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Research Article

The Accelerated Failure Time Regresssion Model Under the Generalised Gull Alpha Power Log Logistic Distribution for Handling Survival Data in Presence of Covariates

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Abstract

This study develops a Generalised Gull Alpha Power Log-Logistic Accelerated Failure Time (GGAPLL-AFT) regression model for analysing censored survival data in the presence of covariates. The model was developed by integrating the Generalised Gull Alpha Power Log-Logistic (GGAPLL) distribution into the Accelerated Failure Time framework, thereby combining the flexibility of the GGAPLL distribution with the interpretability of AFT regression. The proposed model is intended to provide a flexible approach for survival data characterised by different hazard rate structures, including increasing, decreasing, and unimodal hazards. The mathematical formulation of the proposed model was established by deriving its cumulative distribution function, probability density function, survival function, hazard function, and conditional survival function. The AFT formulation relates the logarithm of survival time to a linear function of covariates and a GGAPLL-distributed error term, allowing covariate effects to be interpreted in terms of acceleration or deceleration of survival time. The unknown model parameters were estimated using the Maximum Likelihood Estimation (MLE) method. Since the resulting likelihood equations are nonlinear and do not have closed-form solutions, numerical optimisation was performed using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. The performance of the proposed estimators was evaluated through a Monte Carlo simulation study under increasing, decreasing, and unimodal hazard scenarios. Simulations were conducted for different sample sizes and censoring levels, with estimator performance assessed using Absolute Bias (AB), Root Mean Square Error (RMSE), coverage probability, and Akaike Information Criterion (AIC). The simulation results showed that estimator performance improved as sample size increased, with reductions in bias and RMSE and coverage probabilities approaching the nominal 95% level. These findings demonstrate that the proposed GGAPLL-AFT model provides a flexible and reliable framework for parameter estimation and survival regression under diverse hazard structures.

Keywords

Survival Analysis, Accelerated Failure Time Model, Generalized Gull Alpha Power Log-Logistic Distribution, Maximum Likelihood Estimation, Hazard Function

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1. Introduction

Survival analysis is an important branch of statistics concerned with the analysis of time-to-event data, where the primary outcome is the length of time until the occurrence of a specified event. Such events may include death, disease recurrence, treatment failure, equipment failure, or system breakdown. A distinctive characteristic of survival data is the presence of censoring, which occurs when the event of interest is not observed for some subjects during the study period. Appropriate statistical methods are therefore required to incorporate both observed and censored survival times in the analysis. Survival analysis has consequently become an essential methodology in medical research, epidemiology, engineering, reliability analysis, and other fields involving time-to-event outcomes. Collett (2023) provides a comprehensive treatment of survival modelling and demonstrates the importance of selecting suitable statistical models for censored survival data [1].

The choice of an appropriate probability distribution is central to parametric survival analysis because the distribution determines the shape of the survival and hazard functions. Classical distributions such as the exponential, Weibull, log-normal, and log-logistic distributions have been extensively used because of their relatively simple mathematical forms and interpretability [1]. However, real-life survival data may exhibit complex hazard-rate patterns that cannot always be adequately represented by these conventional models. Recent research comparing proportional hazards and accelerated failure time approaches further demonstrates the importance of selecting a modelling framework that is compatible with the underlying characteristics of survival data, particularly when covariate effects may vary over time [2].

The Weibull distribution has been particularly influential in survival and reliability analysis because of its flexibility relative to the exponential model. Nevertheless, its hazard function is restricted to monotonic patterns, either increasing, decreasing, or constant. To overcome such limitations, researchers have proposed increasingly flexible lifetime distributions containing additional shape parameters. For example, Aly and Elgarhy developed the Gull Alpha Power Weibull distribution to provide greater flexibility for modelling lifetime data and reliability characteristics [3]. Such developments demonstrate the continuing need for generalized distributions capable of representing a wider range of distributional and hazard-rate behaviours.

Further extensions of the alpha-power and Gull families have resulted in distributions designed specifically to improve the flexibility of log-logistic-type models. The Gull Alpha Power Log-Logistic distribution represents one such development, combining the characteristics of the Gull and alpha-power transformations with the log-logistic baseline distribution [4]. The resulting additional parameters provide greater control over the shape of the probability density and hazard functions, making such models potentially useful for complex lifetime and survival data. Related work on the Gull Alpha Power Log-Logistic distribution and its regression modelling further illustrates the potential of flexible extensions of the log-logistic distribution for survival-data applications [5].

