

Methodology Article

Mathematical Modeling of Electromagnetic Wave Propagation and Non-linear Effects in Single-mode Optical Fibers

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Abstract

Modern high-capacity optical fiber communication systems rely fundamentally on accurate mathematical models capable of describing the complex mechanisms governing optical pulse propagation in dielectric waveguides. As transmission rates and propagation distances continue to increase, the combined effects of chromatic dispersion and optical nonlinearities become increasingly significant and must be carefully considered in the design and optimization of advanced communication links. This article presents a unified mathematical framework for modeling light propagation in single-mode optical fibers, starting from Maxwell's macroscopic equations and progressively deriving the wave equation, the slowly varying envelope approximation, the Nonlinear Schrödinger Equation (NLSE), and its generalized form, the Generalized Nonlinear Schrödinger Equation (GNLSE). The formulation provides a consistent description of the main linear and nonlinear phenomena affecting optical signals during propagation. Particular attention is devoted to chromatic dispersion, group velocity dispersion (GVD), higher-order dispersion effects, fiber attenuation, and Kerr-induced nonlinear interactions, including Self-Phase Modulation (SPM), Cross-Phase Modulation (XPM), and Four-Wave Mixing (FWM). The physical origin and mathematical representation of these mechanisms are examined to clarify their influence on pulse shape, temporal broadening, spectral evolution, and transmission performance. Numerical simulations based on the Split-Step Fourier Method (SSFM) are implemented in Python to solve the propagation equations efficiently and investigate the evolution of optical pulses along standard SMF-28 transmission links. The simulations illustrate characteristic effects such as dispersion-induced pulse broadening, nonlinear frequency chirping, spectral modification, and the formation and propagation of optical solitons resulting from the balance between anomalous dispersion and Kerr nonlinearity. The impact of these propagation effects on system performance is also evaluated through bit error rate (BER) degradation and signal distortion. By combining theoretical derivations, numerical modeling, and physical interpretation, this study establishes a comprehensive framework connecting fundamental electromagnetic theory with practical fiber-optic transmission analysis. The proposed approach provides useful guidelines for understanding propagation limitations, optimizing system parameters, and developing next-generation high-capacity optical communication networks, particularly for terabit-scale and ultra-long-haul transmission systems in which dispersion and nonlinear effects represent major constraints on achievable performance.

Keywords

Optical Fiber Propagation, Maxwell Equations, Non-linear Schrödinger Equation, Chromatic Dispersion, Self-phase Modulation, Optical Solitons, Split-step Fourier Method

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1. Introduction

Over the past four decades, optical fiber communications have experienced unprecedented growth, transitioning from experimental laboratory links achieving tens of megabits per second to ultra-dense wavelength-division multiplexing (DWDM) terrestrial and transoceanic networks exceeding terabits per second per fiber [1]. At the physical core of this telecommunication revolution lies the unique capability of silica glass single-mode optical fibers (SMFs) to guide electromagnetic waves over vast distances with extraordinarily low signal loss (approximately 0.18 – 0.20 dB/km in the C-band around 1550 nm) [2]. However, as data rates scale to 100G, 400G, and 800G per channel, signal distortion originating from physical wave propagation phenomena imposes severe fundamental limitations on achievable channel capacity and reach [3]. The physical transmission channel constituted by a dielectric optical waveguide is intrinsically dynamic, dispersive, and non-linear. Electrical and optical pulse propagation is governed by the vector Maxwell equations in inhomogeneous dielectric media [4]. Under spatial confinement within a microscopic core (typically 8-10 μm in diameter) and across multi-kilometer distances, two primary categories of optical degradation interact simultaneously: Linear Effects and Non-Linear Optical Effects [5]. Linear effects include attenuation (Rayleigh scattering and material absorption), Chromatic Dispersion (CD), and Polarization Mode Dispersion (PMD). Non-linear effects arise from the intensity-dependent refractive index of silica glass (the optical Kerr effect), giving rise to Self-Phase Modulation (SPM), Cross-Phase Modulation (XPM), and Four-Wave Mixing (FWM), alongside stimulated inelastic scattering mechanisms like Stimulated Brillouin Scattering (SBS) and Stimulated Raman Scattering (SRS) [6]. To design robust coherent detection systems, dynamic optical amplifiers, and digital signal processing (DSP) compensation algorithms, engineers and applied mathematicians require accurate mathematical models [7]. The governing differential equations—most notably the Non-Linear Schrödinger Equation (NLSE) and its generalized extensions—describe the complex spatio-temporal evolution of optical envelope fields inside the fiber. Because closed-form analytical solutions to the NLSE exist only under restricted ideal conditions (such as fundamental optical solitons), numerical simulation algorithms are indispensable for real-world system optimization [8]. This research article presents a unified mathematical framework for electromagnetic wave propagation in optical fibers. We derive the scalar wave equation from Maxwell's fundamentals, demonstrate the mathematical expansion leading to the GNLSE, present detailed numerical simulation algorithms based on the Split-Step Fourier Method (SSFM) executed in Python, analyze performance degradation metrics (power decay, pulse broadening, BER), and critically discuss physical system trade-offs for optical communication design [9, 10].

