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Research Article

A Bayesian Latent Class Framework for Multiple Imputation of Multivariate Categorical Data Under Ignorable and Non-Ignorable Missingness

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Abstract

Missing data in multivariate categorical datasets are commonly handled under the Missing at Random (MAR) assumption, although this assumption may be inadequate when nonresponse is associated with unobserved response patterns. This study develops a covariate-dependent Bayesian latent class model for multiple imputation under Missing Not at Random (MNAR), denoted by $BLCM_{MNAR}$. The model specifies covariate-dependent latent class membership through multinomial logistic regression, while the missingness indicators are modelled as Bernoulli variables with class-specific response probabilities. The response and missingness processes are thereby associated through their shared dependence on latent class membership. Bayesian estimation is implemented using Gibbs sampling, with Pólya–Gamma data augmentation for the class-membership regression coefficients, and missing responses are generated from the posterior predictive distribution. Finite-sample performance was examined through 500 simulation replications under scenarios with no, moderate, and strong covariate effects at primary sample sizes of 500 and 1,000. Multiple-imputation RMSE was additionally examined across sample sizes of 500, 1,000, 1,500, 2,000, and 2,500 to assess its behaviour with increasing sample size. $BLCM_{MNAR}$ was evaluated alongside a baseline Bayesian latent class model without covariates ($BLCM_0$) and a covariate-dependent MAR formulation ($BLCM_{MAR}$). Across the simulation settings, bias in multiple-imputation estimates was negligible for all three models, while relative RMSE varied by scenario and sample size. $BLCM_{MNAR}$ attained the lowest multiple-imputation RMSE in three of the six primary simulation conditions and recovered the class-specific response probabilities with small bias and RMSE. Recovery of latent class proportions and class-membership coefficients varied across scenarios and sample sizes, and improved point recovery did not uniformly correspond to improved interval calibration. The models were further applied to data from 32,156 women in the 2022 Kenya Demographic and Health Survey (KDHS). All three specifications were fitted with ten latent classes, but their posterior class occupancies differed. The completed-data and imputed-only distributions showed substantial sensitivity to the assumed missingness specification, particularly for skilled antenatal care, skilled delivery, and household decision-making outcomes. $BLCM_0$ and $BLCM_{MAR}$ produced broadly similar imputations for several outcomes, whereas $BLCM_{MNAR}$ produced materially different distributions. Class-specific response probabilities under $BLCM_{MNAR}$ also exhibited substantial heterogeneity across latent classes. These findings demonstrate the importance of accounting for potential nonignorable missingness in Bayesian multiple imputation of multivariate categorical data and provide a framework for assessing the sensitivity of imputations to alternative assumptions about the missingness process.

Keywords

Multiple Imputation, Bayesian latent Class Model, Missing Not at Random, Multivariate Categorical Data, Covariate-dependent Latent Class Model, Latent Class-dependent Missingness

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1. Introduction

Missing data remain one of the most persistent challenges in the statistical analysis of survey, demographic, epidemiological, and social science data. Incomplete observations reduce statistical efficiency and may lead to biased parameter estimates and misleading inferences when missing data are not appropriately handled in the analysis [1, 2]. These challenges are particularly pronounced for multivariate categorical data, where complex dependence structures among variables complicate statistical modelling and inference in the presence of missing observations [3, 4].

Multiple Imputation (MI) has become one of the most widely used approaches for handling incomplete data because it accounts for uncertainty arising from the imputation process while allowing standard complete-data analyses to be performed on multiply imputed datasets [5, 6]. BLCM provide a flexible basis for MI of multivariate categorical data by representing complex associations among observed variables through a finite latent structure, thereby avoiding the explicit specification of numerous higher-order interactions [3, 7]. This makes Bayesian latent class multiple imputation particularly attractive in situations involving several categorical variables, where conventional joint modelling approaches, such as, log-linear models may become increasingly difficult to specify and estimate [8, 9].

Most existing multiple imputation procedures, including Bayesian latent class approaches, are formulated under the MAR assumption, whereby the probability of missingness depends on observed information but not on the missing values after conditioning on the observed data [1, 10]. Under MAR, when the conditions for ignorability are satisfied, inference can be based on the observed-data model without explicitly specifying a separate model for the missingness process, thereby simplifying estimation and imputation [1]. However, the ignorable (MAR) assumption cannot generally be verified from the observed data alone and may be implausible in applications where nonresponse is related to information that remains unobserved. In demographic and health surveys, for example, respondents may be less willing to disclose particular information because of characteristics associated with the underlying responses. In such settings, reliance exclusively on MAR may affect the resulting imputations and subsequent statistical inference [1, 11, 12].

When the probability of missingness depends on information that remains unobserved after conditioning on the observed data, the missingness mechanism is described as MNAR, or nonignorable. Inference under MNAR requires assumptions about the relationship between the data and the missingness process because this relationship cannot generally be identified from the observed data alone [1, 13–15]. There are two commonly used classes of models for handling MNAR data: selection models and pattern-mixture models, which differ in how they factorize the joint distribution of the responses and missingness indicators [16, 17]. Bayesian formulations of these approaches provide a coherent framework for incorporating uncertainty and prior information and have been applied in a range of nonignorable missing-data settings [18–20]. Nevertheless, MNAR inference necessarily relies on assumptions about components of the data-generating process that are not fully informed by the observed data, making model specification and sensitivity assessment crucial [12, 21, 22].

In selection-model formulations, nonignorability is frequently represented by allowing the missingness probability to depend directly on the response, for example,

$$P(R_{ij} = 1 \mid Y_{ij}, \ldots), \tag{1}$$

where Y_{ij} denotes a potentially unobserved response. Such formulations provide a direct representation of outcome-dependent nonresponse but may introduce identification challenges because some parameters governing the association between the missingness process and unobserved outcomes are weakly or not directly informed by the observed data [13, 23, 24]. The challenges may become more pronounced for multivariate categorical data, where separate missingness processes must potentially be specified across several variables and response categories, increasing model dimensionality and sensitivity to specification [25, 26]. Although latent-variable formulations have been incorporated into nonignorable missing-data models, the missingness mechanism in many such approaches remains directly linked to the underlying responses or is developed for specialised modelling settings [17, 27].

Latent Class Models (LCM) provide an alternative representation of this dependence [28]. By introducing a discrete latent variable that captures unobserved heterogeneity in multivariate response patterns, the association between the response process and the missingness process can be represented through their shared dependence on latent class membership [15]. Specifically, rather than requiring the missingness probability to depend directly on a potentially unobserved response, the missingness process may be specified conditional on latent class membership [15]. This creates an indirect association between missingness and the multivariate response distribution through the latent structure. Such a formulation is particularly attractive for multivariate categorical data because the same latent classes used to characterize dependence among the responses can also provide a structured representation of

heterogeneity in missingness. Although latent class structures have been considered in nonignorable missing-data models [15, 27, 29], their integration with covariate-dependent class membership and Bayesian multiple imputation for general multivariate categorical data remains comparatively limited.

This motivates extending Bayesian latent class multiple imputation beyond the ignorable missingness. The covariate-dependent Bayesian latent class model under MAR developed in [30] models multivariate categorical responses while allowing latent class membership probabilities to vary with observed covariates. Under nonignorable missingness, however, modelling the response process alone is insufficient because the missingness process must also be explicitly represented. The present study therefore extends the MAR model developed in [30] to accommodate MNAR mechanism by linking the missingness process to the latent class structure while retaining the covariate-dependent specification of class membership.

Accordingly, the study extends the covariate-dependent Bayesian latent class multiple imputation model from ignorable to nonignorable missingness by introducing a latent-class-dependent model for the missingness indicators. For response variable j, the missingness mechanism is specified through

$$P(R_{ij} = 1 \mid L_i = k). \tag{2}$$

Here, L_i denotes the latent class membership of individual i. Conditional on latent class membership, the missingness probability is modelled without directly conditioning on the potentially unobserved value of Y_{ij}, while dependence between the response and missingness processes is induced through their shared dependence on the latent class. The resulting model accommodates nonignorable missingness while retaining covariate-dependent latent class membership through multinomial logistic regression and enabling posterior multiple imputation within a coherent Bayesian formulation. Its finite-sample performance is evaluated through simulation studies comparing the baseline model without covariates, the covariate-dependent MAR model, and the covariate-dependent MNAR model, and its practical applicability is illustrated using data from the 2022 KDHS.

2. Methodology

2.1. Bayesian Latent Class Model Under Nonignorable Missingness

The covariate-dependent Bayesian latent class model developed under the MAR mechanism in [30], denoted by BLCM_{MAR}, provides the baseline for the nonignorable missingness formulation considered in this study. BLCM_{MAR} represents dependence among multivariate categorical responses through a finite latent class structure while allowing observed covariates to influence latent class membership. The covariate-dependent Bayesian latent class model under MNAR, denoted by $\text{BLCM}_{\text{MNAR}}$, retains the class-membership and item-response components of BLCM_{MAR} and additionally models the missingness process through latent class membership.

Consider a sample of n independent individuals measured on p categorical response variables. Let

$$\mathbf{Y}_i = (Y_{i1}, Y_{i2}, \ldots, Y_{ip}), \qquad i = 1, \ldots, n,$$

denote the response vector for individual i, where Y_{ij} denotes the response to variable j, $j = 1, \ldots, p$, with $Y_{ij} \in \{1, \ldots, S_j\}$, and S_j is the number of categories of the jth response variable. Let $L_i \in \{1, \ldots, K\}$ denote the corresponding latent class membership. Further, let

$$\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iq})^\top$$

denote a vector of fully observed covariates.

Under the covariate-dependent latent class formulation developed in [30], the distribution of the response vector conditional on the covariates is

$$P(\mathbf{Y}_i = \mathbf{y}_i \mid \mathbf{x}_i) = \sum_{k=1}^{K} \pi_k(\mathbf{x}_i) \prod_{j=1}^{p} P(Y_{ij} = y_{ij} \mid L_i = k), \tag{3}$$

where

$$\pi_k(\mathbf{x}_i) = P(L_i = k \mid \mathbf{x}_i)$$

denotes the covariate-dependent probability of membership in latent class k. Equation (3) follows from the local independence assumption, under which the observed responses are conditionally independent given latent class membership [31, 32].

