Engineering Science
2026, Vol. 11, No. 4, pp. 81–95
https://doi.org/10.11648/j.es.20261104.12

SciencePG
Science Publishing Group

Research Article

Indicators of Asymmetry of Bends and Meanders

Vitaly Ivanovich Zamyshlyaev*

Federal State Budgetary Institution "State Hydrological Institute", Saint Petersburg, Russian Federation

Abstract

The question of the parameters characterizing the asymmetry of the bends and the meander deserves special consideration, since most bends and meanders have clearly asymmetrical shapes in the plan, and this fact has been noted by many researchers. In addition, indicators of the asymmetry of bends (meanders) are included in a number of expressions used to approximate the shapes of the axial line of the channel in the plan. This paper discusses some indicators used as asymmetry indices of bends, specifies the parameters included in the formulas determining these indices, and compares them from the standpoint of the completeness of the description of the asymmetry of river bends (meanders) and the convenience of use in analyzing the planimetry of winding channels. It is shown that using a river bend as the structural unit of a meandering channel allows for the unambiguous determination of all coefficients that characterize its degree of asymmetry. At the same time, the use of an alternative concept, where such a structural unit is a meander, makes it difficult or even impossible to use meander asymmetry indicators, which once again confirms the advantages of the "bend" concept over the "meander" concept. The asymmetry indicators are highlighted, which are the most informative from the point of view of the general assessment of the asymmetry of the plan forms of bends, or can be used in constructing approximations describing the planimetry of the channel. Relevant recommendations are provided.

Keywords

Meandering Channel, Systems of Bend Planimetric Indexes, Bend Wavelength, Bend Apex, Entrance Angle, Exit Angle, Bend Wings

1. Introduction

Looking at maps, aerial or satellite images of rivers, it is easy to see that many bends are obviously asymmetrical. The reasons for the asymmetry of river bends have been studied by different authors, and various hypotheses have been put forward to explain this phenomenon, but there is still no consensus among researchers of the riverbed process – a review of works on this topic can be found, for example, in the monograph "River Bends" [1].

The ability to quantitatively assess the nature and degree of asymmetry in meanders and bends is important both for the theoretical study of patterns of change in the plan channel shapes during river meandering, including the search for morphometric relationships between the measuring instruments

*Correspondence: Vitaly Ivanovich Zamyshlyaev (viza51@mail.ru)

This article is an English-language translation of “ПОКАЗАТЕЛИ АСИММЕТРИЧНОСТИ ИЗЛУЧИН И МЕАНДРОВ”, originally published in Russian in Эрозия почв и русловые процессы № 3, с. 42–56, 2025. DOI: https://doi.org/10.71367/3034-4638-2025-2-3-42-56. Permission for this translation was granted by SOIL EROSION AND RIVER CHANNEL PROCESSES.

Received: 4 June 2026; **Accepted:** 10 November 2026; **Published:** 26 November 2026

Copyright: © The Author(s), 2026. Published by Science Publishing Group. This is an **Open Access** article, distributed under the terms of the Creative Commons Attribution 4.0 License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution and reproduction in any medium, provided the original work is properly cited.

describing the planimetry of meandering channels, and for the practical application of the acquired knowledge, since the goal of such research should ultimately be the development of methods for predicting changes in the plan configuration of the channel or at least its morphological units – bends and meanders. Such methods may include approaches related to the construction of approximations of the plan forms of the channel centerline, where the parameters of the approximating curves may include asymmetry indicators. Therefore, the construction of indices that adequately characterize the asymmetry of bends and meanders is a relevant and important task of great theoretical and practical significance. The aim of this work is to analyze existing approaches to the study of the asymmetry of plan forms of a river channel, to describe some indices used as indicators of the asymmetry of bends and meanders, to clarify the parameters included in the formulas that determine these indices, and to compare them from the standpoint of completeness of description and ease of use in the analysis of the planimetry of meandering channels.

2. Analysis of Existing Indicators of Asymmetry

V. I. Zamyshlyaev [2] demonstrated that, when studying the plan configuration of meandering rivers, a number of authors use bends as structural units of the meandering channel, i.e. sections of the channel between two consecutive points of inflection of the center line, and others – meanders, which are understood as either sections of the channel consisting of two adjacent bends, or sections consisting of one bend located in the middle of the meander and two parts (approximately halves) of neighboring bends upstream and downstream. The choice of a structural unit determines, among other things, a significant difference in the system of parameters used to describe the plan forms of bends or meanders, often called the system of indexes. In fact, it is extremely difficult or almost impossible to formulate the concept of symmetry while remaining in the "meander" paradigm, including due to the ambiguity indicated in the definition of a meander. But whatever approach to the definition of a meander is applied, the symmetry of meanders must assume not only the symmetry of its constituent elements, but also that all the inflection points related to these elements must lie on the same straight line, which is not often observed in nature.

As noted above, a bend is typically defined as a section of the channel between *two successive inflection points,* defined as points on the channel's centerline at which the axial curvature changes sign. The first inflection point – upstream – is considered the initial (upper) point of the bend; the second inflection point – downstream – is considered the end (lower) point of the bend. These points – *a* and *b* – are indicated in Figure 1, where the following points are also indicated:

on the centerline of the channel: the apex of the bend *m* is the point farthest from the step line, and the point *n* is the point of intersection of the centerline of the channel with the perpendicular passing through the middle of the step line (the median perpendicular);

on the step line: point *c* is the middle of the bend step and point *d* represents the projection of the bend apex *m* onto the step line (this point, by analogy with a triangle, can be called the base of the bend apex).

In the following, unless otherwise stated, we will assume that in all the figures shown the flow moves from left to right, and, accordingly, we can consider the starting point of the bend to be left, and the ending point to be right.

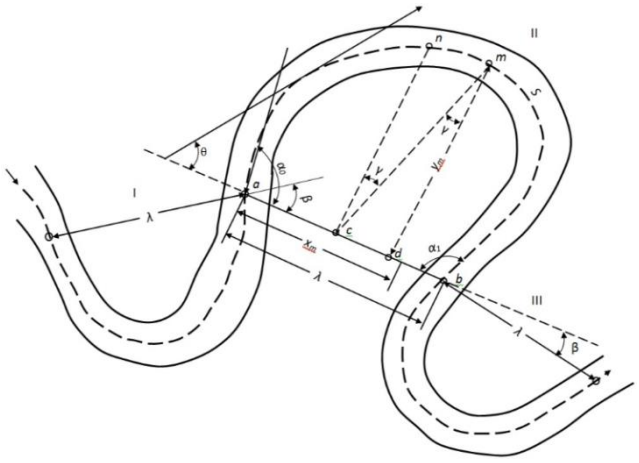


Figure 1. *Basic parameters of plan bend shape.*

Within the "bend" paradigm, the concept of asymmetry can be formulated with much greater precision, and various indices have been proposed by various authors as its indicator. For example, N. E. Kondratiev [3] developed one of the most detailed systems of plan morphological measures of meandering channels, based on the "bend" concept. In this system, *asymmetry bend* is expressed through the coefficient A_K (some notations in the text and figure captions in this article have been changed to maintain consistency compared to the notations used by the authors in their works):

$$A_K = \frac{2x_m - \lambda}{2y_m} \quad (1)$$

where the parameters included in this formula are shown in Figure 1: λ is the bend step, equal to the distance along a straight line between the initial *a* and end *b* points of the bend (step line); y_m – the height (span) of the bend, equal to the distance from the top of the bend *m* to the step line; x_m – the distance between the top point of the bend *a* and the point *d*; S – the length of the bend, equal to the length of the centerline of the channel between the inflection points; the entry angle α_0 and the exit angle α_1 are the angles formed by the tangent to the bend step line at the initial and end points of the bend, re-