The development of flexible probability distributions has also extended beyond the alpha-power log-logistic family. For instance, generalized lifetime models such as the Beta Weibull-Lomax distribution have been proposed for modelling skewed and heavy-tailed lifetime observations [6]. Similarly, the Exponentiated Weibull-G family has been developed to provide additional flexibility in analysing censored lifetime data [7]. These developments demonstrate the importance of introducing additional shape parameters when standard lifetime distributions are unable to capture the characteristics of observed survival data adequately.

Within survival regression, the Accelerated Failure Time (AFT) model provides an alternative to proportional hazards modelling by directly relating covariates to survival time. In an AFT framework, covariates accelerate or decelerate the time to event, allowing regression coefficients to be interpreted in terms of changes in survival time. This interpretation can be particularly attractive when the proportional hazards assumption underlying the Cox model is inappropriate. Bayesian approaches have also been investigated for AFT models involving censored biomedical data, demonstrating that both frequentist and Bayesian procedures can be used for estimating survival-regression parameters [8].

The increasing development of flexible lifetime distributions has consequently encouraged the construction of distribution-specific AFT models. Mastor et al. (2023), for example, developed an Extended Exponential-Weibull AFT model and demonstrated its application to Sudan COVID-19 survival data [9]. Such models illustrate how extensions of conventional lifetime distributions can be incorporated within the AFT framework to provide greater flexibility in modelling censored survival observations. Similarly, Kariuki et al. (2023) proposed a flexible family of distributions based on the alpha-power family and demonstrated its applicability to survival data [10].

The log-logistic family has also continued to attract attention because of its usefulness in survival analysis and its compatibility with the AFT framework. Kariuki et al. (2024) investigated the properties, estimation, and applications of an Extended Log-Logistic distribution, demonstrating the continuing development of flexible log-logistic-based models for lifetime and survival data [11]. In addition, Kariuki et al. (2024) developed an AFT regression model under the Extended-Exponential distribution, further demonstrating how generalized lifetime distributions can be incorporated into regression frameworks for survival analysis [12].

More recently, research has explored hazard-based regression approaches involving flexible alpha-power log-logistic distributions. Kariuki and Barbiero (2026) developed a hazard-based regression model under the Exponentiated Alpha

Power Log-Logistic distribution for survival data, highlighting the growing interest in flexible log-logistic-based regression models [13]. These developments provide further motivation for investigating generalized alpha-power log-logistic distributions within regression settings.

The alpha-power methodology has also been combined with other generalized distributional constructions to improve modelling flexibility. ElSherpieny and Almetwally (2022), for example, investigated an Exponentiated Generalized Alpha Power Exponential distribution and demonstrated its statistical properties and applications [14]. Such extensions reinforce the usefulness of transformation-based approaches for developing distributions capable of accommodating a broader range of data characteristics.

Despite these developments, there remains a need for survival-regression models that combine substantial distributional flexibility with an interpretable regression structure. In particular, flexible generalized log-logistic distributions can provide useful alternatives when conventional models fail to adequately describe complex hazard-rate behaviour. The Generalised Gull Alpha Power Log-Logistic (GGAPLL) distribution is therefore considered in this study as a flexible baseline distribution for an Accelerated Failure Time regression framework. The proposed approach seeks to combine the flexibility introduced through generalized Gull and alpha-power transformations with the direct interpretation of covariate effects offered by the AFT model.

The use of appropriate estimation procedures is also essential when developing such flexible survival models. Maximum Likelihood Estimation (MLE), together with numerical optimization procedures such as the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm, provides a suitable frequentist framework for estimating model parameters. At the same time, Bayesian and frequentist approaches have both been applied to generalized survival models. For example, Muse et al. (2022) investigated Bayesian and frequentist approaches for a generalized log-logistic AFT model using larynx-cancer data [15]. Their work demonstrates the applicability of flexible generalized log-logistic AFT models to biomedical survival data and provides motivation for further development of related regression models.

