American Journal of Theoretical and Applied Statistics
2025; 14(2): 51-60
http://www.sciencepublishinggroup.com/j/ajtas
doi: 10.11648/j.ajtas.20251402.11
ISSN: 2326-8999 (Print); ISSN: 2326-9006 (Online)

Lifetime Distribution Based on Generators for Discrete Mixtures with Application to Lomax Distribution

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To cite this article:
Peter Nyandiri Mecha, Isaac Chumba Kipchirchir, George Odweso Muhua, Joseph Antony Makoteku Ottieno. (2025). Lifetime Distribution Based on Generators for Discrete Mixtures with Application to Lomax Distribution. *American Journal of Theoretical and Applied Statistics, 14*(2), 51-60. https://doi.org/10.11648/j.ajtas.20251402.11

Received: 15 July 2024; **Accepted:** 24 December 2024; **Published:** 13 March 2025

Abstract: In various application areas, data is frequently collected and analyzed using basic statistical distributions such as exponential, Poisson, and gamma distributions. However, these traditional distributions often fail to adequately capture the inherent heterogeneity present in real-world data. This limitation highlights the need for more flexible distributions that can address these complexities. Such distributions can be generated through techniques like reparameterization, generalization, compounding, and mixing. This paper focuses on deriving generators for survival functions of discrete mixtures using minimum and maximum order statistic distributions. The approach leverages the probability generating function (PGF) techniques of mixing distributions, including zero-truncated Poisson, shifted geometric, zero-truncated binomial, zero-truncated negative binomial, and logarithmic series distributions. Specifically, the derived generator was applied to Lomax distributions to construct survival functions. Consequently, the probability density function (PDF) and failure rate of the resulting discrete mixtures were also obtained. Furthermore, the paper examines the shapes of the PDF and failure rate for discrete mixtures derived from the zero-truncated Poisson distribution. Notably, the failure rates of discrete mixtures generated using minimum and maximum order statistics from the Lomax distribution exhibited distinct behaviors. The failure rate for the minimum order statistic was observed to decrease, while the failure rate for the maximum order statistic showed a combination of non-decreasing and bathtub-shaped patterns.

Keywords: Generators, Order Statistic, Discrete, Mixing, Distribution, Mixture

1. Introduction

Combining or mixing multiple distributions to create a new one is a method used in constructing probability distributions. The resulting combined distribution is referred to as a mixture. There are three main types of mixture distributions discussed in the literature: finite, continuous, and discrete. This paper focuses on discrete mixtures, which can be derived from compound distributions or order statistic distributions.

Recently, many researchers have explored the technique of introducing parameters through the compounding of different distributions to develop new lifetime distributions. A foundational contribution in this area was made by Adamidis and Loukas [1], which paved the way for a new class of lifetime distributions. Examples include Kus [9], who introduced the exponential-Poisson distribution, Tahmasbi and Rezaei [11], who developed the exponential-logarithmic distribution, and Hajebi et al. [7], who proposed the exponential-negative binomial distribution. Additionally, Mecha et al. [17] constructed generators for discrete mixtures using the probability generating function approach and applied these models to the exponential distribution.

Al-Zaharani and Sangor [3] examined the maximum order statistic from the Lomax (Pareto II) distribution as the conditional distribution and used the zero-truncated Poisson distribution as the mixing distribution. This specific discrete mixture is known as the Lomax-Poisson distribution.

The aim of this paper is to build upon the work of

Al-Zaharani and Sangor by utilizing the generator method developed by Mecha et al. [17] to create discrete mixtures based on minimum and maximum order statistics, with applications to the Lomax distribution.

The paper is structured as follows: Section 2 derives generators for discrete mixtures based on the minimum order statistic, with applications discussed in Section 3. In Section 4, generators based on the maximum order statistic are derived, followed by applications in Section 5. The conclusion is presented in Section 6.

2. Generators for Discrete Mixtures Based on Minimum Order Statistics

Let $X_1, X_2, ..., X_N$ be independent identically distributed continuous random variables where N is also a random variable independent of $X_i's$

Suppose

$$Z = \min(X_1, X_2, ..., X_N) \tag{1}$$

Then the cumulative distribution function (cdf) of Z given N is

$$F(z|n) = 1 - [1 - G(z)]^n \tag{2}$$

where G(z) is the cdf of the parent distribution.

