	$x_1 \le x_{1h}, x_2 \le x_{2h}, .., x_k \le x_{kh}$
Standardized factors	$\mathrm{S} = \frac{(x - x_d)}{(x_h - x_d)}$	$S_j = \frac{(x_j - x_{jd})}{(x_{jh} - x_{jd})}, j = 1, 2$	$S_j = \frac{(x_j - x_{jd})}{(x_{jh} - x_{jd})}, j = 1, 2, \ldots, k$
Location	$\mu(\mathrm{x}) = \mu(\mathrm{S}) = \beta_0 + \beta_1 S$	$\mu(\mathrm{x}) = \mu(\mathrm{S}) = \beta_0 + \beta_1 S_1 + \beta_2 S_2$	$\mu(\mathrm{x}) = \mu(\mathrm{S}) = \beta_0 + \beta_1 S_1 + \beta_2 S_2 + \cdots + \beta_k S_k$
Experimental region	$\mathrm{S} \le 1$	$\mathrm{S}_1 \le 1, S_2 \le 1$	$\mathrm{S}_1 \le 1, S_2 \le 1, \ldots, S_k \le 1$

7. Critical Discussion

In this section, we provide an analytical review of some important papers published on the optimal design of Accelerated Life Test plans. Figure 2 illustrates the frequency of papers published on accelerated life test designs over the last six decades. Additionally, we highlight the prominent journals where ALT studies are typically published. Furthermore, Table 3 highlights the characterization of one-factor, two-factor, and multi-factor accelerated life test plans.

Table 4. *Review of Some Important Works.*

Author(s)	Lifetime Distribution	Life-Stress Model	Censoring	Research Contributions
Meeker [15]	Weibull and lognormal	Inverse power law	Type I	Minimizes the variance of some estimator.
Escobar & Meeker [17]	Extreme value	Inverse power law	Type II	Methods for continuous and discrete ALT, important properties and expression for BLUE and variance for estimator.
Yum & Choi [19]	Exponential	Inverse power law	Type I	Optimal ALT for exponentially distributed lifetime with type I censoring & periodic inspection.
Seo & Yum [20]	Weibull	Inverse power law	Type I	Failure mechanism of test units, optimal plan involving two stress levels, optimal inspection time.
Bai & Chung [21]	Exponential	Inverse power law	Type I	Optimal ALT plan where failed items are replaced with new ones.
Bai et al. [27]	Lognormal		Type I	Optimal design of Partially ALT.
Meeter & Meeker [29]	Extreme value	Inverse power law	Type I	Optimal ALT, minimizing asymptotic variance for Weibull model.
Ahmad et al. [30]	Rayleigh	Inverse power law	Type I	Optimal ALT for the case of Rayleigh failure distribution.
Islam & Ahmad [31]	Weibull	Inverse power law	Type I	Optimal design of ALT for the case of Weibull failure distribution and sensitivity analysis.
Yang & Jin [32]	Extreme value	Inverse power law	Different censoring	Accelerating life test for Weibull under different censoring.
Park & Yum [78]	Exponential	Eyring	Type I	Optimal ALT in case where two cases have interaction effect.
Ahmad & Islam [35]	Burr type XII	Inverse power law	Type I	Optimal ALT for Burr Type XII, procedure, sample size.
Tang et al. [37]	Exponential	Inverse power law	Type I & Type II	Planning of ALT for two parameter exponential distribution with two stresses.
Tang et al. [38]	Weibull	Arrhenius	Type I	ALT planning with three constant stress levels.
Yang & Tse [41]	Exponential	Inverse power law	Progressive type I	ALT with progressive type I censoring
Pascual & Montepiedra [42]	Lognormal & Weibull	Arrhenius	Type I	Expression for asymptotic distribution of MLEs of ALT.
Ahmad et al. [43]	Exponentiated Weibull	Inverse power law	Type I	Analysis of optimal ALT plans for Exponentiated Weibull.
Pascual [44]	Weibull	Inverse power law	Type I	ALT plans where risks follow Weibull distribution.
Ahmad et al. [46]	Burr type III	Inverse power law	Type I	ALT design for periodic inspection with type I censoring where failures follow Burr type III.
Ahmad [52]	Generalized exponential	Inver power law	Type I	ALT for generalized exponential with type I censoring.
Liao & Elsayed [54]	Log location scale/ Weibull	Inverse power law	Type I	Lognormal based ALT and equivalency of various ALT plans considering different stresses.
Ahmad et al. [57]	Burr Type X	Inverse power law	Type I	Optimal ALT for Burr type X. for period inspection & Type I censoring.