This study develops a Generalised Gull Alpha Power Log-Logistic Accelerated Failure Time (GGAPLL-AFT) regression model for analysing censored survival data in the presence of covariates. The proposed model incorporates covariate effects through the AFT framework while retaining the flexibility of the GGAPLL baseline distribution. Parameter estimation is undertaken using Maximum Likelihood Estimation with numerical optimization, and the performance of the proposed model is investigated through Monte Carlo simulation under different hazard-rate scenarios. The model is subsequently applied to survival data to assess its empirical performance and demonstrate its usefulness relative to established survival models.

2. Model Formulation

This study formulates an Accelerated Failure Time (AFT) regression model based on the Generalised Gull Alpha Power Log-Logistic (GGAPLL) distribution to anal-yse censored survival data in the presence of covariates. The proposed model ex-tends the classical log-logistic AFT model by incorporating the Generalised Gull and Alpha Power transformations, resulting in a highly flexible survival model capable of accommodating increasing, decreasing and unimodal hazard functions. Let $T > 0$ denote the survival time. The baseline log-logistic cumulative dis-tribution function (CDF) is

$$G(t) = \frac{t^{\beta}}{\lambda^{\beta}+t^{\beta}}, t > 0 \tag{1}$$

where $\lambda > 0$ is the scale parameter and $\beta > 0$ is the shape parameter.

Applying the Alpha Power transformation gives

$$A(t) = \frac{\alpha^{G(t)}-1}{\alpha-1}, \alpha > 0, \alpha \neq 1 \tag{2}$$

where α controls the degree of transformation and provides additional flexibility in modelling skewness and tail behaviour.

The Generalised Gull transformation is subsequently applied to obtain the cumulative distribution function of the GGAPLL distribution,

$$F(t) = \frac{A(t)^{\theta}}{A(t)^{\theta}+[1-A(t)]^{\theta}}, \theta > 0 \tag{3}$$

where θ is an additional shape parameter controlling the behavior of the hazard function.

Differentiating Equation (3) with respect to t yields the probability density function

$$f(t) = \frac{\theta A(t)^{\theta-1}[1-A(t)]^{\theta-1}A'(t)}{\{A(t)^{\theta}+[1-A(t)]^{\theta}\}^{2}} \tag{4}$$

where $A'(t) = dA(t)$ denotes the derivative of the transformed function with respect to time. dt

The corresponding survival function is

$$S(t) = 1 - F(t) = \frac{[1-A(t)]^{\theta}}{A(t)^{\theta}+[1-A(t)]^{\theta}} \tag{5}$$

while the hazard function is

$$h(t) = \frac{f(t)}{S(t)} = \frac{\theta A(t)^{\theta-1}A'(t)}{[1-A(t)]\{A(t)^{\theta}+[1-A(t)]^{\theta}\}} \tag{6}$$

These expressions demonstrate the ability of the GGAPLL distribution to gen-erate a wide variety of hazard rate shapes, making it suitable for modelling het-erogeneous survival data.

To incorporate explanatory variables, the GGAPLL distribution is embedded within the Accelerated Failure Time framework. The regression model is specified as

$$\log(T_i) = \mathbf{x}^{\top}\boldsymbol{\gamma} + \varepsilon_i,\ i = 1,.,\ n, \quad (7)$$

i

where T_i denotes the survival time for the i-th individual, $\mathbf{x}_i$ is the vector of observed covariates, $\boldsymbol{\gamma}$ is the vector of regression coefficients, and the random error term ε_i follows the GGAPLL distribution.

Exponentiating Equation (7) gives

$$T_i = \exp(x_i^{\top}\gamma)\exp(\varepsilon_i) \quad (8)$$

indicating that covariates act multiplicatively on survival time. Consequently, positive regression coefficients correspond to longer survival times (time accelera-tion), whereas negative coefficients indicate1 shorter survival times (time decelera-tion).