For categorical response j, define the class-specific item-response probabilities by

$$\pi_{kjs} = P(Y_{ij} = s \mid L_i = k), \qquad s = 1, \ldots, S_j, \tag{4}$$

subject to

$$\sum_{s=1}^{S_j} \pi_{kjs} = 1.$$

The class-specific response distribution can therefore be expressed as

$$P(\mathbf{Y}_i = \mathbf{y}_i \mid L_i = k) = \prod_{j=1}^{p} \prod_{s=1}^{S_j} (\pi_{kjs})^{I(Y_{ij}=s)}, \tag{5}$$

where I_{ijs} is an indicator variable defined as

$$I_{ijs} = \begin{cases} 1, & \text{if } Y_{ij} = s, \\ 0, & \text{otherwise,} \end{cases}$$

for $i = 1, \ldots, n$, $j = 1, \ldots, p$, and $s = 1, \ldots, S_j$.

The covariate-dependent class-membership probabilities are modelled using a baseline-category multinomial logistic regression [31, 33, 34]:

$$\pi_k(\mathbf{x}_i) = \frac{\exp(\mathbf{x}_i^\top \boldsymbol{\beta}_k)}{\sum_{r=1}^{K} \exp(\mathbf{x}_i^\top \boldsymbol{\beta}_r)}, \qquad k = 1, \ldots, K, \tag{6}$$

where $\boldsymbol{\beta}_k$ is the regression coefficient vector associated with latent class k. For identifiability, one class is selected as the baseline by fixing

$$\boldsymbol{\beta}_1 = \mathbf{0}.$$

The multinomial logistic formulation ensures that the class-membership probabilities are positive and sum to one for every covariate structure [35]. Consequently, the covariates influence the multivariate response distribution indirectly by altering the probabilities of latent class membership and, hence, the mixture weights assigned to the class-specific response distributions. Conditional on latent class membership, the item-response probabilities do not depend on the covariates and are therefore invariant across covariate patterns [31].

For this purpose, define the missingness indicator

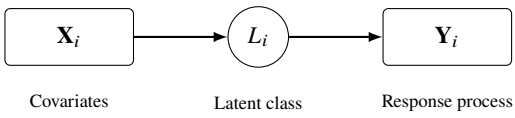
$$R_{ij} = \begin{cases} 1, & \text{if } Y_{ij} \text{ is observed,} \\ 0, & \text{if } Y_{ij} \text{ is missing,} \end{cases} \tag{7}$$

and let

$$\mathbf{R}_i = (R_{i1}, R_{i2}, \ldots, R_{ip})$$

denote the corresponding missingness pattern for individual i. The distinction between the BLCM_{MAR} and $\text{BLCM}_{\text{MNAR}}$ formulations is illustrated in Figure 1.

(a) BLCM_{MAR}



(b) $\text{BLCM}_{\text{MNAR}}$

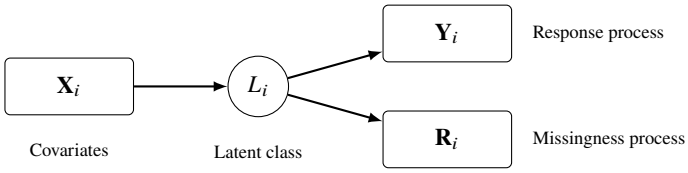


Figure 1. *Conceptual representation of $BLCM_{\text{MAR}}$ and $BLCM_{\text{MNAR}}$.*

As illustrated in Figure 1, the class-membership and item-response components remain unchanged under $\text{BLCM}_{\text{MNAR}}$. The additional component is a missingness model in which the probability that an item is observed is allowed to vary across latent classes. Conditional on latent class membership, the missingness indicators are assumed to be independent Bernoulli random variables,

$$R_{ij} \mid L_i = k \sim \text{Bernoulli}(\rho_{jk}), \tag{8}$$

where

$$\rho_{jk} = P(R_{ij} = 1 \mid L_i = k), \qquad 0 < \rho_{jk} < 1,$$

denotes the class-specific probability that response j is observed. so that,

$$P(\mathbf{R}_i = \mathbf{r}_i \mid L_i = k) = \prod_{j=1}^{p} (\rho_{jk})^{r_{ij}} (1 - \rho_{jk})^{1 - r_{ij}}. \tag{9}$$

Under this formulation, the item responses and missingness indicators are conditionally independent given latent class membership. Thus, rather than specifying the missingness probability directly as a function of a potentially unobserved item response, the association between the response and missingness processes is represented through their shared dependence on L_i [36]. The resulting dependency structure can therefore be represented as

$$\mathbf{X}_i \longrightarrow L_i \longrightarrow (\mathbf{Y}_i, \mathbf{R}_i).$$

Although the response and missingness processes are conditionally independent given latent class membership, the resulting missingness mechanism can be nonignorable after marginalizing over the unobserved latent class. In particular,

$$P(R_{ij} = 1 \mid \mathbf{Y}_i, \mathbf{x}_i) = \textstyle\sum_{k=1}^{K} P(R_{ij} = 1 \mid L_i = k)\, P(L_i = k \mid \mathbf{Y}_i, \mathbf{x}_i) \tag{10}$$

and therefore

$$P(R_{ij} = 1 \mid \mathbf{Y}_i, \mathbf{x}_i) = \sum_{k=1}^{K} \rho_{jk}\, P(L_i = k \mid \mathbf{Y}_i, \mathbf{x}_i). \tag{11}$$

Since the posterior class-membership probabilities $P(L_i = k \mid \mathbf{Y}_i, \mathbf{x}_i)$ depend on the multivariate response pattern, the probability of missingness may depend on components of $\mathbf{Y}_i$ that are themselves unobserved. Consequently, in general,

$$P(\mathbf{R}_i \mid \mathbf{Y}_{i,\text{obs}}, \mathbf{Y}_{i,\text{mis}}, \mathbf{x}_i) \neq P(\mathbf{R}_i \mid \mathbf{Y}_{i,\text{obs}}, \mathbf{x}_i), \tag{12}$$

which characterizes a nonignorable MNAR missingness mechanism [1, 10]. Combining the class-membership, item-response, and missingness components gives the complete-data joint distribution for individual i:

$$P(\mathbf{Y}_i = \mathbf{y}_i, \mathbf{R}_i = \mathbf{r}_i, L_i = k \mid \mathbf{x}_i) = \underbrace{P(L_i = k \mid \mathbf{x}_i)}_{\text{Class membership model}} \underbrace{P(\mathbf{Y}_i = \mathbf{y}_i \mid L_i = k)}_{\text{Item response model}} \underbrace{P(\mathbf{R}_i = \mathbf{r}_i \mid L_i = k)}_{\text{Missingness model}}. \tag{13}$$

Marginalizing over the unobserved latent class gives

$$P(\mathbf{y}_i, \mathbf{r}_i \mid \mathbf{x}_i) = \sum_{k=1}^{K} P(L_i = k \mid \mathbf{x}_i) P(\mathbf{y}_i \mid L_i = k) P(\mathbf{r}_i \mid L_i = k). \tag{14}$$

Substituting Equations (5), (6), and (9) into Equation (14) yields

$$P(\mathbf{y}_i, \mathbf{r}_i \mid \mathbf{x}_i) = \sum_{k=1}^{K} \pi_k(\mathbf{x}_i) \left[\prod_{j=1}^{p} \prod_{s=1}^{S_j} (\pi_{kjs})^{I(y_{ij}=s)} \right] \times \left[\prod_{j=1}^{p} (\rho_{jk})^{r_{ij}} (1 - \rho_{jk})^{1-r_{ij}} \right]. \tag{15}$$

Equation (15) shows how the response and class-membership components of BLCM_{MAR} are augmented by the class-specific missingness component to obtain $\text{BLCM}_{\text{MNAR}}$.

The response vector $\mathbf{Y}_i$ generally contains both observed and missing components and can be partitioned as

$$\mathbf{Y}_i = (\mathbf{Y}_{i,\text{obs}}, \mathbf{Y}_{i,\text{mis}}).$$

The observed-data likelihood for individual i is therefore obtained by marginalizing the joint model over all possible values of the missing responses:

$$P(\mathbf{y}_{i,\text{obs}}, \mathbf{r}_i \mid \mathbf{x}_i) = \sum_{\mathbf{y}_{i,\text{mis}}} P(\mathbf{y}_{i,\text{obs}}, \mathbf{y}_{i,\text{mis}}, \mathbf{r}_i \mid \mathbf{x}_i). \tag{16}$$

Using Equation (15), this becomes

$$P(\mathbf{y}_{i,\text{obs}}, \mathbf{r}_i \mid \mathbf{x}_i) = \sum_{\mathbf{y}_{i,\text{mis}}} \sum_{k=1}^{K} \pi_k(\mathbf{x}_i) \left[\prod_{j=1}^{p} \prod_{s=1}^{S_j} (\pi_{kjs})^{I(y_{ij}=s)} \right] \times \left[\prod_{j=1}^{p} (\rho_{jk})^{r_{ij}} (1 - \rho_{jk})^{1-r_{ij}} \right]. \tag{17}$$

The missingness component in Equation (17) depends on the observed missingness pattern $\mathbf{r}_i$ and latent class k, but not directly on the values of $\mathbf{y}_{i,\text{mis}}$. It is therefore constant with respect to the summation over the missing response values. Moreover, for each item with $r_{ij} = 0$, the corresponding item-response component sums to one:

$$\sum_{y_{ij}} \prod_{s=1}^{S_j} (\pi_{kjs})^{I(y_{ij}=s)} = \sum_{s=1}^{S_j} \pi_{kjs} = 1. \tag{18}$$

Consequently, the response component of the observed-data likelihood contains only the probabilities corresponding to the observed responses. The individual contribution to the observed-data likelihood is therefore

$$P(\mathbf{y}_{i,\text{obs}}, \mathbf{r}_i \mid \mathbf{x}_i) = \sum_{k=1}^{K} \pi_k(\mathbf{x}_i) \left[\prod_{j:r_{ij}=1} \prod_{s=1}^{S_j} (\pi_{kjs})^{I(y_{ij}=s)} \right] \times \left[\prod_{j=1}^{p} (\rho_{jk})^{r_{ij}} (1 - \rho_{jk})^{1-r_{ij}} \right]. \tag{19}$$

Equation (19) constitutes the principal likelihood extension relative to BLCM_{MAR}. Under MAR, the missingness mechanism is ignorable under the required conditions and does not explicitly contribute to the response-model likelihood. Under $\text{BLCM}_{\text{MNAR}}$, the observed missingness pattern contributes through the class-specific observation probabilities ρ_{jk}, thereby providing additional information about latent class membership.

