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Pricing Strategy in the Manufacturing Sector: An Integrated Modeling Framework

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Abstract

Pricing in the manufacturing industry constitutes a fundamentally complex optimization problem arising from the interaction of production economics, market demand behavior, operational constraints, and competitive market conditions. Unlike service-oriented or digital industries, where marginal costs are often negligible and scalability is relatively unconstrained, manufacturing systems operate under significant structural and economic limitations. These include high fixed capital investments, non-linear variable cost structures, finite production capacities, inventory holding costs, setup and changeover costs, lead-time constraints, and stochastic supply chain dynamics. As a result, pricing decisions in manufacturing environments cannot be effectively reduced to simple heuristics, historical pricing practices, or static cost-plus approaches. Instead, they require mathematically grounded optimization frameworks that explicitly incorporate both internal production economics and external market forces. The complexity of manufacturing pricing is further amplified by the interdependence between price, demand, production quantity, capacity utilization, and inventory levels. A change in selling price can influence market demand, which subsequently affects production requirements, resource utilization, inventory accumulation, and overall operating costs. Similarly, fluctuations in raw material prices, labor costs, energy consumption, transportation expenses, and supplier reliability can significantly alter the optimal pricing strategy. Therefore, an effective pricing model must account for these interconnected variables while simultaneously satisfying operational and financial constraints. Competitive interactions also play an important role, particularly in markets where firms continuously adjust prices in response to competitors, customer preferences, and changing market conditions. This paper develops a comprehensive mathematical framework for pricing optimization in complex manufacturing environments. The proposed formulation integrates multiple dimensions of the manufacturing decision-making process, including fixed and variable cost structures, demand behavior, production capacity constraints, competitive pricing interactions, inventory dynamics, resource utilization, and time-dependent market effects. The framework can accommodate both deterministic and stochastic components, thereby enabling the analysis of pricing decisions under uncertainty in demand, supply, production costs, and market conditions. The objective is to determine an optimal pricing strategy that balances profitability, demand generation, capacity utilization, inventory requirements, and operational feasibility. By representing pricing as a constrained optimization problem, the framework provides a systematic alternative to conventional pricing methodologies. It enables manufacturers to evaluate different pricing scenarios, quantify the impact of operational and market variables, and identify economically viable pricing strategies. Consequently, the proposed framework provides a rigorous mathematical basis for analyzing, optimizing, and supporting strategic pricing decisions across diverse manufacturing systems.

Keywords

Pricing Strategy, Manufacturing Systems, Cost Structure, Demand Modeling, Dynamic Pricing, Competition, Inventory Optimization

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1. Introduction

At the core of manufacturing pricing lies the relationship between price and demand. Demand is typically modeled as a decreasing function of price, reflecting consumer sensitivity and substitution effects. However, in real-world manufacturing environments, demand is influenced not only by price but also by product quality, availability, delivery timelines, and competitive offerings. Consequently, demand functions must be interpreted as aggregated representations of multiple latent variables, each contributing to observed purchasing behavior. The sensitivity of demand to price changes, often captured through elasticity measures, plays a central role in determining optimal pricing strategies [1]. Highly elastic demand environments impose strict limits on price increases, whereas relatively inelastic markets allow firms to exercise greater pricing power.

Simultaneously, the cost structure of manufacturing systems introduces additional complexity into the pricing problem. Fixed costs, arising from capital investments in machinery, infrastructure, and technology, create a baseline financial burden that must be amortized over production volumes [2]. Variable costs, including raw materials, labor, and energy, introduce a direct dependency between production output and cost [3]. Furthermore, non-linear cost effects, such as diminishing returns to scale, machine wear-and-tear, and congestion effects, lead to increasing marginal costs at higher levels of production [4, 23]. These factors collectively necessitate a pricing strategy that ensures not only coverage of variable costs in the short term but also recovery of fixed costs over the long term.

The interplay between demand and cost structures gives rise to the central objective of profit maximization. In a mathematical sense, profit is defined as the difference between revenue and total cost, where revenue itself is a function of price and demand. Let us denote:

$$\pi(p) = R(p) - C(q) \quad \text{where} \quad q = D(p) \tag{1}$$

Here, $\pi(p)$ is profit, $R(p) = p \cdot D(p)$ is revenue, and $C(q)$ is total cost. The optimal pricing decision, therefore, emerges from balancing the trade-off between higher prices (which increase revenue per unit but reduce demand) and lower prices (which stimulate demand but reduce per-unit margins) [5, 27]. This trade-off can be expressed through the first-order condition:

$$\frac{d\pi}{dp} = D(p) + p\frac{dD}{dp} - \frac{dC}{dq} \cdot \frac{dD}{dp} = 0 \tag{2}$$

Rearranging yields the classical condition that marginal revenue (MR) equals marginal cost (MC) [6]:

$$MR(p) = MC(q) \quad \text{where} \quad MR(p) = p\left(1 + \frac{1}{E_d}\right) \tag{3}$$

and $E_d = \frac{dD}{dp} \cdot \frac{p}{D}$ is the price elasticity of demand.

This trade-off is further complicated by operational constraints such as limited production capacity, inventory holding costs, and supply chain disruptions [7, 24]. Capacity constraints, in particular, introduce inequality conditions into the optimization problem:

$$D(p) \leq Q_{\max} \tag{4}$$

restricting the feasible set of pricing decisions and often leading to non-linear solution spaces. When capacity is binding, the optimal price must be adjusted upward to ration demand, leading to a constrained optimum characterized by the Kuhn-Tucker conditions [8]:

$$\begin{gathered} \frac{d\pi}{dp} + \lambda \cdot \frac{d}{dp}\left(D(p) - Q_{\max}\right) = 0, \quad \lambda \geq 0, \\ \lambda\left(D(p) - Q_{\max}\right) = 0. \end{gathered} \tag{5}$$

where λ is the Lagrange multiplier representing the shadow price of capacity.

Beyond internal considerations, manufacturing firms must also account for competitive dynamics. In markets characterized by multiple competing firms, pricing decisions are interdependent. The price set by one firm influences not only its own demand but also the demand faced by competitors [9, 25, 26]. This interdependence can be modeled through competitive demand functions:

$$D_i(p_i, p_{-i}) = a_i - b_i p_i + \sum_{j \neq i} \gamma_{ij}(p_j - p_i) \tag{6}$$

where p_i is firm i's price, p_{-i} represents competitors' prices, and γ_{ij} captures cross-price sensitivity [10, 11]. Such formulations introduce strategic behavior into the pricing problem, where firms must anticipate and respond to competitor actions, often leading to equilibrium-based pricing outcomes characterized by the Nash equilibrium condition [12, 13]:

$$p_i^* = \arg\max_{p_i} \pi_i(p_i, p_{-i}^*) \quad \forall i \tag{7}$$

Another critical dimension of manufacturing pricing is the role of inventory. Inventory levels act as both a buffer and a constraint, influencing pricing decisions in subtle but significant ways [14, 15, 33]. Let I_t denote inventory at time t. The inventory dynamics follow:

$$I_{t+1} = I_t + Q_t - D(p_t) \tag{8}$$

where Q_t is production quantity. Excess inventory creates pressure to reduce prices in order to accelerate sales and avoid holding costs, while limited inventory enables firms to increase prices due to scarcity. This dynamic introduces a feedback loop between pricing and inventory, which can be captured by a state-dependent pricing policy [16]:

$$p_t = \pi(I_t, \theta_t) \tag{9}$$

where θ_t represents other market conditions.

In recent years, advancements in data analytics and computational methods have enabled more sophisticated approaches to pricing in manufacturing [17, 28]. Firms increasingly rely on data-driven models to estimate demand, forecast cost fluctuations, and simulate market scenarios [18, 29, 30]. These models facilitate dynamic pricing strategies, where prices are continuously adjusted in response to real-time information about demand, inventory, and external market conditions [19, 20, 31]. From a mathematical perspective, dynamic pricing extends the static optimization framework into a time-dependent setting, where the objective is to maximize cumulative discounted profit over a planning horizon [21]:

$$\max_{\{p_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \left[p_t D_t(p_t) - C_t(D_t(p_t)) \right] \tag{10}$$

where $\beta \in (0, 1]$ is the discount factor.

This paper develops a comprehensive mathematical framework for manufacturing pricing that integrates these diverse elements into a unified structure [22, 32]. The remainder of the paper is organized as follows. Section 2 develops the cost structure of manufacturing systems, including fixed costs, variable costs, capacity constraints, and learning effects. Section 3 presents demand modeling approaches, from linear and nonlinear specifications to competition and uncertainty. Section 4 formulates profit optimization, deriving elasticity-based pricing rules and capacity-constrained solutions. Section 5 extends the analysis to competitive environments, including duopoly, Bertrand, and Cournot competition. Section 6 incorporates inventory dynamics and their implications for pricing. Section 7 develops dynamic pricing models using optimal control and dynamic programming. Section 8 presents an integrated framework that combines all these elements. Section 9. Section 10 concludes with strategic implications and directions for future research.