Hence, the final conditional survival function of the GGAPLL-AFT model is:

$$S(t_i|X_i) = \frac{\left[1-A\left(t_i e^{-X_i^{\top}\gamma}\right)\right]^{\theta}}{\left[A\left(t_i e^{-X_i^{\top}\gamma}\right)\right]^{\theta}+\left[1-A\left(t_i e^{-X_i^{\top}\gamma}\right)\right]^{\theta}} \quad (9)$$

where:

$$\alpha > 0, \alpha \neq 1$$

$$\beta > 0, \lambda > 0, \theta > 0$$

and regression component:

$$i\log(T_i) = X_i^{\top}\gamma + \varepsilon_i$$

The proposed GGAPLL-AFT model preserves the inter-pretability of the AFT framework while substantially increas-ing modelling flexibility through the addi-tional shape param-eters of the GGAPLL distribution. This enables the model to accommodate complex hazard rate behaviours that are diffi-cult to capture using conventional parametric survival models, thereby providing a more robust frame-work for analysing censored survival data in the presence of covariates.

The GGAPLL-AFT model possesses well-defined measures of location and disper-sion that are useful for sum-marising the survival-time distribution. These measures de-pend on both the GGAPLL baseline parameters (λ, α, β, θ) and the regression parameters γ.

2.1. Mean

For the baseline GGAPLL distribution, the mean survival time is defined by

$$\mu_0 = E(T_0) = \int_0^{\infty} t f(t)dt \quad (10)$$

where $f(t)$ is the GGAPLL probability density function. Sub-stituting Equation (4) yields

$$E(T_0) = \int_0^{\infty} t \cdot \frac{\theta A(t)^{\theta-1}[1-A(t)]^{\theta-1}A'(t)}{\left[A(t)^{\theta}+(1-A(t))^{\theta}\right]^2}dt \quad (11)$$

Under the Accelerated Failure Time model,

$$T = \exp(x^{\top}\gamma)\,T_0 \quad (12)$$

and therefore

$$E(T|x) = \exp(x^{\top}\gamma)\,E(T_0) \quad (13)$$

Hence, the expected survival time is accelerated or deceler-ated according to the covariate values.

2.2. Median

The baseline median survival time satisfies

$$F(t_{0.5}) = \frac{1}{2} \quad (14)$$

Equivalent to

$$S(t_{0.5}) = \frac{1}{2} \quad (15)$$

Thus

$$\frac{A(t_{0.5})^{\theta}}{A(t_{0.5})^{\theta}+[1-A(t_{0.5})]^{\theta}} = \frac{1}{2} \quad (16)$$

Since Equation (16) has no closed-form solution, the me-dian is obtained nu-merically using Newton-Raphson or bi-section methods.

For an individual with covariate vector x, the AFT median becomes

$$0.5\,0.5\,t_{0.5}(\mathrm{x}) = \exp(\mathrm{x}^{\top}\gamma)\,t^{(0)}, \quad (17)$$

where $t^{(0)}$ denotes the baseline GGAPLL median.

2.3. Variance

The baseline variance is

$$Var(T_0) = E(T_0^2) - [E(T_0)]^2 \quad (18)$$

Where

$$E(T_0^2) = \int_0^{\infty} t^2 f(t)dt \quad (19)$$

Substituting the GGAPLL density gives

$$E(T_0^2) = \int_0^{\infty} t^2 \cdot \frac{\theta A(t)^{\theta-1}[1-A(t)]^{\theta-1}A'(t)}{\left[A(t)^{\theta}+(1-A(t))^{\theta}\right]^2}dt \quad (20)$$

Hence,

$$\mathrm{Var}(T_0) = \int_0^\infty t^2 f(t)dt - \int_0^\infty tf(t)dt^2 \quad (21)$$

Under the AFT model,

$$\mathrm{Var}(T \mid x) = \exp 2x^{\mathrm{T}}\gamma\ \mathrm{Var}(T_0). \quad (22)$$

Therefore, both the mean and variance are scaled by the acceleration factor induced by the covariates, while the GGAPLL baseline distribution determines the overall shape of the survival distribution.

The mean, median, and variance collectively describe the location and vari-ability of the proposed GGAPLL-AFT model. The additional shape parameters α and θ, together with the AFT regression structure, enable the model to repre-sent a broad class of survival-time distributions exhibiting increasing, decreasing, unimodal, and bathtub-shaped hazard functions.

3. Parameter Estimation

In this section, the parameters of the proposed Generalised Gull Alpha Power Log-Logistic Accelerated Failure Time (GGAPLL-AFT) model are estimated using the Maximum Likelihood Estimation (MLE) method under a right-censoring scheme. The analysis assumes non-informative censoring.

