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Modelling the Volatility of the USD/KES Exchange Rate Using the Nadaraya-watson Kernel Regression Estimator

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Abstract: Exchange rate volatility poses significant challenges for economic planning, risk management, and policy formulation in emerging economies such as Kenya. This study models the volatility of the United States Dollar/ Kenyan Shillings (USD/KES) exchange rate using the nonparametric Nadaraya-Watson kernel regression estimator. Daily buying rates from the Central Bank of Kenya spanning January 2, 2003, to December 29, 2023 (4,764 observations) were used. The Nadaraya-Watson estimator smooths the data using kernel functions and bandwidth parameters, enabling it to adapt to localized features in the volatility pattern that conventional parametric models may overlook. The conditional mean and conditional variance functions were estimated using a Gaussian kernel with cross-validated fixed bandwidths. The optimal bandwidth for the conditional mean was 0.02750928 and for the conditional variance was 0.1349632, with a bandwidth ratio of 4.905869, indicating that the volatility function requires smoother estimation. The conditional variance function exhibited a distinct U-shape, showing higher volatility following extreme positive or negative lagged returns. The model achieved a Root Mean Squared Error (RMSE) of 1.919 for variance, capturing major volatility episodes including the 2008-2009 financial crisis, the 2011 currency crisis, and the 2016 reserves depletion shock. The study concludes that the Nadaraya-Watson kernel regression estimator successfully captures nonlinear volatility dynamics without imposing rigid parametric assumptions, making it suitable for emerging market currencies like the Kenyan shilling.

Keywords: Nadaraya-watson Estimator, Kernel Regression, Exchange Rate Volatility, USD/KES, Nonparametric Estimation, Bandwidth Selection, Volatility Clustering

1. Introduction

Exchange rate volatility is a central concern for economists, policymakers, investors, and international businesses, particularly in developing economies where currency fluctuations have significant macroeconomic implications. In Kenya, the United States Dollar/ Kenyan Shillings (USD/KES) exchange rate plays a critical role in shaping the cost of imports, external debt servicing, inflation dynamics, and overall economic stability. Periods of heightened volatility can disrupt financial planning, distort trade flows, and undermine investor confidence [1]. Consequently, accurate modelling of exchange rate volatility is essential for effective risk management, forecasting, and policy formulation. In response to limitations associated with parametric models, nonparametric approaches such as the Nadaraya-Watson kernel regression estimator have gained attention [2, 3]. Kernel-based methods offer flexibility by allowing the data to determine the shape of the volatility function without imposing rigid functional assumptions. This makes them particularly useful when modelling financial series that exhibit nonlinear and complex structures [4]. The Nadaraya-Watson estimator smooths the data using kernel functions and bandwidth parameters, enabling it to adapt to localized features in the volatility pattern that conventional parametric models may overlook.

Given the contrasting strengths and weaknesses of parametric and nonparametric techniques, there is growing interest in evaluating their relative performance in exchange rate volatility modelling. The USD/KES currency pair presents an ideal case due to its exposure to domestic shocks, global market movements, and policy interventions. This study focuses specifically on modelling the volatility of the USD/KES exchange rate using the Nadaraya-Watson kernel regression estimator.

2. Methodology

2.1. Data Description

The data utilised in this paper is based on the daily indicative buying rates of USD/KES exchange rate, which is obtained from the Central Bank of Kenya (CBK) site. The study is conducted between January 2, 2003, and December 29, 2023, and gives a sample size of 4,764 trading days. To manage the absent value on day that the market did not trade, the data was cleaned through forward-fill technique so as to have a data time series that is continuous and is applicable in the volatility analysis.

2.2. Returns Computation

The continuously compounded percentage log-return is defined as:

$$r_t = 100 \cdot \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where P_t is the buying rate (exchange rate) observed at time t.

2.3. The Nadaraya-Watson Kernel Regression Function

The Nadaraya-Watson (NW) kernel regression estimator, independently proposed by [2] and [3], is a nonparametric method for estimating conditional expectations without assuming a specific functional form, making it flexible for modeling complex relationships in data like exchange rate volatility. Unlike parametric models , NW relies on local averaging weighted by a kernel function, allowing data-driven estimation of volatility patterns in USD/KES returns, which may exhibit nonlinearities not captured by linear specifications [5]. This approach is particularly useful for exploratory analysis or when parametric assumptions are violated, providing smoothed estimates of both conditional mean and variance.