For a sample of n independent individuals, the full observed-data likelihood is

$$\mathcal{L}(\boldsymbol{\Theta} \mid \mathbf{Y}_{\text{obs}}, \mathbf{R}, \mathbf{X}) = \prod_{i=1}^{n} P(\mathbf{y}_{i,\text{obs}}, \mathbf{r}_i \mid \mathbf{x}_i; \boldsymbol{\Theta}), \tag{20}$$

where

$$\boldsymbol{\Theta} = (\boldsymbol{\beta}, \boldsymbol{\Pi}, \boldsymbol{\rho})$$

is a collection of the regression coefficients governing covariate-dependent latent class membership, the class-specific item-response probabilities, and the class-specific response-probability parameters, respectively, where $\rho_{jk} = P(R_{ij} = 1 \mid L_i = k)$ denotes the class-specific probability that response j is observed. This likelihood provides the basis for the Bayesian prior specification and posterior estimation of the nonignorable (MNAR) model in the subsequent sections.

2.2. Prior Distributions

Prior distributions are assigned to the unknown model parameters $\boldsymbol{\Theta}$. The prior specification follows standard Bayesian

formulations for multinomial and regression parameters, with an additional prior assigned to the class-specific response-probability parameters introduced under $\text{BLCM}_{\text{MNAR}}$ [37, 38].

For each response variable j and latent class k, let

$$\boldsymbol{\pi}_{kj} = (\pi_{kj1}, \ldots, \pi_{kjS_j}),$$

where S_j denotes the number of categories for response variable j. Independent Dirichlet priors are assigned to the class-specific item-response probabilities,

$$\boldsymbol{\pi}_{kj} \sim \text{Dirichlet}(1, \ldots, 1), \qquad j = 1, \ldots, p, \quad k = 1, \ldots, K. \tag{21}$$

This specification assigns equal prior weight to all response categories within each latent class while satisfying the simplex constraint

$$\sum_{s=1}^{S_j} \pi_{kjs} = 1.$$

For the multinomial logistic regression coefficients governing covariate-dependent latent class membership, independent multivariate normal priors are assigned,

$$\boldsymbol{\beta}_k \sim N(\mathbf{0}, \Sigma_\beta), \qquad k = 2, \ldots, K, \tag{22}$$

where $\Sigma_\beta = 10\mathbf{I}$, with $\boldsymbol{\beta}_1 = \mathbf{0}$ corresponding to the baseline latent class [31, 39]. The zero-mean normal prior provides regularization of the covariate effects while allowing substantial prior dispersion [37].

The $\text{BLCM}_{\text{MNAR}}$ formulation additionally introduces the class-specific response-probability parameters

$$\rho_{jk} = P(R_{ij} = 1 \mid L_i = k),$$

which represent the probability that response j is observed for an individual belonging to latent class k. Given the Bernoulli model for the missingness indicators, independent conjugate Beta priors are assigned to the response-probability parameters [37]:

$$\rho_{jk} \sim \text{Beta}(0.5, 0.5), \qquad j = 1, \ldots, p, \quad k = 1, \ldots, K. \tag{23}$$

The symmetric $\text{Beta}(0.5, 0.5)$ prior has prior mean 0.5 and places relatively greater prior mass near the boundaries of the unit interval. The Beta prior retains conjugacy with the Bernoulli missingness model [37].

Assuming prior independence among the parameter blocks, the joint prior distribution factorizes as

$$P(\boldsymbol{\Theta}) = P(\boldsymbol{\beta})P(\boldsymbol{\Pi})P(\boldsymbol{\rho}). \tag{24}$$

Combining the observed-data likelihood in Equation (20) with the joint prior distribution through Bayes' theorem yields

$$P(\boldsymbol{\Theta} \mid \mathbf{Y}_{\text{obs}}, \mathbf{R}, \mathbf{X}) \propto \mathcal{L}(\boldsymbol{\Theta} \mid \mathbf{Y}_{\text{obs}}, \mathbf{R}, \mathbf{X})P(\boldsymbol{\Theta}), \tag{25}$$

which forms the basis for posterior estimation of the nonignorable (MNAR) model.

2.3. Posterior Estimation

Posterior inference for $\text{BLCM}_{\text{MNAR}}$ is based on the joint posterior distribution in Equation (25). Unlike the corresponding MAR formulation in [30], the posterior distribution under $\text{BLCM}_{\text{MNAR}}$ explicitly conditions on the observed missingness pattern $\mathbf{R}$ through the class-specific response-probability parameters $\boldsymbol{\rho}$.

Direct analytical evaluation of the posterior distribution is generally intractable because of the latent class structure and the multinomial logistic model governing covariate-dependent class membership [30, 37]. Posterior inference is performed using Markov Chain Monte Carlo (MCMC) methods, specifically a Gibbs sampling algorithm with data augmentation [40, 41]. At each iteration, the latent class memberships, class-specific item-response probabilities, regression coefficients, and class-specific response-probability parameters are sequentially updated conditional on the most recent values of the remaining quantities. For the multinomial logistic class-membership model, Pólya–Gamma data augmentation is employed to facilitate conditionally Gaussian updates of the regression coefficients [42].

A key distinction from the MAR formulation in [30] arises in the posterior updating of latent class membership. Under the nonignorable model, posterior class allocation depends jointly on the covariate-dependent class-membership probabilities, the observed response pattern, and the observed missingness pattern. Consequently, the missingness indicators contribute information about latent class membership through the class-specific response-probability parameters ρ_{jk}.

The item-response probabilities have conjugate Dirichlet updates [7, 30], while the class-specific response-probability parameters have conjugate Beta updates under the Bernoulli missingness model.

Following an initial burn-in period, posterior draws are retained to approximate the joint posterior distribution. Posterior summaries, including posterior means, posterior standard deviations, and credible intervals, are obtained from the retained draws [37]. The retained posterior draws also provide the basis for posterior predictive generation of the missing responses, as described in the following section.

2.4. Multiple Imputation Procedure

Multiple imputation follows the Bayesian posterior predictive framework described in [30], with the posterior predictive

distribution under $\text{BLCM}_{\text{MNAR}}$ additionally accounting for the nonignorable missingness process. Let $\mathbf{Y}^{obs}$ and $\mathbf{Y}^{mis}$ denote the observed and missing components of the response matrix, respectively, and let $\mathbf{R}$ denote the corresponding missingness indicator matrix. Under $\text{BLCM}_{\text{MNAR}}$, the posterior predictive distribution of the missing responses explicitly conditions on the observed missingness pattern:

$$P(\mathbf{Y}^{mis} \mid \mathbf{Y}^{obs}, \mathbf{R}, \mathbf{X}) = \int P(\mathbf{Y}^{mis} \mid \boldsymbol{\Theta}, \mathbf{Y}^{obs}, \mathbf{R}, \mathbf{X}) \times P(\boldsymbol{\Theta} \mid \mathbf{Y}^{obs}, \mathbf{R}, \mathbf{X})\, d\boldsymbol{\Theta}. \tag{26}$$

In contrast to the MAR formulation in [30], the observed missingness pattern $\mathbf{R}$ contributes to the posterior allocation of individuals to latent classes through the class-specific response-probability parameters ρ_{jk}. Consequently, the missingness pattern influences the class-specific predictive distribution from which the missing responses are imputed.

The posterior predictive integral in Equation (26) is evaluated by Monte Carlo integration using retained posterior draws from the Gibbs sampler, following the general procedure described in [30]. Under $\text{BLCM}_{\text{MNAR}}$, each retained draw is obtained from the joint posterior distribution conditional on the observed responses, missingness pattern, and covariates:

$$\boldsymbol{\Theta}^{(t)} \sim P\left(\boldsymbol{\Theta} \mid \mathbf{Y}^{obs}, \mathbf{R}, \mathbf{X}\right), \qquad t = 1, \ldots, m. \tag{27}$$

Conditional on each retained posterior state, the missing responses are generated using the item-response probabilities corresponding to the sampled latent class. Specifically, for an imputable response y_{ij}, if $L_i^{(t)} = k$, then

$$y_{ij}^{mis(t)} \sim \text{Categorical}\left(\pi_{kj1}^{(t)}, \ldots, \pi_{kjS_j}^{(t)}\right). \tag{28}$$

Each selected posterior draw is used to generate one completed dataset. From the $m = 500$ retained posterior draws, $m_I = 100$ approximately equally spaced draws are selected, with each selected draw used to generate one completed dataset, yielding $m_I = 100$ multiply imputed datasets. This posterior predictive procedure propagates uncertainty in the model parameters and missing responses into the completed datasets [5, 30].

Each of the $m_I = 100$ completed datasets is analysed separately, and the resulting estimates and variances are combined using Rubin's rules [5, 30]. The combining procedure accounts for within- and between-imputation variability. Under $\text{BLCM}_{\text{MNAR}}$, the completed datasets additionally reflect information from the observed missingness pattern through the class-specific response-probability parameters. The completed datasets are subsequently used to evaluate inferential performance in the simulation study and to examine the empirical implications of the alternative model specifications in the application to the 2022 KDHS data.

3. Simulation Study

3.1. Simulation Design

The simulation design extends that used for the MAR formulation in [30]. The common data-generating structure and simulation settings were retained to facilitate comparison with the earlier formulation, while the present study additionally introduced a nonignorable missingness mechanism and evaluated $\text{BLCM}_{\text{MNAR}}$. Accordingly, three models were considered: the conventional Bayesian latent class model without covariates, BLCM_0; the covariate-dependent model under MAR, BLCM_{MAR}; and the covariate-dependent model under MNAR, $\text{BLCM}_{\text{MNAR}}$.

As in [30], three scenarios representing different levels of covariate influence on latent class membership were considered: no covariate effects (Scenario A), moderate covariate effects (Scenario B), and strong covariate effects (Scenario C). Each scenario was evaluated at sample sizes $n = 500$ and $n = 1000$. The principal extension concerned the missingness process. Under MAR, missingness was generated according to the previously specified ignorable mechanism, whereas under MNAR, observation probabilities were allowed to vary across latent classes through the class-specific parameters ρ_{jk}. This design enabled evaluation of the three model specifications across the simulation conditions considered in the study. The simulation design and implementation settings are summarized in Table 1.

Table 1. *Simulation design and implementation settings.*

Component	Specification
Latent classes (K)	3
Response variables	6 categorical variables, 3 categories each
Covariates	$X_1, \ldots, X_6$
Primary sample sizes	500, 1000

Component	Specification
Additional MI sample sizes	1500, 2000, 2500
Covariate-effect scenarios	None (A), moderate (B), strong (C)
Variables with missing values	Y_2, Y_3
Missingness mechanisms	MAR and MNAR
MAR missingness rates	Approximately 10% for Y_2 and 20% for Y_3
MNAR specification	Class-specific observation probabilities ρ_{jk}
Simulation replications	500
Posterior iterations	1,500
Multiply imputed datasets	100
Models compared	$BLCM_0$, $BLCM_{MAR}$, $BLCM_{MNAR}$
Software	R version 4.5.2

The primary simulation comparisons were conducted at $n = 500$ and $n = 1000$. For MI inference performance, an additional sample-size analysis was conducted at $n = 1500$, 2000, and 2500 to examine the behaviour of Root Mean Squared Error (RMSE) with increasing sample size.