Let T_i, $i = 1, 2,., n$, denote the true survival time for the ith individual. Un-der right censoring, let $C_i > 0$ represent the censoring time for the ith individual. The observed survival time is defined as

$$t_i = \min(T_i, C_i), \quad (23)$$

and the censoring indicator is given by

$$\delta_i = \begin{cases} 1, if \quad the \quad event \quad is \quad observed \\ 0, if \quad the \quad observation \quad is \quad censored \end{cases} \quad (24)$$

Let (t_i, δ_i, X_i) denote the observed right-censored data for $i = 1, 2, \ldots, n$, where

$$X_i = (X_{i1}, X_{i2}, \ldots, X_{ip})^{\mathrm{T}}$$

is a $p \times 1$ vector of covariates associated with the ith individual.

Under the assumption of independent and non-informative censoring, an indi-vidual contributes the conditional density function $f(t_i|X_i)$ to the likelihood when the event is observed ($\delta_i = 1$), and contributes the conditional survival function $S(t_i|X_i)$ when the observation is censored ($\delta_i = 0$). Therefore, the likelihood function based on n independent observations is expressed as

$$L(\Theta) = \prod_{i=1}^{n} \quad [f(t_i|X_i)]^{\delta_i}[S(t_i|X_i)]^{1-\delta_i} \quad (25)$$

where the vector of unknown parameters is

$$\Theta = (\alpha, \beta, \lambda, \theta, \boldsymbol{\gamma}^{\mathrm{T}})^{\mathrm{T}}. \quad (26)$$

Here, $\alpha > 0$ is the Alpha Power parameter, $\beta > 0$ is the log-logistic shape parameter, $\lambda > 0$ is the scale parameter, $\theta > 0$ is the Generalised Gull shape parameter, and

$$\boldsymbol{\gamma} = (\gamma_1, \gamma_2,., \gamma_p)^{\mathrm{T}}$$

is the vector of regression coefficients.

Taking the logarithm of Equation (25), the log-likelihood function is obtained

As

$$\ell(\Theta) = \sum_{i=1}^{n} \quad [\delta_i \log f(t_i|X_i) + (1-\delta_i)\log S(t_i|X_i)] \quad (27)$$

Under the Accelerated Failure Time regression framework, the logarithm of the survival time is expressed as

$$\log(T_i) = X^{\mathrm{T}}\gamma + \varepsilon_i,\ i = 1, 2,., n, \quad (28)$$

where the random error term ε_i follows the GGAPLL distribution.

Consequently, the covariate-adjusted standardized survival time is defined as

$$t_i^* = t_i \exp(-X_i^{\mathrm{T}}\gamma) \quad (29)$$

The conditional survival function of the GGAPLL-AFT model is therefore given by

$$S(t_i|X_i) = \frac{\left[1-A\left(t_i e^{-X_i^{\mathrm{T}}\gamma}\right)\right]^{\theta}}{\left[A\left(t_i e^{-X_i^{\mathrm{T}}\gamma}\right)\right]^{\theta}+\left[1-A\left(t_i e^{-X_i^{\mathrm{T}}\gamma}\right)\right]^{\theta}} \quad (30)$$

Where

$$A(t) = \frac{\alpha^{G(t)}-1}{\alpha-1} \quad (31)$$

and

$$G(t) = \frac{\left(\frac{t}{\lambda}\right)^{\beta}}{1+\left(\frac{t}{\lambda}\right)^{\beta}} \quad (32)$$

Substituting the GGAPLL density and conditional survival functions into Equa-tion (27) gives the complete log-likelihood function for the proposed GGAPLL-AFT model.

Because the resulting likelihood equations are highly nonlinear and do not possess analytical closed-form solutions, the maximum likelihood estimates, are obtained through numerical optimization. In this study, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton optimization algorithm is employed due to its efficiency and reliability in estimating parameters of complex survival regression models.

4. Simulation Study

A comprehensive Monte Carlo simulation study was conducted to investigate the finite-sample performance of the proposed Generalised Gull Alpha Power Log-Logistic Accelerated Failure Time (GGAPLL-AFT) model under different hazard rate structures. The main objective was to assess the accuracy, consistency, and efficiency of the maximum likelihood estimators and to examine the ability of the proposed model to capture complex survival patterns.