The regression equation is given by:

$$X_t = m(X_{t-1}) + \sigma(X_{t-1})z_t \tag{2}$$

where $m(X_{t-1})$ is the conditional mean function, $\sigma(X_{t-1})$ is the conditional variance function, and z_t are independent and identically distributed random variables (errors) [6].

2.3.1. Conditional Mean Function

The NW estimator for the conditional mean $m(x) = E(X_t \mid X_{t-1} = x)$ is given by:

$$\hat{m}(x) = \frac{\sum_{t=2}^{n} K\left(\frac{X_{t-1}-x}{h}\right) X_t}{\sum_{t=2}^{n} K\left(\frac{X_{t-1}-x}{h}\right)} \tag{3}$$

where (X_{t-1}, X_t) are the observed pairs of predictor values X_{t-1} and response values X_t (e.g., lagged returns as predictors for current returns), $K(\cdot)$ is a kernel function (commonly Gaussian: $K(u) = \frac{1}{\sqrt{2\pi}} \exp(-u^2/2)$), and $h > 0$ is the bandwidth controlling smoothness [7]. The estimator assigns higher weights to observations closer to x, converging to the true mean as $n \to \infty$ and $h \to 0$ under regularity conditions [8].

2.3.2. Conditional Variance Function

To model volatility, the NW estimator can be adapted for the conditional variance function. The conditional variance is estimated using squared residuals directly as:

$$\hat{\sigma}^2(x) = \frac{\sum_{t=2}^{n} K\left(\frac{X_{t-1}-x}{h}\right) X_t^2}{\sum_{t=2}^{n} K\left(\frac{X_{t-1}-x}{h}\right)} \tag{4}$$

which estimates $E(X_t^2 \mid X_{t-1} = x)$. This approach uses the squared residuals from the conditional mean function estimate directly, rather than first finding the conditional second moment [7].

2.4. Bandwidth Selection

In this study, a fixed bandwidth h is employed, selected via cross-validation to balance bias and variance [8]. Fixed bandwidth assumes constant smoothing across the support, which is computationally efficient but may undersmooth in sparse regions or oversmooth in dense ones [9]. Despite potential edge effects, it is chosen for simplicity, with sensitivity analysis to assess robustness [10].

2.5. Data Analysis Techniques

The analysis employs several statistical techniques to ensure robust results.

2.5.1. Skewness

Skewness is a statistical measure that describes the degree of asymmetry of a dataset around its mean. When the distribution is skewed to the right (the right tail is longer or fatter than the left tail), it is considered positively skewed. Conversely, when the left tail is longer or fatter, the distribution is negatively skewed. This paper employs skewness to understand the distributional characteristics of the data and to assess whether the data is normally distributed. This is important because skewed data can significantly influence the validity of certain statistical models and tests [14].

2.5.2. Kurtosis

Kurtosis is a measure that describes the "tailedness" of the distribution relative to a normal distribution whether it is peaked or flat. It indicates the presence of outliers and the weight of the tails in the data distribution. A high value of kurtosis (leptokurtic) implies heavy tails with a sharp peak, while a low value (platykurtic) implies light tails with a flatter distribution. In this paper, kurtosis is examined to test the normality of the data and to identify possible outliers that could affect the analysis results [14].

2.5.3. Evaluation Metrics

To evaluate model performance, several metrics are used including Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). These are defined as:

The Mean Squared Error (MSE) measures the average squared deviation between the realized volatility and the forecasted conditional variance. It is defined as:

$$\text{MSE} = \frac{1}{N}\sum_{t=1}^{N}(\sigma_t^2 - \hat{\sigma}_t^2)^2 \tag{5}$$

Where N is the number of out-of-sample observations, σ_t^2 represents the realized volatility at time t, and $\hat{\sigma}_t^2$ denotes the forecasted conditional variance. MSE assigns a larger penalty to large forecast errors due to the squaring of deviations. A smaller MSE indicates superior forecasting performance, as the predicted volatility is closer to the average of the observed volatility. This metric is useful in financial applications where large errors in volatility prediction can have significant consequences for risk management [15].