2. Cost Structure in Manufacturing Systems

2.1. Overview and Fundamental Relationships

The cost structure of a manufacturing system forms the foundational basis for all pricing decisions. Unlike service-based or digital industries, where marginal costs are often negligible, manufacturing systems are characterized by complex and multi-layered cost functions. These costs arise from capital-intensive infrastructure, labor requirements, raw material dependencies, and operational inefficiencies. Consequently, pricing decisions must be derived with a detailed understanding of how costs behave across different levels of production.

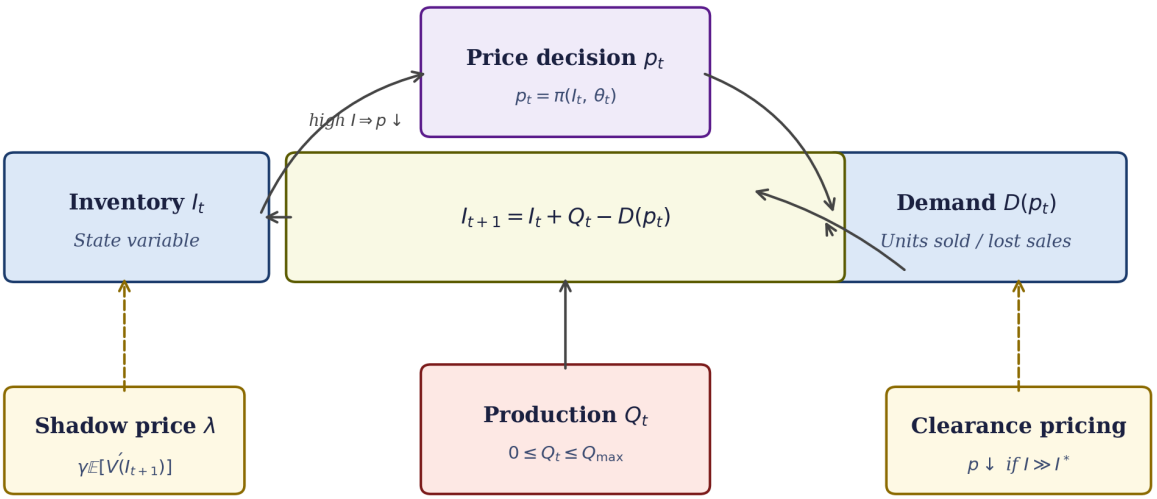


Figure 1. Inventory Loop.

In a formal sense, the total cost of production can be expressed as a function of output quantity. Let q denote the quantity of goods produced. The total cost function, $C(q)$, captures the aggregate expenditure required to produce q units of output. The structure of this function determines not only the minimum feasible price but also the shape of profit-maximizing pricing strategies.

A general representation of the cost function in manufacturing systems is given by:

$$C(q) = F + VC(q) \tag{11}$$

where F represents fixed costs and $VC(q)$ represents variable costs that depend on output. While this decomposition appears simple, both components exhibit complex behavior in real-world manufacturing environments.

The relationship between cost functions and pricing can be understood through the following fundamental identities. The average cost (AC) and marginal cost (MC) are defined as:

$$AC(q) = \frac{C(q)}{q}, \quad MC(q) = \frac{dC(q)}{dq} \tag{12}$$

For a firm to remain viable in the long run, the price must satisfy:

$$p \geq AC(q) \quad \text{for some } q \geq 0 \tag{13}$$

In the short run, the firm can operate as long as price covers average variable cost:

$$p \geq AVC(q) = \frac{VC(q)}{q} \tag{14}$$

These conditions define the feasible pricing region.

2.2. Fixed Cost Structure

Fixed costs are those that remain invariant with respect to the level of production, at least within a given operational range. These costs are incurred regardless of whether the firm produces zero units or operates at full capacity. Mathematically, fixed costs are constant with respect to q:

$$\frac{dF}{dq} = 0 \tag{15}$$

Examples of fixed costs include expenditures on plant construction, machinery acquisition, technological infrastructure, and long-term contractual obligations such as leases and salaried personnel.

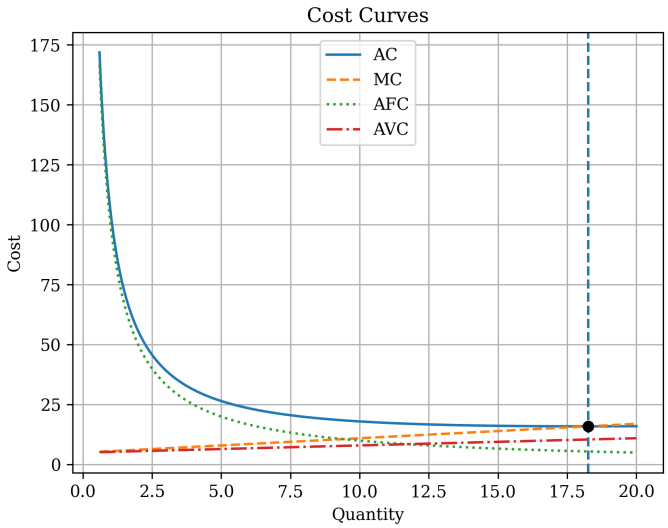


Figure 2. *Cost Curves.*

From a pricing perspective, fixed costs introduce a critical requirement: over the long run, total revenue must exceed total costs to ensure sustainability. However, fixed costs do not directly influence short-term pricing decisions in marginal analysis. Instead, they affect pricing indirectly by influencing the scale at which production must operate to achieve profitability.

An important concept associated with fixed costs is the *average fixed cost* (AFC), defined as:

$$AFC(q) = \frac{F}{q} \tag{16}$$

As production increases, the average fixed cost declines, reflecting economies of scale. This phenomenon creates a strong incentive for firms to increase production volume, as spreading fixed costs over a larger output reduces per-unit cost. The rate at which AFC declines is given by:

$$\frac{d(AFC)}{dq} = -\frac{F}{q^2} < 0 \tag{17}$$

2.3. Variable Cost Structure

Variable costs depend directly on the level of output and are influenced by factors such as raw material consumption, labor input, and energy usage. Unlike fixed costs, variable costs exhibit dynamic behavior and may not scale linearly with production.

A common representation of variable costs is:

$$VC(q) = cq + \beta q^2 \tag{18}$$

where c represents the linear marginal cost component and βq^2 captures nonlinear effects. The quadratic term reflects

inefficiencies that arise at higher production levels, such as equipment wear, labor fatigue, and coordination complexity.

The marginal cost (MC), defined as the derivative of total cost with respect to output, is given by:

$$MC(q) = \frac{dC}{dq} = c + 2\beta q \tag{19}$$

This expression indicates that marginal cost increases with output when $\beta > 0$. Such increasing marginal costs are typical in manufacturing systems where capacity constraints and inefficiencies become more pronounced at higher production levels. The rate of increase is:

$$\frac{d(MC)}{dq} = 2\beta \tag{20}$$

2.4. Average Cost and Its Implications

The average cost (AC) is defined as the total cost per unit of output:

$$AC(q) = \frac{C(q)}{q} = \frac{F}{q} + c + \beta q \tag{21}$$

This function exhibits a characteristic U-shape, driven by two opposing effects:

1) The term $\frac{F}{q}$ decreases with increasing q, reflecting economies of scale.
2) The term βq increases with q, reflecting diseconomies of scale.

The minimum point of the average cost curve represents the most efficient scale of production. To find this point, we set the derivative to zero:

$$\frac{d(AC)}{dq} = -\frac{F}{q^2} + \beta = 0 \tag{22}$$

Solving yields the efficient scale:

$$q^*_{AC} = \sqrt{\frac{F}{\beta}} \tag{23}$$

At this quantity, average cost is minimized:

$$AC_{\min} = c + 2\sqrt{F\beta} \tag{24}$$

Pricing strategies must account for this point, as producing significantly below or above this level leads to inefficiencies.

2.5. Economies and Diseconomies of Scale

Economies of scale arise when increasing production leads to a reduction in average cost. Mathematically, this occurs when:

$$\frac{dAC}{dq} < 0 \quad \Leftrightarrow \quad q < \sqrt{\frac{F}{\beta}} \tag{25}$$

Conversely, diseconomies of scale occur when:

$$\frac{dAC}{dq} > 0 \quad \Leftrightarrow \quad q > \sqrt{\frac{F}{\beta}} \tag{26}$$

The elasticity of average cost with respect to output provides a scale economy measure:

$$\varepsilon_{AC,q} = \frac{d(AC)}{dq} \cdot \frac{q}{AC} = \frac{\beta q^2 - F}{F + cq + \beta q^2} \tag{27}$$

When $\varepsilon_{AC,q} < 0$, economies of scale exist; when $\varepsilon_{AC,q} > 0$, diseconomies prevail.

The transition between these regimes is critical for pricing decisions. Firms operating in the economies-of-scale region may adopt aggressive pricing strategies to increase volume, while firms in the diseconomies region must raise prices to compensate for rising costs.

2.6. Capacity Constraints and Cost Behavior

Manufacturing systems are inherently constrained by production capacity. Let Q_{max} denote the maximum feasible output. As production approaches this limit, costs often increase disproportionately due to overtime labor, expedited shipping, and equipment stress.