3.2. Data-Generating Process

The generation of the complete multivariate categorical data followed the procedure described in [30]. Six trichotomous covariates with equal category probabilities were generated independently, followed by covariate-dependent latent class membership using a multinomial logistic regression model. Three levels of covariate influence on latent class membership were considered: none (Scenario A), moderate (Scenario B), and strong (Scenario C). Conditional on latent class membership, six trichotomous response variables were generated under the local independence assumption. The regression coefficients defining the three scenarios and the class-specific response probabilities were unchanged from those reported in [30].

For the MAR reference condition, missingness was imposed on Y_2 and Y_3 using the same covariate- and response-dependent logistic models described in [30]. The intercepts were calibrated to produce average missingness levels of approximately 10% for Y_2 and 20% for Y_3, respectively.

Simulation-based evaluation of imputation methods under MNAR requires explicit specification of the process used to generate nonignorable missingness, and recent studies have emphasized the importance of considering alternative MNAR generation strategies [43].

The present study additionally considers a nonignorable missingness mechanism based on class-specific response probabilities.

3.3. MNAR mechanism

Under the MNAR mechanism, missingness was generated conditional on latent class membership. For the two response variables subject to missingness, Y_2 and Y_3, the missingness indicators were generated according to

$$R_{ij} \mid L_i = k \sim \text{Bernoulli}(\rho_{jk}), \qquad j \in \{2,3\}, \quad k = 1,\ldots,K, \tag{29}$$

where

$$\rho_{jk} = P(R_{ij} = 1 \mid L_i = k)$$

denotes the class-specific probability that response j is observed.

The class-specific response probabilities were fixed at

$$(\rho_{21}, \rho_{22}, \rho_{23}) = (0.95, 0.85, 0.75) \tag{30}$$

for Y_2, and

$$(\rho_{31}, \rho_{32}, \rho_{33}) = (0.90, 0.75, 0.60) \tag{31}$$

for Y_3. These specifications allow the response probabilities to vary across latent classes, with greater class-specific variation for Y_3.

Because latent class membership governs both the distribution of the categorical responses and the class-specific response probabilities, variation in ρ_{jk} across classes induces an association between the response and missingness processes through their shared latent class structure, providing the nonignorable missingness setting used to evaluate $BLCM_{MNAR}$ [44, 45].

4. Results and Discussions

4.1. Simulation Results and Discussion

This section presents the simulation results for $BLCM_0$, $BLCM_{MAR}$, and the proposed $BLCM_{MNAR}$ under MAR and MNAR missingness mechanisms. Model performance is examined in terms of multiple imputation performance, latent class

recovery, and regression coefficient recovery across the three simulation scenarios. Under the MNAR mechanism, recovery of the class-specific observation probabilities ρ_{jk} is additionally assessed for $\text{BLCM}_{\text{MNAR}}$.

4.1.1. Multiple Imputation (MI) Performance

The MI inference performance was evaluated using bias, RMSE, posterior stability, and empirical 95% coverage across the three simulation scenarios. Table 2 extends the comparison reported in [30] by including $\text{BLCM}_{\text{MNAR}}$ alongside BLCM_0 and BLCM_{MAR}.

Table 2. *Multiple imputation performance across simulation scenarios.*

Scenario	*n*	Method	Bias	RMSE	Stability	Cov. 95%
A	500	BLCM_0	0.0000	0.0089	0.0185	0.9000
A	500	BLCM_{MAR}	-0.0000	0.0089	0.0188	0.8889
A	500	$\text{BLCM}_{\text{MNAR}}$	0.0000	*0.0067*	0.0259	0.9889
A	1000	BLCM_0	0.0000	0.0053	0.0152	0.9778
A	1000	BLCM_{MAR}	0.0000	*0.0052*	0.0153	0.9889
A	1000	$\text{BLCM}_{\text{MNAR}}$	-0.0000	0.0061	0.0168	0.9556
B	500	BLCM_0	0.0000	0.0085	0.0174	0.9667
B	500	BLCM_{MAR}	0.0000	*0.0079*	0.0172	0.9667
B	500	$\text{BLCM}_{\text{MNAR}}$	-0.0000	0.0081	0.0214	0.9778
B	1000	BLCM_0	0.0000	0.0064	0.0114	0.9556
B	1000	BLCM_{MAR}	0.0000	0.0068	0.0114	0.9333
B	1000	$\text{BLCM}_{\text{MNAR}}$	-0.0000	*0.0061*	0.0128	0.9778
C	500	BLCM_0	0.0000	0.0086	0.0230	0.9444
C	500	BLCM_{MAR}	0.0000	0.0087	0.0236	0.9333
C	500	$\text{BLCM}_{\text{MNAR}}$	0.0000	*0.0080*	0.0260	0.9333
C	1000	BLCM_0	-0.0000	0.0056	0.0146	0.9222
C	1000	BLCM_{MAR}	0.0000	0.0059	0.0144	0.9444
C	1000	$\text{BLCM}_{\text{MNAR}}$	-0.0000	*0.0050*	0.0170	0.9889

Note. Bias, RMSE, posterior stability, and empirical 95% coverage are averaged across the target category-probability parameters. The lowest RMSE for each scenario and sample size is shown in bold.

Across all scenarios and sample sizes, bias was negligible for the three models, and differences in MI performance were therefore primarily reflected in RMSE, posterior stability, and empirical coverage. $\text{BLCM}_{\text{MNAR}}$ attained the lowest RMSE in three of the six simulation conditions, although relative performance varied with the strength of the covariate effects and sample size.

Under Scenario A, where covariates did not influence latent class membership, $\text{BLCM}_{\text{MNAR}}$ attained the lowest RMSE at $n = 500$, whereas BLCM_{MAR} attained the lowest RMSE at $n = 1000$. The lower RMSE for $\text{BLCM}_{\text{MNAR}}$ at the smaller sample size was accompanied by greater posterior variability, with a stability measure of 0.0259 compared with 0.0185 and 0.0188 for BLCM_0 and BLCM_{MAR}, respectively. At $n = 1000$, the differences among the three models were smaller.

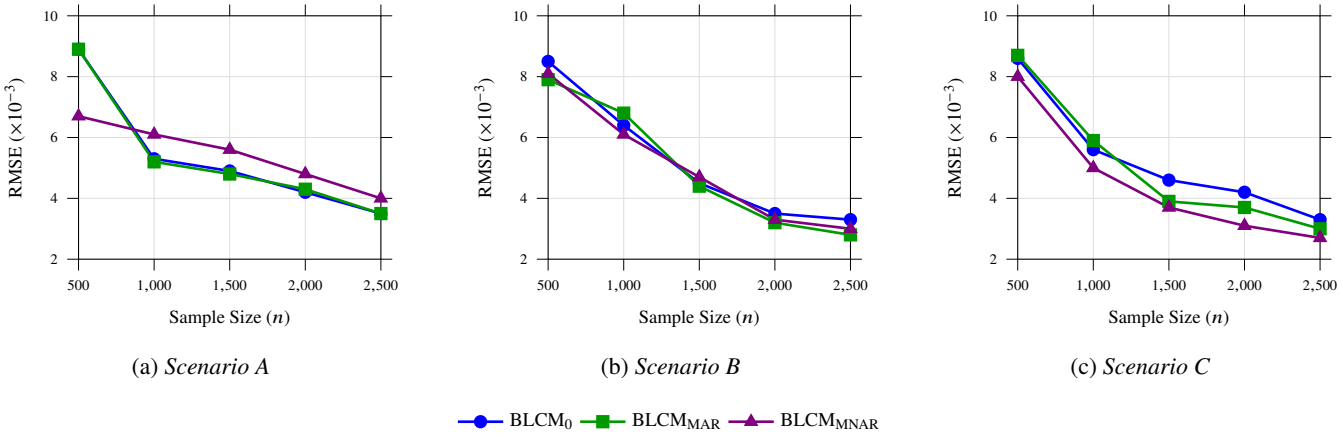
Under Scenario B, characterized by moderate covariate effects, BLCM_{MAR} attained the lowest RMSE at $n = 500$, although $\text{BLCM}_{\text{MNAR}}$ performed similarly. At $n = 1000$, $\text{BLCM}_{\text{MNAR}}$ attained the lowest RMSE, decreasing from 0.0081 at $n = 500$ to 0.0061. Thus, the two covariate-dependent formulations showed broadly comparable estimation accuracy under moderate covariate effects, with their relative performance varying by sample size.

The clearest distinction occurred under Scenario C, where covariate effects were strongest. $\text{BLCM}_{\text{MNAR}}$ attained the lowest RMSE at both sample sizes, with values of 0.0080 and 0.0050 for $n = 500$ and $n = 1000$, respectively. At $n = 1000$, the corresponding RMSE values were 0.0056 for BLCM_0 and 0.0059 for BLCM_{MAR}. Under the simulation conditions considered, the relative improvement in RMSE for $\text{BLCM}_{\text{MNAR}}$ was therefore most apparent when the covariate effects on latent class membership were strongest.

Posterior stability generally improved as the sample size increased from 500 to 1000 for all three models. $\text{BLCM}_{\text{MNAR}}$ exhibited somewhat greater posterior variability in several conditions, particularly at the smaller sample size. This pattern may reflect the additional uncertainty associated with jointly estimating the class-specific missingness parameters and the latent class model, consistent with the greater complexity of latent-class models that explicitly incorporate a nonignorable missingness process [44]. The reduction in the stability measures at the larger sample size suggests that this additional variability became less pronounced as sample size increased.

Empirical 95% coverage was generally close to the nominal level across the simulation conditions. $\text{BLCM}_{\text{MNAR}}$ attained coverage between 0.9333 and 0.9889, while BLCM_0 and BLCM_{MAR} also showed generally modest deviations from the nominal level. Thus, the observed differences in estimation accuracy were not accompanied by a systematic deterioration in interval coverage for $\text{BLCM}_{\text{MNAR}}$.

To further examine the behaviour of MI inference as sample size increased, an additional sample-size analysis was conducted using $n = 1500$, 2000, and 2500, supplementing the primary simulation settings of $n = 500$ and $n = 1000$. Figure 2 presents the resulting RMSE patterns across the five sample sizes for the three simulation scenarios.