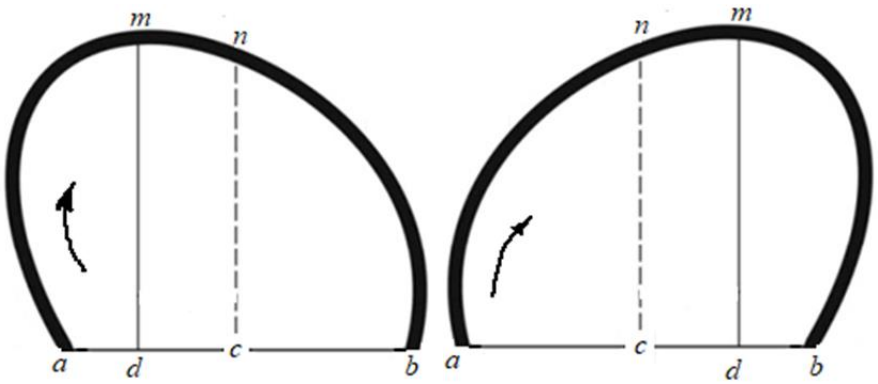
spectively (considered positive); $\alpha = \alpha_0 + \alpha_1$ is the bend turning angle; β is the angle of conjugation of bends, equal to the angle formed by the step lines of adjacent bends (considered positive if the entry angle of the lower bend is greater than the exit angle of the upper bend when the step line deviates to the right regarding the flow, and negative otherwise when the step line deviates to the left regarding the flow). Also, parameter θ in Figure 1 denotes the angle formed by the tangent to the centerline and the step line. Note that the "step of a bend" is called both the segment of the straight line, connecting the inflection points in the bend, and the distance between them.

Coefficient A_K can take on both positive and negative values or be equal to zero. If point *d* lies between points *a* and *c*, this means that $x_m < \lambda/2$. Moreover, by definition, if point *d* lies to the left of point *a* on the extension of the step line, then x_m takes on negative values. Thus, according to formula (1), if point *d* is to the left of point *c*, then $A_K < 0$, and if it is to the right, then $A_K > 0$.

Since the discrepancy between the projection of the vertex *m* on the step line (point *d)* and the mark of the middle of the bend step (point *c*) is obvious evidence of the asymmetry of the bend, we will call the bends for which the point *d* is located to the left of the point *c* "skewed upstream", and the bends for which the point *d* is located to the right of the point *c* "skewed downstream" (examples of such bends are shown in Figure 2*a* and 2*b* accordingly). If the bends are so strongly asymmetrical that point *d* goes beyond the bend step, then we will call such bends "tilted upstream" if point *d* lies to the left of the initial point of the bend *a,* and "tilted downstream" if point *d* lies to the right of the end point of the bend *b*.

Taking into account these definitions, it can be stated that:

1) *if* $A_K = 0$, then $x_m = \lambda/2$, the points *d* and *c* coincide;
2) *if* $A_K < 0$, then $x_m < \lambda/2$, the point *d* is located to the left of point *c* and the bend is "skewed" upstream (to the left);
3) *if* $A_K > 0$, then $x_m > \lambda/2$, the point *d* is located to the right of point *c* and the bend is "skewed" downstream (to the right).



(a) (b)

Figure 2. *Examples of skewed bends: (a) – skewed upstream (to the left); (b) – skewed downstream (to the right).*

For symmetrical bends $x_m = \lambda/2$, and therefore $A_K = 0$. Although the converse, strictly speaking, is not true, i.e., from the condition $A_K = 0$ it does not follow that the bend is absolutely symmetrical (this issue is discussed in more detail below), but with the value of the coefficient A_K equal to zero, the shape of the bends is generally close to symmetrical.

It is easy to see that the coefficient A_K is equal to the tangent of the angle γ formed by the perpendicular to the step line passing through its midpoint *c* (the perpendicular median) and the straight line connecting this midpoint with the apex of the bend *m* (see Figure 1):

$$A_K = \operatorname{tg}\gamma. \tag{2}$$

From elementary geometric considerations it is obvious that the angle γ defined in the above manner is equal to the angle formed by the perpendicular lowered from the apex of the bend *m* to the step line and the segment connecting the apex of the bend *m* with the point *c*, so if for some reasons it is preferable to express the asymmetry coefficient A_K via angle then any of the specified angles can be used. Note that for symmetrical bends, these angles are equal to zero. Based on the typical shapes of river bends, it can be assumed that in most cases, the absolute value of the coefficient A_K less than 1, but we are not aware of any studies on the statistics of this value, and, as will be shown below, this circumstance does not matter significantly.

To express the degree of asymmetry of the bends, in our opinion, it is more convenient to use a simpler indicator of asymmetry A_Z, whose value is determined by the position of point *d* on the step line:

$$A_Z = 0.5 - \frac{x_m}{\lambda}. \tag{3}$$

Based on the values of this index, one can obtain a fairly

complete idea of the plan shape of the bend:

1) *at* $A_Z = 0$, the bend is conditionally symmetrical, $x_m = \lambda/2$;
2) *at* $A_Z > 0$ the bend is skewed upstream (to the left), and if $A_Z > 0.5$, then the bend is tilted – point *d* is to the left of point *a*;
3) *at* $A_Z < 0$, the bend is skewed downstream (to the right), and if $A_Z < -0.5$, then the bend is tilted upwards – point *d* is to the right of point *b*.

Comparing the indices A_K and A_Z, it can be noted that although the sign of the coefficient A_K can be used to determine the direction of the bend, the absolute value of this coefficient does not add any significant information about the possible shape of the bend. This is explained by the fact that the formula for A_K contains the height of the bend, y_m as a normalizing term, the value of which is determined by the degree of development of the bend and is not known in advance. V. A. Savitsky [4] also pointed out this drawback of the coefficient A_K – the entry of the bend height y_m in its structure, noting that this coefficient can take on the same values for different combinations of x_m / λ and y_m / λ, and therefore it can be used to compare the degree of asymmetry of only the bends with the same relative height. At the same time the bend height y_m does not have any effect on the value of the coefficient A_Z, and therefore this index seems to be more convenient to use than the index A_K. It is easy to show that A_Z and A_K are connected by a simple relationship:

$$A_Z = -A_K \frac{y_m}{\lambda} \tag{4}$$

eliminating, just, the influence of the bend height y_m on the value of the coefficient A_Z, which, definitely, can be attributed to its advantages.