Therefore, survival function of Z given N is given by

$$\begin{aligned} S(z|n) &= 1 - F(z|n) \\ &= [1 - G(z)]^n \end{aligned} \tag{3}$$

and

$$\begin{aligned} S(z) &= \sum_{n=1}^{\infty} S(z|n) p_n \\ &= \sum_{n=1}^{\infty} [1 - G(z)]^n p_n \\ &= E[1 - G(z)]^N . \end{aligned} \tag{4}$$

Next, let

$$\phi_N(s) = E\left[s^N\right] \tag{5}$$

be the pgf of N, then from equations (4) and (5)

$$S(z) = \phi_N[1 - G(z)] \tag{6}$$

which is the pgf of N at $1 - G(z)$, i.e, survival function of the parent distribution.

The survival function of a discrete mixture, based on the minimum order statistic, represents the probability of generating N according to the survival function of the underlying parent distribution.

The pdf is given by

$$f(z) = \frac{-dS}{dz}, \quad z > 0 \tag{7}$$

and the hazard function given by

$$h(z) = \frac{f(z)}{S(z)}, \quad z > 0. \tag{8}$$

For the Lomax (Pareto II) distribution, the pdf and cdf are given below.

$$g(z) = \alpha\beta (1 + \beta z)^{-(\alpha+1)} \tag{9}$$

$$G(z) = 1 - (1 + \beta z)^{-\alpha} \tag{10}$$

Therefore

$$S(z) = \phi_N\left[(1 + \beta z)^{-\alpha}\right] \tag{11}$$

2.1. Poisson Generators Based on Minimum Order Statistic

The zero-truncated Poisson distribution with parameter θ. the probability mass function (pmf) is

$$p_n = \frac{\theta^n e^{-\theta}}{1 - e^{-\theta}}, \quad n = 1, 2, 3, ..., \quad \theta > 0, \tag{12}$$

the probability generating function (pgf) is

$$\phi_N(s) = \frac{e^{\theta s} - 1}{e^{\theta} - 1} \tag{13}$$

and from equation (6) the survival function is

$$S(z) = \frac{e^{\theta[1-G(z)]} - 1}{e^{\theta} - 1} \tag{14}$$

consequently, the pdf and hazard function are given by

$$f(z) = \frac{\theta g(z) e^{\theta[1-G(z)]}}{e^{\theta} - 1}, \tag{15}$$

$$h(z) = \frac{\theta g(z)}{1 - e^{-\theta[1-G(z)]}} \tag{16}$$

Therefore, substituting equations (9) and (10) in the above equations (14), (15) and (16) we obtain Lomax-Poisson distribution for minimum order statistic as

$$f(z) = \frac{\theta\alpha\beta (1 + \beta z)^{-(\alpha+1)} e^{\theta(1+\beta z)^{-\alpha}}}{e^{\theta} - 1} \quad \alpha, \theta\beta > 0, z > 0 \tag{17}$$

$$s(z) = \frac{e^{\theta(1+\beta z)^{-\alpha}} - 1}{e^{\theta} - 1} \tag{18}$$

$$h(z) = \frac{\theta\alpha\beta\,(1+\beta z)^{-(\alpha+1)}}{1 - e^{-\theta(1+\beta z)^{-\alpha}}} \tag{19}$$