(a) *Scenario A* (b) *Scenario B* (c) *Scenario C*

Figure 2. *RMSE across increasing sample sizes for $BLCM_0$, $BLCM_{\text{MAR}}$, and $BLCM_{\text{MNAR}}$ under the three simulation scenarios.*

Figure 2a shows that, under Scenario A, RMSE generally decreased as sample size increased for all three models. $\text{BLCM}_{\text{MNAR}}$ attained the lowest RMSE at $n = 500$, whereas BLCM_0 and BLCM_{MAR} attained lower values from $n = 1000$ onwards. The latter two models remained nearly indistinguishable across the larger sample sizes, consistent with the absence of covariate effects on latent class membership in this scenario.

Under Scenario B (Figure 2b), RMSE decreased with increasing sample size for all three models. Their relative performance remained close across the range considered. $\text{BLCM}_{\text{MNAR}}$ attained the lowest RMSE at $n = 1000$, whereas BLCM_{MAR} attained slightly lower values at $n = 500$, 1500, 2000, and 2500. At $n = 2500$, the RMSE values were 0.0033, 0.0028, and 0.0030 for BLCM_0, BLCM_{MAR}, and $\text{BLCM}_{\text{MNAR}}$, respectively, indicating similar estimation accuracy for the two covariate-dependent formulations.

The clearest distinction among the models occurred under Scenario C (Figure 2c). Although RMSE decreased with increasing sample size for all three models, $\text{BLCM}_{\text{MNAR}}$ attained the lowest value at every sample size considered. Its RMSE decreased from 0.0080 at $n = 500$ to 0.0027 at $n = 2500$, compared with 0.0033 for BLCM_0 and 0.0030 for BLCM_{MAR} at $n = 2500$. Thus, under the simulation conditions considered, the relative improvement in RMSE for $\text{BLCM}_{\text{MNAR}}$ was most apparent in Scenario C, where the covariate effects on latent class membership were strongest.

Overall, RMSE declined with increasing sample size for all three models, indicating improved estimation accuracy as sample size increased. The relative performance of the models nevertheless varied across the three simulation scenarios. In particular, $\text{BLCM}_{\text{MNAR}}$ maintained the lowest RMSE across the full range of sample sizes under Scenario C, whereas the differences among the models were less systematic under Scenarios A and B.

4.1.2. Latent Class Proportion Recovery

Latent class proportion recovery was evaluated using the bias and RMSE of the estimated latent class proportions across the three simulation scenarios. Table 3 extends the comparison reported in [30] by including $\text{BLCM}_{\text{MNAR}}$ alongside BLCM_0 and BLCM_{MAR}.

Table 3. Latent class proportion recovery across simulation scenarios.

Scenario	*n*	Method	Bias	RMSE
A	500	$BLCM_0$	0.0000	*0.0410*
A	500	$BLCM_{MAR}$	-0.0000	0.0920
A	500	$BLCM_{MNAR}$	0.0000	0.1282
A	1000	$BLCM_0$	0.0000	0.0450
A	1000	$BLCM_{MAR}$	-0.0000	0.0983
A	1000	$BLCM_{MNAR}$	0.0000	*0.0415*
B	500	$BLCM_0$	0.0000	*0.0543*
B	500	$BLCM_{MAR}$	0.0000	0.0827
B	500	$BLCM_{MNAR}$	0.0000	0.1101
B	1000	$BLCM_0$	-0.0000	0.0444
B	1000	$BLCM_{MAR}$	-0.0000	0.0368
B	1000	$BLCM_{MNAR}$	-0.0000	*0.0228*
C	500	$BLCM_0$	0.0000	0.0399
C	500	$BLCM_{MAR}$	-0.0000	*0.0324*
C	500	$BLCM_{MNAR}$	0.0000	0.0413
C	1000	$BLCM_0$	0.0000	0.0277
C	1000	$BLCM_{MAR}$	-0.0000	*0.0231*
C	1000	$BLCM_{MNAR}$	-0.0000	0.0286

Note. Bias and RMSE are averaged across the three latent class proportions. The lowest RMSE within each scenario and sample size is shown in bold.

Across all scenarios and sample sizes, bias was negligible for the three models, and differences in latent class proportion recovery were therefore primarily reflected in the RMSE values. The relative performance of $BLCM_{MNAR}$ varied considerably with sample size and the strength of the covariate effects.

Under Scenario A, $BLCM_{MNAR}$ exhibited the highest RMSE at $n = 500$, but its RMSE decreased substantially from 0.1282 to 0.0415 as the sample size increased to $n = 1000$. At the larger sample size, $BLCM_{MNAR}$ attained the lowest RMSE among the three models. A similar sample-size pattern occurred under Scenario B: its RMSE decreased from 0.1101 at $n = 500$ to 0.0228 at $n = 1000$, with the latter being lower than the corresponding values of 0.0444 for $BLCM_0$ and 0.0368 for $BLCM_{MAR}$. These results indicate substantially improved class proportion recovery for $BLCM_{MNAR}$ at the larger sample size under Scenarios A and B.

Under Scenario C, the differences among the three models were smaller. $BLCM_{MAR}$ attained the lowest RMSE at both sample sizes, whereas $BLCM_{MNAR}$ produced values of 0.0413 and 0.0286 at $n = 500$ and $n = 1000$, respectively. At $n = 1000$, the corresponding values for $BLCM_0$ and $BLCM_{MAR}$ were 0.0277 and 0.0231, indicating relatively similar class proportion recovery among the three models under the strong covariate-effect scenario.

Overall, the addition of the class-specific missingness process did not produce a uniform improvement in latent class proportion recovery. $BLCM_{MNAR}$ showed comparatively high RMSE at $n = 500$ under Scenarios A and B, followed by substantial reductions at $n = 1000$, where it attained the lowest RMSE in both scenarios. Under Scenario C, $BLCM_{MAR}$ retained the lowest RMSE at both sample sizes, while $BLCM_{MNAR}$ remained close to the other two models at $n = 1000$. Thus, the simulation results suggest that recovery under the nonignorable formulation is sensitive to both sample size and the underlying covariate-effect structure.

4.1.3. Recovery of Class-Membership Coefficients

Recovery of the regression coefficients governing covariate-dependent latent class membership was evaluated using bias, RMSE, posterior stability, empirical coverage probabilities, and credible interval widths. Because these coefficients are estimated only by models with covariate-dependent class membership, the comparison is restricted to $BLCM_{MAR}$ and $BLCM_{MNAR}$.

Table 4. Recovery of class-membership coefficients across simulation scenarios.

Scenario	*n*	Method	Bias	RMSE	Stability
A	500	$BLCM_{MAR}$	-0.0988	*0.4611*	0.4781
A	500	$BLCM_{MNAR}$	-0.1039	0.8014	0.8081
A	1000	$BLCM_{MAR}$	-0.0625	0.4457	0.4399
A	1000	$BLCM_{MNAR}$	-0.0177	*0.1746*	0.1546
B	500	$BLCM_{MAR}$	-0.1139	*0.5696*	0.5455
B	500	$BLCM_{MNAR}$	-0.3111	0.9650	0.9171
B	1000	$BLCM_{MAR}$	-0.1651	0.4725	0.3682
B	1000	$BLCM_{MNAR}$	0.0200	*0.4038*	0.2851
C	500	$BLCM_{MAR}$	-0.1262	0.8713	0.5589
C	500	$BLCM_{MNAR}$	-0.0330	*0.7917*	0.5331
C	1000	$BLCM_{MAR}$	-0.1136	0.7699	0.5657
C	1000	$BLCM_{MNAR}$	-0.1018	*0.7175*	0.6139

Note. Bias, RMSE, and stability are averaged across the class-membership coefficients governing covariate-dependent latent class membership. The lowest RMSE within each scenario and sample size is shown in bold.

Recovery of the class-membership coefficients varied across sample sizes and simulation scenarios. Under Scenarios A and B, $BLCM_{MNAR}$ exhibited higher RMSE and posterior variability than $BLCM_{MAR}$ at $n = 500$, but substantially lower RMSE at $n = 1000$. In Scenario A, its RMSE decreased from 0.8014 to 0.1746, while in Scenario B it decreased from 0.9650 to 0.4038. At $n = 1000$, the corresponding RMSE values for $BLCM_{MAR}$ were 0.4457 and 0.4725, respectively. Posterior stability showed a similar improvement for $BLCM_{MNAR}$, particularly under Scenario A.

Under Scenario C, $BLCM_{MNAR}$ attained lower RMSE than $BLCM_{MAR}$ at both sample sizes. At $n = 500$, the corresponding values were 0.7917 and 0.8713, respectively, while at $n = 1000$ they were 0.7175 and 0.7699. The differences were more modest than those observed under Scenarios A and B. Moreover, although $BLCM_{MNAR}$ had lower RMSE at $n = 1000$, its stability measure was slightly higher than that of $BLCM_{MAR}$, indicating that lower estimation error did not uniformly correspond to lower posterior variability.

Overall, $BLCM_{MNAR}$ did not uniformly improve coefficient recovery at the smaller sample size. Its relative performance improved at $n = 1000$, where it attained lower RMSE than $BLCM_{MAR}$ under all three scenarios. Thus, under the simulation conditions considered, coefficient recovery for the nonignorable formulation was sensitive to both sample size and the strength of the covariate effects.

Empirical coverage probabilities and average credible interval widths were additionally examined to assess uncertainty quantification for the class-membership coefficients. Table 5 presents the results for 90%, 95%, and 99% posterior credible intervals.

Table 5. Coverage and credible interval widths for class-membership coefficients.