This behavior can be incorporated into the cost function by introducing a capacity penalty term:

$$C(q) = F + cq + \beta q^2 + \gamma \cdot \frac{q^k}{(Q_{max} - q)} \tag{28}$$

where γ and k are parameters controlling the severity of capacity-induced cost escalation. As $q \to Q^-_{max}$, the penalty term tends to infinity:

$$\lim_{q \to Q_{max}} \frac{q^k}{Q_{max} - q} = +\infty \tag{29}$$

The marginal cost including capacity effects becomes:

$$MC(q) = c + 2\beta q + \gamma \left[\frac{kq^{k-1}}{Q_{max} - q} + \frac{q^k}{(Q_{max} - q)^2} \right] \tag{30}$$

This formulation captures the intuitive notion that operating near full capacity leads to sharply increasing costs. From a pricing standpoint, this necessitates higher prices to regulate demand and prevent overutilization.

2.7. Supply Chain and Input Cost Volatility

In manufacturing, variable costs are heavily influenced by external factors such as raw material prices, transportation costs, and supplier reliability. These factors introduce stochasticity into the cost function.

Let input cost be represented as a stochastic variable c_t, evolving over time according to a mean-reverting process:

$$dc_t = \kappa(\bar{c} - c_t)dt + \sigma dW_t \tag{31}$$

where κ is the mean-reversion rate, $\bar{c}$ is the long-run mean cost, σ is volatility, and dW_t is a Wiener process. The cost function then becomes time-dependent:

$$C(q,t) = F + c_t q + \beta q^2 \tag{32}$$

This introduces uncertainty into pricing decisions, as firms must anticipate future cost fluctuations when setting prices. The expected future cost at time $t+\tau$ given current cost c_t is:

$$\mathbb{E}[c_{t+\tau}|c_t] = \bar{c} + (c_t - \bar{c})e^{-\kappa\tau} \tag{33}$$

Risk-adjusted pricing strategies often incorporate buffers to account for such volatility. A common approach is to use a certainty-equivalent cost:

$$\tilde{c} = c_t + \lambda \cdot \frac{\sigma^2}{2\kappa} \tag{34}$$

where λ is a risk aversion parameter.

2.8. Multi-Product Cost Structures

Many manufacturing firms produce multiple products, leading to shared costs across production lines. Let q_i denote the output of product i for $i = 1, \ldots, n$. The total cost function becomes:

$$C(q) = F + \sum_{i=1}^{n} c_i q_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} q_i q_j \tag{35}$$

The cross terms β_{ij} for $i \neq j$ capture interactions between products, such as shared machinery or resource competition. The symmetry condition $\beta_{ij} = \beta_{ji}$ ensures that the Hessian matrix is symmetric.

The marginal cost for product i is:

$$MC_i(q) = \frac{\partial C}{\partial q_i} = c_i + \sum_{j=1}^{n}\beta_{ij} q_j \tag{36}$$

This structure complicates pricing, as the cost of producing one product depends on the production levels of others. Optimal pricing must therefore consider the joint optimization of multiple products, leading to the system of first-order conditions:

$$p_i + \sum_{j=1}^{n}\frac{\partial D_j}{\partial p_i}(p_j - MC_j) = 0 \quad \forall i \tag{37}$$

2.9. Learning Effects and Cost Reduction

Over time, manufacturing processes often become more efficient due to learning effects. This can be modeled by allowing marginal cost to decrease with cumulative production. Let $Q^{cum} = \sum_{t=0}^{T} q_t$ denote cumulative output. The learning curve is typically expressed as:

$$c(Q^{cum}) = c_0 (Q^{cum})^{-\theta} \tag{38}$$

where $\theta > 0$ represents the learning rate. The relationship between learning rate L and θ is:

$$L = 1 - 2^{-\theta} \tag{39}$$

The cost reduction from doubling cumulative output is:

$$\frac{c(2Q^{cum})}{c(Q^{cum})} = 2^{-\theta} = 1 - L \tag{40}$$

This implies that costs decline as experience accumulates, enabling firms to reduce prices over time while maintaining profitability. The dynamic pricing problem with learning becomes:

$$\max_{\{p_t, q_t\}} \sum_{t=0}^{T} \beta^t \left[p_t D_t(p_t) - C(q_t; Q_{t-1}^{cum})\right] \tag{41}$$

subject to:

$$Q_t^{cum} = Q_{t-1}^{cum} + q_t \tag{42}$$

Such dynamics are commonly observed in industries like electronics and automotive manufacturing.

2.10. Implications for Pricing Strategy

The structure of the cost function has direct implications for pricing decisions. To see this, consider the profit maximization condition expressed in terms of cost elasticities. The optimal price can be written as:

$$p = \frac{MC(q)}{1 + \frac{1}{E_d}} \cdot \left(1 + \frac{\partial \ln MC}{\partial \ln q} \cdot \frac{1}{E_d}\right)^{-1} \tag{43}$$

This generalized formula shows that:

1) When marginal costs are constant ($\frac{\partial MC}{\partial q} = 0$), the standard markup rule applies.
2) When marginal costs increase with output ($\frac{\partial MC}{\partial q} > 0$), the optimal markup is lower.
3) When marginal costs decrease with output ($\frac{\partial MC}{\partial q} < 0$), the optimal markup is higher.

In essence, pricing in manufacturing is constrained by the need to balance cost recovery with demand stimulation. Firms must carefully analyze cost behavior to determine feasible and optimal pricing strategies.

3. Demand Modeling in Manufacturing Systems

3.1. Overview and Fundamental Relationships

Demand modeling is a fundamental component of pricing strategy in manufacturing systems. It establishes the quantitative relationship between the price of a product and the quantity demanded in the market. In manufacturing environments, demand is influenced not only by price but also by a wide range of external and internal factors, including product quality, availability, brand reputation, delivery timelines, and competitive dynamics.

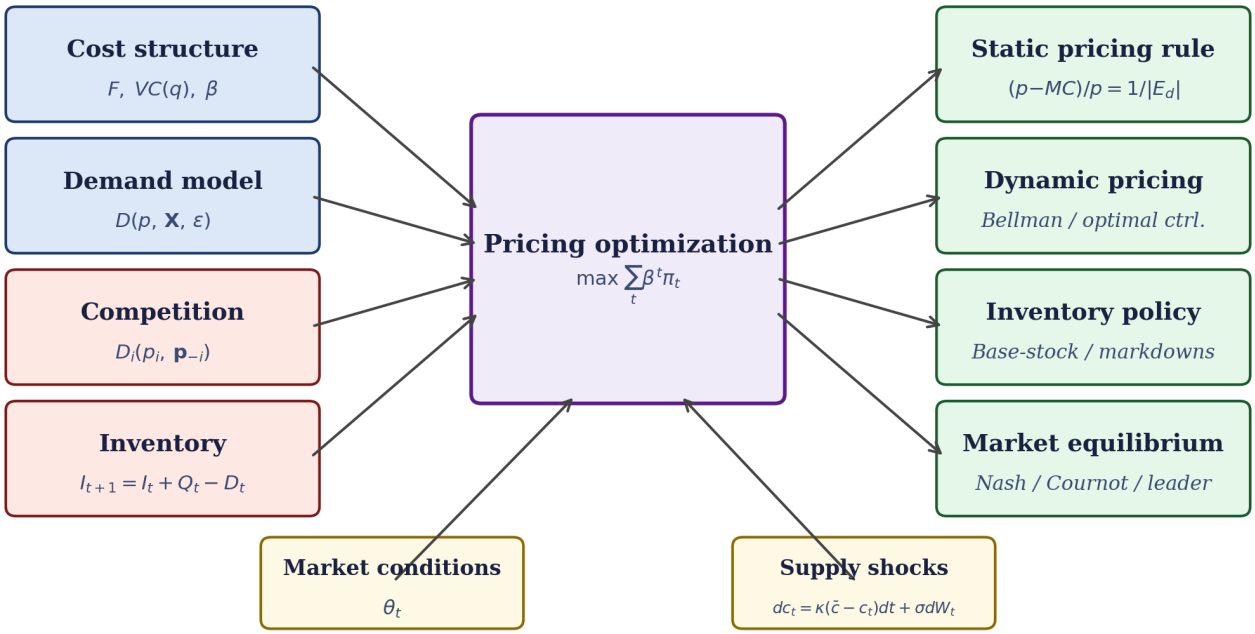


Figure 3. Integrated Framework.

From a mathematical standpoint, demand is represented as a function of price and other relevant variables. Let p denote the price of the product, and let $D(p)$ denote the demand function. The simplest representation of demand assumes that demand is a decreasing function of price:

$$\frac{dD}{dp} < 0 \tag{44}$$

This condition reflects the law of demand, which states that, ceteris paribus, higher prices lead to lower demand.