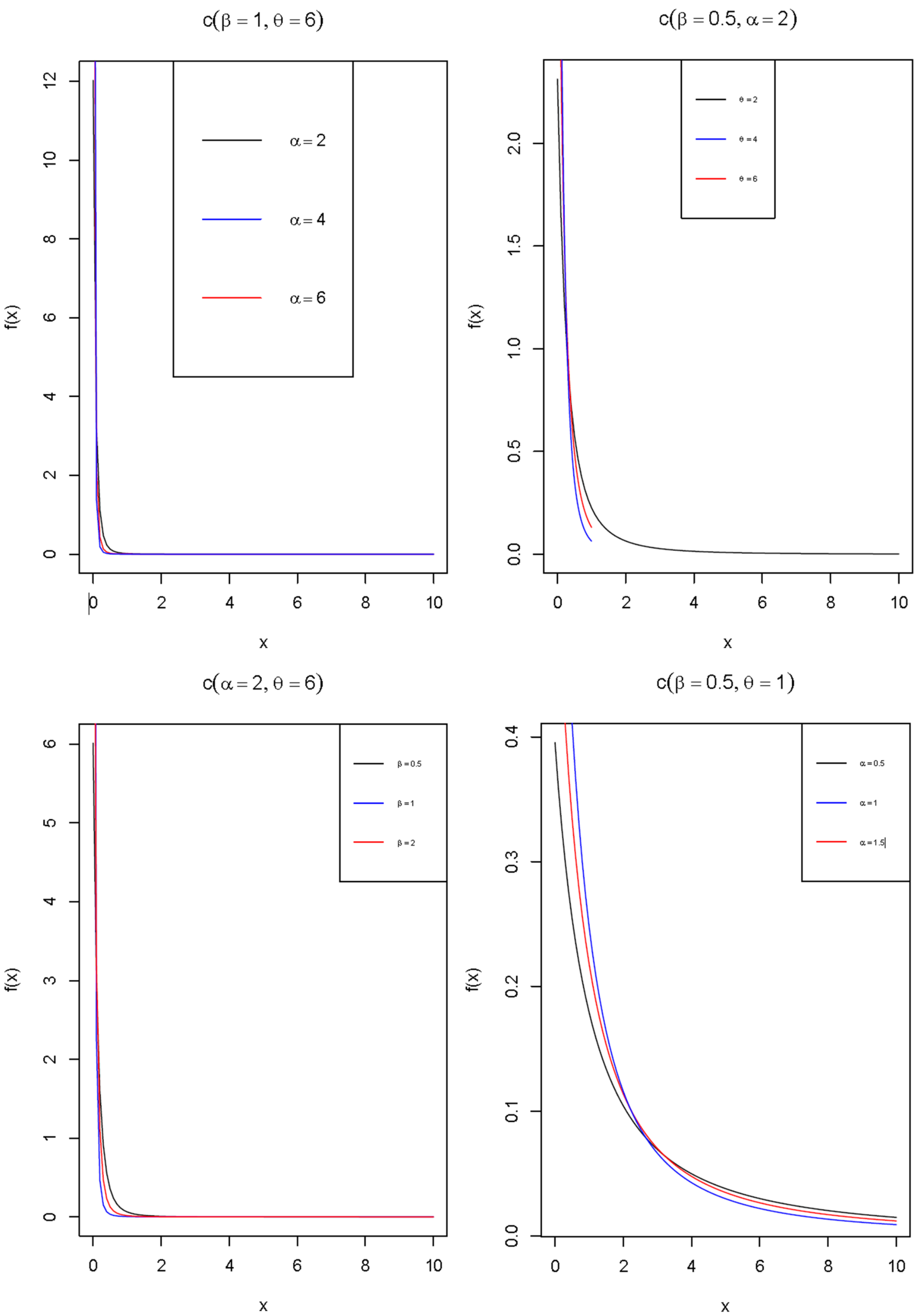

Figure 1. *Plot of the probability density function for varying values of the parameters α, β, and θ.*