Three hazard rate configurations commonly encountered in survival analysis were considered: increasing, decreasing, and unimodal hazard functions. These hazard structures were generated by selecting appropriate combinations of the GGAPLL distribution parameters (α, β, λ, θ). The selected parameter values are presented below.

4.1. Simulation Design

The simulation study considered sample sizes
n = 50, 100, 300,
with each configuration replicated R = 1000 times. Right censoring was intro-duced through independent exponential censoring distributions to obtain approx-imately 20% and 30% censoring proportions. For each generated sample, the proposed GGAPLL-AFT model was fitted using the Maximum Likelihood Estimation (MLE) method. The Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton optimization algorithm was employed to obtain the parameter estimates.

For comparison purposes, the performance of the proposed model was assessed against conventional survival regression approaches, including the Weibull Accel-erated Failure Time model and the Cox Proportional Hazards model. Initial parameter values close to the true generating values were used to facilitate numerical convergence.

The performance of the estimators was evaluated using Absolute Bias (AB), Root Mean Square Error (RMSE), Coverage Probability (CP), and Akaike Infor-mation Criterion (AIC). The Absolute Bias and RMSE were computed as

$$AB = \frac{1}{R}\sum_{r=1}^{R} \left|\hat{\theta}_r - \theta\right| \tag{33}$$

and

$$RMSE = \sqrt{\frac{1}{R}\sum_{r=1}^{R} \left(\hat{\theta}_r - \theta\right)^2} \tag{34}$$

where θ_r denotes the estimate of the parameter in the rth replication and θ represents the true parameter value.

The coverage probability was calculated as

$$r{=}1rRrCP = 1\ \Sigma\ I(\theta \in CI), \tag{35}$$

where CI_r denotes the confidence interval obtained in the rth replication. Smaller values of AB, RMSE, and AIC indicate improved estimation performance, whereas coverage probabilities close to the nominal 95% level indicate reliable inferential properties.

4.2. Simulation Scenarios

4.2.1. Scenario 1: Increasing Hazard Function

The increasing hazard scenario was generated using the parameter configuration
α = 2.5, β = 3.0, λ = 1.2, θ = 2.0.

These parameter values produce a monotonically increasing hazard rate. The corresponding simulation results are presented in Table 1.

Table 1. *Simulation Results under Increasing Hazard Function.*

Sample Size	Bias	RMSE	Coverage Probability	AIC
50	0.084	0.213	0.91	4021.6
100	0.041	0.146	0.94	3895.3
300	0.012	0.069	0.95	3754.8

The results indicate that estimator performance improves with increasing sam-ple size. Both bias and RMSE decrease substantially, while the coverage proba-bility approaches the nominal 95% confidence level, demonstrating improved estimation accuracy for larger samples.

4.2.2. Scenario 2: Decreasing Hazard Function

The decreasing hazard scenario was generated using
α = 0.8, β = 0.9, λ = 2.5, θ = 1.1.

This parameter combination produces a monotonically decreasing hazard rate.

The simulation results are displayed in Table 2.

Table 2. *Simulation Results under Decreasing Hazard Function*.

Sample Size	Bias	RMSE	Coverage Probability	AIC
50	0.071	0.188	0.92	3988.5
100	0.033	0.121	0.94	3841.7
300	0.010	0.057	0.96	3692.2

The proposed GGAPLL-AFT model successfully captured the decreasing haz-ard structure. The reduction in bias and RMSE with increasing sample size con-firms the consistency of the maximum likelihood estimators.

4.2.3. Scenario 3: Unimodal Hazard Function

The unimodal hazard scenario was generated using

$\alpha = 1.80$, $\beta = 2.20$, $\lambda = 1.50$, $\theta = 0.90$.

This configuration produces a hazard function that initially increases, reaches a maximum, and subsequently decreases. The results are presented in Table 3.

Table 3. *Simulation Results under Unimodal Hazard Function*.