In the monograph by R. S. Chalov, A. S. Zavadskiy and A. V. Panin [1], two indicators are used to assess the degree of plan asymmetry of free bends. These indicators are determined by the values of the bend parameters tied to point *d* and point *c* – (see Figure 3, borrowed from the mentioned work). One of these indicators is the ratio of the difference in the length l_u of the upper and lower l_d wings of the bend to the total length of the bend $S = l_u + l_d$, called by the authors the index of wing asymmetry A_W:

$$A_W = (l_u - l_d) / S. \tag{5}$$

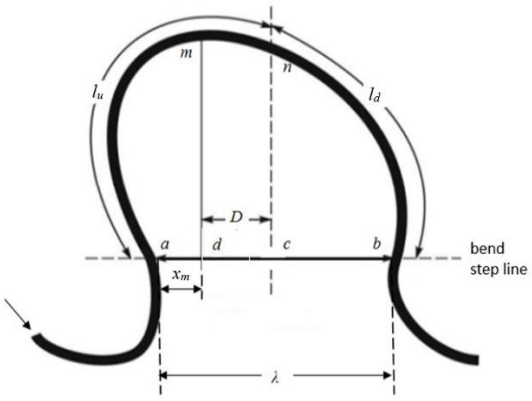


Figure 3. *Morphometric parameters used in the monograph [1] to assess the asymmetry of bends in plan.*

The length of the upper wing of the bend l_u is defined as the distance along the centerline of the channel from the upper inflection point *a* to the point *n*, at which the median perpendicular, called the geometric axis of the bend in the work, intersects with the center line of the channel; accordingly, the length of the lower wing of the bend l_d is the distance from point *n* to the lower inflection point *b*. As follows from formula (5), the absolute value of the wing asymmetry index A_W is always less than 1.

The second index of bend asymmetry introduced by the authors is called the apex asymmetry A_a and is defined as follows:

$$A_a = 2D / \lambda, \tag{6}$$

where λ is the bend pitch, and *D* is the distance between points *c* and *d*.

Based on the processing of a significant amount of field data, the authors found that in most cases the signs of the asymmetry indicators A_a and A_W coincided, and their values are connected by a fairly close linear relationship (correlation coefficient of the order of 0.8-0.9) and are approximated by the regression equation

$$A_W = 0.3\, A_a. \tag{7}$$

Regarding the index of vertex asymmetry A_a, the authors point out that its absolute value can exceed 1 if the bend is

very much "tilted" upstream or downstream, and the projection of the apex falls on the continuation of the step line. At the same time, the authors assume that the index A_a, as well as the index of wing asymmetry A_W, has a positive value if the bend apex is shifted relative to the median perpendicular upstream and a negative value – when it is shifted down. However, the sign of the wing asymmetry index A_W is uniquely determined by the sign of the numerator in expression (5) and, therefore, no additional assumptions need to be made about it. The explanation of the sign of A_a means that the made assumption in fact is referring to the parameter D that can be either positive or negative (λ is by definition a positive value). Thus, this assumption means that the value *of* D is considered positive if the point d lies to the left of the point c (as in Figure 3, where the flow moves from left to right), and is considered negative otherwise, i.e. when the point d lies to the right of the point c. Accordingly, the index of apex asymmetry index A_a behaves in a similar way, and its sign depends on where the point d is located in relation to the point c.

For symmetric bends where the median perpendicular line is the line of symmetry, both indicators are equal to zero, i.e.

$$A_a = A_W = 0.$$

At the same time, the authors note that since the shapes of bends sometimes have rather complex outlines, it is not always possible to reliably assess the degree of asymmetry with only one of the indicators they introduced. Thus, the equality of one of the asymmetry criteria to zero does not guarantee the symmetry of the bend shape; in some cases, the symmetrical location of the bend apex is observed when the lengths of the upper and lower wings differ, and vice versa. Therefore, to characterize the asymmetry of bends, according to the authors, both indicators should be used simultaneously, and it is possible to judge the symmetry of the channel bend only if both indicators are equal to zero.

The proposed index of vertex asymmetry A_a is very close in meaning to the asymmetry coefficient A_K described by formula (1) from the system of plan morphological meters by N. E. Kondratiev [3]. Indeed, if $x_m < \lambda/2$, then the following equations hold for a positive value of D:

$$D = \lambda/2 - x_m \text{ or } 2D = \lambda - 2\,x_m.$$

If $x_m > \lambda/2$, then the value of D, which has a negative value by due to the assumption made by the authors, D is determined as follows:

$$D = x_m - \lambda/2 \text{ or } 2D = \lambda - 2\,x_m.$$

Thus, from formula (1) and the last two equalities, it is easy to see that the index of apex asymmetry A_a and the coefficient of asymmetry of the bend A_K have opposite signs and are related by the relation:

$$A_a = -2\,A_K\,(y_m / \lambda). \tag{8}$$

Comparing the latter equality with expression (4), we also have:

$$A_a = 2A_Z. \tag{9}$$

To take into account the asymmetry of bends V. A. Savitsky [4] proposed the asymmetry index A_F, based on a comparison of the areas under the upper and lower wings of the bends (see Figure 4):

$$A_F = \frac{F_d - F_u}{F_d + F_u} = \frac{F_d - F_u}{F}, \tag{10}$$

where F_u and F_d are the areas under the upper and lower wings of the bend, $F = F_u + F_d$ is the total area of the figure bounded by the center line of the bend and its pitch. Figure 4 shows that the sign of the index A_F is opposite to the signs A_W, A_a and A_Z, and coincides with the sign A_K. V. A. Savitsky analyzed field data on 123 bends of the Yamal rivers and found that 60% of these rivers had a negative A_F index and 40% had a positive one. He also obtained the following data on the distribution of absolute values of the index A_F:

1) the amount of bends with values of $|A_F| \le 0.12$ is 40%;
2) the amount of bends with values of $0.12 \le |A_F| \le 0.36$ is 51.5%;
3) the amount of bends with values of $0.36 \le |A_F| \le 0.60$ is 8.5%.

No bends with values of $|A_F| > 0.60$ were detected. The author proposed to call the bends in the first interval slightly asymmetrical, those in the second – moderately asymmetrical, and those in the third – strongly asymmetric. It is difficult to say at this point how valid this proposal is – an analysis of a much more extensive data set is required, including the analysis of other asymmetry indices. Since the point of separation of the upper and lower wings of the bend V. A. Savitsky chose the point of intersection of the median perpendicular with the centerline of the channel (point n), there is reason to believe that there may be a high correlation between the A_F index and the index A_W, which constructs the bend wings in a similar manner; however, this assumption has not been tested using natural data.