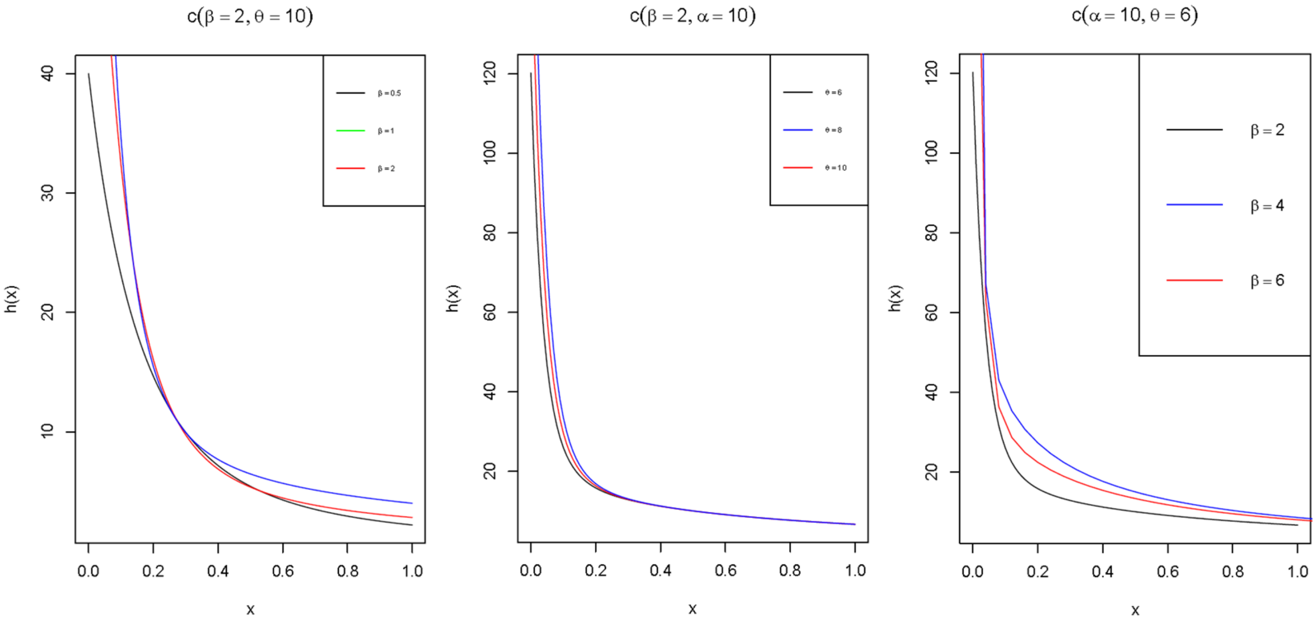

Figure 2. *Plot of the hazard function for various values of the parameters α, β, and θ.*

2.2. Binomial Generators Based on Minimum Order Statistic

The zero-truncated binomial distribution with index $m = 1, 2, 3, \ldots$ and parameter $\dfrac{\theta}{1+\theta}$, where $\theta > 0$.

The pmf is

$$p_n = \frac{\binom{m}{n}\theta^n}{(1+\theta)^m - 1}, \quad n = 1, 2, ..., m, \quad \theta > 0, \tag{20}$$

the pgf is

$$\phi_N(s) = \frac{(1+\theta s)^m - 1}{(1+\theta)^m - 1}, \tag{21}$$

From equation (6), the survival function can be expressed as:

$$S(z) = \frac{[1+\theta-\theta G(z)]^m - 1}{(1+\theta)^m - 1}$$

therefore, the pdf and hazard function are given by

$$f(z) = \frac{m\theta g(z)\,[1+\theta-\theta G(z)]^{m-1}}{(1+\theta)^m - 1}, \tag{22}$$

$$h(z) = \frac{m\theta g(z)\,[1+\theta-\theta G(z)]^{m-1}}{[1+\theta-\theta G(z)]^m - 1}. \tag{23}$$

By substituting equations (9) and (10) in the above equations (21), (22) and (23) we obtain Lomax-Binomial distribution for minimum order statistic as

$$S(z) = \frac{\left[1+\theta\,(1+\beta z)^{-\alpha}\right]^m - 1}{(1+\theta)^m - 1} \tag{24}$$

$$f(z) = \frac{m\theta\alpha\beta\,(1+\beta z)^{-(\alpha+1)}}{(1+\theta)^m - 1}\left[1+\theta\,(1+\beta z)^{-\alpha}\right]^{m-1} \tag{25}$$

$$h(z) = \frac{m\theta\alpha\beta\,(1+\beta z)^{-(\alpha+1)}}{\left[1+\theta\,(1+\beta z)^{-\alpha}\right]\left\{1-\left[1+\theta\,(1+\beta z)^{-\alpha}\right]^{-m}\right\}} \tag{26}$$

2.3. Geometric Generators Based on Minimum Order Statistic

The shifted geometric distribution with parameter $1-\theta$, where $0<\theta<1$, the pmf is

$$p_n = \theta^{n-1}(1-\theta), \quad n=1,2,... \quad 0<\theta<1, \tag{27}$$

the pgf is

$$\phi_N(s) = \frac{(1-\theta)s}{1-\theta s}, \tag{28}$$

From equation (6), the survival function can be expressed as:

$$S(z) = \frac{(1-\theta)[1-G(z)]}{[1-\theta+\theta G(z)]} \tag{29}$$

Consequently, the pdf and hazard function are given by

$$f(z) = \frac{(1-\theta)g(z)}{[1-\theta+\theta G(z)]^2}, \tag{30}$$

$$h(z) = \frac{g(z)}{[1-\theta+\theta G(z)][1-G(z)]}. \tag{31}$$

Similarly, substituting equations (9) and (10) in the above equations (29), (30) and (31) we obtain Lomax-Binomial distribution for minimum order statistic as

$$S(z) = \frac{(1-\theta)(1+\beta z)^{-\alpha}}{1-\theta[1+\beta z]^{-\alpha}} \tag{32}$$

$$f(z) = \frac{\alpha\beta(1-\theta)(1+\beta z)^{-(\alpha+1)}}{[1-\theta(1+\beta z)^{-\alpha}]^2} \tag{33}$$

$$h(z) = \frac{\alpha\beta}{(1+\beta z)[1-\theta(1+\beta z)^{-\alpha}]} \tag{34}$$