Scenario	*n*	Method	Cov_{90}	Cov_{95}	Cov_{99}	$Width_{90}$	$Width_{95}$	$Width_{99}$
A	500	$BLCM_{MAR}$	0.8762	0.9143	0.9571	1.7567	2.0939	2.7538
A	500	$BLCM_{MNAR}$	0.9048	0.9286	0.9667	2.4462	2.9157	3.8345
A	1000	$BLCM_{MAR}$	0.8905	0.9524	0.9857	0.9081	1.0824	1.4235
A	1000	$BLCM_{MNAR}$	0.9238	0.9619	0.9857	0.9920	1.1824	1.5550
B	500	$BLCM_{MAR}$	0.7619	0.8143	0.8762	1.3112	1.5628	2.0554
B	500	$BLCM_{MNAR}$	0.6571	0.7238	0.8381	1.4574	1.7371	2.2845
B	1000	$BLCM_{MAR}$	0.4429	0.4762	0.5952	0.5930	0.7068	0.9295
B	1000	$BLCM_{MNAR}$	0.4952	0.5476	0.6429	0.5583	0.6655	0.8752
C	500	$BLCM_{MAR}$	0.4714	0.5524	0.6286	1.0820	1.2897	1.6961

Scenario	*n*	Method	Cov_{90}	Cov_{95}	Cov_{99}	$Width_{90}$	$Width_{95}$	$Width_{99}$
C	500	$BLCM_{MNAR}$	0.4571	0.5429	0.6190	1.0027	1.1951	1.5717
C	1000	$BLCM_{MAR}$	0.4524	0.4952	0.5714	0.6169	0.7353	0.9671
C	1000	$BLCM_{MNAR}$	0.4190	0.4810	0.5286	0.5981	0.7128	0.9375

Under Scenario A, empirical coverage was reasonably close to the nominal levels for both models, particularly for the 95% credible intervals. $BLCM_{MNAR}$ attained slightly higher 95% coverage than $BLCM_{MAR}$ at both sample sizes, although its intervals were also wider. The difference in interval width was particularly pronounced at $n = 500$, where the average 95% widths were 2.9157 and 2.0939 for $BLCM_{MNAR}$ and $BLCM_{MAR}$, respectively. At $n = 1000$, the corresponding widths decreased to 1.1824 and 1.0824.

Under Scenario B, coverage was below the nominal levels for both models. At $n = 500$, $BLCM_{MAR}$ attained higher 95% coverage (0.8143 versus 0.7238), whereas at $n = 1000$, $BLCM_{MNAR}$ attained the higher value (0.5476 versus 0.4762). Despite narrower intervals at $n = 1000$, coverage remained substantially below the nominal level for both models.

A similar pattern of undercoverage occurred under Scenario C. The 95% coverage probabilities were 0.5524 and 0.5429 at $n = 500$, and 0.4952 and 0.4810 at $n = 1000$, for $BLCM_{MAR}$ and $BLCM_{MNAR}$, respectively. Interval widths decreased with sample size for both models, but this narrowing was not accompanied by improved coverage.

Overall, the results distinguish point recovery from interval calibration. Although $BLCM_{MNAR}$ attained lower RMSE than $BLCM_{MAR}$ under all three scenarios at $n = 1000$, this improvement did not consistently translate into coverage closer to the nominal levels. In particular, substantial undercoverage persisted under Scenarios B and C for both models. Under Scenario C, $BLCM_{MNAR}$ attained lower RMSE than $BLCM_{MAR}$ at both sample sizes, but exhibited slightly lower empirical coverage across the 90%, 95%, and 99% credible intervals. This pattern may partly reflect the additional complexity associated with jointly estimating the class-specific missingness parameters ρ_{jk} together with the other model parameters [44]. Thus, under the simulation conditions considered, improved point recovery did not necessarily imply improved uncertainty quantification for the class-membership coefficients.

4.1.4. Recovery of MNAR Missingness Parameters

$BLCM_{MNAR}$ introduces class-specific observation probabilities, ρ_{jk}, which govern the missingness mechanism conditional on latent class membership. Specifically, ρ_{jk} represents the probability that response variable j is observed for an individual belonging to latent class k. Recovery of these parameters therefore provides a direct assessment of the model's ability to estimate the class-dependent nonignorable missingness mechanism.

Recovery of the ρ_{jk} parameters was evaluated using bias, RMSE, and stability across simulation replications. The corresponding results, averaged across the class-specific observation probabilities, are presented in Table 6.

Table 6. *Recovery of class-specific observation probabilities under the MNAR mechanism.*

Scenario	*n*	Bias	RMSE	Stability
A	500	-0.0136	0.0496	0.0431
A	1000	-0.0063	0.0466	0.0399
B	500	-0.0077	0.0468	0.0403
B	1000	-0.0019	0.0464	0.0344
C	500	-0.0030	0.0479	0.0305
C	1000	-0.0005	0.0459	0.0330

Note. Bias, RMSE, and stability are averaged across the ρ_{jk} parameters governing the class-dependent missingness mechanism. Lower stability values indicate less variability in parameter estimates across simulation replications.

Across all simulation scenarios and sample sizes, $BLCM_{MNAR}$ recovered the class-specific observation probabilities with relatively small bias. Bias ranged from −0.0136 to −0.0005 and generally decreased in magnitude as the sample size increased. In particular, bias approached zero at $n = 1000$ under Scenarios B and C, indicating little systematic deviation between the estimated and true class-specific observation probabilities.

The RMSE values were small and relatively consistent across the simulation settings, ranging from 0.0459 to 0.0496. Under Scenario A, RMSE decreased from 0.0496 at $n = 500$ to 0.0466 at $n = 1000$, while corresponding reductions from 0.0468 to 0.0464 and from 0.0479 to 0.0459 were observed under Scenarios B and C, respectively. The similarity of the RMSE values across the three scenarios suggests that recovery of the class-specific observation probabilities was not substantially affected by the strength of the covariate effects governing latent class membership.

The stability measure also remained low across the simulation settings, ranging from 0.0305 to 0.0431, indicating limited variability in the estimated observation probabilities across simulation replications. Stability improved with increasing sample size under Scenarios A and B, with the measure decreasing from 0.0431 to 0.0399 and from 0.0403 to 0.0344, respectively. Under Scenario C, the stability measure increased slightly from 0.0305 at $n = 500$ to 0.0330 at $n = 1000$; however, the difference was small and both values indicated relatively consistent estimation across replications.

Overall, the small bias, low RMSE, and limited variability across simulation replications indicate that $\text{BLCM}_{\text{MNAR}}$ adequately recovered the class-specific observation probabilities across the simulation settings. Recovery was also relatively consistent across scenarios with no, moderate, and strong covariate effects. These findings provide direct evidence, under the simulation conditions considered, that the proposed model can recover the additional parameters used to represent the class-dependent nonignorable missingness process.

4.2. Application of the Models to the 2022 KDHS Data

The Bayesian latent class multiple imputation models were applied to data from the 2022 KDHS to examine their empirical implications in the presence of multivariate categorical nonresponse. Three specifications were considered: the baseline model without covariates in the latent class membership model (BLCM_0), the covariate-dependent model under MAR (BLCM_{MAR}), and the covariate-dependent model under MNAR ($\text{BLCM}_{\text{MNAR}}$). The application examined the posterior latent class structure, covariate effects on class membership, the sensitivity of the resulting imputations to alternative assumptions about the missingness process, and the class-specific response probabilities under $\text{BLCM}_{\text{MNAR}}$.

4.2.1. Data and Missingness Structure

The analysis included 32,156 women from the 2022 KDHS individual recode dataset [46]. Missing observations were distinguished as either structural, arising from questionnaire skip patterns or respondent ineligibility, or imputable, corresponding to missing responses among respondents eligible to provide information. Table 7 summarizes the missingness structure of the eight response variables included in the analysis.

Table 7. Missingness structure of target response variables in the 2022 KDHS.

Variable	Structural	Eligible	Imputable	Imputable (%)
Decision health	10,048	22,108	3,796	17.2
Decision purchases	11,068	21,088	2,776	13.2
Decision visits	10,606	21,550	3,238	15.0
Skilled ANC	17,738	14,418	4,115	28.5
Skilled delivery	17,738	14,418	4,115	28.5
Fertility preference	18,053	14,103	0	0.0
Unmet need	20,816	11,340	0	0.0
Contraceptive use	5,258	26,898	0	0.0

Five of the eight response variables contained imputable missing values. Among eligible respondents, the proportion requiring imputation ranged from 13.2% for decision-making on major household purchases to 28.5% for skilled Antenatal Care (ANC) and skilled delivery. For fertility preference, unmet need, and contraceptive use, all missing observations were structural, and these variables therefore required no imputation. The imputable missingness pattern was non-monotone, with missing responses occurring across different variables without a common hierarchical ordering.

Observed-data logistic regression diagnostics indicated that imputable missingness in each of the five affected variables was associated with multiple observed respondent characteristics, showing that the observed covariates contained information

relevant to the missingness process. These associations, however, cannot distinguish MAR from MNAR, because possible dependence on unobserved responses cannot be determined from the observed data alone. Accordingly, $BLCM_{MAR}$ and $BLCM_{MNAR}$ were fitted as alternative missingness specifications, allowing the sensitivity of the resulting imputations to the assumed missingness process to be examined.

4.2.2. Model Implementation and Convergence Assessment

The number of fitted latent classes was determined separately for each model using a posterior occupancy-based procedure. Starting from a deliberately large number of classes, the number of fitted classes was progressively reduced while monitoring posterior class occupancy until all fitted classes exhibited nonzero posterior occupancy. This procedure resulted in $K = 10$ for $BLCM_0$, $BLCM_{MAR}$, and $BLCM_{MNAR}$. The final chains comprised 4,000, 8,000, and 30,000 iterations for $BLCM_0$, $BLCM_{MAR}$, and $BLCM_{MNAR}$, respectively, with corresponding burn-in periods of 1,600, 5,500, and 28,000 iterations. A thinning interval of $d = 2$ was used for all models, and 500 posterior draws were retained. From these draws, 100 approximately equally spaced posterior draws were selected, with each draw used to generate one completed dataset, yielding 100 multiply imputed datasets per model.

Convergence was assessed using observed-data log-likelihood traces together with numerical stability diagnostics. Figure 3 presents the post-burn-in log-likelihood traces over the model-specific diagnostic regions. Across the three models, the traces fluctuated around relatively stable levels without evidence of sustained systematic drift, supporting the selected burn-in periods. $BLCM_{MNAR}$ required a substantially longer burn-in period before reaching a stable log-likelihood region than $BLCM_0$ and $BLCM_{MAR}$, and was correspondingly more computationally demanding when fitted to the 2022 KDHS data.

