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Research Article

Interpretation of the Calculation Problem of Power Loss in Long-distance Transmission Lines——Starting from the Serious Errors in Calculation Methods in High School Physics Textbooks

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Abstract

This article points out the erroneous calculations of power loss and voltage loss in long-distance transmission lines in high school physics textbooks; Based on the basic circuit structure of long-distance power transmission, provide a series of correct functional relationships between power transmission power, line loss power and loss voltage, transmission line current and transformer ratio, load resistance, transmission line resistance, and the relationship between power supply voltage and transmission voltage, and discuss related issues based on these functional relationships.

Keywords

Long-Distance Power Transmission, Loss of Power, Loss of Voltage, Incorrect Calculation, Power Factor

1. Background of the Problem

Regarding the calculation and analysis of power loss and voltage loss in long-distance transmission lines, physics textbooks in high school and various versions have always adopted similar methods, which analyze or calculate conclusions based on the relationship between transmission power, transmission current, and transmission voltage. During the 1980s, textbooks [1, 2] only used this as a qualitative analysis without quantitative calculations, and there seemed to be no abnormal phenomena in the teaching process. However, after entering this century, some textbooks [3-5] almost all provided specific cases for quantitative calculations, but due to the lack of consideration of the matching relationship between transmission line resistance and load and transmitted power in the calculation process, the calculation results seriously violated the law of conservation of energy, resulting in a ridiculous phenomenon.

In this regard, Earlier literature [6, 7] pointed out this erroneous phenomenon and discussed in detail the issue of power loss in transmission system lines as a function of load and other factors. However, the commonly used new textbooks [8-11] have not corrected the errors of previous textbooks in response to this phenomenon, but have also made the same mistakes.

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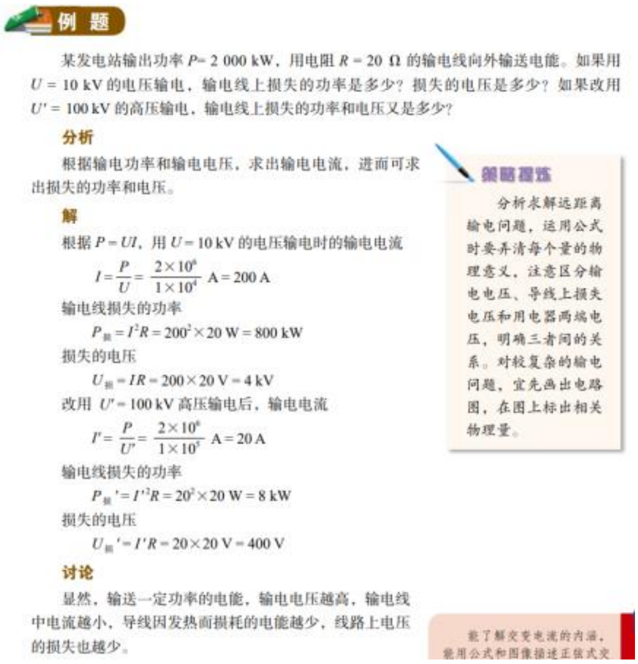
2. Serious Errors in High School Physics Textbooks

The example cases attached on page 70 of the Physics Selective Compulsory Volume 2 of the first edition of the general high school textbook published by Shandong Science and Technology Press in December 2019, as shown in Figure 1, the analytical thinking of this kind of case exposes the errors of high school physics textbooks in the calculation of transmission power, line loss power and loss voltage in long-distance transmission system.



例 题

某发电站输出功率 $P=2\ 000$ kW，用电阻 $R=20\ \Omega$ 的输电线向外输送电能。如果用 $U=10$ kV 的电压输电，输电线上损失的功率是多少？损失的电压是多少？如果改用 $U'=100$ kV 的高压输电，输电线上损失的功率和电压又是多少？