2.4. Zero-truncated Negative Binomial Generators

The zero-truncated negative binomial distribution with parameters $\alpha>0$ and $1-\theta$, where $0<\theta<1$, the pmf is

$$p_n = \frac{\binom{\alpha+n-1}{n}\theta^n}{(1-\theta)^{-\alpha}-1}, \quad \text{for} \quad n=1,2,..., \quad 0<\theta<1, \alpha>0, \tag{35}$$

the pgf is

$$\phi_N(s) = \frac{(1-\theta s)^{-\alpha}-1}{(1-\theta)^{-\alpha}-1} \tag{36}$$

From equation (6), the survival function can be expressed as:

$$S(z) = \frac{[1-\theta+\theta G(z)]^{-\alpha}-1}{(1-\theta)^{-\alpha}-1} \tag{37}$$

Therefore, the pdf and hazard function are given by

$$f(z) = \frac{\alpha\theta g(z)[1-\theta+\theta G(z)]^{-\alpha-1}}{(1-\theta)^{-\alpha}-1}, \tag{38}$$

$$h(z) = \frac{\alpha\theta g(z)}{[1-\theta+\theta G(z)]\{1-[1-\theta+\theta G(z)]^{\alpha}\}}. \tag{39}$$

Therefore, substituting equations (9) and (10) in the above equations (37), (38) and (39) we obtain Lomax-Binomial distribution for minimum order statistic as

$$S(z) = \frac{\left[1-\theta(1+\beta z)^{-\alpha}\right]^{-a}-1}{(1-\theta)^{-a}-1} \tag{40}$$

$$f(z) = \frac{a\theta\alpha\beta(1+\beta z)^{-(\alpha+1)}}{(1-\theta)^{-a}-1}\left\{1-\theta(1+\beta z)^{-\alpha}\right\}^{-(a+1)} \tag{41}$$

$$h(z) = \frac{a\theta\alpha\beta(1+\beta z)^{-(\alpha+1)}}{[1-\theta(1+\beta z)^{-\alpha}]\left\{1-\left[1-\theta(1+\beta z)^{-\alpha}\right]^{a}\right\}} \tag{42}$$

2.5. Logarithmic Series Generators

For a logarithmic series distribution with parameter θ, $0<\theta<1$, the pmf is

$$p_n = \frac{\theta^n}{-n\log(1-\theta)}, \tag{43}$$

the pgf is

$$\phi_N(s) = \frac{\log(1-\theta s)}{\log(1-\theta)} \tag{44}$$

From equation (6), the survival function can be expressed as:

$$S(z) = \frac{\log(1-\theta+\theta G(z))}{\log(1-\theta)}$$

Consequently, the pdf and hazard function are given by

$$f(z) = \frac{\theta g(z)\left[1-\theta+\theta G(z)\right]^{-1}}{-\log(1-\theta)}, \tag{45}$$

$$h(z) = \frac{\left[1-\theta+\theta G(z)\right]^{-1}}{-\log(1-\theta+\theta G(z))} \tag{46}$$

Therefore, substituting equations (9) and (10) in the above equations (45), (46) and (47) we obtain Lomax-Binomial distribution for minimum order statistic as

$$S(z) = \frac{\log[1-\theta(1+\beta z)^{-\alpha}]}{\log(1-\theta)} \tag{47}$$

$$f(z) = \frac{\theta\alpha\beta(1+\beta z)^{-(\alpha+1)}\left[1-\theta(1+\beta z)^{-\alpha}\right]^{-1}}{-\log(1-\theta)} \tag{48}$$

$$h(z) = \frac{\theta\alpha\beta(1+\beta z)^{-(\alpha+1)}\left[1-\theta(1+\beta z)^{-\alpha}\right]^{-1}}{-\log\left[1-\theta(1+\beta z)^{-\alpha}\right]} \tag{49}$$

3. Generators for Discrete Mixtures Based on Maximum Order Statistic

Let

$$Y = \max(X_1, X_2, ..., X_N) \tag{50}$$

Then the cdf of Y given $N = n$ is

$$F(y|n) = [G(y)]^n \tag{51}$$

where $G(y)$ denotes the cumulative distribution function of the parent distribution.