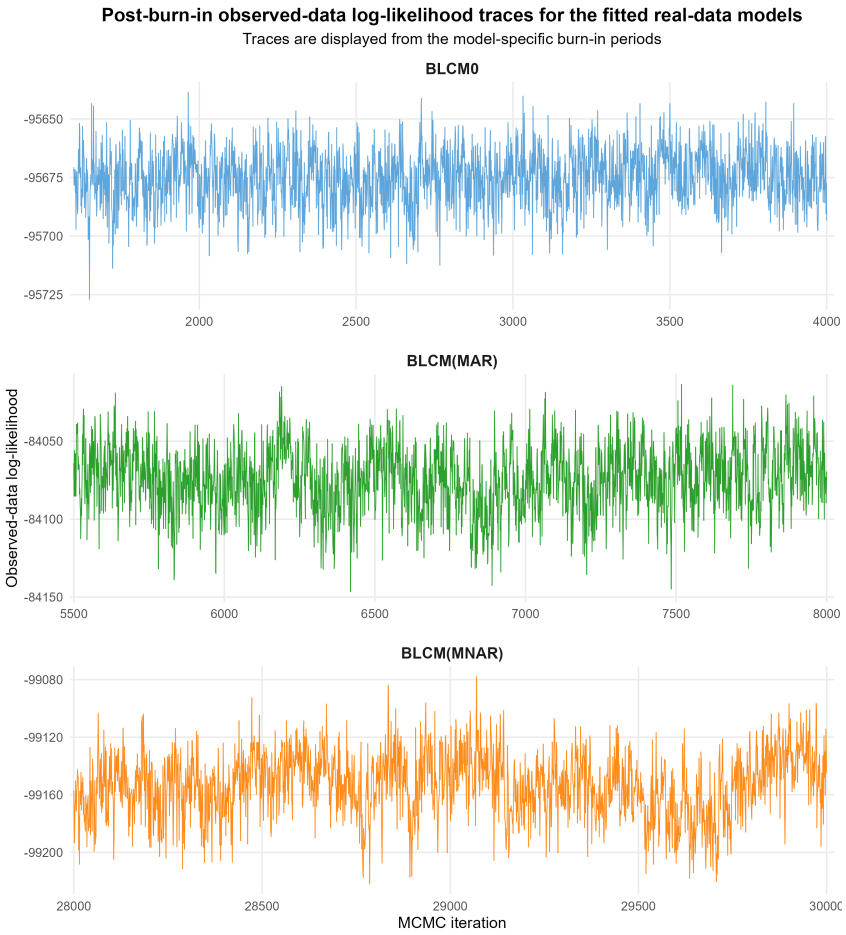


Figure 3. Post-burn-in observed-data log-likelihood traces for $BLCM_0$, $BLCM_{MAR}$, and $BLCM_{MNAR}$. The traces are displayed from the model-specific burn-in periods of 1,600, 5,500, and 28,000 iterations, respectively.

4.2.3. Latent Class Structure

Although all three models were fitted with $K = 10$, their posterior allocation of respondents across the latent classes differed. Table 8 presents the posterior mean class occupancies under the three specifications.

Table 8. Posterior mean latent class occupancies under the fitted models.

Class	$BLCM_0$	$BLCM_{MAR}$	$BLCM_{MNAR}$
Class 1	7.17	*0.86*	*0.74*
Class 2	8.00	12.39	9.05
Class 3	11.14	8.43	5.21
Class 4	12.64	8.12	14.70
Class 5	10.09	10.16	8.70
Class 6	23.80	12.15	19.85
Class 7	*1.04*	16.56	7.66
Class 8	5.30	12.04	13.42
Class 9	20.67	*4.53*	8.27
Class 10	*0.15*	14.77	12.40

Note. Entries are posterior mean occupancy percentages, averaged over the retained posterior draws. Bold values denote posterior mean occupancies below 5%. The 5% threshold is used as an interpretive criterion and was not used to determine the fitted value of K.

For interpretation of the effective latent structure, a 5% minimum posterior mean occupancy criterion was adopted. Following [47], the 5% threshold was treated as a practical benchmark for identifying classes with sufficient representation for substantive interpretation rather than as an absolute statistical threshold, since smaller classes may still represent meaningful population subgroups. Under this criterion, $BLCM_0$ and $BLCM_{MAR}$ each exhibited eight effectively represented classes, whereas $BLCM_{MNAR}$ exhibited nine. Under $BLCM_0$, Classes 7 and 10 fell below the threshold; under $BLCM_{MAR}$, Classes 1 and 9 fell below it; and under $BLCM_{MNAR}$, only Class 1 had a posterior mean occupancy below 5%. These results indicate that the alternative model specifications supported different distributions of posterior mass across the fitted components.

4.2.4. Covariate Effects on Latent Class Membership

Covariate effects on latent class membership were examined for $BLCM_{MAR}$ and $BLCM_{MNAR}$. Table 9 reports the 10 largest absolute posterior effects from each model among coefficients whose 95% credible intervals excluded zero.

Table 9. Selected posterior covariate effects on latent class membership.

Model	Contrast	Covariate effect	Mean	SD	2.5%	97.5%
$BLCM_{MAR}$						
$BLCM_{MAR}$	Class 5 vs Class 1	Marital: living with partner	5.89	0.78	3.78	6.82
$BLCM_{MAR}$	Class 4 vs Class 1	Region: Tana River	-5.79	1.32	-9.26	-3.89
$BLCM_{MAR}$	Class 6 vs Class 1	Region: Marsabit	5.42	0.58	4.29	6.49
$BLCM_{MAR}$	Class 7 vs Class 1	Marital: married	-4.77	0.31	-5.29	-4.13
$BLCM_{MAR}$	Class 3 vs Class 1	Marital: living with partner	4.54	0.59	3.23	5.57
$BLCM_{MAR}$	Class 7 vs Class 1	Parity	-4.36	0.22	-4.75	-3.91
$BLCM_{MAR}$	Class 8 vs Class 1	Region: Marsabit	4.34	0.64	3.17	5.52
$BLCM_{MAR}$	Class 10 vs Class 1	Region: Turkana	4.33	0.86	2.17	5.72
$BLCM_{MAR}$	Class 9 vs Class 1	Region: Vihiga	-4.31	1.72	-8.71	-1.58
$BLCM_{MAR}$	Class 7 vs Class 1	Region: Marsabit	4.23	0.68	2.91	5.43

Model	Contrast	Covariate effect	Mean	SD	2.5%	97.5%
$BLCM_{MNAR}$						
$BLCM_{MNAR}$	Class 3 vs Class 1	Marital: separated	14.23	0.39	13.59	14.99
$BLCM_{MNAR}$	Class 9 vs Class 1	Marital: married	-10.48	0.57	-11.82	-9.43
$BLCM_{MNAR}$	Class 3 vs Class 1	Marital: widowed	9.46	0.32	8.94	10.06
$BLCM_{MNAR}$	Class 9 vs Class 1	Marital: separated	9.34	0.24	8.87	9.83
$BLCM_{MNAR}$	Class 9 vs Class 1	Age	9.20	0.65	8.21	10.49
$BLCM_{MNAR}$	Class 3 vs Class 1	Marital: divorced	8.88	0.21	8.48	9.29
$BLCM_{MNAR}$	Class 8 vs Class 1	Marital: separated	-8.36	1.91	-11.00	-4.57
$BLCM_{MNAR}$	Class 7 vs Class 1	Region: Kilifi	-7.31	1.06	-8.85	-5.07
$BLCM_{MNAR}$	Class 3 vs Class 1	Parity	-6.62	1.17	-8.37	-4.40
$BLCM_{MNAR}$	Class 10 vs Class 1	Region: Homa Bay	-6.29	0.73	-8.70	-5.27

Note. The table reports the 10 largest absolute posterior covariate effects within each covariate-dependent model among coefficients whose 95% credible intervals excluded zero. Positive coefficients indicate increased log-odds of membership in the specified class relative to Class 1, whereas negative coefficients indicate decreased log-odds. For categorical covariates, coefficients are interpreted relative to the corresponding reference category.

Marital status and region were prominent among the largest posterior covariate effects in both models. Under $BLCM_{MAR}$, large effects were associated with marital status and several regional contrasts, including Tana River, Marsabit, Turkana, and Vihiga. Under $BLCM_{MNAR}$, marital-status contrasts were particularly prominent, while age, parity, and regional effects also appeared among the largest coefficients. These results indicate substantial heterogeneity in latent class membership associated with demographic and geographic characteristics. Because the latent classes were estimated separately under the two model specifications, coefficient magnitudes are interpreted within each model rather than as direct effect-size comparisons between corresponding class labels across models.

4.2.5. Sensitivity of Imputation to the Missingness Specification

Sensitivity to the missingness specification was first assessed by comparing the observed-case distributions with the mean completed-data distributions under the alternative model specifications. Table 10 shows the resulting category distributions for the five response variables with imputable missing values.

Table 10. *Observed and posterior mean completed-data category distributions under the fitted models.*

Variable	Category	Observed	$BLCM_0$	$BLCM_{MAR}$	$BLCM_{MNAR}$
	Husband/partner alone	16.27	16.33	14.89	16.88
	Other	0.09	0.10	0.08	3.59
Decision health	Respondent alone	38.52	38.36	34.83	35.33
	Respondent and husband/partner	44.95	45.04	50.06	40.43
	Someone else	0.17	0.18	0.15	3.77
	Husband/partner alone	22.90	22.91	21.20	22.61
	Other	0.12	0.13	0.11	2.52
Decision purchases	Respondent alone	18.53	18.59	16.81	18.88
	Respondent and husband/partner	58.15	58.07	61.59	53.24
	Someone else	0.30	0.31	0.30	2.74
	Husband/partner alone	18.62	18.57	16.62	19.14
	Other	0.04	0.05	0.04	2.74
Decision visits	Respondent alone	22.83	22.89	19.94	22.35
	Respondent and husband/partner	58.40	58.39	63.31	52.76

Variable	Category	Observed	$BLCM_0$	$BLCM_{MAR}$	$BLCM_{MNAR}$
Skilled ANC	Someone else	0.10	0.10	0.09	3.02
	No	3.83	3.76	3.72	14.66
	Yes	96.17	96.24	96.28	85.34
Skilled delivery	No	0.48	0.51	0.49	13.08
	Yes	99.52	99.49	99.51	86.92

Note. Entries are percentages. The observed column reports category proportions among non-missing observed responses. The model columns report posterior mean category proportions in the completed datasets, combining originally observed and imputed values and averaged across the multiple imputed datasets. Only response variables with imputable missing values are reported.

For skilled delivery, 99.52% of the observed responses were in the "Yes" category. The corresponding completed-data proportions remained very similar under $BLCM_0$ (99.49%) and $BLCM_{MAR}$ (99.51%), but decreased to 86.92% under $BLCM_{MNAR}$. A similar pattern was observed for skilled antenatal care, for which the observed proportion in the "Yes" category was 96.17%, compared with completed-data proportions of 96.24%, 96.28%, and 85.34% under $BLCM_0$, $BLCM_{MAR}$, and $BLCM_{MNAR}$, respectively. Differences were also evident in the household decision-making variables. In particular, the completed-data proportion for joint decision-making was lower under $BLCM_{MNAR}$ than under $BLCM_{MAR}$ for all three decision-making outcomes. Overall, $BLCM_0$ and $BLCM_{MAR}$ produced completed-data distributions that remained comparatively close to the observed-case distributions, whereas larger distributional shifts were evident under $BLCM_{MNAR}$.