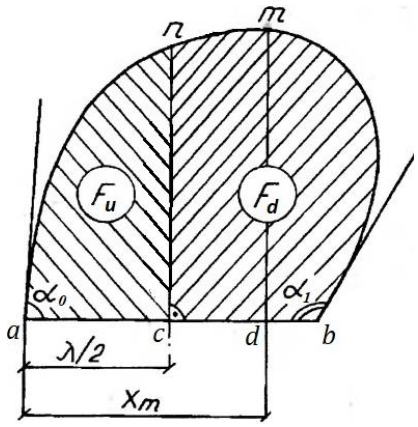


Figure 4. *Bend sketch: to the derivation of the asymmetry index A_F.*

When constructing the bend asymmetry indices discussed above, various approaches were implemented, in which the bend asymmetry is characterized in one of three ways:

1) ratios of the lengths of some characteristic segments on the step line (indices A_K, A_Z, and A_a).
2) the difference between the length of the upper and lower wings of the bend, related to the total length of the bend (index A_W);
3) ratios between the areas under the upper and lower wings of the bend (index A_F).

In the second of these approaches, it is not necessary to include the difference in the length of the bend wings in the structure of the asymmetry index; it is enough to use the length of only one of the wings. Following this idea, we would suggest using the asymmetry index of the bend A_S, which is similar in structure to the index A_Z, but instead of the ratio of the lengths of segments on the step line, use the ratio of the length of the upper wing of the bend to its full length:

$$A_S = 0,5 - \frac{S_u}{S}. \quad (11)$$

where S_u is the length of the bend section from its beginning to its apex (point m). Accordingly, the fraction on the right-hand side of the formula indicates what proportion of the total length of the bend is its upstream part. In this form, the structure of the index A_S fully corresponds to the structure of the index A_Z, which characterizes the ratio of the length x_m to the step λ. The index A_S is a variable sign, like all the indices discussed above, and its sign will coincide with the sign of the indices A_W and A_Z, and the values will lie in the range

$$-0,5<A_S<+0,5. \quad (12)$$

For symmetrical bends, $A_S = 0$. It can be assumed that due to the conceptual identity of the formulas defining the indices A_Z and A_S, their numerical values will show a high correlation, but this assumption requires verification by actual data.

Let us now consider what indicators of asymmetry were proposed for analyzing the planimetry of a meandering channel in cases where a meander is taken as a morphological unit. It should be noted that of the two approaches to the definition of the term "meander" (see [2]), none of the works devoted to the analysis of the asymmetry of meanders was given the definition of the asymmetry index for meanders consisting of a sequence of two adjacent bends. This is quite understandable, since most often adjacent bends do not have exactly the same plan shapes and, accordingly, their asymmetry indicators do not necessarily coincide, but then a natural question arises: which index of the two bends included in the meander should be used as an indicator of the asymmetry of the entire meander. For this reason, in all the works discussed below, meanders are considered to be sections of the channel consisting of one bend located in the middle of the meander and two parts of adjacent bends upstream and downstream. The differences in the subsequent analysis in the reviewed works were as follows: either the asymmetry of the meander was estimated or as an asymmetry of only its central bend – this was done by Ielpi et al. [5] and Lopez Dubon et al. [6], or the upper and lower halves (wings) of meanders, i.e., the lengths between successive meander apexes, were considered, and these values were used in expressions that serve as indicators of meander asymmetry – this was done by Limaye [7] and Carson & Lapointe [8].

In the work of Ielpi et al. [5] the asymmetry index A is used, which is the ratio of the difference in the wing lengths of the bend included in the meander to its length – the meaning of the parameters used in this indicator is illustrated by the sketch borrowed from the authors' work and shown in Figure 5:

$$A = \frac{L_u - L_d}{L_u + L_d}, \quad (13)$$

where Lu is the length of the upper and Ld is the length of the lower bend wing. Note that these lengths Lu and Ld are defined by the authors as the distance along the centerline to the apex of the bend, and not to the point of intersection of the median perpendicular with the centerline of the channel, as is done when determining the asymmetry index of wings AW. Moreover, the authors define the apex as the point at which the curvature of the axial line reaches its maximum, and not the point that is at the maximum distance from the step line. The authors also note that A is in the range $-1 \le A \le +1$, but the basis for this estimate is unclear: all variables on the right-hand side of formula (13) are greater than zero, and therefore there are no points within the bend at which the index A in absolute value can be equal to 1.

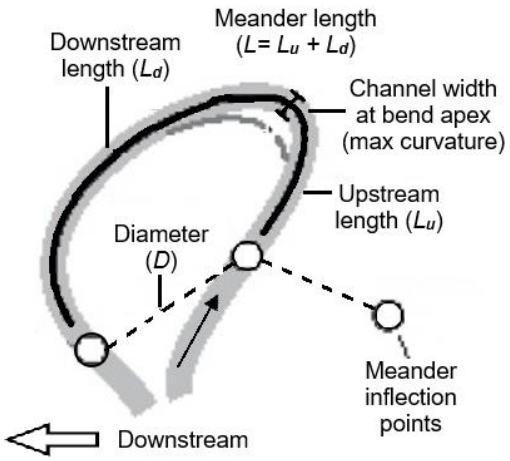


Figure 5. *Illustration from [5] for determining the coefficient of bend asymmetry A (flow from right to left).*

The indicator A^* from Limaye [7] has a similar structure, only the concept of bend wings is interpreted differently by the author: if Ielpi et al. [5], when talking about the asymmetry of meanders, in fact, use parameters related to the bend, then Limaye [7] takes the distances L_u^* и L_d^* between successive peaks on the channel section as wing lengths identified as a square wave – see Figure 6*(a)* taken from the author's work:

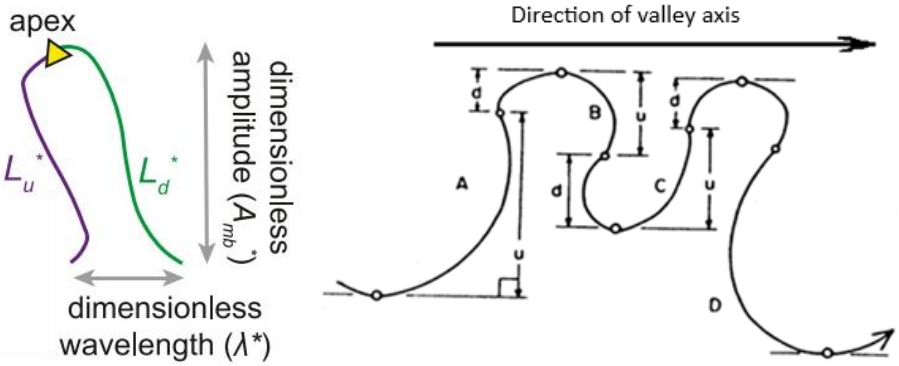
$$A^* = \frac{L_u^* - L_d^*}{L_u^* + L_d^*} \tag{14}$$

It is easy to see that the structure of the last two indicators is similar not only to each other, but also to the structure of the A_W index in the work of Chalov et al. [1].