Proposition 3.1. The cumulative distribution function of a discrete mixture derived from the maximum order statistic is the probability generating function of N evaluated at the cumulative distribution function point of the parent distribution.

Proof. The cdf of Y

$$\begin{aligned} F(y) &= \sum_{n=1}^{\infty} F(y|n)p_n \\ &= \sum_{n=1}^{\infty} [G(y)]^n p_n \\ &= E\,[G(y)]^N \end{aligned} \tag{52}$$

Considering equation (5),

$$F(y) = \phi_N\left[G(y)\right]. \tag{53}$$

Thus, the survival function is

$$S(z) = 1 - \phi_N\left[G(y)\right] \tag{54}$$

The following proposition presents the generators for discrete mixtures derived from the maximum order statistic for the six mixing distributions.

Proposition 3.2. For $y > 0$, the survival function $S(y)$, the probability density function $f(y)$, and the hazard function $h(y)$ for the discrete mixture of the maximum order statistic are as follows:

(i). The zero-truncated Poisson distribution with parameter θ, where $\theta > 0$:

$$S(y) = \frac{1-e^{-\theta[1-G(y)]}}{1-e^{-\theta}} \tag{55}$$

$$f(y) = \frac{\theta g(y) e^{-\theta[1-G(y)]}}{1-e^{-\theta}}, \tag{56}$$

$$h(y) = \frac{\theta g(y)}{e^{\theta[1-G(y)]}-1} \tag{57}$$

(ii). The zero-truncated binomial distribution with index $m = 1, 2, 3, \ldots$ and parameters $\dfrac{\theta}{1+\theta}$,, $\theta > 0$:

$$S(y) = \frac{(1+\theta)^m - [1+\theta G(y)]^m}{(1+\theta)^m - 1} \tag{58}$$

$$f(y) = \frac{m\theta g(y)\left[1+\theta G(y)\right]^{m-1}}{(1+\theta)^m - 1}, \tag{59}$$

$$h(y) = \frac{m\theta g(y)\left[1+\theta G(y)\right]^{m-1}}{(1+\theta)^m - [1+\theta G(y)]^m} \tag{60}$$

(iii). The shifted geometric distribution with parameter $1-\theta$, $0 < \theta < 1$:

$$S(y) = \frac{1-G(y)}{1-\theta G(y)} \tag{61}$$

$$f(y) = \frac{(1-\theta)\,g(y)}{[1-\theta G(y)]^2}, \tag{62}$$

$$h(y) = \frac{(1-\theta)\,g(y)}{[1-G(y)][1-\theta G(y)]} \tag{63}$$

(iv). The zero-truncated negative binomial distribution with parameters where $\alpha > 0$ and $1-\theta$, $0 < \theta < 1$:

$$S(y) = \frac{(1-\theta)^{-\alpha} - [1-\theta G(y)]^{-\alpha}}{(1-\theta)^{-\alpha} - 1} \tag{64}$$

$$f(y) = \frac{\alpha\theta g(y)\,[1-\theta G(y)]^{-\alpha-1}}{(1-\theta)^{-\alpha} - 1}, \tag{65}$$

$$h(y) = \frac{\alpha\theta g(y)\,[1-\theta G(y)]^{-\alpha-1}}{(1-\theta)^{-\alpha} - [1-\theta G(y)]^{-\alpha}} \tag{66}$$

(v). The negative binomial distribution with parameters $r = 1, 2, 3, 4, ...$ and $1-\theta$, $0<\theta<1$:

$$S(y) = 1 - \left[\frac{(1-\theta)\,G(y)}{1-\theta G(y)}\right]^r \tag{67}$$

$$f(y) = \frac{r\,(1-\theta)^r\, g(y)[G(y)]^{r-1}}{[1-\theta G(y)]^{r+1}}, \tag{68}$$

$$h(y) = \frac{r\,(1-\theta)^r\, g(y)[G(y)]^{r-1}}{[1-\theta G(y)]\,\{[1-\theta G(y)]^r - (1-\theta)^r[G(y)]^r\}} \tag{69}$$

(vi). For logarithmic series distribution with parameter $1-\theta$, $0<\theta<1$:

$$S(y) = \frac{\log(1-\theta) - \log[1-\theta G(y)]}{\log(1-\theta)}, \tag{70}$$

$$f(y) = \frac{\theta g(y)}{[-\log(1-\theta)][1-\theta G(y)]} \tag{71}$$

4. Discrete Mixtures Based on Maximum Order Statistic from Lomax Distribution

By utilizing the generators derived in Section 4 for the Lomax distribution, we arrive at the results presented in the following proposition.