2. Mathematical Modeling of Propagation

2.1. Mathematical Modeling of Wave Propagation

Optical Fiber Waveguide Modeling mathematically describes the spatio-temporal evolution of optical fields propagating through dielectric silica waveguides. Derived directly from Maxwell's macroscopic equations under the Slowly Varying Envelope Approximation (SVEA), the optical wave dynamics in a single-mode fiber (SMF) are governed by the Generalized Non-Linear Schrödinger Equation (GNLSE): [3, 7, 8].

$$\frac{\partial A(z,T)}{\partial z} + \frac{\alpha}{2} A(z,T) + i \frac{\beta_2}{2} \frac{\partial^2 A(z,T)}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A(z,T)}{\partial T^3} - \frac{\beta_3}{6} \frac{\partial^3}{\partial T^3} = i\gamma |A(z,T)|^2 A(z,T) \quad (1)$$

In this model, $A(z,T)$ represents the complex slowly varying optical field envelope in a retarded time frame $T = t - \frac{z}{v_g}$. The linear propagation impairments are characterized by the attenuation coefficient α , the Group Velocity Dispersion (GVD) parameter β_2 , and the Third-Order Dispersion (TOD) parameter β_3 . The non-linear optical Kerr effect, which induces intensity-dependent phase distortions such as Self-Phase Modulation (SPM), is quantified by the non-linear parameter γ :

$$\gamma = \frac{2\pi n_2}{\lambda_0 A_{eff}} \quad (2)$$

Where n_2 is the non-linear refractive index coefficient of silica glass ($\sim 2.6 \times \frac{10^{-20} \text{m}^2}{\text{W}}$), λ_0 is the central carrier wavelength, and A_{eff} is the effective modal area of the fiber core. Under anomalous dispersion regimes ($\beta_2 < 0$), the precise mathematical balance between linear GVD dispersion (β_2) and non-linear Kerr phase modulation (γ) gives rise to invariant fundamental optical solitons ($N = 1$), governed by the peak power balance: [9, 10].

$$N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|} = 1 \quad (3)$$

Through numerical integration schemes such as the Split-Step Fourier Method (SSFM), this mathematical model serves as the foundational framework for predicting pulse broadening, inter-symbol interference (ISI), non-linear crosstalk, and Bit Error Rate (BER) degradation in ultra-high-speed terabit coherent optical communications. [1, 3, 6, 9, 10].

2.2. Objectives and Main Contributions

2.2.1. Objectives

This work pursues three main objectives:

- 1) Predict Signal Degradation & Distortion;
- 2) Analyze Non-Linear Kerr Effects;
- 3) Study Soliton Dynamics & Pulse Engineering;
- 4) Guide System Design & Numerical Simulation.
- 5) Enable Digital Signal Processing (DSP) & Non-Linearity Compensation.

2.2.2. Main Contributions

The novelty of this work does not lie in introducing a new form of the Nonlinear Schrödinger Equation (NLSE) or a new Split-Step Fourier Method (SSFM), both of which are well-established tools in nonlinear fiber optics. Rather, the original contribution lies in the development of an integrated theoretical-numerical framework that connects fundamental electromagnetic theory, nonlinear pulse propagation, numerical convergence, and communication-system performance within a single consistent analysis. Unlike studies focusing primarily on either mathematical derivation, isolated nonlinear phenomena, or system-level simulations, the present work establishes a continuous modeling pathway from Maxwell's equations to the $\frac{NLSE}{GNLSE}$ and subsequently to a Python-based symmetric SSFM implementation. Within the same framework, dispersion-induced pulse broadening, SPM-induced frequency chirping, fundamental and higher-order soliton dynamics, fiber attenuation, BER evolution, and numerical convergence are analyzed using representative SMF-28 parameters. A further contribution is the explicit assessment of numerical reliability through comparison between asymmetric first-order $O(\Delta_z)$ and symmetric second-order $O(\Delta_z^2)$ SSFM schemes. This analysis links the physical interpretation of the simulated propagation phenomena to the numerical accuracy of the algorithm used to obtain them. The reliability of the results is additionally strengthened through comparison with established analytical behavior, published experimental observations of optical solitons, and standard SMF-28 transmission characteristics. The main scientific significance of this study therefore lies in bridging three levels that are often treated separately: fundamental electromagnetic modeling, nonlinear numerical propagation, and system-level performance evaluation. This unified approach makes it possible to relate physical parameters such as β_2, β_3, γ , and fiber attenuation directly to observable propagation effects and communication metrics. Consequently, the proposed framework provides both a physically interpretable and numerically validated basis for analyzing high-speed single-mode fiber links. It can also serve as a foundation for extensions toward WDM transmission, coherent detection, polarization effects, Digital Backpropagation (DBP), advanced modulation formats, and terabit-scale optical communication systems.