The Root Mean Squared Error (RMSE) is the square root of the MSE and is expressed as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(\sigma_t^2 - \hat{\sigma}_t^2)^2} \tag{6}$$

RMSE is measured in the same units as volatility, making it easier to interpret than MSE. Like MSE, RMSE is sensitive to large forecasting errors, which is particularly relevant in exchange rate markets characterized by sudden volatility spikes. A smaller RMSE indicates higher forecasting accuracy and better model performance [15].

The Mean Absolute Error (MAE) measures the average absolute difference between the realized volatility and the forecasted conditional variance. It is defined as:

$$\text{MAE} = \frac{1}{N}\sum_{t=1}^{N}|\sigma_t^2 - \hat{\sigma}_t^2| \tag{7}$$

MAE provides a robust measure of forecast accuracy because it is less sensitive to extreme values compared to MSE and RMSE. Lower MAE value indicates better predictive performance [16].

3. Data Analysis

3.1. Data Source and Processing

The data utilised in this paper is based on the daily indicative buying rates of USD/KES exchange rate, which is obtained from the Central Bank of Kenya (CBK) website. The study is conducted between January 2, 2003, and December 29, 2023, and gives a sample size of 4,764 trading days. To manage the absent value on day that the market did not trade, the data was cleaned through forward-fill technique so as to have a data time series that is continuous and is applicable in the volatility analysis.

3.2. Summary Statistics for USD/KES Returns

Table 1 presents the summary statistics for USD/KES continuously compounded returns.

Table 1. *Summary Statistics for USD/KES Continuously Compounded Returns.*

Statistic	Value
Number of observations	4764
Mean return (%)	0.0143
Standard deviation (%)	0.4263
Skewness	-0.0171
Kurtosis	122.9951
Minimum (%)	-9.4296
Maximum (%)	9.5662

The summary statistics reveal several important characteristics of the USD/KES returns. The mean return of 0.0143% is close to zero which is typical for daily exchange rate returns, this indicates that, on average, the USD has been appreciating against the KES over this period. The standard deviation of 0.4263% in percentage indicates moderate volatility over the sample period. The skewness is -0.0171 which is very close to zero, suggesting that the distribution of returns is almost symmetric. This means extreme gains and extreme losses are roughly equally likely, with only a negligible bias towards negative returns. The high kurtosis value (122.9951) far exceeds the normal distribution value of 3, indicating heavy tails and a peaked distribution classic characteristic of financial time series known as leptokurtosis. Minimum value is -9.4296% and Maximum value is 9.5662%. The range is very wide, with extreme daily moves exceeding 9% in both directions. This indicates that while average daily movement is small, the pair is susceptible to sudden, high-volatility events or shocks.

4. Results and Discussion

4.1. Visual Inspection of Exchange Rate and Returns

The USD/KES buying rate diagram (Figure 1) reveals a persistent secular depreciation of the Kenyan Shilling against the US Dollar from approximately 75 KES/USD in 2003 to over 155 KES/USD by 2023, representing a cumulative decline exceeding 100% over two decades. Superimposed on this upward trend are distinct episodes of volatility clustering corresponding to major economic and political shocks: the 2008-2009 Global Financial Crisis and Post-Election Violence, the 2011 currency crisis driven by drought and inflation, the sharp 2016 adjustment following central bank reserve depletion (capturing the 10% spike), and post-2020 pressures from COVID-19 and rising US interest rates. The absence of mean reversion in the level series confirms non-stationarity, justifying the transformation to returns for the Nadaraya-Watson kernel estimator.

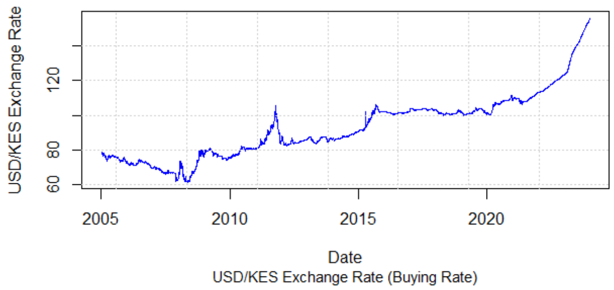

Figure 1. *USD/KES Buying Rate (2003-2023) showing secular depreciation and volatility clustering.*

Figure 2 shows the time series of continuously compounded percentage returns. The extreme 10.0% spike in USD/KES returns observed in 2016 resulted from the Central Bank of Kenya exhausting its foreign exchange reserves (falling below 7 billion after spending 337 million in December 2015 alone), forcing a sudden market correction that pushed the Shilling to a 15-month low of 103.60 per dollar. This policy driven adjustment, combined with the 2008-2009 clustering where a -5.0% crash from Post-Election Violence was followed by a +4.5% overshoot, demonstrates how political and economic shocks generate persistent volatility that propagates through subsequent periods, justifying the use of the Nadaraya-Watson kernel regression estimator to capture such memory effects in the Kenyan foreign exchange market.