Proposition 4.1. For $y > 0$, the survival function $S(y)$, the probability density function $f(y)$, and the hazard function $h(y)$ of the discrete mixture, derived from the maximum order statistic of the Lomax distribution with parameter $\lambda > 0$, are as follows;

(i). The zero-truncated Poisson distribution with parameter θ, where $\theta > 0$.

$$S(y) = \frac{1 - e^{-\theta(1+\beta y)^{-\alpha}}}{(1-e^{-\theta})} \tag{72}$$

$$f(y) = \frac{\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}\, e^{-\theta(1+\beta y)^{-\alpha}}}{(1-e^{-\theta})}, \tag{73}$$

$$h(y) = \frac{\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}}{e^{\theta(1+\beta y)^{-\alpha}} - 1} \tag{74}$$

as obtained by Al-Zahrani et al (2014).

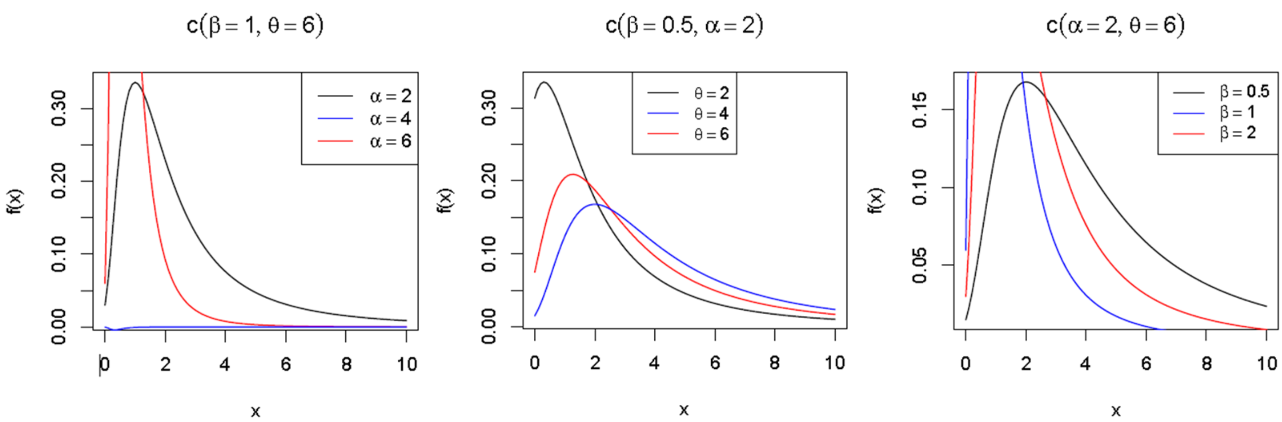

Figure 3. *Plot of probability density function for different values of the parameters α,β and θ.*

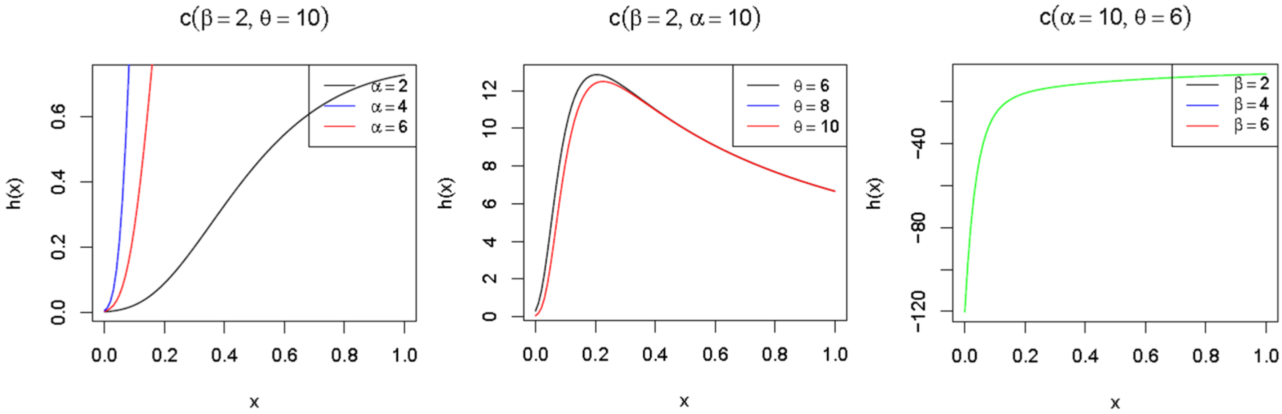

Figure 4. *Plot of hazard function for different values of the parameters α, β and θ.*