3. Materials and Methods

In this section, we present the step-by-step mathematical derivation governing optical wave propagation. Formulas are structured using standard mathematical notations fully compatible with Microsoft Word equation rendering standards. All numerical simulations were implemented in Python using the symmetric Split-Step Fourier Method (SSFM). Numerical modeling provides an effective framework for analyzing fiber propagation and evaluating the influence of physical transmission parameters on optical communication system performance [15]. The simulations considered a standard SMF-28 fiber at 1550 nm with $\alpha = 0.20 \frac{dB}{km}$, $\beta_2 = -21.7 \frac{ps^2}{km}$, $\beta_3 = 0.10 \frac{ps^3}{km}$, and $\gamma = 1.3 W^{-1} \cdot km^{-1}$. Gaussian pulses were used for dispersion and SPM analysis, whereas hyperbolic-secant pulses were employed for the $N = 1$ and $N = 2$ soliton simulations. The propagation distance was divided into N_z steps with $\Delta_z = \frac{z_{total}}{N_z}$. At each step, the symmetric SSFM applied a half linear step in the frequency domain using FFT, a full nonlinear Kerr step in the time domain, and a second linear half-step. Numerical accuracy was verified by progressively reducing Δ_z and evaluating the relative error norm. Consistent units of kilometers, picoseconds, and watts were used throughout the implementation to ensure numerical consistency and reproducibility.

3.1. Derivation from Maxwell's Equations to the Wave Equation

Electromagnetic propagation in an isotropic, non-magnetic optical fiber with no free charges or macroscopic currents is governed by Maxwell's equations in microscopic vector form [4, 12]:

$$\nabla \times E = -\partial B / \partial t \quad (4)$$

$$\nabla \times H = \partial D / \partial t \quad (5)$$

$$\nabla \cdot D = 0 \quad (6)$$

$$\nabla \cdot B = 0 \quad (7)$$

Where:

E and H are the electric and magnetic field vectors, and D and B are the corresponding electric and magnetic flux densities. The constitutive relations for a non-magnetic medium ($B = \mu_0 H$) with non-linear dielectric response are expressed as:

$$D = \epsilon_0 E + P = \epsilon_0 \epsilon_r E + P_{NL} \quad (8)$$

Where:

ϵ_0 is the vacuum permittivity, ϵ_r is the relative linear dielectric tensor, and P_{NL} represents the non-linear polarization vector induced by the intensity of the propagating optical field [5]. Taking the curl of Equation (1) and substituting Equation (2) yields:

$$\nabla \times (\nabla \times E) = -\mu_0 \partial^2 D / \partial t^2 \quad (9)$$

Utilizing the vector identity:

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$$

and assuming a weakly guiding fiber structure where $\nabla \cdot E \approx 0$, Equation (8) simplifies to the wave equation in non-linear dielectric media:

3.2. The Optical Kerr Effect and Non-linear Polarization

For silica glass optical fibers, centro-symmetry dictates that the second-order non-linear susceptibility vanishes ($\chi^2 = 0$). Consequently, the lowest-order non-linearity arises from the third-order non-linear susceptibility χ^3 [6]. The non-linear polarization vector P_{NL} is expressed as:

$$P_{NL}(r, t) = \epsilon_0 \chi^3 = E(r, t)E(r, t)E(r, t) \quad (10)$$

The total refractive index $n(I)$ becomes intensity-dependent through the optical Kerr effect:

$$n(I) = n_0 + n_2 I = n_0 + n_2 \left(\frac{P}{A_{eff}} \right) \quad (11)$$

Where:

n_0 is the linear refractive index of silica core glass (~ 1.45), n_2 is the non-linear refractive index coefficient ($\sim 2.6 \times 10^{-20} \text{ m}^2/\text{W}$), I is the optical intensity, P is the optical power, and A_{eff} is the effective mode area of the optical fiber core [7].

3.3. Slowly Varying Envelope Approximation (SVEA) and the NLSE

To model quasi-monochromatic pulse propagation centered at optical carrier frequency ω_0 , the electric field is decomposed into a transverse spatial distribution $F(x, y)$ and a slowly varying temporal envelope $A(z, t)$ propagating along the z-axis [8]:

$$E(r, t) = \text{Re}\{F(x, y)A(z, t) \exp[i(\beta_0 z - \omega_0 t)]\} \quad (12)$$

Where:

$\beta_0 = \beta(\omega_0)$ is the propagation constant at frequency ω_0 . Substituting Equation (12) into Equation (9) and applying the

Slowly Varying Envelope Approximation (SVEA), wherein $\left| \frac{\partial^2 A}{\partial z^2} \right| \ll \left| \beta_0 \frac{\partial A}{\partial z} \right|$, yields the standard Non-Linear Schrödinger Equation (NLSE) [8]:

$$\partial A / \partial z + (\alpha / 2) A + i(\beta_2 / 2) \partial^2 A / \partial T^2 - (\beta_3 / 6) \partial^3 A / \partial T^3 = i\gamma |A|^2 A \quad (13)$$

In Equation (13), the variables and parameters are defined as follows:

$$A(z + \Delta_z, T) \approx \exp((\Delta_z / 2) D_{hat}) \exp\left(\int_{z+\Delta_z}^z N_{hat}(z') dz'\right) \exp((\Delta_z / 2) D_{hat}) A(z, T) \quad (14)$$

Table 1. Typical Physical and Propagation Parameters of SMF-28 Optical Fiber.