M. A. Carson and M. F. Lapointe [8] introduce the meander asymmetry index A_M, expressed as a percentage and based on the ratio of the traverses of bends in meanders, which has the form:

$$A_M = 100\frac{u}{u+d}, \tag{15}$$

where the values *of* u and d are proposed to be understood from the illustration borrowed from this work, shown in Figure 6*(b)*. On the one hand, this index is structurally close to the last four indicators of asymmetry, but on the other hand, from the above figure, it is not quite clear what exactly the authors call a meander – apparently, the authors do not have a completely traditional approach to considering meanders: they take the section of the riverbed between two consecutive peaks as the structural unit for analysis, i.e. half of meander in one of the traditional interpretations of the term "meander" (see [2]). The only thing that can be said with certainty about this index is that its value lies in the range $0< A_M <100\%$; other details on the structure of formula (15) cannot be disclosed.



(a) (b)

Figure 6. *Illustrations for determining the asymmetry coefficients of bends: (a) – from [7]; (b) – from [8].*

Another approach to determining the asymmetry index of a meander (and, in fact, a bend) A_L was applied in the work of Lopez Dubon et al [6]. This index has the same structure as the above skewness indices A_W, A, and A^* (see Figure 7, borrowed from [6]), but the numerator is not the length of the wings of the bends, but the length of the segments l_u^* and l_d^* connecting the corresponding inflection points with the vertex, which is the point most distant from the bend pitch line:

$$A_L = \frac{l_u^* - l_d^*}{S} \tag{16}$$

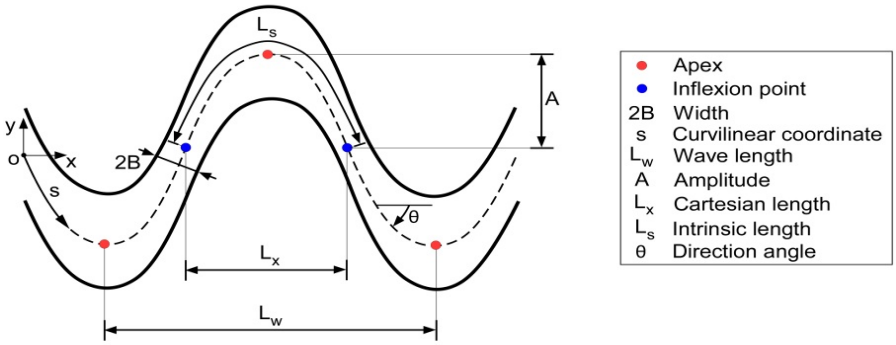


Figure 7. *Illustration from [6] to illustrate the definition of the meander asymmetry index A_L.*

It is obvious that the indices A, A^* and A_L, as well as A_W, A_Z and A_S, can have positive and negative values, but, however, due to the difference in the definition of wing lengths, the signs of these indices will not necessarily coincide for all bends.

The above indices used to describe the asymmetry of plan forms of river bends and meanders, except for the A_F index, are based on the ratios of the values of linear elements of these morphological units, most often on the difference in the lengths of the upper and lower wings of the bend, and these wings are defined differently, but still mainly as sections of the centerline of the channel between points inflection and some characteristic points on the axial line:

1) the intersection point of the median perpendicular with the axial one is the values of l_u and l_d in formula (5).
2) the point of maximum axial curvature is the values of L_u and L_d in formula (13).
3) the point of maximum distance from the axis of the meandering belt is the values L_u^* and L_d^* in formula (14).

or even as line segments l_u^* and l_d^* connecting the inflection points with the apex of the bend in formula (16). All these indexes are dimensionless, and in all cases the normalizing parameter is the total length of the bend, except for the index A^*, where the meander length is used for normalization.

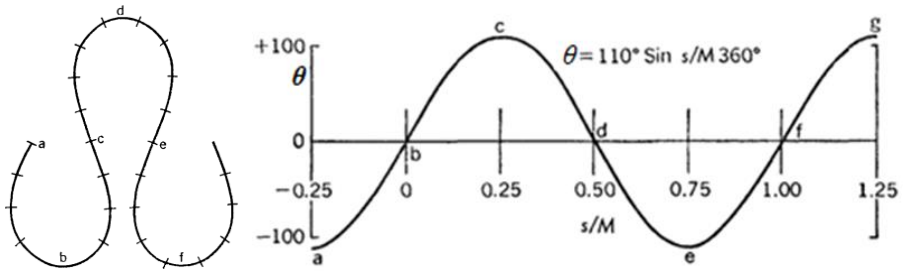
3. Application of Asymmetry Indices to Describe the Planimetry of Bends/Meanders

Since the analysis of planimetry of meandering rivers in all cases is based on the study of the behavior of the centerline of the channel, most often its curvature, which by definition is derived from the directional angle in natural coordinates, indicators of asymmetry of bends or meanders can be expressed not only in terms of their linear or areal elements: a fairly natural approach is to study the behavior of the specified directional angle and the use of its values at some characteristic points to determine various meters of the plan bend shape, including asymmetry indices.

One of the most well-known implementations of such approaches to describing the centerline of a channel is the work of W. Langbein and L. Leopold [9], in which the authors, solving the problem of obtaining the most likely plan shape of a channel between two fixed points with a given length and direction at the initial point, at which the sum of the squares of the changes in the directional angles at each subsequent section of the channel is minimized, obtained an expression for the angle of inclination of the tangent to the centerline within the meander:

$$\theta = \omega \sin(2\pi s / M) \tag{17}$$

where θ is the angle of inclination of the tangent to the axis; M is the arc length of the meander; $s \in [0,M]$ is the natural coordinate; ω is the maximum absolute value of the angle θ reached at the points $s/M = 0.25$ and $s/M = 0.75$ (see Figure 8, borrowed from the cited work). The authors called the resulting curve a "sine-generated curve" (SGC), noting the fact that within the meander, the angle of inclination of the tangent to the axis changes according to the sinusoidal law.



(a) (b)

Figure 8. *(a): plan shape of the "sine-generated curve" (SGC) at ω = 110°; (b): graph of the dependence of the angle θ (in degrees) on the dimensionless intrinsic coordinate s/M within the meander.*

The meander in Figure 8 is considered to be the section of the channel from point "b" to point "f", and the bends that are the sections of the channel between the inflection points are three in this figure: from "a" to "c", from "c" to "e" and from "e" to "g". For bends, instead of expression (12), it is more convenient to use a relationship of the following type:

$$\theta = \omega \cos(\pi s / S) \tag{18}$$

where S is the arc length of the bend (from point "c" to point "e"); s for the bend, respectively, lies in the interval $s \in [0,S]$; ω, as before, is the maximum value of the angle θ reached at the point $s/S = 0$. Since for the bend the horizontal axis coincides with the bend step line, then ω means the bend entry angle.