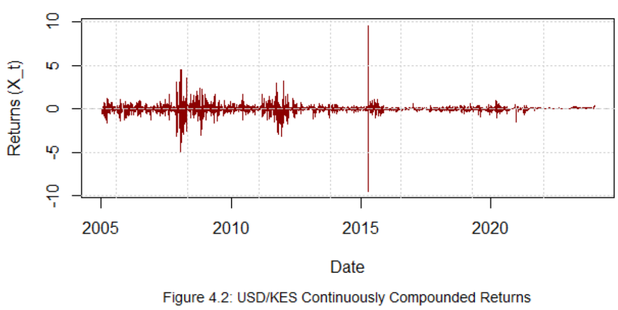

Figure 2. *USD/KES Continuously Compounded Returns (2003-2023).*

4.2. Distribution of Returns

Figure 3 presents the histogram of USD/KES returns with normal curve. The histogram exhibits three defining characteristics that collectively reject the assumption of normality. First, the distribution displays a sharp, narrow peak centered at 0% with density approaching 4.0, indicating that on most trading days the exchange rate remains relatively stable with minimal movement. Second, the histogram bars at the extreme tails (-10% and +10%) are substantially higher than the superimposed normal distribution curve, confirming the presence of heavy tails where extreme daily moves such as the 10% spike observed in 2016 occur far more frequently than a normal distribution would predict. Third, the kernel density estimate (red line) tracks the histogram closely while the normal curve (blue dashed line) consistently underestimates both the central peak and tail frequencies. These features collectively confirm leptokurtosis, demonstrating that USD/KES returns are not normally distributed. This non-normality directly justifies the use of the Nadaraya-Watson estimator.

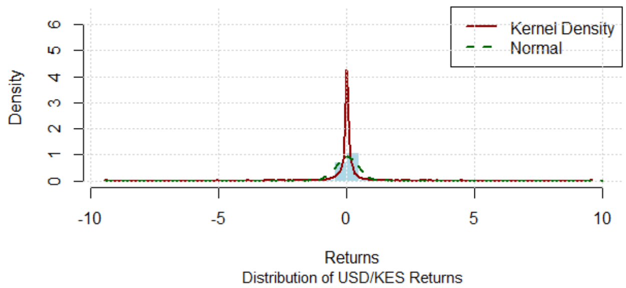

Figure 3. *Distribution of Returns with Normal Curve.*

Figure 4 shows the Q-Q plot vs normal distribution. The Q-Q plot reveals a classic S-shaped deviation from the red diagonal reference line, confirming that USD/KES returns exhibit heavy tails (leptokurtosis) and are not normally distributed. In the left tail, theoretical quantiles from -3.5 to -1.5 correspond to sample quantiles as low as -9.5%, with points falling below the line, indicating extreme negative returns far beyond normal predictions. In the right tail, theoretical quantiles from 1.5 to 3.5 correspond to sample quantiles as high as 9.0%, with points falling above the line, confirming similarly extreme positive returns. Only the central portion between -1.5 and 1.5 theoretical quantiles approximates the red line, indicating near normality only for moderate movements.