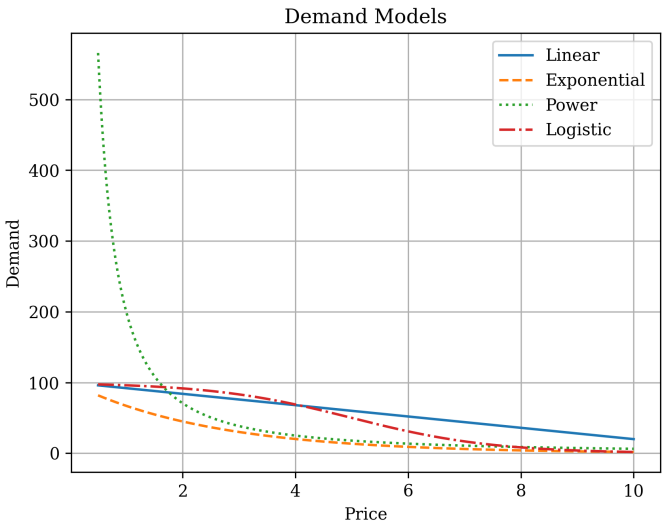


Figure 4. Demand Models.

The relationship between demand and price can be characterized by the price elasticity of demand:

$$E_d(p) = \frac{dD}{dp} \cdot \frac{p}{D(p)} \tag{45}$$

Since $dD/dp < 0$, $E_d(p) < 0$. The magnitude $|E_d(p)|$ indicates the responsiveness of demand to price changes.

3.2. Linear Demand Model

The most commonly used demand model in manufacturing is the linear demand function:

$$D(p) = a - bp \tag{46}$$

where:

1) $a > 0$ represents the maximum market demand when price is zero (market size)
2) $b > 0$ represents the sensitivity of demand to price (slope parameter)

The inverse demand function, expressing price as a function of quantity, is:

$$p(q) = \frac{a}{b} - \frac{1}{b}q \tag{47}$$

The price elasticity for the linear model is:

$$E_d(p) = -\frac{bp}{a - bp} \tag{48}$$

This elasticity varies with price: $|E_d(p)|$ increases as price increases. At the choke price $p = a/b$, demand is zero and elasticity is undefined (infinite). At $p = a/(2b)$, $|E_d| = 1$, representing unit elasticity.

Implications:

1) When $p = 0$, demand is at its maximum value a
2) When $p = \frac{a}{b}$, demand drops to zero
3) Revenue $R(p) = p(a - bp)$ is maximized at $p = a/(2b)$

This model is particularly useful for commodity goods where demand behavior is relatively predictable and homogeneous.

3.3. Nonlinear Demand Models

In many manufacturing contexts, demand does not follow a linear pattern. Instead, it may exhibit nonlinear characteristics due to saturation effects, brand differentiation, and customer segmentation.

3.3.1. Quadratic Demand

$$D(p) = a - bp - cp^2 \tag{49}$$

with $a > 0$, $b > 0$, $c > 0$. The derivative is:

$$\frac{dD}{dp} = -b - 2cp < 0 \tag{50}$$

This model captures accelerating demand decline at higher price levels. The elasticity is:

$$E_d(p) = -\frac{(b + 2cp)p}{a - bp - cp^2} \tag{51}$$

3.3.2. Exponential Demand

$$D(p) = Ae^{-kp} \tag{52}$$

where $A > 0$ is the maximum demand (at $p = 0$) and $k > 0$ is the price sensitivity parameter. The derivative is:

$$\frac{dD}{dp} = -kAe^{-kp} = -kD(p) \tag{53}$$

A key feature of the exponential model is constant elasticity:

$$E_d = \frac{dD}{dp} \cdot \frac{p}{D} = -kp \tag{54}$$

This form is useful for modeling highly price-sensitive markets where the percentage change in demand is proportional to the absolute change in price.

3.3.3. Constant Elasticity (Power) Demand

$$D(p) = \alpha p^{-\varepsilon} \tag{55}$$

where $\varepsilon > 0$ is the constant price elasticity (in absolute value). This model satisfies:

$$\frac{dD}{dp} = -\varepsilon\alpha p^{-\varepsilon-1} = -\varepsilon\frac{D(p)}{p} \tag{56}$$

Thus, $E_d = -\varepsilon$ constant. The inverse demand function is:

$$p(q) = \left(\frac{q}{\alpha}\right)^{-1/\varepsilon} \tag{57}$$

This model is widely used in industrial organization due to its analytical tractability.

3.3.4. Logistic Demand (Saturation Model)

$$D(p) = \frac{K}{1 + e^{\alpha(p - p_0)}} \tag{58}$$

where:

1) K is the maximum market size (saturation level)
2) p_0 is the inflection point (price at which demand is $K/2$)
3) $\alpha > 0$ controls the steepness of the curve

The derivative is:

$$\frac{dD}{dp} = -\frac{\alpha K e^{\alpha(p-p_0)}}{(1+e^{\alpha(p-p_0)})^2} = -\alpha D(p)\left(1 - \frac{D(p)}{K}\right) \tag{59}$$

This model captures market saturation and is particularly useful for products with adoption cycles, where demand is relatively inelastic at very low and very high prices but elastic in the intermediate range.

3.4. Price Elasticity of Demand and Revenue Implications

The relationship between price, elasticity, and revenue is fundamental to pricing decisions. Revenue $R(p) = pD(p)$ has derivative:

$$\frac{dR}{dp} = D(p) + p\frac{dD}{dp} = D(p)\left(1 + E_d(p)\right) \tag{60}$$

Since $E_d(p) < 0$, we have:

1) If $|E_d| < 1$ (inelastic demand), $1 + E_d > 0$, so $dR/dp > 0$: raising price increases revenue.
2) If $|E_d| > 1$ (elastic demand), $1 + E_d < 0$, so $dR/dp < 0$: raising price decreases revenue.
3) If $|E_d| = 1$ (unit elastic), $dR/dp = 0$: revenue is maximized.

The revenue-maximizing price satisfies $E_d(p) = -1$.

3.5. Demand Under Competition

In competitive markets, demand depends not only on the firm's price but also on competitor pricing. A general formulation for firm i in an n-firm market is:

$$D_i(p_i, p_{-i}) = a_i - b_i p_i + \sum_{j \neq i} \gamma_{ij}(p_j - p_i) \tag{61}$$

where $\gamma_{ij} \geq 0$ represents the cross-price sensitivity between firms i and j. The own-price derivative is:

$$\frac{\partial D_i}{\partial p_i} = -b_i - \sum_{j \neq i} \gamma_{ij} < 0 \tag{62}$$

The cross-price derivative for $j \neq i$ is:

$$\frac{\partial D_i}{\partial p_j} = \gamma_{ij} > 0 \tag{63}$$

This formulation captures substitution effects between competing products. The elasticity with respect to own price is:

$$E_{ii} = \frac{\partial D_i}{\partial p_i} \cdot \frac{p_i}{D_i} = -\frac{(b_i + \sum_{j \neq i} \gamma_{ij}) p_i}{D_i} \tag{64}$$

The cross-price elasticity is:

$$E_{ij} = \frac{\partial D_i}{\partial p_j} \cdot \frac{p_j}{D_i} = \frac{\gamma_{ij} p_j}{D_i} \tag{65}$$

3.6. Multi-Factor Demand Models

In realistic manufacturing scenarios, demand depends on multiple variables beyond price:

$$D(p, X) = a - bp + \sum_{k=1}^{m} \theta_k X_k \tag{66}$$

where $X = (X_1, \ldots, X_m)$ represents additional factors such as:

1) Product quality Q: $\partial D/\partial Q > 0$
2) Marketing expenditure M: $\partial D/\partial M > 0$ (with diminishing returns)
3) Delivery time L: $\partial D/\partial L < 0$
4) Brand reputation B: $\partial D/\partial B > 0$

The marginal effect of each factor on demand is given by θ_k. The complete differential of demand is:

$$dD = -b\, dp + \sum_{k=1}^{m} \theta_k\, dX_k \tag{67}$$

This formulation shows that pricing decisions must consider the substitutability between price and non-price factors. For example, if the firm improves quality, it may be able to increase price without losing demand, provided:

$$\frac{dp}{dQ} \leq \frac{\theta_Q}{b} \tag{68}$$

3.7. Stochastic Demand Modeling

Demand in manufacturing is inherently uncertain. To account for this, demand can be modeled as a random variable:

$$D(p) = \bar{D}(p) + \epsilon \tag{69}$$

where:

1) $\bar{D}(p)$ is the expected demand (deterministic component)
2) ϵ is a random disturbance term with $\mathbb{E}[\epsilon] = 0$ and $\mathrm{Var}(\epsilon) = \sigma^2(p)$

Common specifications for the error term include:

1) Additive: $D(p) = a - bp + \epsilon$, $\epsilon \sim N(0, \sigma^2)$
2) Multiplicative: $D(p) = (a - bp)e^{\epsilon}$, $\epsilon \sim N(0, \sigma^2)$
3) Logit: $D(p) = \frac{e^{a-bp}}{1+e^{a-bp}} + \epsilon$

The probability that demand exceeds a given threshold D_0 is:

$$\mathbb{P}(D(p) > D_0) = 1 - F_\epsilon(D_0 - \bar{D}(p)) \tag{70}$$

where F_ϵ is the cumulative distribution function of ϵ.

3.8. Time-Dependent Demand

Demand often varies over time due to seasonality, economic cycles, and market trends:

$$D_t(p) = a_t - b_t p \tag{71}$$

where parameters evolve over time. A common specification is:

$$a_t = a_0 + \alpha \sin\left(\frac{2\pi t}{T_s}\right) + \beta t \tag{72}$$

where T_s is the seasonal period, α captures seasonal amplitude, and β captures trend.