Author(s)	Lifetime Distribution	Life-Stress Model	Censoring	Research Contributions
Zhu & Elsayed [59]	Weibull	Linear inverse power law	Type I & II	ALT plans under multiple stresses.
Xu et al. [64]	Weibull	Inverse power law	Type II	ALT using Fuzzy theory.
Gao et al. [66]	Weibull	nonlinear stress-life	Type I	Time censored ALT under type I censoring.
Huang & Wu [67]	Exponential	Arrhenius law	Type II	Optimal sample size allocation for ALT with multiple level constant stress.
Dey & Nassar [69]	Exponentiated Lindley	Inverse power law	Right	Nine different classical methods of estimation under ALT.
Fan & Wang [79]	Exponential	Inverse power law	Type I	Comparison between CSALT and SSALT.
Ayasse & Seo [71]	Lognormal	Inverse power law	Right	A practical method to find an optimal design of experiments for ALTs.
Kumar et al. [72]	Generalized inverse Lindley	Inverse power law	Right	ALT for generalized inverse Lindley distribution.
Wu et al. [73]	Exponential	Inverse power law	Type II	Interval estimation of Scale and location parameters based on ALT.
Smit et al. [74]	Weibull	Generalized Eyring	Type I & II	Bayesian ALT for the case of Eyring-Weibull model.

Of the papers on optimal design of accelerated life tests listed in Table 4, the majority of authors employed the inverse power law as the life-stress model, with most studies considering the Weibull family as the lifetime distribution. However, Type-I censoring was commonly used in accelerated life tests, while very few authors employed Type-II or right censoring in their studies.

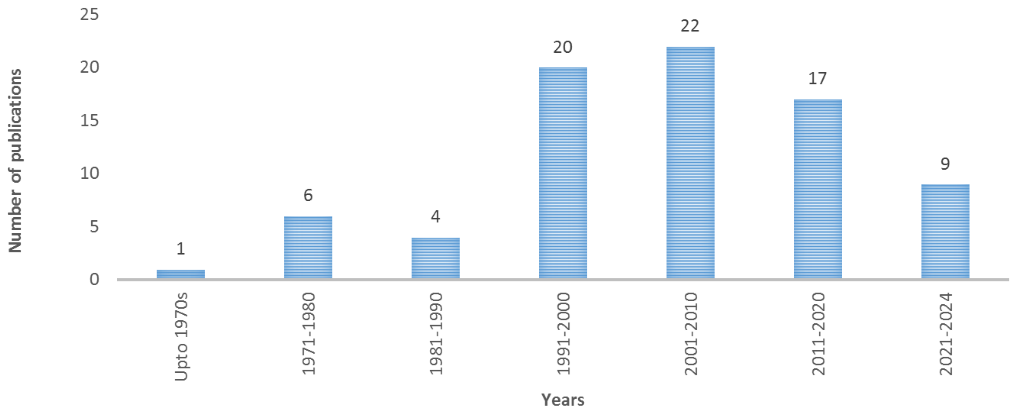

Figure 2. *Trends of publication in optimal design of ALT in Six decade.*

8. Conclusions

This paper presented a comprehensive study of methods for optimal accelerated life test (ALT) plans, along with an analytical discussion on ALT. We provided a list of key journals that have published over 75 percent of the research papers on accelerated life test designs. We also presented a flowchart outlining the process of accelerated life test (ALT) planning.

In conclusion, the proposed review on accelerated life test plans may offer valuable guidelines for researchers in selecting appropriate problems to estimate the lifetime of highly reliable products or materials.

Abbreviations

ALT	Accelerated Life Test
CSALT	Constant Stress Accelerated Life Test
SSALT	Step-Stress Accelerated Life Test
AF	Acceleration Factor
PALT	Partially Accelerated Life Test
MLE	Maximum Likelihood Estimation
BLUE	Best Linear Unbiased Estimator

Author Contributions

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Conflicts of Interest

The authors declare no conflicts of interest.

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Research Fields

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