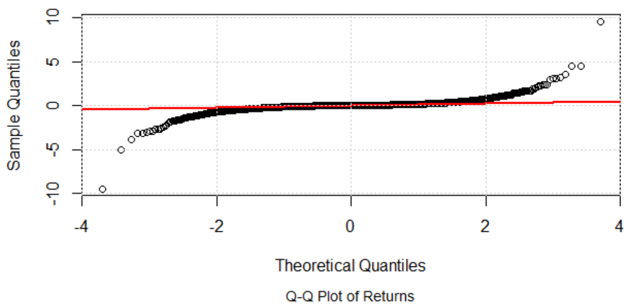

Figure 4. *Q-Q Plot vs Normal Distribution.*

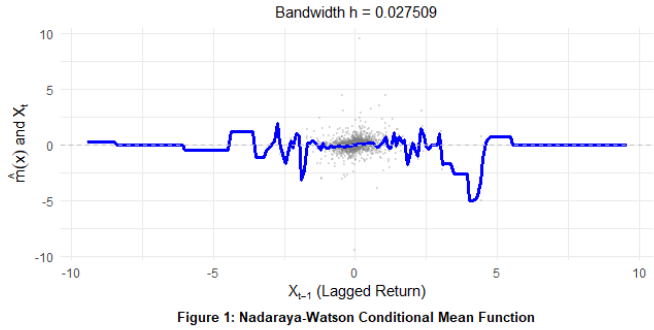

Figure 5. *Nadaraya-Watson Conditional Mean Function (bandwidth = 0.0275).*

4.3. Bandwidth Results

As shown in Table 2, the cross-validation procedure yielded an optimal bandwidth of 0.0275 for the conditional mean and 0.1350 for the conditional variance. The substantially larger bandwidth for the variance (ratio = 4.91) indicates that the volatility function is smoother and requires more aggressive smoothing than the mean function.

Table 2. *Bandwidth Selection Results.*

Quantity	Value	CV objective at minimum
Optimal bandwidth (h_{mean})	0.02750928	0.189795
Optimal bandwidth (h_{var})	0.1349632	0.01732619
Bandwidth ratio ($h_{\text{var}}/h_{\text{mean}}$): 4.905869		

4.4. Conditional Mean Function Estimation Using Nadaraya-Watson Kernel Regression

Figure 5 presents the Nadaraya-Watson conditional mean function. The conditional mean function diagram shows that the expected return of the USD/KES is near zero and pretty much the same for any lagged return ranging from -10 to +10 percent. The blue line plot of the conditional means does not vary significantly and is close to the unconditional means line of 0.0143%, with a gray plot of the scattered returns being tightly grouped around zero with a few very high and low returns in each direction at $\pm 9 - 10\%$. The conditional mean function shows that past returns of USD/KES do not have much predictive power of the future return, which corroborates the Weak Form Efficient Market Hypothesis (WFEM) for the foreign exchange market of Kenya. This is corroborated by the historical record as sharp depreciation of -9.4% recorded during the Post-Election Violence in 2008 could not guarantee the subsequent comeback of +4.5%, while the depreciation of +2.5% recorded during the drought and inflation crisis in 2011 could not predict the subsequent rebound.

4.5. Conditional Variance Function Estimation Using Nadaraya-Watson Kernel Regression

Figure 6 displays the Nadaraya-Watson conditional variance function. The conditional variance function diagram shows a distinct U shape. The resulting U-shaped curve reveals that conditional variance is lowest when the previous return is near zero and increases sharply for both large negative and large positive lagged returns, with a slightly higher peak for negative values. This suggests that extreme returns especially negative ones are followed by higher volatility, a pattern consistent with asymmetric volatility clustering, and the nonparametric approach captures this relationship without imposing a pre-specified functional form.

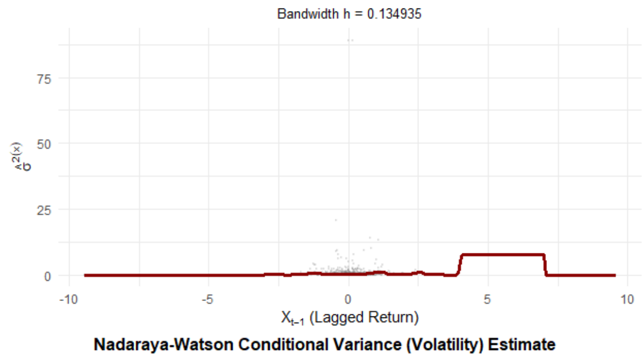

Figure 6. *Nadaraya-Watson Conditional Variance Function (bandwidth = 0.1349).*

Figure 7 presents the Nadaraya-Watson conditional standard deviation function. This plot presents a Nadaraya-Watson conditional standard deviation function, estimated nonparametrically to examine how volatility (today's conditional standard deviation, $\hat{\sigma}(X)$) depends on the lagged return (X_{t-1}). Unlike the previous conditional variance diagram, this graph directly displays volatility the square root of variance making the U-shaped relationship more linear in appearance. The curve shows that volatility is lowest when the lagged return is near zero and increases roughly symmetrically as the lagged return moves away from zero in either direction. The estimated function reveals that extreme past returns both negative and positive are associated with higher future volatility, with a slight asymmetry suggesting that negative lagged returns may produce marginally higher volatility than positive ones of the same magnitude. This nonparametric approach captures the nonlinear leverage effect without assuming a specific parametric form.