Alternatively, demand may follow a stochastic process such as geometric Brownian motion:

$$da_t = \mu a_t dt + \sigma a_t dW_t \tag{73}$$

3.9. Segmented Demand and Price Discrimination

Manufacturing firms often serve multiple customer segments with different price sensitivities. Let $D_i(p)$ denote demand for segment i:

$$D_i(p) = a_i - b_i p \tag{74}$$

Total demand is:

$$D(p) = \sum_{i=1}^{n} D_i(p) = A - Bp \tag{75}$$

where $A = \sum a_i$ and $B = \sum b_i$.

If the firm can price discriminate (charge different prices to different segments), the optimal prices are:

$$p_i^* = \frac{a_i + b_i c}{2b_i} \tag{76}$$

where c is marginal cost. The profit gain from price discrimination over uniform pricing is:

$$\Delta\pi = \frac{1}{4}\sum_{i=1}^{n} \frac{(a_i - b_i c)^2}{b_i} - \frac{(A - Bc)^2}{4B} \tag{77}$$

This gain is positive whenever segments have different price sensitivities (b_i vary).

3.10. Inventory-Linked Demand

Demand can also depend on inventory availability due to stockout avoidance behavior or promotional effects:

$$D(p, I) = (a - bp)(1 + \delta I) \tag{78}$$

where $\delta > 0$ captures the positive effect of visible inventory on demand (e.g., in retail settings). Alternatively, a more realistic formulation includes a stockout penalty:

$$D(p, I) = \begin{cases} a - bp & \text{if } I \geq a - bp \\ I & \text{if } I < a - bp \end{cases} \tag{79}$$

This captures demand truncation due to limited availability.

4. Profit Optimization in Manufacturing Pricing

4.1. Overview and Fundamental Relationships

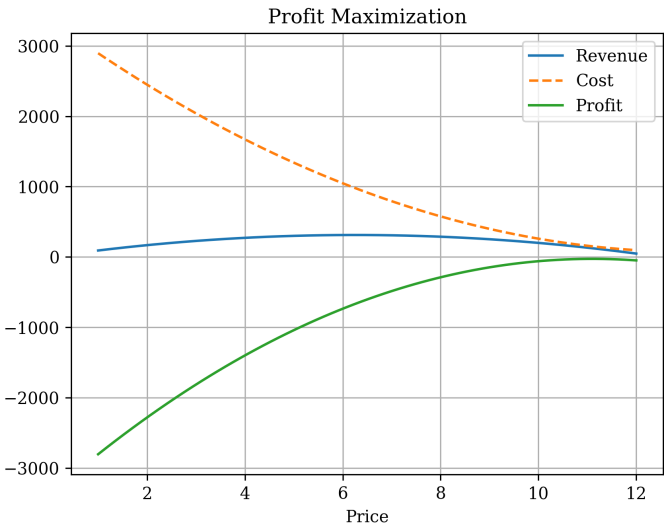


Figure 5. *Plot for Profit Maximization*

Profit optimization lies at the core of pricing strategy in manufacturing systems. While cost structures determine the feasibility of production and demand modeling captures market behavior, profit optimization integrates these elements into a unified decision-making framework. The objective of the firm is to determine the price that maximizes profit while respecting operational constraints and market conditions.

Let p denote the unit price and $D(p)$ denote the demand function. Let $C(q)$ denote the cost function, where $q = D(p)$ represents the quantity sold. The profit function is defined as:

$$\Pi(p) = p \cdot D(p) - C(D(p)) \tag{80}$$

The pricing problem is therefore formulated as:

$$\max_{p \geq 0} \Pi(p) \tag{81}$$

This formulation captures the fundamental trade-off between price and demand: increasing price raises revenue per unit but reduces quantity sold, while decreasing price increases demand but lowers margins.

4.2. First-Order Conditions and the Lerner Index

The first-order condition for profit maximization is:

$$\frac{d\Pi}{dp} = D(p) + p\frac{dD}{dp} - \frac{dC}{dq}\frac{dD}{dp} = 0 \tag{82}$$

Rearranging:

$$p - MC(q) = -\frac{D(p)}{dD/dp} \tag{83}$$

Dividing both sides by p and using the definition of price elasticity $E_d = (dD/dp)(p/D)$:

$$\frac{p - MC(q)}{p} = -\frac{1}{E_d} = \frac{1}{|E_d|} \tag{84}$$

The left-hand side is the Lerner Index of market power, which measures the relative markup over marginal cost. This fundamental relationship shows that:

1) The optimal markup is inversely proportional to the absolute price elasticity of demand
2) More elastic demand ($|E_d|$ large) implies smaller markups
3) Less elastic demand ($|E_d|$ small) allows larger markups
4) In perfectly competitive markets ($|E_d| \to \infty$), $p = MC$

4.3. Profit Maximization with Linear Demand and Linear Cost

Assume linear demand:

$$D(p) = a - bp \tag{85}$$

and linear cost:

$$C(q) = F + cq \tag{86}$$

where c is constant marginal cost. Substituting:

$$\Pi(p) = p(a - bp) - [F + c(a - bp)] \tag{87}$$

Expanding:

$$\Pi(p) = ap - bp^2 - F - ca + cbp \tag{88}$$

Taking derivative:

$$\frac{d\Pi}{dp} = a - 2bp + cb \tag{89}$$

Setting equal to zero:

$$a - 2bp + cb = 0 \tag{90}$$

Solving:

$$p^* = \frac{a + cb}{2b} \tag{91}$$

The optimal quantity is:

$$q^* = a - bp^* = a - b\left(\frac{a + cb}{2b}\right) = \frac{a - cb}{2} \tag{92}$$

Maximum profit is:

$$\Pi^* = p^*q^* - F - cq^* = \frac{(a - cb)^2}{4b} - F \tag{93}$$

The markup over marginal cost is:

$$p^* - c = \frac{a + cb}{2b} - c = \frac{a - cb}{2b} \tag{94}$$

The Lerner index is:

$$\frac{p^* - c}{p^*} = \frac{a - cb}{a + cb} \tag{95}$$

4.4. Profit Maximization with Quadratic Cost

Now assume quadratic cost:

$$C(q) = F + cq + \beta q^2 \tag{96}$$

with $\beta > 0$ (increasing marginal cost). Using linear demand $D(p) = a - bp$, profit is:

$$\Pi(p) = p(a - bp) - F - c(a - bp) - \beta(a - bp)^2 \tag{97}$$

Expanding:

$$\Pi(p) = ap - bp^2 - F - ca + cbp - \beta(a^2 - 2abp + b^2p^2) \tag{98}$$

$$\Pi(p) = ap - bp^2 - F - ca + cbp - \beta a^2 + 2\beta abp - \beta b^2p^2 \tag{99}$$

Grouping terms:

$$\Pi(p) = -p^2(b + \beta b^2) + p(a + cb + 2\beta ab) - (F + ca + \beta a^2) \tag{100}$$

Taking derivative:

$$\frac{d\Pi}{dp} = -2(b + \beta b^2)p + (a + cb + 2\beta ab) = 0 \tag{101}$$

Solving:

$$p^* = \frac{a + cb + 2\beta ab}{2b(1 + \beta b)} \tag{102}$$

The marginal cost at the optimum is:

$$MC(q^*) = c + 2\beta q^* = c + 2\beta(a - bp^*) \tag{103}$$

It can be verified that $p^* > MC(q^*)$, with the markup decreasing in β.

4.5. Capacity-Constrained Optimization

Manufacturing systems operate under capacity constraints:

$$D(p) \leq Q_{max} \tag{104}$$

Thus, the optimization problem becomes:

$$\max_p \Pi(p) \quad \text{s.t.} \quad D(p) \leq Q_{max} \tag{105}$$

The Lagrangian for this problem is:

$$\mathcal{L}(p, \lambda) = \Pi(p) + \lambda(Q_{max} - D(p)) \tag{106}$$

where $\lambda \geq 0$ is the Lagrange multiplier (shadow price of capacity). The Karush-Kuhn-Tucker (KKT) conditions are:

$$\frac{d\Pi}{dp} - \lambda \frac{dD}{dp} = 0 \tag{107}$$

$$\lambda(Q_{max} - D(p)) = 0 \tag{108}$$

$$\lambda \geq 0, \quad Q_{max} - D(p) \geq 0 \tag{109}$$

There are two cases:

Case 1: Capacity not binding ($D(p) < Q_{max}$). Then $\lambda = 0$ and the unconstrained optimum applies.

Case 2: Capacity binding ($D(p) = Q_{max}$). Then $\lambda > 0$ and the optimal price satisfies:

$$p = MC(q) + \frac{D(p)}{-dD/dp} - \lambda \frac{dD/dp}{-dD/dp} = MC(q) + \frac{1}{|E_d|} \cdot p - \lambda \tag{110}$$

Since $\lambda > 0$, the binding capacity constraint reduces the markup compared to the unconstrained case, because the firm cannot expand output to meet demand.