分析

根据输电功率和输电电压，求出输电电流，进而可求出损失的功率和电压。

解

根据 $P=UI$，用 $U=10$ kV 的电压输电时的输电电流

$$I=\frac{P}{U}=\frac{2\times10^6}{1\times10^4}\ \text{A}=200\ \text{A}$$

输电线损失的功率

$$P_{损}=I^2R=200^2\times20\ \text{W}=800\ \text{kW}$$

损失的电压

$$U_{损}=IR=200\times20\ \text{V}=4\ \text{kV}$$

改用 $U'=100$ kV 高压输电后，输电电流

$$I'=\frac{P}{U'}=\frac{2\times10^6}{1\times10^5}\ \text{A}=20\ \text{A}$$

输电线损失的功率

$$P_{损}'=I'^2R=20^2\times20\ \text{W}=8\ \text{kW}$$

损失的电压

$$U_{损}'=I'R=20\times20\ \text{V}=400\ \text{V}$$

讨论

显然，输送一定功率的电能，输电电压越高，输电线中电流越小，导线因发热而损耗的电能越少，线路上电压的损失也越少。

分析求解远距离输电问题，运用公式时要弄清每个量的物理意义，注意区分输电电压、导线上损失电压和用电器两端电压，明确三者间的关系。对较复杂的输电问题，宜先画出电路图，在图上标出相关物理量。

能了解交变电流的内涵，能用公式和图像描述正弦式交

Figure 1. Example and Analysis of Textbook.

2.1. Original Question Analysis and Presentation

[Example] A certain power station transmits power $P_0 = 2000$ kW, and uses a resistance $R_L = 20\Omega$ transmission line to transmit electrical energy outward. What is the power loss on the transmission line if a voltage of $U = 10$kV is used for power transmission? What is the voltage loss? If we switch to high voltage transmission with $U_1 = 100$kV, what would be the power and voltage loss on the transmission line?

[Analysis] Based on the transmission power and voltage, the transmission current can be calculated, and then the lost power and voltage can be determined.

[Solution] According to $P_0 = IU$, the transmission current when using $U = 10$kV voltage transmission

$$I = \frac{P_0}{U} = \frac{2\times10^6}{1\times10^4}\text{A} = 200\text{A} \tag{1}$$

Power loss of transmission lines

$$\Delta P = I^2 R_L = 200^2 \times 20\text{W} = 800\,\text{kW} \tag{2}$$

Lost voltage

$$\Delta U = I\,R_L = 200\times 20\text{V} = 4\text{kV} \tag{3}$$

When switching to $U_1 = 100$kV high-voltage transmission, the transmission current

$$I_1 = \frac{P_0}{U_1} = \frac{2\times10^6}{1\times10^5}\text{A} = 20\text{A} \tag{4}$$

Power loss of transmission lines

$$\Delta P_1 = I_1^2 R_L = 20^2 \times 20\text{W} = 8\text{kW} \tag{5}$$

Lost voltage

$$\Delta U_1 = I_1 R_L = 20\times 20\text{V} = 400\text{V} \tag{6}$$

Obviously, when transmitting a certain amount of power, the higher the transmission voltage, the smaller the current in the transmission line, and the less energy is lost due to heat generation in the wire, resulting in less voltage loss on the line.

2.2. Error Exposure and Root Causes

From the analytical approach to the above problems, it can be concluded that the power loss of a transmission line is directly proportional to the square of the transmission power of the power source and inversely proportional to the square of the transmission voltage; The loss voltage of transmission lines is directly proportional to the transmitted power of the power source and inversely proportional to the transmitted voltage. On the surface, this analysis may seem to have no problem, but with a little deeper research, its errors become apparent. To clarify the issue, a question is added on the basis of the original question: What is the power and voltage loss on the transmission line if $U_2 = 1$kV voltage transmission is used instead?

At this point, according to the above idea, it is easy to calculate the transmission current

$$I_2 = \frac{P_0}{U_2} = \frac{2\times10^6}{1\times10^3}\text{A} = 2\times10^3\text{A} \tag{7}$$

Power loss of transmission lines

$$\Delta P_2 = I_2^2 R_L = 2000^2 \times 20\text{W} = 80000\,\text{kW} \tag{8}$$

Lost voltage

$$\Delta U_2 = I_2 R_L = 2000 \times 20\,\text{V} = 40\,\text{kV} \tag{9}$$

It is not difficult to see that the power loss of transmission lines is much greater than the transmission power, and the voltage loss is also much higher than the transmission voltage. In fact, according to this approach, as long as the transmission voltage is lower than a certain value (for example, in this problem, it can be calculated that the transmission voltage is lower than $2\sqrt{10}\,\text{kV}$), the loss power of the transmission line is greater than the transmission power, and the loss voltage is also higher than the transmission voltage. Moreover, the lower the transmission voltage, the greater the power loss of the transmission line compared to the transmission power, and the higher the voltage loss compared to the transmission voltage. Obviously, this seriously violates the law of conservation of energy and is a rather absurd result.