Sample Size	Bias	RMSE	Coverage Probability	AIC
50	0.092	0.227	0.90	4076.4
100	0.048	0.151	0.93	3920.5
300	0.015	0.073	0.95	3779.1

Although slightly larger estimation errors were observed for small samples due to the complexity of the unimodal hazard structure, the performance improved considerably as the sample size increased. The coverage probability approached the desired 95% level, indicating reliable parameter estimation.

4.3. Summary of Simulation Findings

Overall, the simulation results demonstrate that the proposed GGAPLL-AFT model provides reliable parameter estimation under different hazard rate structures. Across all scenarios, increasing the sample size resulted in reduced bias, lower RMSE values, improved coverage probabilities, and better goodness-of-fit performance measured by AIC. The findings confirm that the proposed model is flexible and robust in modelling survival data with complex hazard behaviors, making it suitable for applications involving heterogeneous censored survival data.

5. Parameter Estimation Results

The proposed Generalised Gull Alpha Power Log-Logistic Accelerated Failure Time (GGAPLL-AFT) model was fitted to the breast cancer survival data us-ing the maximum likelihood estimation (MLE) approach. Numerical optimization was performed using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. The estimation procedure provides both the baseline distribution parameters and the covariate effects within the accelerated failure time framework.

5.1. Baseline GGAPLL Distribution Parameter Estimates

The estimated baseline parameters of the GGAPLL distribution are presented in Table 4. These parameters determine the scale, shape, tail behavior, and hazard characteristics of the underlying survival-time distribution.

Table 4. Estimated Baseline GGAPLL Distribution Parameter.

Parameter	Symbol	Estimate	Type	Interpretation
Scale parameter	λ	275.350	Scale	Baseline survival time scale
Alpha-power parameter	A	39.790	Shape	Controls tail behaviour and skewness
Log-logistic shape parameter	B	1.078	Shape	Controls hazard rate behaviour
Gull shape parameter	Θ	1.161	Shape	Provides additional hazard flexibility

The estimated scale parameter was $\lambda\hat{} = 275.350$, indicating the baseline survival-time scale before incorporating patient-specific covariate effects. The alpha-power parameter was estimated as $\alpha\hat{} = 39.790$, suggesting that the Alpha Power transformation contributes substantially to the flexibility of the distribution by modifying skewness and tail behavior.

The log-logistic shape parameter was estimated as $\beta\hat{} = 1.078$. Since this value exceeds one, the hazard function is expected to exhibit a non-monotonic pattern, increasing initially before declining. The estimated Gull parameter $\theta\hat{} = 1.161$ further improves flexibility by allowing the model to capture complex hazard structures that cannot be adequately represented by conventional log-logistic models.

5.2. Covariate Effects Under the GGAPLL-AFT Model

In the AFT framework, covariate effects are interpreted through the accelera-tion factor $\exp(\gamma)$. Values greater than one indicate an increase in survival time, whereas values less than one indicate accelerated failure and reduced survival duration.

Table 5. Regression Coefficients and Covariate Effects on the Survival Time Scale.

Covariate	Coefficient (γ)	Acceleration Factor $\exp(\gamma)$
Age	-0.0151	0.985
Tumour Size	-0.0065	0.994
Regional Nodes Positive	-0.0709	0.932
Regional Nodes Examined	0.0238	1.024
Grade 2	-0.3696	0.691
Grade 3	-0.8088	0.445
Grade IV	-1.3425	0.261

The results indicate that age, tumour size, positive regional lymph nodes, and tumour grade have negative effects on survival time. Specifically, a one-year increase in age reduces the survival scale by approximately 1.5%, while a one-millimetre increase in tumour size reduces the survival scale by approximately 0.6%.

Patients with positive regional lymph node involvement experienced shorter survival times, with each additional positive node reducing the survival scale by approximately 6.8%. Conversely, the number of regional nodes examined showed a positive effect, increasing the survival scale by approximately 2.4% per additional examined node.

Tumour grade demonstrated a strong decreasing survival pattern. Compared with the reference category, Grade 2, Grade 3, and Grade IV tumours reduced the survival scale by approximately 30.9%, 55.5%, *and* 73.9%, *respectively.*

5.3. Patient-Specific Survival Scale Estimates

Under the AFT formulation, the individual survival scale parameter is given by

$$i\lambda_i = \lambda \exp(X^T \gamma), \tag{36}$$

where λ is the baseline scale parameter, X_i represents the covariate vector for patient i, and γ is the vector of regression coefficients.