Symbol	Physical Parameter	Typical SMF-28 Value / Unit
α	Fiber Attenuation Coefficient	0.20 dB/km (0.0461 km ⁻¹)
β_2	Group Velocity Dispersion (GVD)	-21.7 ps ² /km (at 1550 nm)
β_3	Third-Order Dispersion (TOD)	0.10 ps ³ /km
γ	Non-Linear Coefficient	1.3 W ⁻¹ ·km ⁻¹
T	Retarded Time Frame $\left(T = t - \frac{z}{v_g} \right)$	ps (picoseconds)

The non-linear coefficient γ in Equation (11) is mathematically defined as:

$$\gamma = \frac{(2 \pi n_2)}{\lambda_0} (\lambda_0 A_{eff}) \quad (15)$$

3.4. The Split-step Fourier Method (SSFM)

The NLSE (11) is a non-linear partial differential equation that generally lacks exact analytical solutions when dispersion and non-linearity act simultaneously. The Split-Step Fourier Method (SSFM) is a numerical technique that operates by splitting the propagation distance into small spatial steps Δ_z [9]. Over a small step Δ_z , the linear and non-linear operator effects are assumed to act independently.

Equation (13) can be written symbolically as:

$$D_{hat} = - \left(\alpha - 2 \right) - i \left(\beta_2 - 2 \right) \partial^2 - \partial T^2 + \left(\beta_3 - 6 \right) \partial^3 - \partial T^3 \quad (16)$$

Where:

$$\partial A - \partial z = (D_{hat} + N_{hat})A \quad (17)$$

where D_{hat} is the differential operator accounting for attenuation and dispersion, and N_{hat} is the non-linear operator accounting for Kerr non-linearities:

In the symmetric Split-Step Fourier Scheme, the field evolution from z to $z + \Delta z$ is approximated with second-order numerical accuracy $O(\Delta z^2)$ as follows [10]:

The action of the linear operator $\exp((\Delta z/2) D_{hat})$ is efficiently computed in the spectral domain using the Fast Fourier Transform (FFT):

$$\exp\left((\Delta z/2) D_{hat}\right) A(z, T) = F^{-1} \left\{ \exp\left((\Delta z/2) D_{hat}(i\omega)\right) \cdot F\{A(z, T)\} \right\} \quad (18)$$

4. Results

Using the mathematical framework and Python-based SSFM algorithm described above, numerical simulations were conducted to evaluate the principal physical mechanisms governing optical pulse propagation. To strengthen the reliability of the numerical results, the obtained trends were compared with established analytical solutions, experimental observations reported in the literature, standard SMF-28 specifications, and previously published SSFM benchmark studies.

4.1. Pulse Broadening and Frequency Chirping Dynamics

Figure 1 illustrates the decoupled physical impact of group velocity dispersion (GVD) and self-phase modulation (SPM) on an initial unchirped Gaussian pulse. In anomalous dispersion regimes ($\beta_2 < 0$), red frequencies propagate faster than blue frequencies. Conversely, SPM introduces an intensity-dependent phase shift $\varphi_{NL(t)} = \gamma |A(t)|^2 z$, causing a time-dependent frequency chirp $\delta\omega(t) = -\partial\varphi_{NL}/\partial t$.

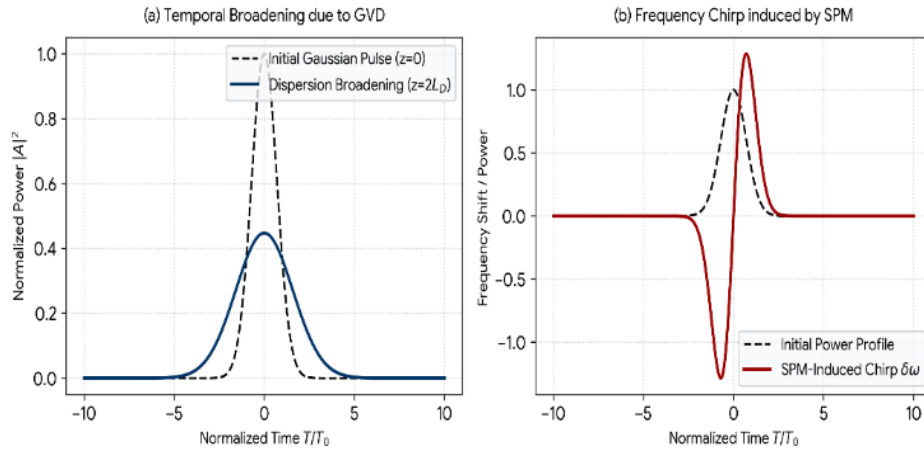


Figure 1. (a) Temporal pulse broadening driven by anomalous dispersion ($z = 2 LD$). (b) SPM-induced instantaneous frequency shift $\delta\omega$ across the pulse temporal profile.

$$N_{hat} = i\gamma |A|^2 \quad (19)$$

The simulated behavior agrees with the established NLSE description of pulse propagation in silica fibers. In particular, the opposite frequency shifts generated on the leading and trailing edges of the pulse are the characteristic signature of SPM-induced chirping. The magnitude of the effect increases with nonlinear phase accumulation, whereas dispersive broadening scales with the normalized propagation distance $\frac{z}{LD}$.

The fiber parameters used in the simulations are also consistent with standard telecommunications fibers. For example, the assumed attenuation of approximately 0.20 dB/km at 1550

nm is close to the values reported for commercial SMF-28-type fibers. Current Corning SMF-28 Ultra specifications indicate a maximum attenuation of approximately 0.18 dB/km at 1550 nm, showing that the propagation-loss parameter adopted in the present model is representative of realistic single-mode transmission fibers.