4.6. Multi-Product Profit Optimization

For multiple products with interdependent demand:

$$\Pi(p) = \sum_{i=1}^{n} p_i D_i(p) - C(D(p)) \tag{111}$$

The first-order conditions for each product i are:

$$\frac{\partial \Pi}{\partial p_i} = D_i + \sum_{j=1}^{n} p_j \frac{\partial D_j}{\partial p_i} - \sum_{j=1}^{n} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i} = 0 \tag{112}$$

This can be rewritten as:

$$D_i + \sum_{j=1}^{n} (p_j - MC_j) \frac{\partial D_j}{\partial p_i} = 0 \tag{113}$$

In matrix form, let p be the price vector, MC the marginal cost vector, and J the Jacobian matrix with entries $J_{ij} = \partial D_j / \partial p_i$. The optimality condition is:

$$D(p) + J^T(p - MC) = 0 \tag{114}$$

This system of equations must be solved simultaneously for the optimal prices.

4.7. Profit Optimization Under Uncertainty

When demand is uncertain, the firm maximizes expected profit:

$$\max_p \ \mathbb{E}[\Pi(p)] = p\bar{D}(p) - \mathbb{E}[C(\bar{D}(p) + \epsilon)] \tag{115}$$

If cost is linear, $C(q) = F + cq$, then:

$$\mathbb{E}[\Pi(p)] = p\bar{D}(p) - F - c\bar{D}(p) \tag{116}$$

The uncertainty does not affect the optimal price because expected profit is linear in demand. However, if cost is convex ($C''(q) > 0$), by Jensen's inequality:

$$\mathbb{E}[C(\bar{D} + \epsilon)] \geq C(\bar{D}) \tag{117}$$

This implies that uncertainty reduces expected profit and may affect optimal pricing. For quadratic cost $C(q) = F + cq + \beta q^2$:

$$\mathbb{E}[C(\bar{D} + \epsilon)] = F + c\bar{D} + \beta(\bar{D}^2 + \sigma^2) \tag{118}$$

The optimal price then satisfies:

$$p^* = \frac{a + cb + 2\beta ab}{2b(1 + \beta b)} + \frac{2\beta b\sigma^2}{2b(1 + \beta b)} = p_0^* + \frac{\beta\sigma^2}{1 + \beta b} \tag{119}$$

where p_0^* is the optimal price under certainty. Thus, uncertainty with convex costs leads to higher optimal prices as a risk premium.

4.8. Dynamic Profit Optimization

Over a discrete-time horizon:

$$\max_{\{p_t\}_{t=0}^{T}} \sum_{t=0}^{T} \gamma^t \left[p_t D_t(p_t) - C_t(D_t(p_t)) \right] \tag{120}$$

where $\gamma = 1/(1+r)$ is the discount factor and r is the interest rate. The Euler equation for this problem (ignoring state variables) is:

$$\frac{d\Pi_t}{dp_t} + \gamma \frac{d\Pi_{t+1}}{dp_t} = 0 \tag{121}$$

where the second term captures intertemporal effects (e.g., through inventory or learning).

5. Competition Modeling in Manufacturing Pricing

5.1. Overview and Strategic Interdependence

In manufacturing industries, pricing decisions are rarely made in isolation. Firms operate in competitive markets where the pricing strategy of one firm directly influences the demand and profitability of others. As a result, pricing must be analyzed within a strategic framework that accounts for the actions and reactions of competing firms.

Competition modeling provides a mathematical structure for analyzing such interactions. It extends the basic demand and profit models by incorporating competitor behavior, leading to equilibrium-based pricing outcomes. These models are particularly important in manufacturing sectors characterized by product similarity, low switching costs, and intense price competition.

5.2. Duopoly Pricing Framework

Consider a market with two firms, indexed by $i \in \{1, 2\}$. Let p_i denote the price set by firm i, and let $D_i(p_i, p_j)$ denote its demand, where $j \neq i$ represents the competing firm.

A common demand specification is the linear differentiated products model:

$$D_i(p_i, p_j) = a - bp_i + \gamma(p_j - p_i) \tag{122}$$

where:

1) $a > 0$ represents baseline demand
2) $b > 0$ captures own-price sensitivity
3) $\gamma > 0$ captures cross-price sensitivity (substitutability)

The own-price derivative is:

$$\frac{\partial D_i}{\partial p_i} = -(b + \gamma) < 0 \tag{123}$$

The cross-price derivative is:

$$\frac{\partial D_i}{\partial p_j} = \gamma > 0 \tag{124}$$

The degree of product substitutability is measured by $\sigma = \gamma/(b+\gamma) \in [0, 1)$, where $\sigma = 0$ indicates independent products and $\sigma \to 1$ indicates nearly perfect substitutes.

5.3. Best Response Functions and Nash Equilibrium

Each firm seeks to maximize its profit:

$$\Pi_i(p_i, p_j) = p_i D_i(p_i, p_j) - C_i(D_i) \tag{125}$$

Assuming linear cost $C_i(q) = c_i q$:

$$\Pi_i = p_i[a - bp_i + \gamma(p_j - p_i)] - c_i[a - bp_i + \gamma(p_j - p_i)] \tag{126}$$

Simplifying:

$$\Pi_i = (p_i - c_i)[a + \gamma p_j - (b + \gamma)p_i] \tag{127}$$

The first-order condition with respect to p_i is:

$$\frac{\partial \Pi_i}{\partial p_i} = [a + \gamma p_j - (b + \gamma)p_i] - (b + \gamma)(p_i - c_i) = 0 \tag{128}$$

Solving for p_i yields the best response function:

$$p_i = \frac{a + \gamma p_j + (b + \gamma)c_i}{2(b + \gamma)} \tag{129}$$

This can be written as:

$$p_i = \frac{a + \gamma p_j}{2(b + \gamma)} + \frac{c_i}{2} \tag{130}$$

The best response function shows that:

1) p_i is increasing in p_j (strategic complements) because $\partial p_i / \partial p_j = \gamma/[2(b+\gamma)] > 0$
2) p_i is increasing in own cost c_i
3) p_i is increasing in market size a

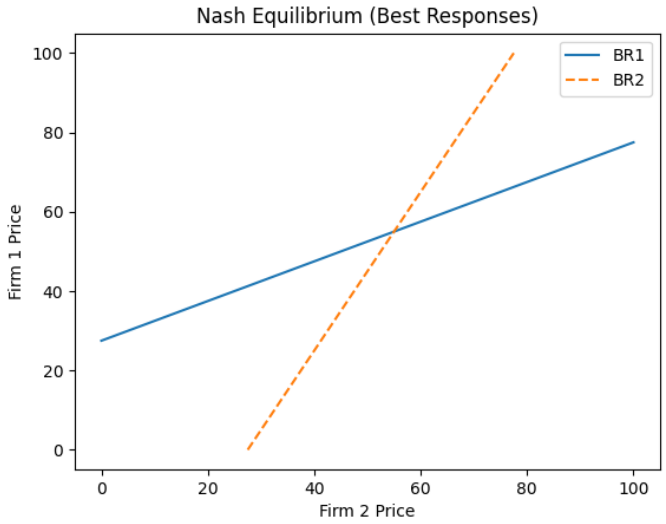


Figure 6. Nash Equilibrium

The Nash equilibrium is found by solving the system of best response functions:

$$p_1 = \frac{a+\gamma p_2+(b+\gamma)c_1}{2(b+\gamma)}, \quad p_2 = \frac{a+\gamma p_1+(b+\gamma)c_2}{2(b+\gamma)} \tag{131}$$

Solving simultaneously:

$$p_1^* = \frac{a}{2b+\gamma} + \frac{(b+\gamma)(2b+\gamma)c_1 + \gamma(b+\gamma)c_2}{4b(b+\gamma)+\gamma^2} \tag{132}$$

$$p_2^* = \frac{a}{2b+\gamma} + \frac{(b+\gamma)(2b+\gamma)c_2 + \gamma(b+\gamma)c_1}{4b(b+\gamma)+\gamma^2} \tag{133}$$

For symmetric costs ($c_1 = c_2 = c$):

$$p^* = \frac{a}{2b+\gamma} + \frac{(b+\gamma)(2b+\gamma)+\gamma(b+\gamma)}{4b(b+\gamma)+\gamma^2}c \tag{134}$$

which simplifies to:

$$p^* = \frac{a}{2b+\gamma} + \frac{b+\gamma}{2b+\gamma}c \tag{135}$$

The equilibrium markup over marginal cost is:

$$p^* - c = \frac{a-(b+\gamma)c}{2b+\gamma} \tag{136}$$

Note that as γ increases (products become more substitutable), the markup decreases, approaching the competitive outcome as $\gamma \to \infty$.

5.4. Bertrand Competition with Homogeneous Products

When products are identical (perfect substitutes), $\gamma \to \infty$ and the demand for each firm becomes perfectly elastic at the market price. In the standard Bertrand model with constant marginal cost c, the unique Nash equilibrium is:

$$p_1^* = p_2^* = c \tag{137}$$

with zero profits. This is the Bertrand paradox: with just two firms, price equals marginal cost as in perfect competition. The intuition is that any firm can undercut its rival to capture the entire market, leading to a price war until price reaches marginal cost.