The curvature of the line K at a point is the derivative with respect to the natural coordinate, i.e. $K = \partial\theta / \partial s$; accordingly, the expression for K within the bend has the form:

$$K = \frac{1}{\rho} = -\frac{\pi}{S}\omega \sin \pi \frac{s}{S} \tag{19}$$

where ρ is the radius of axial curvature at the point s / S.

The problem of river meanders in the formulation by W. Langbein and L. Leopold, is identical to the problem from the class of random walks, which was studied by Von Schelling in 1951 and 1964 [10, 11], who obtained a similar expression for the directional angle. It is interesting to note that even earlier – at the end of the 19th century – L. Fargue [12] used two types of curves to approximate the plan forms of the Garonne River bends, in which the curvature varied in one case according to a linear law, and in the other – according to a sinusoidal law. Thus, the approximation of W. Langbein and L. Leopold expressed by dependence (19) is essentially reduced to the second method of description used by L. Fargue (the expression for curvature as a cosine function of s is given with reference to L. Fargue in the work of S. Lelyavsky [13].

When describing the centerline of the channel in terms of the behavior of the directional angle θ as a function of the natural coordinate s measured along the channel axis from the beginning of the bend ($s \in [0, S]$), it can be shown [14] that the following equation is a necessary and sufficient condition for the symmetry of the bend at any points:

$$\Theta(s) = -\Theta(S-s) \tag{20}$$

(for meanders, this condition will have a slightly different form, depending on what is meant by a meander).

Many researchers, though not all (e.g., [8]), believe that the SGC is a satisfactory approximation of the plan shape of many natural bends, but it describes only symmetric bends for which equality (20) holds. To describe the plan shape of asymmetrical meanders, the so-called " Kinoshita curve " [15] is often used, which can be reduced to the following form [16]:

$$\theta = \omega \sin\left(\frac{2\pi s}{M}\right) + \omega^3 \left(J_s \cos\left(3\frac{2\pi s}{M}\right) - J_f \sin\left(3\frac{2\pi s}{M}\right)\right) \tag{21}$$

where J_s and J_f are the skewness and flatness coefficients respectively. Although the exact expressions for these coefficients are not given, it can be shown that

$$J_s \sim D^* / S \tag{22}$$

$$J_f \sim S / y_m \tag{23}$$

where D^* is the distance between the midpoint of the step and the projection of the maximum axial curvature point on the step line, S is the length, and y_m is the span of the bend. It should be noted that expression (21) does not satisfactorily describe all asymmetric bends.

N. E. Kondratiev [3] also selected ways to describe the plan shape of the channel's centerline. A description of the method he proposed would take up a lot of space and therefore is not given here. Note only that the application of this method involves the generation of some empirical parameters that characterize, in particular, the asymmetry of the bend. It was shown by V.I. Zamyshlyaev [17] that the approach proposed by N. E. Kondratiev allows one to describe the behavior of the directional angle on a bend using the following expression:

$$\theta = \frac{\alpha}{2}\left[\left(\cos\pi\frac{s}{S} - \frac{\sin 2\pi\frac{s}{S}\sin\pi\frac{s}{S}}{14 - 18\cos^2\frac{s_*}{S}}\right) - E_Z\right], \quad (24)$$

where s^* denotes the point on the axis where the curvature has the maximum value. This expression, in fact, includes two indicators that characterize the asymmetry of the bend:
index s^*/S, which has a structure similar to the indicator A_S in formula (14), if the S_u value L_u from formula (10) is used as S_u;

Asymmetry coefficient E_Z, proposed by V. I. Zamyshlyaev [17].

$$E_Z = \frac{\alpha_0 - \alpha_1}{\alpha_0 + \alpha_1} = \frac{\alpha_0 - \alpha_1}{\alpha}. \quad (25)$$

This simple coefficient of asymmetry of bends E_Z, like most of the indices discussed above, has a positive value for bends sloping downstream, a negative value for bends sloping upstream, and zero value for symmetrical bends.

M. M. Gendel'man [18] proposed the coefficient E_G, which characterizes the asymmetry of the bend and is also based on the values of its entry and exit angles:

$$E_G = \frac{\alpha_0}{\alpha_1} \quad (26)$$

It is obvious that in all cases $E_G > 0$, and for symmetrical bends $E_G = 1$, for $E_G < 1$ bends are slanted downstream, and at $E_G > 1$ – upstream. It is also easy to see the connection between the indices E_Z and E_G:

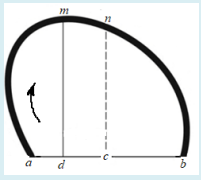
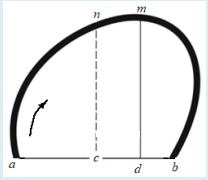
$$\begin{cases} E_Z = \dfrac{E_G - 1}{E_G + 1}, \\ E_G = \dfrac{1 + E_Z}{1 - E_Z} \end{cases} \quad (27)$$

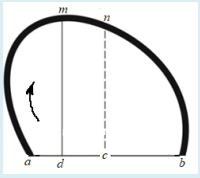
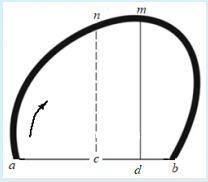
therefore, the value of one of them can be used to set the value of the other.

4. Result

A summary of all the bend asymmetry indices considered in this paper is given in Table 1. From this table, it is easy to see that the most informative indicators are A_a and A_Z, since they allow us to determine not only the direction of the slope of the bends-down or upstream, but also the degree of this asymmetry, to detect that the bend is skewed. As we have shown, there is also a clear relationship between these indices, expressed by formula (9).