The root cause of this error lies in the fact that the current in the transmission line is not unilaterally determined by the transmitted power and voltage, but is closely related to factors such as the resistance, load, and transformer ratio of the transmission line.

3. Determining Factors of Transmission Power

In order to have a clear understanding of the determining factors of the transmission power and line loss power of the transmission system. In this paper, the ideal transmission line (the transmission line has no resistance) and the non-ideal transmission line (the transmission line has resistance) are respectively analyzed and discussed through the ideal transformer as the transmission link.

Firstly, it should be clarified that a transformer is an electrical energy transmission component that integrates self inductance and mutual inductance based on the principle of electromagnetic induction. According to literature [12], the working process of transformers has a rigorous mathematical logic relationship; Under no-load conditions, the current in both the primary and secondary coils of an ideal transformer is zero.

3.1. Ideal Transmission Line

The basic structure of an ideal transmission line is shown in Figure 2, which includes three circuits. The current and related physical quantity symbols of each circuit have been indicated in the diagram. Let the transformation ratio of the step-up transformer be $k_1 = \frac{n_2}{n_1}$ ($k_1 > 1$), the transformation ratio of the step-down transformer be $k_2 = \frac{n_2'}{n_1'}$ ($k_2 < 1$), and the load be a pure resistor component.

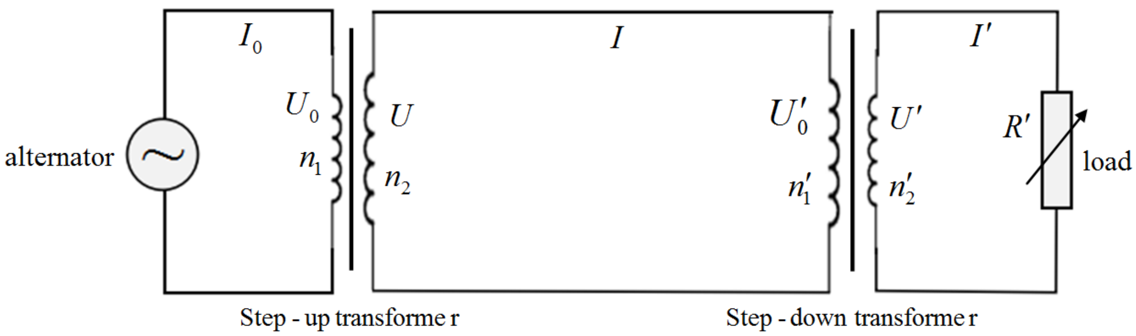


Figure 2. *Idesl Transmissino Line.*

Due to the fact that ideal transformers belong to pure inductive components, the power factor of their primary and secondary coils is 0, while the power factor of pure resistive loads is 1. Therefore, the transmission power of the generator (i.e. active power, the same below) is equal to the power consumed by the load, and there is no loss in the transmission line. According to the law of conservation of energy, there are

$$P_0 = I_0 \cdot U_0 = I \cdot U = I \cdot U_0' = I' \cdot U' = I'^2 R = \frac{U'^2}{R'} \tag{10}$$

According to the transformation ratio relationship of the transformer, $I_0 = k_1 I$ 、 $I = k_2 I'$ 、 $U = U_0' = k_1 U_0$ 、 $U' = k_2 U_0' = k_2 U$, So, equation (10) can be rewritten as

$$P_0 = k_1^2 k_2^2 \cdot \frac{U_0^2}{R'} = \frac{k_2^2}{R'} U^2 \tag{11}$$

This indicates that in an ideal transmission line, the output power of the generator is determined by the generator (power source) voltage or transmission voltage, transformer ratio, and load size.

According to equation (11), it can be seen that under ideal transmission line conditions, the transmission power of the generator (power source) is proportional to the square of the transmission voltage (or power source voltage), proportional to the square of the transformer ratio, and inversely proportional to the resistance value of the load resistor; When the transmission system is in an unloaded ($R' \to \infty$) state, the output power of the generator is equal to zero.

3.2. Non Ideal Transmission Lines

The basic structure of a non ideal transmission line is shown in Figure 3, which differs from Figure 2 only in the presence of resistance R_L in the transmission line between two ideal transformers. For ease of comparison, the physical quantity symbols are labeled with the same symbols as in Figure 2.