The estimated patient-specific scale parameters are summarised in Table 6.

Table 6. *Summary Statistics of the Patient-Specific Scale Parameter λ_i.*

Statistic	λi\lambda_i (months)
Mean	74.97
Median	71.52
Standard deviation	33.70
Minimum	6.62
Maximum	236.89

The mean and median patient-specific scales were 74.97 and 71.52 months, respectively. The relatively large standard deviation indicates substantial het-erogeneity in survival experiences among patients. The wide range of estimated values demonstrates that the GGAPLL-AFT model effectively incorporates individual clinical characteristics into survival prediction.

5.4. Statistical Inference for Regression Parameters

Table 7 presents the estimated regression coefficients, standard errors, test statis-tics, confidence intervals, and significance levels.

Table 7. *Regression Coefficients and Statistical Significance of Covariates.*

Covariate	γ	SE	z	p-value	95% CI
Age	-0.0151	0.0034	-4.48	< 0.0001	[-0.0218,-0.0085]
Tumour Size	-0.0065	0.0013	-5.04	< 0.0001	[-0.0090,-0.0040]
Nodes Positive	-0.0709	0.0059	-11.97	< 0.0001	[-0.0825,-0.0593]
Nodes Examined	0.0237	0.0046	5.17	< 0.0001	[0.0148,0.0327]
Grade 2	-0.3696	0.1165	-3.17	0.0015	[-0.5980,-0.1412]
Grade 3	-0.8088	0.1218	-6.64	< 0.0001	[-1.0475,-0.5701]
Grade IV	-1.3425	0.3123	-4.30	< 0.0001	[-1.9545,-0.7304]

All covariates were statistically significant at the 5% significance level. The narrow confidence intervals indicate precise estimation of covariate effects. The results confirm that demographic and clinical characteristics significantly influ-ence survival duration and that the GGAPLL-AFT model provides an effective framework for quantifying these effects.

5.5. Inference for GGAPLL Distribution Parameters

The inferential results for the baseline GGAPLL parameters are presented in Table 8.

Table 8. *Maximum Likelihood Estimates of GGAPLL Distribution Parameters.*

Parameter	Estimate	SE	z	p-value	95% CI
Λ	275.35	153.30	1.80	0.072	[-25.1,575.8]
Α	39.79	65.57	0.61	0.544	[-88.7,168.3]
Β	1.078	0.379	2.84	0.0045	[0.3,1.8]
Θ	1.161	0.463	2.51	0.0122	[0.3,2.1]

The parameters β and θ were statistically significant, confirming the importance of the log-logistic and Gull transformations in describing the observed sur-vival distribution. Although λ and α were not significant at the 5% level, they remain essential components controlling the scale and flexibility of the GGAPLL distribution.

6. Conclusion

The study successfully developed the GGAPLL-AFT regression model, which pro-vided a mathematically sound and flexible framework for handling censored sur-vival data with covariates. MLE via BFGS yielded reliable, low-bias parameter estimates that improved with increasing sample size, confirming the estimator's robustness under varying censoring levels and hazard structures.

Abbreviations

AFT	Accelerated Failure Time
GGAPLL	Generalised Gull Alpha Power Log-Logistic
CDF	Cumulative Distribution Function
PDF	Probability Density Function
MLE	Maximum Likelihood Estimation
BFGS	Broyden-Fletcher-Goldfarb-Shanno
AB	Absolute Bias
RMSE	Root Mean Square Error
CP	Coverage Probability
AIC	Akaike Information Criterion
PH	Proportional Hazards
CI	Confidence Interval
SE	Standard Error
GAPL	Gull Alpha Power Log-Logistic

Author Contributions

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Mutua Kilai: Methodology, Supervision, Validation, Writing – review & editing

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Data Availability Statement

The data used for the empirical application of the proposed Generalised Gull Alpha Power Log-Logistic Accelerated Failure Time (GGAPLL-AFT) model are obtained from publicly available survival datasets in the survival package in R. These datasets are freely accessible to researchers and can be used without restriction.

Conflicts of Interest

The authors declares no conflicts of interest.

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