If firms have different marginal costs ($c_1 < c_2$), the equilibrium is:

$$p_1^* = c_2 - \epsilon, \quad p_2^* = c_2 \tag{138}$$

where ϵ is arbitrarily small. The low-cost firm captures the entire market and earns profit $(c_2 - c_1) \cdot D(c_2)$, while the high-cost firm sells nothing.

5.5. Bertrand Competition with Differentiated Products

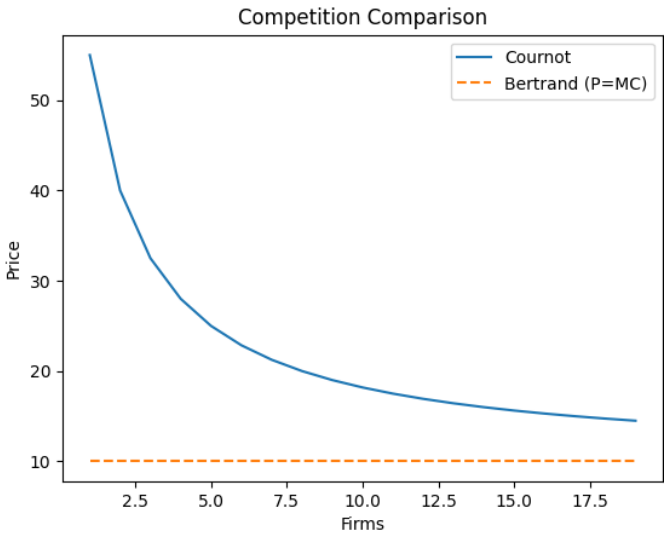


Figure 7. Competition Comparsion

With differentiated products, firms can maintain positive markups even with price competition. The equilibrium conditions from the duopoly analysis apply. The Lerner index for each firm in equilibrium is:

$$\frac{p_i^* - c_i}{p_i^*} = \frac{1}{|E_{ii}|}\left(1 - \sum_{j \neq i} \frac{E_{ij}}{E_{ii}} \cdot \frac{p_j^* - c_j}{p_j^*}\right) \tag{139}$$

where E_{ii} is own-price elasticity and E_{ij} is cross-price elasticity. This shows that markups are reduced by the presence of close substitutes.

5.6. Cournot Competition (Quantity-Based)

In some manufacturing settings, firms compete in quantities rather than prices. This is appropriate when capacity constraints are important or when firms set production levels first and then let the market determine price.

Let q_i denote output of firm i. Total market quantity is $Q = \sum_i q_i$. Price is determined by inverse demand:

$$p = a - bQ \tag{140}$$

Profit for firm i is:

$$\Pi_i = (a - bQ)q_i - c_i q_i \tag{141}$$

The first-order condition for firm i is:

$$\frac{\partial \Pi_i}{\partial q_i} = a - bQ - bq_i - c_i = 0 \tag{142}$$

This gives:

$$a - b(q_i + Q_{-i}) - bq_i - c_i = 0 \quad \Rightarrow \quad a - 2bq_i - bQ_{-i} - c_i = 0 \tag{143}$$

where $Q_{-i} = \sum_{j \neq i} q_j$. The best response function is:

$$q_i = \frac{a - c_i - bQ_{-i}}{2b} \tag{144}$$

For symmetric costs ($c_i = c$ for all i), the Nash equilibrium quantity per firm is:

$$q^* = \frac{a - c}{b(n+1)} \tag{145}$$

Total quantity is:

$$Q^* = \frac{n(a - c)}{b(n+1)} \tag{146}$$

Equilibrium price is:

$$p^* = a - bQ^* = a - \frac{n(a-c)}{n+1} = \frac{a + nc}{n+1} \tag{147}$$

As $n \to \infty$, $p^* \to c$ (perfect competition). The Lerner index is:

$$\frac{p^* - c}{p^*} = \frac{1}{n} \tag{148}$$

This shows that market power decreases with the number of firms.

5.7. Comparison of Bertrand and Cournot Equilibria

For the same demand and cost parameters, Bertrand competition yields lower prices and higher quantities than Cournot competition. For duopoly with linear demand and constant marginal cost:

$$p_B = c \quad \text{(homogeneous products)} \tag{149}$$

$$p_C = \frac{a + 2c}{3} > c \tag{150}$$

With differentiated products, the comparison depends on the degree of substitutability. As $\gamma \to 0$ (independent products), both models yield the monopoly price. As $\gamma \to \infty$ (perfect substitutes), Bertrand yields $p \to c$ while Cournot yields $p \to (a + 2c)/3$.

5.8. Price Leadership Models

In many manufacturing industries, dominant firms act as price leaders. Consider a market with a leader (firm 1) and a follower (firm 2). The follower behaves as a price taker, setting $p_2 = p_1$ (if products are homogeneous) or choosing a best response. The leader anticipates the follower's response.

For homogeneous products, the leader faces residual demand:

$$D_{res}(p) = D(p) - S_f(p) \tag{151}$$

where $S_f(p)$ is the follower's supply function. If the follower has constant marginal cost c_2 and capacity constraint K_2, then:

$$S_f(p) = \begin{cases} 0 & \text{if } p < c_2 \\ [0, K_2] & \text{if } p = c_2 \\ K_2 & \text{if } p > c_2 \end{cases} \tag{152}$$

The leader maximizes profit on the residual demand, leading to a price typically above the competitive level.

5.9. Dynamic Competition

Competition evolves over time. In repeated games, firms may sustain collusive outcomes through trigger strategies. The grim trigger strategy (cooperate as long as all have cooperated; defect forever if anyone defects) sustains collusion if the discount factor satisfies:

$$\gamma > \frac{\pi^D - \pi^C}{\pi^D - \pi^N} \tag{153}$$

where π^C is collusive profit, π^D is deviation profit, and π^N is Nash equilibrium profit. For Bertrand competition with homogeneous products:

$$\gamma > \frac{1}{2} \tag{154}$$

Thus, patient enough firms can sustain monopoly pricing through tacit collusion.

6. Inventory-Based Pricing in Manufacturing Systems

6.1. Overview and Dynamic Coupling

Inventory plays a critical role in manufacturing pricing decisions. Unlike purely demand-driven pricing frameworks, manufacturing firms must account for the availability of physical goods, storage constraints, and inventory-related costs. Inventory levels introduce a dynamic coupling between production, pricing, and demand, making pricing decisions inherently state-dependent.

Let I_t denote the inventory level at the beginning of period t. The evolution of inventory is:

$$I_{t+1} = I_t + Q_t - D(p_t) \tag{155}$$

where Q_t is production quantity and $D(p_t)$ is demand. The firm incurs holding costs hI_t (where $h > 0$ is per-unit holding cost) and may face stockout costs when demand exceeds available inventory.

6.2. Profit Function with Inventory Costs

The period profit function including inventory costs is:

$$\begin{aligned}\Pi_t = {} & p_t \min(D(p_t), I_t + Q_t) - C(Q_t) - hI_t \\ & - s \cdot \max(0, D(p_t) - I_t - Q_t)\end{aligned} \tag{156}$$

where s is the per-unit stockout penalty (including lost goodwill). For simplicity, assuming production can be adjusted to meet demand up to capacity, we set $Q_t = D(p_t)$ (make-to-order) and the profit simplifies to:

$$\Pi_t = p_t D(p_t) - C(D(p_t)) - hI_t \tag{157}$$

but inventory now evolves as $I_{t+1} = I_t - D(p_t)$ (if no production), which is problematic. More realistically, with production decisions, we have a two-variable control problem.

6.3. Optimal Pricing with Inventory Constraints

The pricing problem with inventory is a dynamic programming problem. Let $V(I_t)$ be the value function (maximum expected present value of future profits) starting with inventory I_t. The Bellman equation is:

$$V(I_t) = \max_{p_t, Q_t} \{p_t D(p_t) - C(Q_t) - hI_t + \gamma \mathbb{E}[V(I_{t+1})]\} \tag{158}$$

subject to:

$$I_{t+1} = I_t + Q_t - D(p_t) \geq 0 \tag{159}$$

and $0 \leq Q_t \leq Q_{\max}$.

The first-order condition with respect to p_t is:

$$\frac{\partial}{\partial p_t} [p_t D(p_t) - \gamma \mathbb{E}[V'(I_{t+1})] D(p_t)] = 0 \tag{160}$$

This gives:

$$p_t + \frac{D(p_t)}{dD/dp_t} = MC(Q_t) + \gamma \mathbb{E}[V'(I_{t+1})] \tag{161}$$

The term $\gamma \mathbb{E}[V'(I_{t+1})]$ is the shadow value of inventory at time $t+1$. This shows that the optimal price depends on the marginal value of future inventory: higher future inventory value (scarcity) leads to higher current prices.

6.4. Base-Stock and Order-Up-To Policies

A common heuristic is the base-stock policy: maintain inventory at a target level I^*. The production decision is:

$$Q_t = \max(0, I^* - I_t + D(p_t)) \tag{162}$$

This is equivalent to an order-up-to policy where $I_{t+1} = I^*$ whenever possible. With this policy, the pricing problem reduces to choosing p_t to balance current profit against the cost of achieving the target inventory.