(ii). For zero-truncated binomial distribution with index $m = 1, 2, 3, \ldots$ and $\frac{\theta}{1+\theta}$, $\theta > 0$:

$$S(y) = \frac{(1+\theta)^m - \left[1 + \theta\left(1 - (1+\beta y)^{-\alpha}\right)\right]^m}{(1+\theta)^m - 1}, \tag{75}$$

$$f(y) = \frac{m\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}}{(1+\theta)^m - 1}\left[1 + \theta(1 - (1+\beta y)^{-\alpha})\right]^{m-1}, \tag{76}$$

$$h(y) = \frac{m\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}\left[1 + \theta\left(1 - (1+\beta y)^{-\alpha}\right)\right]^{m-1}}{(1+\theta)^m - \left[1 + \theta\left(1 - (1+\beta y)^{-\alpha}\right)\right]^m} \tag{77}$$

(iii). Shifted geometric distribution with parameter $1-\theta$, where $0 < \theta < 1$:

$$S(y) = \frac{(1+\beta y)^{-\alpha}}{1 - \theta\left[1 - (1+\beta y)^{-\alpha}\right]}, \tag{78}$$

$$f(y) = \frac{(1-\theta)\,\alpha\beta\,(1+\beta y)^{-(\alpha+1)}}{\left[1 - \theta\left(1 - (1+\beta y)^{-\alpha}\right)\right]^2}, \tag{79}$$

$$h(y) = \frac{\alpha\beta(1-\theta)}{(1+\beta y)\left[1 - \theta\left(1 - (1+\beta y)^{-\alpha}\right)\right]} \tag{80}$$

(iv). Zero-truncated negative binomial distribution with parameters $\alpha > 0$ and $1-\theta$, where $0 < \theta < 1$:

$$S(y) = \frac{(1-\theta)^{-a} - \left[1 - \theta\left(1 - (1+\beta y)^{-\alpha}\right)\right]^{-a}}{(1-\theta)^{-a} - 1}, \tag{81}$$

$$f(y) = \frac{a\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}}{(1-\theta)^{-a} - 1}\left[1 - \theta\left(1 - (1+\beta y)^{-\alpha}\right)\right]^{-(a+1)}, \tag{82}$$

$$h(y) = \frac{a\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}\,(1-\theta)^{a}}{\left[1-\theta\left(1-(1+\beta y)^{-\alpha}\right)\right]\left\{\left[1-\theta\left(1-(1+\beta y)^{-\alpha}\right)\right]^{a}-(1-\theta)^{a}\right\}}. \tag{83}$$

(v). logarithmic series distribution with parameter $1-\theta$, where $0<\theta<1$:

$$S(y) = \frac{\log(1-\theta)-\log[1-\theta\left(1-(1+\beta y)^{-\alpha}\right)]}{\log(1-\theta)}, \tag{84}$$

$$f(y) = \frac{\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}\left[1-\theta\left(1-(1+\beta y)^{-\alpha}\right)\right]^{-1}}{-\log(1-\theta)}, \tag{85}$$

$$h(y) = \frac{\theta\alpha\beta\,(1+\beta y)^{-(\alpha+1)}}{\log[1-\theta\left(1-(1+\beta y)^{-\alpha}\right)]-\log(1-\theta)} \tag{86}$$

5. Conclusion

We have developed generators for discrete mixtures derived from order statistics of continuous distributions using the probability generating function (pgf) technique. Although previous research has explored discrete mixtures of order statistics from distributions like Lomax, these studies did not employ the generator and pgf approach.

In this paper, we focus on five discrete mixing distributions. However, other distributions such as discrete phase-type distributions, Poisson, binomial, and negative binomial mixtures could also be applied.

The generator method for creating probability distributions became widely recognized following the introduction of the beta generator by Eugene et al. and Jones, with the exponentiated generator being a specific example. In this paper, we present new generators designed to construct distributions, specifically discrete mixtures based on minimum and maximum order statistics.

Conflicts of Interest

The authors declare no conflicts of interest.

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