Table 1. *Indicators of asymmetry of bends.*

Asymmetry index (in order of presentation in the article)	Values for: Skewed / tilted upwards	Skewed / tilted downwards	Author(s)
			
$A_K = \dfrac{2x_m - \lambda}{2y_m} = \operatorname{tg}\gamma$	AK <0 / AK <0	AK >0 / AK >0	N. E. Kondratiev [3]
$A_Z = 0,5 - \dfrac{x_m}{\lambda}$	AZ > 0 / AZ > 0,5	AZ < 0 / AZ < – 0,5	V. I. Zamyshlyaev
$A_W = \dfrac{l_u - l_d}{l_u + l_d} = \dfrac{l_u - l_d}{S}$	1>AW >0 / 1>AW >0	–1<AW<0 / –1<AW <0	R. S. Chalov, A. S. Zavadskiy, A. V. Panin [1]

Asymmetry index (in order of presentation in the article)	Values for Skewed / tilted upwards 	Skewed / tilted downwards 	Author(s)
$Aa = \frac{2D}{\lambda}$	Aa >0 / Aa >1	Aa <0 / Aa <–1	
$A_F = \frac{F_d - F_u}{F_d + F_u} = \frac{F_d - F_u}{F}$	AF <0 / AF <0	AF >0 / AF >0	V. A. Savitsky [4]
$A_S = 0.5 - \frac{S_u}{S}$	AS >0 / AS >0	AS <0 / AS <0	V. I. Zamyshlyaev
$A = \frac{L_u - L_d}{L_u + L_d}$	A>0 / A >0	A <0 / A <0	A. Ielpi [5]
$A^* = \frac{L_u^* - L_d^*}{L_u^* + L_d^*}$	N/A	N/A	A. B. Limaye [7]
$A_M = 100\frac{u}{u + d}$ 0<AM <100%	N/A	N/A	M. A. Carson, M. F. Lapointe [8]
$A_L = \frac{l_u^* - l_d^*}{S}$	N/A	N/A	S. Lopez Dubon, A. Sgarabotto, S. Lanzoni [6]
$E_Z = \frac{\alpha_0 - \alpha_1}{\alpha}$	EZ >0 / EZ >0	EZ <0 / EZ <0	V. I. Zamyshlyaev [17]
$E_G = \frac{\alpha_0}{\alpha_1}$	EG >1	EG <1	M. M. Gendel'man [18]

5. Discussion

The situation is different when the meander is used as a morphological unit in analyzing the planimetric characteristics of a meandering channel. For a meander as a single morphological unit, the concept of asymmetry as a whole does not make sense (which is noted, for example, by Limaye, A. B. [7], and therefore most often the indices of asymmetry of meanders are used as indices of asymmetry of bends that are part of meanders, which, in fact, can serve as evidence of the advantage of the approach, based on the use of the concept of "bend", over the "meander". In general, since a channel section consisting of two adjacent bends or parts of three adjacent bends is considered a meander, perhaps a more representative indicator of meander asymmetry would be an index based on the ratios of not only wing lengths, but also other parameters related to the bends (or their parts) that make up the meander, for example, the average values of radii of curvature, bend angles, etc., or combinations thereof. An important factor that must be considered in understanding the causes of asymmetry is most likely the fact that the pitch lines of adjacent meanders or their parts that make up the meander do not lie on a straight line but rather form some angle(s) with each other. It is quite possible that this is the main reason for the asymmetry of free

bends and meanders. A more in-depth discussion of these issues, however, requires further study.

The convenience of using coefficients expressed in terms of the angle of inclination of the tangent to the centerline of the channel at certain points, as in the case of the E_Z indicator, where such angles are used at the initial and end points of the bend, can be justified by the following considerations.

V. I. Zamyshlyaev [14] developed a mathematical model of plan deformations of meandering channels, in which the equation of plan deformation of a channel was obtained and studied, which has the following form in a simplified form (lower indices mean differentiation according to the corresponding variables):

$$\theta_T + \left[V^2 \int_0^s \theta_s^2 ds + f(T) \right] \theta_s = -V^2 \theta_{ss}, \qquad (28)$$

where V is the average flow velocity over the cross-section; $f(T)$ is a function determined by the type of boundary conditions; T is a dimensionless time associated with the real physical time t by the formula:

$$T = \varepsilon \frac{V^2}{\lambda^2} t \qquad (29)$$

ε is an empirical coefficient that characterizes the shore's compliance to erosion. It was shown that, under certain boundary conditions, a good approximation to the solution of equation (28) is a function having the form SGC:

$$\theta(s,T) = \omega(T) \cos\left(\pi \frac{s}{S(T)} \right) \qquad (30)$$

where $\omega(T)$ and $S(T)$ are determined by the following ordinary differential equations:

$$\begin{cases} \dfrac{d\omega}{dT} = \pi^2 V^2 \dfrac{\omega}{S^2} \\ \dfrac{dS}{dT} = \pi^2 V^2 \dfrac{\omega^2}{2S} \end{cases} \qquad (31)$$

In fact, it can be shown [14] that the solution of the plan channel deformation equation (28) is not only a function (30), but for any integer m also a function of the form

$$\theta(s,T) = \frac{\omega_0}{2} + \sum_{n=1}^{m} \omega_n \cos\left(\pi n \frac{s}{S_n} \right) \qquad (32)$$

It is obvious that such a Fourier expansion of the function $\theta(s,T)$ is capable, with a sufficient number of harmonics m, of describing the plan shapes of practically any bends (of course, corresponding to the conditions under which the Fourier transform is applicable). Knowing the initial shape of the bend, it is possible to determine the values of the coefficients ω_n, and the more terms in this sum, the more accurate the approximation will be. However, for most regular bends, the values of the first few harmonics will be quite sufficient for a good approximation of their plan shape. V. I. Zamyshlyaev [17] outlined the expressions for these coefficients for values up to $m = 4$. These coefficients are expressed through the values of the angle of inclination of the tangent θ at different points along the bend. In particular, for $m = 4$, the coefficients ω_n are expressed through the values of the angle $\theta(s)$ at the following points of the bend: $s = 0$, $s = S0/3$, $s = S0/2$, $s = 2S0/3$, $s = S0$, where $S0$ is the length of the bend at the initial moment of time. Moreover, these coefficients ω_n have the same structure as the index E_Z. There may be other approaches where the ratio of the entry and exit angles is important, which is why they may be of independent interest when studying the asymmetry of bends.

6. Conclusion And Recommendations

The above analysis showed that there are many indexes that can be used to get an idea of the nature and degree of asymmetry of bends. It can be noted that most of these indicators are based on the ratios of certain linear elements that make up the bend, mainly on the ratios of segments into which the step of the bend is divided by projections of certain points on the axis –indices A_K, A_a, A_Z; or on the ratios of the lengths of the wings of the bend, determined in various ways – indices A_W, A, A^*, A_L, A_M, and A_S. Also considered is an index based on the ratios of the areas under the bend wings A_F, and two indicators based on the values of the bend entry and exit angles – E_Z and E_G, which are related by the dependence (27). Note that for symmetrical bends index A_M = 50%, index E_G = 1, and all other asymmetry indices for symmetrical bends are equal to zero.

The factor of asymmetry of bends and meanders is important for understanding the patterns of changes in their plan shapes. In addition, the asymmetry indices, along with other parameters, are used in some methods of approximating the channel centerline of the meandering rivers and subsequently analyzing its curvature. Due to the increasing automation of methods for processing cartographic materials and satellite images, such an analysis can be based on processing large amounts of data, which, of course, will increase the reliability of the obtained knowledge about the regularities of the meandering process, but the consideration of such works goes far beyond the scope of this article.