6.5. Clearance Pricing and Markdowns

When inventory is excessive, firms may use clearance pricing to accelerate sales. A common clearance pricing model is:

$$p(I) = p_0 - \kappa (I - I^*)_+ \tag{163}$$

where $\kappa > 0$ is the clearance sensitivity. The revenue from clearance is:

$$R_{clear} = \int_{I^*}^{I} p(I') dI' = p_0 (I - I^*) - \frac{\kappa}{2} (I - I^*)^2 \tag{164}$$

The optimal clearance depth can be derived from dynamic programming.

6.6. Stockout Pricing and Rationing

When inventory is low, firms may increase prices to ration demand. A simple rationing price is:

$$p(I) = p_0 + \frac{\eta}{I} \tag{165}$$

for $I > 0$, where $\eta > 0$ determines the scarcity premium. This ensures that as $I \to 0^+$, price becomes very high, preventing stockouts.

6.7. Stochastic Inventory Systems

With stochastic demand $D(p) = \bar{D}(p) + \epsilon$, inventory follows:

$$I_{t+1} = \max(0, I_t + Q_t - D(p_t)) \tag{166}$$

The Bellman equation becomes:

$$V(I_t) = \max_{p_t, Q_t} \{\mathbb{E}[p_t D(p_t) - C(Q_t)] - hI_t + \gamma \mathbb{E}[V(I_{t+1})]\} \tag{167}$$

This is a stochastic dynamic programming problem that can be solved numerically using value iteration or policy iteration.

7. Dynamic Pricing in Manufacturing Systems

7.1. Overview and Time-Dependent Optimization

Dynamic pricing extends the traditional static pricing framework by allowing prices to vary over time in response to changing market conditions, inventory levels, cost fluctuations, and competitive behavior. In manufacturing systems, dynamic pricing is particularly important due to the presence of inventory constraints, demand uncertainty, and time-varying production costs.

Unlike static pricing, where a single price is chosen to maximize profit, dynamic pricing involves determining a sequence of prices $\{p_t\}_{t=0}^{T}$ over a planning horizon. The objective is to maximize cumulative profit while accounting for intertemporal trade-offs.

7.2. Optimal Control Formulation

Consider continuous time with inventory dynamics:

$$\frac{dI(t)}{dt} = -D(p(t)), \quad I(0) = I_0, \quad I(T) \geq 0 \tag{168}$$

The firm chooses the price path $p(t)$ to maximize:

$$\max_{p(t)} \int_0^T e^{-rt} \left[p(t)D(p(t)) - hI(t)\right] dt \tag{169}$$

where r is the discount rate. This is an optimal control problem with state variable $I(t)$ and control $p(t)$. The Hamiltonian is:

$$H(I, p, \lambda, t) = e^{-rt}[pD(p) - hI] - \lambda D(p) \tag{170}$$

where $\lambda(t)$ is the costate variable (shadow price of inventory). The optimality condition $\partial H/\partial p = 0$ gives:

$$e^{-rt}\left[D(p) + p\frac{dD}{dp}\right] - \lambda \frac{dD}{dp} = 0 \tag{171}$$

Simplifying:

$$p(t) + \frac{D(p)}{dD/dp} = e^{rt}\lambda(t) \tag{172}$$

The costate equation is $\dot{\lambda} = -\partial H/\partial I = e^{-rt}h$, so:

$$\lambda(t) = \lambda(T)e^{rt} - \frac{h}{r}(e^{rt} - e^{rT}) \tag{173}$$

The transversality condition $\lambda(T)I(T) = 0$ determines the terminal condition.

7.3. Discrete-Time Dynamic Programming

In discrete time, with state $S_t = (I_t, \theta_t)$ where θ_t captures market conditions, the Bellman equation is:

$$V(S_t) = \max_{p_t} \{\Pi_t(S_t, p_t) + \gamma \mathbb{E}[V(S_{t+1})|S_t, p_t]\} \tag{174}$$

The optimal policy is:

$$p_t^* = \pi^*(S_t) = \arg\max_{p_t} \{\Pi_t + \gamma \mathbb{E}[V(S_{t+1})]\} \tag{175}$$

7.4. Learning and Exploration-Exploitation Trade-off

When demand parameters are unknown, the firm faces a trade-off between exploration (learning about demand) and exploitation (maximizing current profit). This can be formulated as a Bayesian dynamic programming problem.

Let θ be unknown parameters with prior distribution $P(\theta)$. After observing demand D_t at price p_t, the posterior is updated via Bayes' rule:

$$P(\theta|D_t) \propto P(D_t|p_t,\theta)P(\theta) \tag{176}$$

The value function incorporates learning:

$$V(S_t,P_t) = \max_{p_t}\left\{\mathbb{E}_{\theta\sim P_t}[\Pi_t(p_t,\theta)] + \gamma\mathbb{E}_{D_t}[V(S_{t+1},P_{t+1})]\right\} \tag{177}$$

where P_t is the belief distribution at time t.

8. Integrated Model for Manufacturing Pricing

8.1. Unified State Space Representation

We now integrate all previous components into a unified framework. The system state at time t is:

$$S_t = (I_t, c_t, \theta_t, p_{-i,t}) \tag{178}$$

where:

1) I_t: inventory level
2) c_t: marginal cost (may be stochastic)
3) θ_t: demand parameters (may be unknown)
4) $p_{-i,t}$: competitors' prices

The pricing policy is:

$$p_t = \pi(S_t) \tag{179}$$

8.2. Integrated Demand and Cost Functions

Demand is:

$$D_t(p_t, p_{-i,t}, \theta_t) = a(\theta_t) - b(\theta_t)p_t + \sum_{j\neq i}\gamma_{ij}(p_{j,t} - p_t) \tag{180}$$

Cost is:

$$C_t(Q_t, c_t) = F + c_tQ_t + \beta Q_t^2 + \gamma_c\frac{Q_t^k}{Q_{\max} - Q_t} \tag{181}$$

where $Q_t = \min(D_t, I_t + Q_t^{prod})$ and Q_t^{prod} is production quantity.

8.3. Dynamic Optimization Problem

The firm solves:

$$\max_{\pi}\mathbb{E}\left[\sum_{t=0}^{T}\gamma^t\Pi_t(S_t,p_t)\right] \tag{182}$$

subject to:

$$I_{t+1} = I_t + Q_t^{prod} - D_t \tag{183}$$

$$Q_t^{prod} \leq Q_{\max} \tag{184}$$

$$|p_t - p_{t-1}| \leq \Delta \quad \text{(price adjustment constraint)} \tag{185}$$

8.4. Solution Approaches

This integrated model is typically solved using numerical methods:

1) Dynamic programming for low-dimensional state spaces
2) Reinforcement learning (Q-learning, policy gradients) for high-dimensional problems
3) Model predictive control for real-time applications

The Bellman optimality equation is:

$$V(S) = \max_{p}\left\{\Pi(S,p) + \gamma\int V(S')f(S'|S,p)dS'\right\} \tag{186}$$

where f is the transition kernel.

9. Limitations and Future Research

This study presents a structured analytical framework for pricing decisions in manufacturing environments. The framework can be further enriched by incorporating broader demand specifications that capture evolving customer preferences and market conditions. Future research can integrate stochastic and data-driven demand models to support continuous learning from observed market responses. The inclusion of inventory levels, setup costs, lead times, supply-chain conditions, and multi-period production planning can provide a more comprehensive representation of manufacturing operations. Further extensions can consider multi-product markets, heterogeneous consumer preferences, and dynamic interactions among competing firms. Machine learning and reinforcement learning techniques can also be integrated to develop adaptive pricing strategies that respond to changing demand and competitive conditions. Finally, empirical studies using real-world manufacturing datasets and industry case studies can provide valuable insights into the practical applicability, robustness, and scalability of the proposed framework.

10. Conclusion

This paper has developed a comprehensive mathematical framework for pricing in manufacturing systems, integrating cost structures, demand modeling, profit optimization, competitive interactions, inventory dynamics, and dynamic pricing strategies. The analysis demonstrates that manufacturing pricing cannot be reduced to simple heuristics but requires a multi-layered approach grounded in economic theory, operational analysis, and strategic decision-making.

The key contributions of this work include: (i) a formal characterization of manufacturing cost structures with fixed, variable, capacity-dependent, and learning components; (ii) demand models that account for linear and nonlinear behaviors, competition, uncertainty, and time-dependence; (iii) profit optimization formulations incorporating elasticity-based pricing rules, capacity constraints, and risk considerations; (iv) inventory-based pricing frameworks that capture the dynamic coupling between supply and demand; (v) dynamic pricing models using optimal control and dynamic programming; and (vi) an integrated framework that unifies these dimensions into a tractable optimization problem.

The findings highlight that effective pricing in manufacturing requires firms to consider not only marginal costs and demand elasticity but also capac ity limitations, inventory holding costs, competitive responses, and intertemporal trade-offs. Firms that adopt mathematically grounded pricing models are better equipped to navigate uncertainty, respond to market changes, and achieve sustainable profitability.

Abbreviations

AC	Average Cost
AFC	Average Fixed Cost
KKT	Karush-Kuhn-Tucker
MC	Marginal Cost
MR	Marginal Revenue

Author Contributions

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Conflicts of Interest

The author declares no conflicts of interest.

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