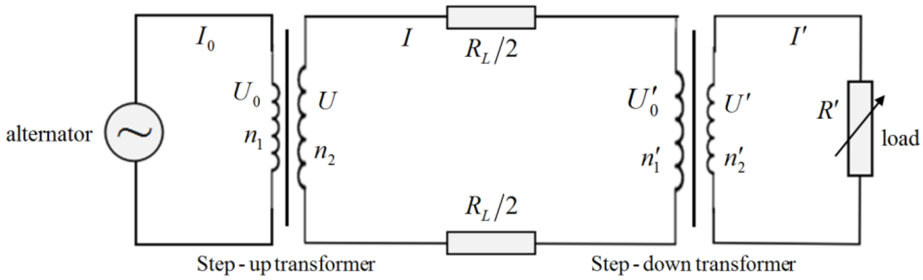


Figure 3. *Nonideal Transmissino Line.*

In this non ideal transmission line, the output power (active power) of the generator is equal to the power consumed by the load and the power lost by the resistance of the transmission line. According to the law of conservation of energy, there are

$$P_0 = I_0 \cdot U_0 = I \cdot U = I \cdot U_0' + I^2 R_L = I' \cdot U' + I^2 R_L = I'^2 R' + I^2 R_L \tag{12}$$

Similarly, based on the transformation ratio relationship of the transformer, $I_0 = k_1 I$ 、 $I = k_2 I'$ 、 $U = k_1 U_0 = U_0' + I R_L$ 、 $U' = k_2 U_0'$, Therefore, equation (12) can be rewritten as

$$P_0 = \underbrace{\frac{k_1^2 k_2^2 R' U_0^2}{\left(R' + k_2^2 R_L\right)^2}}_{\text{Load consumption power}} + \underbrace{\frac{k_1^2 k_2^4 R_L U_0^2}{\left(R' + k_2^2 R_L\right)^2}}_{\text{Line loss power}} = \frac{k_1^2 k_2^2}{R' + k_2 R_L} U_0^2 = \frac{1}{\dfrac{R'}{k_2^2} + R_L} U^2 \tag{13}$$

This expression indicates that in non ideal transmission lines, the output power of the generator is determined by the generator (power source) voltage or transmission voltage, transformer ratio, and the resistance of the load and transmission line.

According to equation (13), it is not difficult to understand that in non ideal transmission lines, the transmission power of the generator (power source) is proportional to the second power of the transmission voltage (or power source voltage), proportional to the second power of the step-up transformer ratio, monotonically increases with the increase of the second power of the step-down transformer ratio, and monotonically decreases with the increase of the resistance of the transmission line and load resistance (or monotonically increases with the decrease of the resistance of the transmission line and load resistance); When the transmission system is in an unloaded ($R' \to \infty$) state, the output power of the generator is equal to zero.

4. Correct Calculation of Power Loss and Voltage Loss

4.1. Loss of Power

In order to facilitate the discussion of the dependence between the power loss of transmission lines and the transmission power and voltage, we use ΔP to represent the power loss of transmission lines in (13). According to equation (13), we can obtain

$$\Delta P = \frac{\Delta P}{P_0} \cdot P_0 = \frac{\dfrac{k_1^2 k_2^4 R_L U_0^2}{(R' + k_2^2 R_L)^2}}{\dfrac{k_1^2 k_2^2 R' U_0^2}{(R' + k_2^2 R_L)^2} + \dfrac{k_1^2 k_2^4 R_L U_0^2}{(R' + k_2^2 R_L)^2}} \cdot P_0 = \frac{1}{1 + \dfrac{R'}{k_2^2 R_L}} \cdot P_0 = \frac{k_2^4 R_L}{(k_2^2 R_L + R')^2} U^2 = \frac{k_1^2 k_2^4 R_L}{(k_2^2 R_L + R')^2} U_0^2 \tag{14}$$

Obviously, combining equations (13) and (14), it can be concluded that the power loss of a transmission line is not only related to the transmission power or transmission voltage, power supply voltage, but also to the transmission line resistance, load resistance, and transformer ratio, and the power loss cannot be greater than the transmission power. Under the condition of constant load, transmission line resistance, and transformer ratio, the loss power of the transmission line is proportional to the square of the transmission voltage (or the square of the power supply voltage).