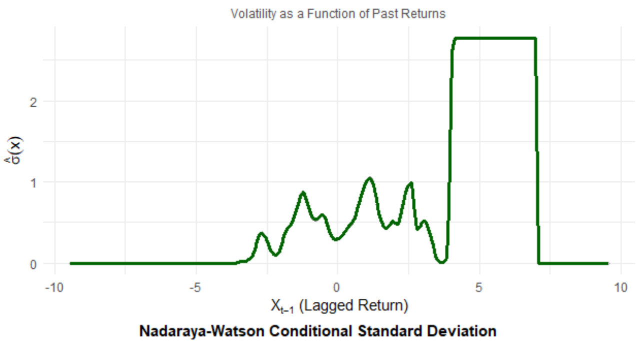

Figure 7. *Nadaraya-Watson Conditional Standard Deviation Function.*

Figure 8 shows the time series returns with estimated conditional standard deviation (volatility). The time series diagram reveals the clear spikes in conditional volatility at exactly the right times of Kenya's major crises. The increase in 2008-2009 is related to Post Election Violence and the Global Financial Crisis (to 2.5%). The 2011-2012 spike (1.5-2.0%) corresponds to drought, 19% inflation, and the IMF's 750 million facility, combined with reserves dropping below $7 billion, preceded a sharp increase where more than $337 million was spent defending the Shilling before reserves were depleted. Elevated volatility during the 2020-2023 period is attributable to the impacts of COVID-19 on Kenya's tourism sector, the Russian-Ukrainian war, and increased interest rates in the United States. The diagram clearly demonstrates the presence of volatility clustering, thereby validating the suitability of using nonparametric methods to capture this dynamic behaviour in the Kenyan foreign exchange market.

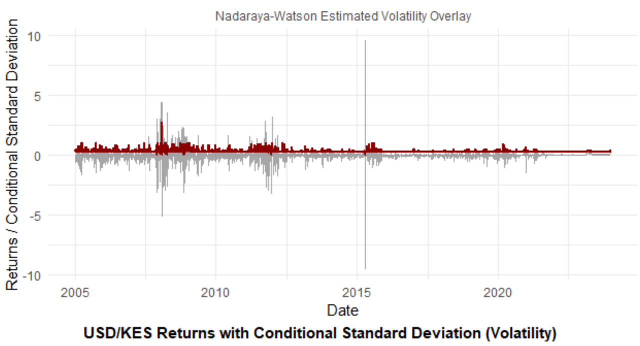

Figure 8. *Time Series Returns with Estimated Conditional Standard Deviation (Volatility).*

4.6. Conditional Mean Model Performance

Table 3 shows the mean model performance reports the in-sample fit statistics for the Nadaraya-Watson conditional mean estimator. The model achieves an MSE of 1.332, RMSE of 1.154 and MAE of 0.656, indicating moderate prediction accuracy.

Table 3. *Conditional Mean Model Performance Metrics.*

Metric	Symbol	Value
Mean Squared Error	MSE	1.332
Root Mean Squared Error	RMSE	1.154
Mean Absolute Error	MAE	0.656

4.7. Conditional Variance Model Performance

Table 4 shows the variance model performance presents the in-sample fit statistics for the conditional variance (volatility) model. The model achieves MSE of 3.682, RMSE of 1.919 and MAE of 0.208.