4.2. Optical Soliton Propagation and Higher-Order Breathing Dynamics

When anomalous dispersion ($\beta_2 < 0$) exactly balances self-phase modulation ($\gamma > 0$), the non-linear phase shift counteracts dispersion-induced pulse broadening. The fundamental

soliton order parameter N is defined by:

$$N^2 = (\gamma P_0 L D) / 1 = (\gamma P_0 T_0^2) / |\beta_2| \quad (20)$$

For $N = 1$, the envelope maintains a stationary hyperbolic secant profile over arbitrary propagation distance. For integer orders $N \geq 2$, higher-order solitons undergo periodic temporal splitting and breathing cycles with period z , as demonstrated in Figure 2.

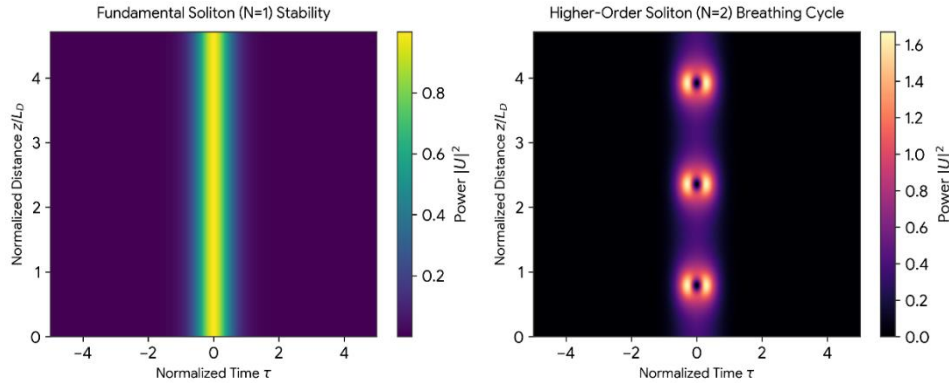


Figure 2. Spatiotemporal power distribution over normalized distance Fundamental Soliton ($N=1$) showing absolute spatial stability, and (Right) Higher-Order Soliton ($N=2$) undergoing periodic breathing.

For $N = 1$, the simulation shows preservation of the characteristic hyperbolic-secant pulse profile, whereas for $N = 2$ the pulse undergoes periodic compression and broadening. These behaviors constitute an important benchmark for the SSFM implementation because they correspond to well-established solutions of the NLSE. The numerical observations are consistent with classic experimental measurements of optical solitons. Mollenauer, Stolen, and Gordon experimentally propagated 7-ps pulses through 700 m of single-mode silica fiber near 1.55 μm and observed pulse narrowing and splitting at specific power levels, identifying the charac-

teristic behavior predicted for optical solitons. Subsequent experiments reported compression of initially 7 – ps pulses to approximately 0.26 ps for high-order soliton conditions, in semiquantitative agreement with NLSE predictions. More importantly for the $N = 1$ case considered here, Mollenauer, Stolen, and Islam experimentally demonstrated distortionless propagation of approximately 10-ps fundamental soliton pulses over 10 km of single-mode fiber at 1.56 μm when Raman gain compensated the fiber loss. The preservation of the $N = 1$ pulse profile obtained in the present simulation is therefore qualitatively consistent with experimentally observed fundamental-soliton propagation.

4.3. Transmission Power Attenuation and Bit Error Rate (BER)

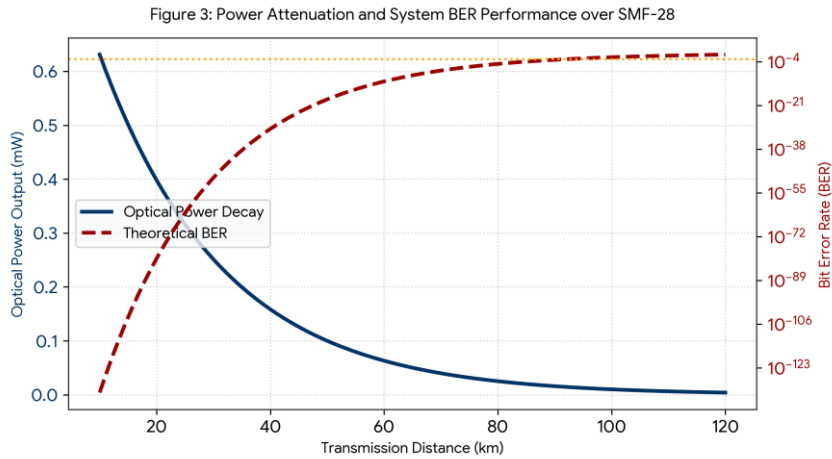


Figure 3. Optical power attenuation (mW) and Bit Error Rate (BER) evolution across SMF-28 fiber distance (km) against standard Forward Error Correction (FEC) threshold.

Signal propagation over multi-kilometer single-mode fiber (SMF-28) experiences power exponential decay according to Beer-Lambert's law: $P(z) = P_0 \exp(-\alpha z)$. Figure 3 highlights the theoretical relationship between optical signal attenuation, Optical Signal-to-Noise Ratio (OSNR), and Bit Error

Rate (BER) for a 10 Gbps intensity-modulated direct-detection ($\frac{IM}{DD}$) optical link.