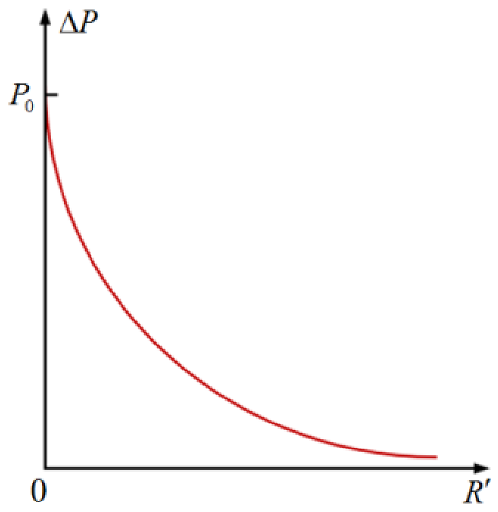


Figure 4. *Variation of Line Loss Power with Load Resistance.*

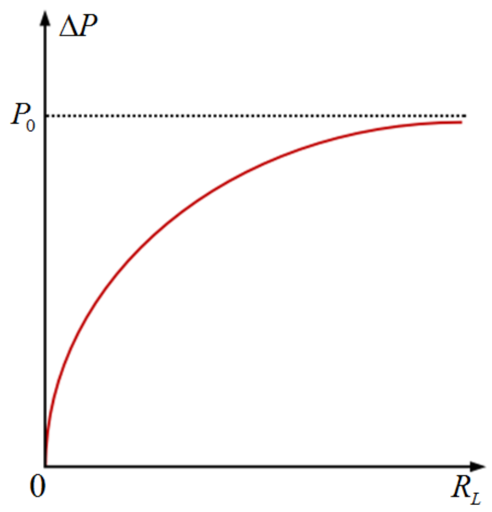


Figure 5. *Variation of Line Loss Power with Line Resistance.*

It is not difficult to see from equation (14) that if the resistance of the transmission line remains constant, when the load resistance $R' = 0$ (short circuit in the transmission system), the power loss of the line is equal to the transmitted power; When the load resistance $R \to \infty$ (transmission system unloaded) is applied, the power loss of the line is equal to zero, as shown in Figure 4. If the load resistance remains constant, when the transmission line resistance $R_L = 0$, the line loss power is equal to zero. When the transmission line resistance $R_L \to \infty$ is constant, the transmission line loss power is equal to the transmitted power, as shown in Figure 5.

4.2. Voltage Loss

The calculation of the loss voltage of the transmission line is also not difficult to obtain according to the ratio relationship of the transformer

$$\Delta U = IR_L = \frac{k_1 k_2^2}{k_2^2 + \frac{R'}{R_L}} \cdot U_0 = \frac{1}{1 + \frac{R'}{k_2^2 R_L}} \cdot U = \frac{k_2 R_L \sqrt{k_2 R_L + R'}}{k_2^2 R_L + R'} \cdot \sqrt{P_0} \quad (15)$$

This indicates that the loss voltage of transmission lines is related to the resistance of transmission lines, load resistance, and transformer ratio, and the loss voltage cannot be greater than the transmission voltage.

From equation (15), it is easy to see that if the resistance of the transmission line remains constant, when the load resistance $R' = 0$ (short circuit in the transmission system), the line loss voltage is equal to the transmission voltage; When the load resistance $R' \to \infty$ (transmission system unloaded) is present, the line loss voltage is equal to zero, as shown in Figure 6. If the load resistance remains constant, when the transmission line resistance $R_L = 0$, the line loss voltage is equal to zero. When the transmission line resistance $R_L \to \infty$ is reached, the line loss voltage is equal to the transmission voltage, as shown in Figure 7.