The analysis showed that the use of a bend as a structural unit of a meandering channel makes all coefficients characterizing the degree of its asymmetry unambiguously definable. At the same time, the use of an alternative concept, where such

a structural unit is a meander, makes it difficult or even impossible to use meander asymmetry indicators, which once again confirms the advantages of the "bend" concept over the "meander" concept. It is shown that the A_a and A_Z indices are the most informative for the overall assessment of the asymmetry of bends. At the same time, the E_Z indicator, along with indicators having a similar structure, can be used to describe the planimetry of meandering channels. Thus, when studying the plan forms of meandering channels, including when determining the degree of their asymmetry, it is recommended to determine first of all the indicators A_a or A_Z, and the index E_Z.

Nomenclature

a	initial (upstream) point of the bend (upstream point of the axial inflection)
b	the final (downstream) point of the bend (the downstream point of the axial bend)
m	the apex of the bend (the point farthest from the step line)
n	the point of intersection of the channel centerline with the perpendicular passing through the middle of the step line (median perpendicular);
Points on the bend step line	
c	the middle of the bend step
d	base of the bend apex (projection of the bend apex m onto the step line)
Linear parameters (bend meters)	
λ	bend step (pitch)
ρ	radius of curvature of the axial line at the point
D	the distance between the middle of the step and the base of the bend apex
K	curvature of the axial line at a point
M	arc length of the meander
s	natural coordinate
S	bend length
SGC	Sine-Generated Curve
x_m	the distance between the apex of the bend and the base of the summit
y_m	height (span) of the bend
Angles (bend meters)	
α_0	bend entry angle
α_1	bend exit angle
α	bend turning angle
β	bend conjugation angle
γ	is the angle between the median perpendicular and the line connecting the middle of the step with the apex of the bend
θ	is the angle between the tangent to the axis and the step line
ω	is the maximum absolute value of the angle θ
Area parameters	
F	total area of the bend
F_d	the area under the lower wing of the bend
F_u	area under the upper wing of the bend
Indicators asymmetries expressed through linear parameters	
A, A^*, A_a, A_F, A_K, A_L, A_M, A_S, A_W, A_Z,	Indicators asymmetries expressed through angles
E_G, E_Z	Linear parameters included in the asymmetry indices
d	length of the lower element (in index A_M)
l_d	length of the lower wing of the bend (in index A_W)
l_u	the length of the upper wing of the bend (in index A_W)
l_d^*	the length of the lower segment (in index A_L)
l_u^*	the length of the upper segment (in index A_L)
L_d	length of the lower wing of the bend (in index A)
L_u	the length of the upper wing of the bend (in index A)
L_d^*	the length of the lower wing of the bend (in index A^*)

L_u^*	the length of the upper wing of the bend (in index A^*)
S_u	the length of the section from the beginning of the bend to the top (in index A_S)
u	the length of the upper element (in index A_M)
Other designations	
ε	is an empirical coefficient characterizing the susceptibility of the bank to erosion
π	the number π
ω_n	coefficients of the Fourier series expansion
D^*	the distance between the middle of the step and the projection onto the step line of the point of maximum curvature of the axial
$f(T)$	is a function determined by the type of boundary conditions
J_s and J_f	coefficients of skewness and flattening for the Kinoshita curve
s^*	is the point on the axis at which the curvature has its maximum value
$S0$	the length of the bend at the initial moment of time
T	dimensionless time
V	is the average flow velocity over the cross-section;
a, b, c, d, e, f, g	points of the axial line in Figure 8

Author Contributions

Vitaly Ivanovich Zamyshlyaev: Conceptualization, Formal Analysis, Investigation, Methodology, Visualization, Writing – original draft, Writing – review & editing

Conflicts of Interest

The author declares no conflicts of interest.

References

[1] Chalov R. S., Zavadsky A. S., Panin A. V. River meanders. [Scientific editor Chalov, R. S.] Moscow University Press, 2004. 371 p. (In Russian).

[2] Zamyshlyaev V. I. "Meander" and "bend" as morphological units of planimetry of a meandering channel: an analysis of existing concepts // Soil erosion and river channel processes (in print). 2025 (In Russian).

[3] Kondratiev N. E. Hydromorphological basis of calculations of free meandering. *Tr. GGI* [Proceedings of the State Hydrological Institute], 1968, iss. 155, P. 5–38. (In Russian).

[4] Savitsky V. A. On the asymmetry of river bends // Geomorphology. 1995. No. 3. P. 73-79. (In Russian).

[5] Ielpi A., Lapôtre M. G. A., Finotello A., Ghinassi M. Planform-asymmetry and backwater effects on river-cutoff kinematics and clustering // Earth Surface Processes and Landforms. Vol. 46, Iss2. 2021. P.357-370. https://doi.org/10.1002/esp.5029

[6] Lopez Dubon, S., Sgarabotto, A., Lanzoni, S. A curvature-based framework for automated classification of meander bends // Water Resources Research, 61, 2025. https://doi.org/10.1029/2024WR037583

[7] Limaye, A. B. A geometric algorithm to identify river meander bends: 2. Test for characteristic shapes. Journal of Geophysical Research: Earth Surface, 130, e2024JF007909. 2025. https://doi.org/10.1029/2024JF007909

[8] Carson, M. A., Lapointe, M. F. The inherent asymmetry of river meander planform // Journal of Geology, 91(1), 1983. P. 41–55.

[9] Langbein, W. B., Leopold L. B. River meanders—Theory of minimum variance, U.S. Geol. Surv. Prof. Pap., 422H, 1966. P. 1–19.

[10] Von Schelling, H. Most frequent particle paths in a plane // Am. Geophys. Union Trans., Vol. 32, 1951, P. 222-226.

[11] Von Schelling, H. Most frequent random walks // Gen. Elec. Co. Rept. 64GL92, Schenectady, N. Y. 1964

[12] Fargue, L. 1908. La forme du lit des rivières à fond mobile, Paris: Gauthier-Villars.

[13] Lelyavsky S. N. Introduction to river hydraulics. – L.: Gidrometeoizdat, 1961. 233 p. (In Russian).

[14] Zamyshlyaev V. I. Mathematical model of plan deformation of river channels // *Tr. GGI* [Proceedings of the State Hydrological Institute], 1983, iss. 288, P. 56–72. (In Russian).

[15] Kinoshita, R. Investigation of channel deformation in Ishi-kari River // Dep. Sci. and Tech., Bur. of Resour., Tech. Rep. 13, Tokyo, Japan. 1961. 174 p.

[16] Abad, J. D. Cfd simulations of asymmetric kinoshita-generated meandering bends // The XXXI International Association of Hydraulic Engineering and Research (IAHR) Congress / September 11–16, Seoul, Korea, 2005. 10 p.

[17] Zamyshlyaev V. I. Mathematical modeling of plan re-formations of meandering rivers channels. Dissertation for the degree of Candidate of Technical Sciences. 1982. 184 p. (In Russian).

[18] Gendel'man, M. M. On the forms of river meanders // Collection of works on hydrology, No. 17, 1982. - P. 197-207.