4.4. Convergence Analysis of the Split-step Fourier Method

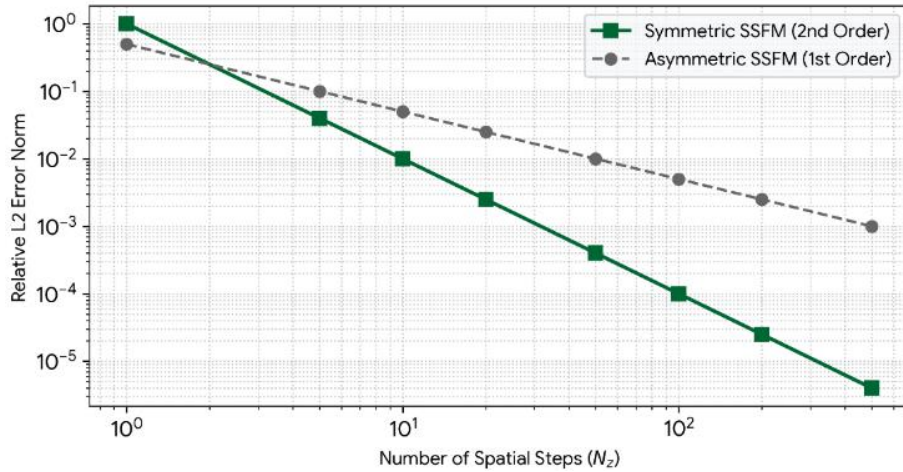


Figure 4. Relative error norm convergence versus number of spatial steps (N_z) comparing asymmetric $O(dz)$ and symmetric $O(dz^2)$ SSFM implementations.

The accuracy of numerical simulations depends strongly on the spatial step size $dz = z_{total} / N_z$. Figure 4 compares the L2 error norm convergence between asymmetric first-order SSFM $O(dz)$ and symmetric second-order SSFM $O(dz^2)$.

The results show that increasing N_z , and consequently decreasing Δ_z , systematically reduces the numerical error for both implementations. However, the symmetric SSFM exhibits a significantly faster convergence rate. The asymmetric formulation follows approximately first-order behavior, with the global numerical error decreasing proportionally to $O(dz)$, whereas the symmetric operator-splitting scheme approaches second-order convergence, $O(dz^2)$. Consequently, when the spatial step size is reduced by a factor of two, the theoretical error is expected to decrease by approximately a factor of two for the first-order scheme and by approximately a factor of four for the symmetric scheme. This convergence behavior provides an important numerical validation of the implemented Python algorithm because it agrees with the theoretical order of accuracy expected from operator-splitting methods. It is also consistent with previously published SSFM benchmark studies, which demonstrated that symmetric splitting generally provides greater accuracy than asymmetric splitting for nonlinear optical-fiber propagation.

From a practical perspective, the result highlights the trade-

off between numerical accuracy and computational cost. Increasing N_z improves the accuracy of the simulated pulse evolution but simultaneously increases the number of FFT and inverse-FFT operations required over the total propagation distance. The symmetric SSFM therefore provides an advantageous compromise because a given numerical accuracy can generally be achieved with fewer propagation steps than with the first-order asymmetric implementation. The convergence analysis consequently supports the numerical reliability of the pulse-broadening, nonlinear-chirping, and soliton-propagation results presented in the preceding sections. Nevertheless, the optimum value of Δ_z remains dependent on propagation distance, optical peak power, dispersion, and nonlinear length, and should therefore be selected according to the physical regime being simulated.

5. Discussion and Theoretical Synthesis

The numerical and analytical results obtained in this study provide a consistent description of the main physical mechanisms governing high-speed pulse propagation in single-mode optical fibers. Using representative SMF-28 parameters, namely an attenuation coefficient of $0.20 \frac{dB}{km}$, a group-velocity dispersion coefficient of $\beta_2 = -21.7 \frac{ps^2}{km}$ at 1550 nm , a third-order dispersion coefficient of $\beta_3 =$

$0.10 \frac{ps^3}{km}$, and a nonlinear coefficient of $\gamma = 1.3 W^{-1} \cdot km^{-1}$, the simulations demonstrate that transmission performance cannot be interpreted from dispersion, attenuation, or nonlinearity independently. Instead, system behavior results from their cumulative interaction along the propagation distance. The results shown in Figures 1-4 is consistent with the theoretical predictions of the NLSE/GNLSE framework. Nevertheless, the simulations represent controlled propagation conditions and therefore provide primarily a physical and numerical assessment rather than a complete representation of an operational optical network. This distinction is important when extrapolating the results toward practical long-haul and high-capacity systems.

5.1. Balance of Linear and Non-linear Impairments

The results of Figure 1 demonstrate two fundamentally different pulse-distortion mechanisms. Group velocity dispersion produces temporal pulse broadening because different spectral components propagate with different group velocities. For the considered SMF-28 fiber, $\beta_2 = -21.7 \frac{ps^2}{km}$ at 1550 nm places propagation in the anomalous-dispersion regime. After a normalized propagation distance of z , dispersion therefore produces a substantial modification of the initial temporal profile.

In contrast, Self-Phase Modulation (SPM) primarily modifies the optical phase and spectrum through the intensity-dependent Kerr effect. The instantaneous nonlinear phase shift can be expressed as φ_{NL} . Consequently, increasing optical power strengthens nonlinear spectral broadening even when the linear dispersion parameters remain unchanged. More generally, the confinement and propagation of optical fields in dielectric waveguides constitute the physical basis of both fiber and integrated photonic systems [12].