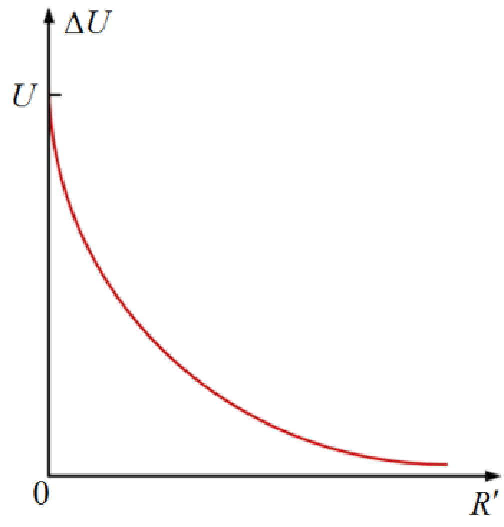


Figure 6. *Variation of Line Loss Voltage with Load Resistance.*

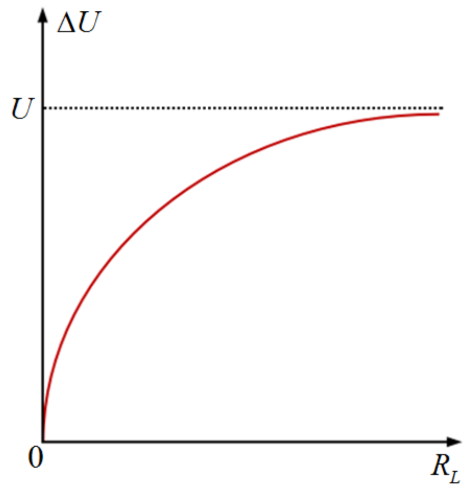


Figure 7. *Variation of Line Loss Voltage with Line Resistance.*

4.3. Transmission Current

Further discuss the relationship between current and power supply voltage or transmission voltage in transmission lines. According to equation (15), the current of the transmission line, i.e. the transmission current, can be obtained as

$$I = \frac{\Delta U}{R_L} = \frac{1}{R_L + \frac{R'}{k_2^2}} \cdot U = \frac{k_1}{R_L + \frac{R'}{k_2^2}} \cdot U_0 = \frac{k_2\sqrt{k_2 R_L + R'}}{k_2^2 R_L + R'} \cdot \sqrt{P_0} \quad (16)$$

This is an expression that determines the magnitude of current in a transmission line.

The formula (16) indicates that under the condition of constant load, transmission line resistance, and transformer ratio, the current of the transmission line is proportional to the transmission voltage (or power supply voltage).

5. Conclusion

(1) In long-distance transmission systems, the transmission power of generators (power sources), the loss power and voltage of transmission lines, and the current of transmission lines are determined by equations (13), (14), (15), and (16), respectively. Any speculation that violates science is not acceptable.

(2) Using equations (1) to (9) to calculate the transmission current, loss power, and loss voltage of a transmission line violates the law of conservation of energy and is an extremely absurd and erroneous measure. The reason for this serious error is that the deployment relationship of the transformer ratio is not considered, that is, the essential factor that the current of the transmission line is determined by the formula (16).

(3) From equations (13) or (14) and (15), it can be seen that when the transmission system is in a short-circuit ($R = 0$) state, the power or voltage transmitted by the generator (power source) is completely lost on the transmission line, which is not only meaningless but also a very dangerous phenomenon.

(4) In long-distance transmission engineering and teaching, it is usually said that "the line current and loss power of the transmission line decrease with the increase of the transmission voltage", which is not contradictory to the statements in equations (16) and (14) that "the current of the transmission line is proportional to the transmission voltage (or power supply voltage)" and "the loss power of the transmission line is proportional to the square of the transmission voltage (or power supply voltage)", respectively._Among them, the former is a comparison between two or more sets of transmission systems with different transformer ratios under equal transmission power conditions, while the latter is a comparison of the same set of transmission systems with different transmission power conditions determined by the transformer ratio. When comparing the power loss or voltage loss of two or more transmission systems, their transmission currents must be calculated separately according to equation (16) and cannot be arbitrarily assumed.

6. Explanation and Reflection

(1) In transmission engineering, an important indicator of power grid renovation is to change the load capacity and transformation ratio structure of transformers by replacing them based on the regional power consumption (load matching) situation.

(2) Given the complexity of power loss issues in transmission lines, related topics can only be qualitatively analyzed and discussed in high school physics textbooks, college entrance exams, and simulation preparations, and should not be quantitatively calculated. However, such propositions can be fully considered in university independent enrollment, strong foundation programs, and Olympic competitions.

(3) The textbook is the model for teaching and learning activities, and its preparation and review must be careful and rigorous to avoid small mistakes, let alone cause serious mistakes in knowledge.

Author Contributions

Huang Shaoshu: Writing – original draft
Feng JunJie: Writing – original draft

Conflicts of Interest

The author declares that there is no conflict of interest.

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Research Fields

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Feng JunJie: Mechanical and Electrical Engineering, Physics and Engineering, Theoretical Mechanics, Aerospace engineering, Atmospheric Physics, Marine Physics