Because completed-data distributions combine originally observed and imputed responses, however, they can attenuate differences arising specifically from the imputation model. Sensitivity was therefore examined more directly by restricting the comparison to observations that were originally imputable-missing. Table 11 reports the posterior mean category distributions among these cases across the 100 multiply imputed datasets.

Table 11. *Posterior mean distributions of imputed values among originally missing cases.*

Variable	Category	$BLCM_0$	$BLCM_{MAR}$	$BLCM_{MNAR}$
Decision health	Husband/partner alone	16.60	8.19	19.78
	Other	0.14	0.06	20.50
	Respondent alone	37.60	17.02	19.95
	Respondent and husband/partner	45.44	74.68	18.63
	Someone else	0.22	0.05	21.14
Decision purchases	Husband/partner alone	22.92	9.96	20.69
	Other	0.17	0.03	18.38
	Respondent alone	19.02	5.46	21.19
	Respondent and husband/partner	57.55	84.26	20.90
	Someone else	0.34	0.28	18.84
Decision visits	Husband/partner alone	18.29	5.28	22.05
	Other	0.09	0.01	17.97
	Respondent alone	23.19	3.59	19.60
	Respondent and husband/partner	58.29	91.07	20.84
	Someone else	0.14	0.05	19.55
Skilled ANC	No	3.57	3.45	41.78
	Yes	96.43	96.55	58.22
Skilled delivery	No	0.59	0.54	44.65
	Yes	99.41	99.46	55.35

Note. Entries are percentages among observations that were originally imputable-missing for the corresponding response variable. Proportions were calculated within each of the 100 multiply imputed datasets and then averaged across imputations.

Restricting the comparison to the originally missing cases revealed substantially larger differences between the model specifications. For skilled delivery, $BLCM_0$ and $BLCM_{MAR}$ assigned 99.41% and 99.46%, respectively, of the originally missing responses to the "Yes" category, compared with 55.35% under $BLCM_{MNAR}$. Correspondingly, 44.65% of the missing skilled-delivery responses were assigned to the "No" category under $BLCM_{MNAR}$, compared with less than 1% under the other two models. A similar pattern was observed for skilled antenatal care: 41.78% of the missing responses were imputed as "No" under $BLCM_{MNAR}$, compared with 3.57% under $BLCM_0$ and 3.45% under $BLCM_{MAR}$.

Differences were also pronounced for the household decision-making variables. Under $BLCM_{MAR}$, the joint-decision category accounted for 74.68%, 84.26%, and 91.07% of imputed responses for decision-making on health care, major purchases, and visits, respectively. Under $BLCM_{MNAR}$, the corresponding proportions decreased to 18.63%, 20.90%, and 20.84%, with the imputed responses distributed more broadly across the five categories. These differences among the originally missing cases account for the direction of the shifts observed in the completed-data distributions, while also showing that the completed-data percentages can mask the magnitude of differences occurring specifically among imputed cases.

Overall, the completed-data and imputed-only comparisons demonstrate sensitivity of the imputation results to the assumed missingness specification, particularly for the maternal-health and household decision-making outcomes. The comparisons quantify the extent to which the imputed and completed-data distributions can change when dependence of the missingness process on the latent response structure is allowed.

4.2.6. MNAR Response-Probability Parameters

To further characterize the missingness process under $BLCM_{MNAR}$, the class-specific response-probability parameters, ρ_{jk}, were examined for the five variables containing imputable missingness. Larger values of ρ_{jk} correspond to higher probabilities that a response is observed within a latent class, whereas smaller values correspond to greater probabilities of missingness. Table 12 summarizes the posterior mean class-specific response probabilities.

Table 12. *Posterior summary of class-specific MNAR response-probability parameters (ρ_{jk}).*

Variable	Minimum	Median	Maximum	Mean
Decision health	0.0002	0.9997	0.9999	0.7952
Decision purchases	0.0003	0.9997	0.9999	0.7953
Decision visits	0.0003	0.9997	0.9999	0.7954
Skilled ANC	0.0002	0.9937	0.9998	0.7136
Skilled delivery	0.0002	0.9935	0.9998	0.7134

Note. Minimum, median, maximum, and mean values are calculated across the class-specific posterior mean response probabilities for each response variable.

The estimated response probabilities exhibited substantial heterogeneity across latent classes. For the three decision-making variables, the class-specific posterior mean response probabilities ranged from values near zero to values near one, while their medians were approximately 0.9997. Skilled antenatal care and skilled delivery displayed a similar pattern, with minimum response probabilities near zero and median probabilities of approximately 0.994. Thus, response probabilities were estimated to be very high in many latent classes but substantially lower in a smaller subset of classes.

This class-specific heterogeneity in response probabilities provides a mechanism through which $BLCM_{MNAR}$ can generate imputations that differ materially from those obtained under $BLCM_0$ and $BLCM_{MAR}$. In particular, the MNAR specification allows the probability that a response is observed to vary according to latent class membership through the class-specific parameters ρ_{jk}, a dependence not included in $BLCM_{MAR}$.

The empirical findings complement the simulation results while serving a different evaluative purpose. In the simulation study, where the true data-generating mechanism and param-

eter values were known, $BLCM_{MNAR}$ was able to recover the class-specific response probabilities with small bias and RMSE, although its relative performance varied across sample sizes and covariate-effect scenarios. In the 2022 KDHS application, the true missingness mechanism is unknown and direct assessment of parameter recovery is therefore not possible. Nevertheless, the substantial heterogeneity in the estimated class-specific response probabilities and the differences between the imputations obtained under $BLCM_{MNAR}$ and the MAR-based specifications demonstrate the practical implications of allowing the response process to depend on latent class membership. Thus, the simulation study provides evidence of parameter recovery under controlled conditions, while application to the 2022 KDHS data illustrates the empirical sensitivity of the resulting inferences to alternative assumptions about the missingness process.

5. Conclusion

This study developed a covariate-dependent Bayesian latent class model for multiple imputation of multivariate categorical data under MNAR and evaluated it alongside the covariate-dependent $BLCM_{MAR}$ formulation and the baseline $BLCM_0$ without covariates. By incorporating observed covariates into the latent class membership model, $BLCM_{MAR}$ extends the conventional $BLCM_0$. $BLCM_{MNAR}$ additionally specifies a class-specific model for the response process to accommodate non-ignorable missingness. Together, these extensions provide a unified Bayesian latent class framework for examining multiple imputation under alternative assumptions about the missingness process.

The simulation study demonstrated that incorporating observed covariates into the latent class membership model generally improved estimation accuracy relative to $BLCM_0$, which did not include covariates in the class-membership model. The models supported recovery of latent class proportions, with estimation performance varying across simulation scenarios and sample sizes. For the covariate-dependent models, recovery of the class-membership coefficients also varied across simulation settings, particularly with respect to estimation accuracy and interval calibration. $BLCM_{MNAR}$ also recovered the class-specific response probabilities, ρ_{jk}, with small bias and RMSE across the simulation settings. The simulation results also showed that imputation performance varied across the alternative missingness specifications..

Application to the 2022 KDHS further illustrated the practical implications of the alternative specifications. Although all three models were fitted with ten latent classes, their posterior class occupancies differed, and observed respondent characteristics were associated with latent class membership under the covariate-dependent models. The distributions of imputed values were sensitive to the assumed missingness specification. $BLCM_0$ and $BLCM_{MAR}$ produced broadly similar imputations for several outcomes, whereas $BLCM_{MNAR}$ produced substantially different distributions, particularly for skilled antenatal care, skilled delivery, and household decision-making outcomes. The class-specific response-probability parameters under $BLCM_{MNAR}$ further indicated substantial heterogeneity in the response process across latent classes.

These findings highlight the importance of examining non-ignorable missingness as part of sensitivity analysis to departures from the MAR assumption. The application to the 2022 KDHS data does not identify the underlying missingness mechanism; rather, it shows the extent to which imputations and resulting distributions may change under alternative assumptions about the missingness process. The proposed framework therefore provides a flexible approach for Bayesian multiple imputation of multivariate categorical data when both covariate-dependent latent class membership and potentially non-ignorable missingness are of interest.

The proposed $BLCM_{MNAR}$ has several limitations that should be considered when interpreting and extending the method. First, as with MNAR models generally, the missingness process cannot be fully identified from the observed data alone; consequently, inference depends on assumptions about the class-specific response process that cannot be empirically verified. The $BLCM_{MNAR}$ formulation should therefore be interpreted as a sensitivity model for assessing the robustness of inferences under $BLCM_{MAR}$ to departures from the MAR assumption. Second, posterior inference may be sensitive to prior specification, particularly where class-specific information is sparse or parameters are weakly identified. Third, the additional response-probability parameters increase the computational burden of $BLCM_{MNAR}$, with this burden expected to increase as the number of latent classes, response variables, covariates, and missing-data patterns increases. Finally, the simulation study evaluated performance under data-generating mechanisms constructed within the latent class framework and therefore does not establish robustness when the true data-generating process arises from a fundamentally different model family. These limitations define important areas for further evaluation and extension of the proposed method.

6. Recommendations for Further Research

Several directions for further research arise from the present study.

1) Future research could examine alternative specifications of the non-ignorable missingness process, including pattern-mixture and shared-parameter formulations, and assess their implications for Bayesian latent class multiple imputation.
2) Given the greater computational demands of $\text{BLCM}_{\text{MNAR}}$, future research could investigate more computationally efficient Bayesian estimation methods, particularly for large and high-dimensional datasets.
3) Future research could assess the robustness of $\text{BLCM}_{\text{MNAR}}$ when the fitted model differs from the true data-generating process, including incorrect specification of the number of latent classes and the class-specific missingness mechanism. This would help determine how such misspecification affects imputation and parameter estimation.
4) Further applications to survey and observational datasets with different missingness patterns and data structures could provide additional evidence on the empirical behaviour of $\text{BLCM}_{\text{MNAR}}$ beyond the 2022 KDHS application.

Abbreviations

ANC	Antenatal Care
BLCM	Bayesian Latent Class Models
KDHS	Kenya Demographic and Health Survey
LCM	Latent Class Models
MAR	Missing at Random
MCMC	Markov Chain Monte Carlo
MI	Multiple Imputation
MNAR	Missing Not at Random
RMSE	Root Mean Squared Error

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Author Contributions

Daniel Fundi Murithi: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualization, Writing – original draft, Writing – review & editing

Josephine Njeri Ngure: Supervision, Validation, Writing – review & editing

Victor Muthama Musau: Supervision, Validation, Writing – review & editing

Data Availability Statement

Additional materials supporting this study, including detailed numerical convergence diagnostics, supplementary simulation results, and additional results from the real-data application, are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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