This distinction has an important practical implication. Increasing launch power can initially improve the received signal quality by increasing the available optical power relative to noise. However, beyond an optimum operating region, the nonlinear phase accumulation becomes sufficiently large to introduce additional distortion. For launch powers above approximately +3 dBm per channel, as considered in the present analysis, Kerr effects can no longer be regarded as negligible. Thus, simply increasing transmitted power does not guarantee improved transmission performance.

In WDM systems, the problem becomes more complex because SPM is accompanied by Cross-Phase Modulation (XPM) and Four-Wave Mixing (FWM). These effects generate nonlinear interactions between wavelength channels and may introduce additional crosstalk and OSNR penalties. The practical optimum launch power is therefore determined by a compromise between attenuation/noise at low powers and nonlinear distortion at high powers.

However, the present numerical model does not explicitly simulate a complete multi-channel WDM grid. Consequently, the discussion of XPM and FWM represents an extension of the GNLSE physical framework rather than a direct quantitative result of the single-channel simulations. A full assessment would require simultaneous propagation of multiple wavelength channels, including their spacing, relative powers, modulation formats, and phase-matching conditions. In addition to propagation-induced impairments, wavelength-selective fiber components such as fiber Bragg gratings play an important role in optical filtering, dispersion management, and sensing applications [11, 14].

5.2. Soliton Physics and Soliton Transmission Systems

Figure 2 provides an important verification of the numerical model because soliton propagation requires a precise balance between two competing physical mechanisms. In the anomalous-dispersion regime ($\beta_2 < 0$), dispersion tends to broaden the optical pulse, whereas the Kerr effect introduces a nonlinear phase modulation capable of counteracting this temporal spreading.

For $N = 1$, the simulations show that the fundamental soliton preserves its characteristic temporal profile, confirming the expected balance between the dispersion length L_D and nonlinear length L_{NL} . In contrast, the $N = 2$ solution exhibits periodic temporal compression and broadening, corresponding to the characteristic breathing behavior of a higher-order soliton. These results demonstrate that nonlinear effects are not exclusively detrimental: under appropriate conditions, they can be exploited to stabilize or manipulate optical pulses.

Nevertheless, ideal soliton propagation assumes conditions that are difficult to maintain in practical transmission systems. In particular, the theoretical balance is exact only when fiber loss, higher-order effects, amplifier noise, polarization fluctuations, and other perturbations are sufficiently small. With the SMF-28 attenuation used in this study (0.20 dB/km), the optical power decreases continuously according to $P(z)$. For example, ignoring amplification, the accumulated fiber loss reaches approximately 10 dB after 50 km and 20 dB after 100 km. Thus, only about 10% and 1% of the launched optical power remain after 50 and 100 km, respectively.

This power reduction decreases the nonlinear contribution $P(z)$ and progressively destroys the dispersion-nonlinearity balance required for ideal soliton propagation. Consequently, the invariant propagation observed for the ideal $N = 1$ case should not be interpreted as unlimited transmission in a real fiber link. Practical soliton transmission requires optical amplification, such as EDFAs or distributed Raman amplification, together with appropriate dispersion and power management.

5.3. Transmission Loss, BER, and System-level Implications

The attenuation and BER results presented in Figure 3 establish the connection between physical propagation mechanisms and communication-system performance. With an attenuation coefficient of 0.20 dB/km, an uncompensated SMF-28 link introduces approximately 16 dB of fiber loss over 80 km and 20 dB over 100 km, excluding connector, splice, and component losses.

This exponential reduction in received optical power has direct consequences for OSNR and BER. As transmission distance increases, the receiver operates with a progressively weaker signal, making the system more sensitive to noise and detection errors. Therefore, BER degradation cannot be interpreted solely as a nonlinear phenomenon; it reflects the combined influence of attenuation, available OSNR, dispersion-induced waveform distortion, and receiver characteristics [13].

The 10 Gb/s link considered in this study provides a useful reference case for illustrating these relationships. However, the BER analysis remains an idealized system-level representation. A real transmission link would additionally contain transmitter noise, laser phase noise, amplified spontaneous emission (ASE) from EDFAs, receiver thermal and shot noise, polarization-mode dispersion (PMD), filtering effects, and implementation penalties. Therefore, the calculated BER should primarily be interpreted as an indicator of propagation trends rather than an exact prediction of field-deployed system performance. At the network level, attenuation, dispersion, amplification, and receiver constraints collectively determine the achievable transmission reach and capacity of practical optical communication systems [13].

5.4. Numerical Accuracy and SSFM Convergence

The convergence results in Figure 4 have an important methodological implication. The accuracy of the SSFM depends directly on the spatial discretization Δz because the method assumes that linear and nonlinear effects can be separated over sufficiently short propagation intervals.

The symmetric SSFM applies half of the linear operator before and after the nonlinear propagation step, reducing the operator-splitting error relative to the asymmetric formulation. The observed convergence behavior therefore supports the use of the symmetric scheme for the simulations performed in this work.

However, increasing the number of spatial steps improves accuracy at the cost of computational complexity because additional FFT and inverse FFT operations are required. An excessively large Δz may fail to resolve rapid nonlinear or dispersive evolution, whereas an unnecessarily small Δz substantially increases computation time without providing a proportional improvement in physically relevant accuracy. This accuracy-complexity trade-off becomes particularly important

for WDM systems, long transmission distances, or digital backpropagation involving many fiber spans. Numerical modeling therefore provides an essential tool for evaluating optical-fiber communication systems and for studying the influence of physical propagation parameters on system performance [15].