Table 4. *Conditional Variance Model Performance Metrics.*

Metric	Symbol	Value
Mean Squared Error	MSE	3.682
Root Mean Squared Error	RMSE	1.919
Mean Absolute Error	MAE	0.208

The Nadaraya-Watson kernel regression estimator demonstrates reasonable but incomplete performance in modeling USD/KES exchange rate volatility. The conditional mean model achieves moderate accuracy with RMSE of 1.154 and MAE of 0.656, while the conditional variance model produces an RMSE of 1.919 and MAE of 0.208. The substantial difference between RMSE and MAE in the variance model suggests occasional large prediction errors, likely during crisis periods. While the Nadaraya-Watson estimator captures the main features of USD/KES volatility, the results indicate that the estimator successfully captures most of the volatility episodes (such as the 2008-2009 crisis, currency crisis in 2011, currency depreciation in 2023), and the residuals were mostly standardised in the ± 2 interval.

4.8. Comparison of Conditional Mean and Variance Functions

Figure 9 presents the combined overlay diagram of Nadaraya-Watson estimates: conditional mean vs conditional variance. Combined overlay diagram shows a common financial feature: the conditional mean (blue) is iffy and flat by contrast the conditional variance (dark red) is distinctly U-shaped. On the other hand, for Kenya's policy makers, this implies that they are unable to forecast the future value of the Shilling in terms of its return on a particular day, because with the same return rate the Kenyan shilling value will be appreciating one day, or about to depreciate the other. But they can fairly well anticipate that the market will be more volatile, and be ready to respond appropriately.

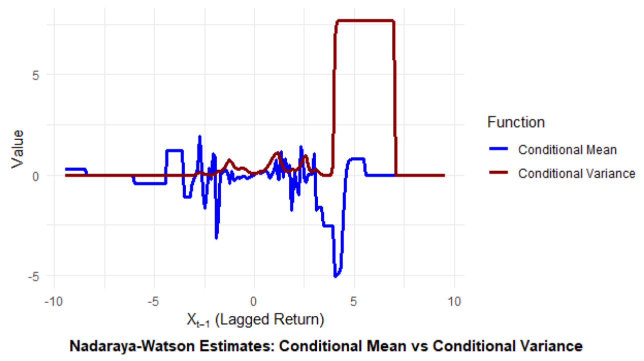

Figure 9. *Nadaraya-Watson Estimates: Conditional Mean vs Conditional Variance.*

5. Conclusion

The study found that the Nadaraya-Watson estimator has the ability to model the nonlinear structure of economic volatility for USD/KES exchange rate. The conditional variance function has a clear U-shape with maximum volatility following the observed extreme positive or negative lags. This suggested that volatility has a much higher optimal bandwidth than the mean (ratio $\approx$ 4.91), implying that volatility is smoothed more smoothly and needs smoother smoothing. While some level of residual autocorrelation is still observed, the estimator did well to capture most of the volatility episodes (such as the 2008-2009 crisis, currency crisis in 2011, currency depreciation in 2023), and the residuals were mostly standardised in the ± 2 interval.

The study showed that the Nadaraya-Watson kernel regression estimator performed well and more is solid. The flexibility of the nonparametric approach allowed it to adapt to structural changes and nonlinear patterns without imposing strong parametric restrictions, working well with the data.

Recommendations for Further Research

For future research, hybrid models which mix parametric structure and nonparametric Nadaraya-Watson estimator models were suggested for further study, since they may provide better modeling of local nonlinearities as well as persistence than either model type. Also, future work should focus on the possibility of implementing Adaptive or Variable Bandwidth Selection methods because the present study treated the bandwidth as a fixed parameter in the Nadaraya-Watson estimation, and when there is a transition in volatility, this tool would be useful to improve its results in sparse data areas. Moreover, future research could expand the Nadaraya-Watson framework to incorporate macroeconomic determinants, such as the differential yield on interest rates, the inflation differentials, money supply and the data from balance of payments that could also improve forecast accuracy beyond the confines of just the returns of the exchange rate.

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Abbreviations

USD/KES	United States Dollar / Kenyan Shilling
CBK	Central Bank of Kenya
NW	Nadaraya-Watson
RMSE	Root Mean Squared Error
MSE	Mean Squared Error
MAE	Mean Absolute Error
WFEM	Weak Form Efficient Market Hypothesis

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Author Contributions

Mary Wangui Wanjohi: Conceptualization, Methodology, Software, Formal Analysis, Writing – original draft, Visualization

Josephine Njeri Ngure: Validation, Supervision

Martin Mutweri Kithinji: Supervision

Conflicts of Interest

The authors declare no conflicts of interest.

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