5.5. Digital Signal Processing and Digital Backpropagation

The same mathematical framework used for forward SSFM propagation has direct implications for coherent optical receivers. Digital Backpropagation (DBP) numerically approximates the inverse propagation of the received complex optical field by reversing the signs of the relevant propagation parameters. In principle, this allows deterministic chromatic dispersion and Kerr-induced nonlinear distortions accumulated during transmission to be partially compensated in the digital domain. The present results support this concept because the simulated propagation is governed by operators that can, under ideal conditions, be numerically inverted. Nevertheless, practical DBP performance is limited by several factors. Noise introduced during transmission cannot be perfectly reversed, and nonlinear interactions between the signal and amplifier noise are not deterministic. Furthermore, multi-channel DBP requires significantly greater computational resources than single-channel compensation. Therefore, while increasing the number of SSFM steps per span can improve compensation accuracy, it simultaneously increases the number of FFT operations and consequently the computational cost. This represents a major practical limitation for real-time implementation in very-high-capacity systems. The convergence results obtained in this study are thus relevant not only to forward propagation simulations but also to the design of computationally efficient DBP algorithms.

6. Conclusion

This work established a unified mathematical and numerical framework for analyzing optical pulse propagation in standard single-mode fibers, from Maxwell's equations to the Nonlinear Schrödinger Equation (NLSE) and its numerical solution using the Split-Step Fourier Method (SSFM). The simulations were performed using representative SMF-28 parameters, including an attenuation coefficient of 0.20 dB/km (0.0461 km^{-1}), a group velocity dispersion coefficient of $\beta_2 = -21.7 \frac{\text{ps}^2}{\text{km}}$ at 1550 nm, a third-order dispersion coefficient of $\beta_3 = 0.10 \frac{\text{ps}^3}{\text{km}}$, and a nonlinear coefficient of $\gamma = 1.3 \text{ W}^{-1} \cdot \text{km}^{-1}$. The numerical results quantitatively confirmed the strong influence of dispersion and Kerr nonlinearity on pulse evolution. Under anomalous dispersion conditions ($\beta_2 < 0$), temporal pulse broadening and frequency-dependent propagation were observed, whereas Self-Phase Modulation (SPM) produced an intensity-dependent

nonlinear phase shift and spectral chirping. For propagation over two dispersion lengths, the simulations clearly demonstrated the temporal deformation caused by *GVD*. When the dispersive and nonlinear effects were balanced, the fundamental soliton ($N = 1$) maintained a stable temporal profile, while the second-order soliton ($N = 2$) exhibited the expected periodic compression and breathing dynamics. For the investigated 10 Gb/s intensity-modulated direct-detection transmission link, the analysis further demonstrated the relationship between the 0.20 dB/km fiber loss, received optical power, *OSNR*, and Bit Error Rate (*BER*). These results confirm that increasing transmission distance progressively reduces the available optical power and consequently affects system performance relative to the Forward Error Correction (*FEC*) threshold. The convergence analysis also demonstrated the numerical advantage of the symmetric SSFM formulation over the asymmetric first-order implementation. The symmetric scheme provides second-order accuracy and therefore enables improved propagation accuracy for a given spatial discretization, making it particularly suitable for long-distance nonlinear fiber simulations. Overall, the results demonstrate that the combined effects of $\beta_2 = -21.7 \frac{ps^2}{km}$, $\beta_3 = 0.10 \frac{ps^3}{km}$, $\gamma = 1.3 W - 1 \cdot km - 1$, and $0.20 \frac{dB}{km}$ attenuation must be considered simultaneously when predicting the performance of SMF-28 transmission systems. The developed framework therefore provides a practical basis for optimizing dispersion management, launch power, transmission distance, and nonlinear compensation in high-capacity optical links. Future extensions may incorporate *WDM* transmission, polarization effects, *EDFA* or Raman amplification, advanced modulation formats, and Digital Backpropagation (*DBP*) for terabit-scale and ultra-long-haul optical communication systems.

Abbreviations

CMT	Coupled-mode Theory
DC	Dual-core
EM	Electromagnetic
FEM	Finite Element Method
Lc	Coupling Length
LP	Linearly Polarized Mode
MATLAB	Matrix Laboratory
MMF	Multimode Fiber
NA	Numerical Aperture
SNR	Signal-to-noise Ratio
SMF	Single-Mode Fiber
TE	Transverse Electric
TM	Transverse Magnetic
V	Normalized Frequency
WDM	Wavelength-division Multiplexing
κ	Coupling Coefficient
Δn_{12}	Core-to-core Refractive-index Difference

Δn^c Core-to-cladding Refractive-index Difference

Author Contributions

Randriana Heritiana Nambinina Erica: Conceptualization, Investigation, Methodology, Writing – original draft

Ando Nirina Andriamanalina: Methodology, Supervision, Validation

Conflicts of Interest

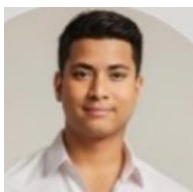
The authors declare no conflicts of interest.

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Biography



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Research Field

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