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Research Article

Competitive Analysis at the Cournot-Nash Equilibrium of an Interconnected Network

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Abstract

The electric power network has long been subject to monopolization, encompassing production, transportation, and distribution sectors. However, recent liberalization efforts have introduced competition into the electricity market. To understand and manage this competition, game theory, a prominent tool in economics, is frequently employed. Specifically, competition within the electricity market has been analyzed through various game-theoretical models, including Bertrand's atomicity, Cournot's homogeneity, and Nash's research competition. These models aim to achieve the Cournot-Nash equilibrium, where each participant in the market makes optimal decisions given the strategies of others. To effectively allocate production and ensure a balance between supply and demand, as well as to maintain the stability of the interconnected network, one has adopted a method that combines Load Flow techniques with game theory principles. This hybrid approach enables a strategic distribution of power production, taking into account the competitive dynamics of the market. By integrating these methodologies, one can address the complexities of competition while ensuring efficient and stable operation of the power grid. This innovative approach not only enhances the management of electricity production and distribution but also fosters a more competitive and resilient power network. Moreover, the application of game theory in this context allows for a deeper understanding of strategic interactions among market participants. It helps in predicting behaviors, formulating strategies, and anticipating market changes, thus providing a robust framework for decision-making. This is particularly crucial in a liberalized market where multiple entities vie for market share and profitability. By employing game-theoretical insights, one can simulate various market scenarios, optimize resource allocation, and enhance overall market efficiency. Furthermore, this approach supports the integration of renewable energy sources by ensuring that their variable nature is accommodated within the grid's operational dynamics. In summary, the intersection of game theory and load flow methods offers a comprehensive solution to the challenges posed by a competitive electricity market, paving the way for a sustainable and efficient energy future.

Keywords

Electricity Market, Competition, Load Flow, Game Theory, Interconnected Network, Cournot-Nash

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The 1980s to 1990s led to decisions to open electric power grids to competition by Americans and Europeans. After a few years, some African countries are concerned with liberalizing the electricity market because the companies in possession of the monopoly find themselves in difficulty because of the soaring price of fuel for thermal power stations and the gradual increase in energy consumers.

Following the liberalization of the electricity market, power plants of foreign companies are connected to the network to increase the power requested in order to ensure user satisfaction and limit the occurrence of tense situations.

The management of competition plays an important role because competitive behavior during the liberalization process is still poorly understood.

1.1. Operating Models of the Electricity Market

The restructuring of electricity markets means, as the term implies, a change in the very structure of the industry, generally aimed at introducing increased competition in the production of electricity. [1, 14]

To this end, and unlike the options present in other economic sectors, several market models can be put in place:

1. A vertically integrated monopoly;
2. Competition in the wholesale market;
3. Competition in the retail market;
4. Competition through an electricity exchange.

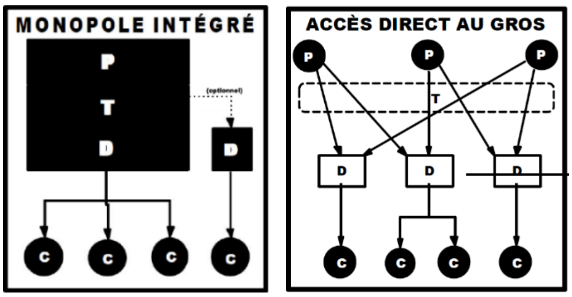

Figure 1. Model of an integrated monopoly and Model of competition on the wholesale market (P: producer, T: transport, D: distributor, C: consumer).

For this study, one chooses the model of competition on the wholesale market: the technical and economic parameters are defined for competition in oligopoly of the electricity market.

1.2. Rules for Electricity Markets Open to Competition

Competition needs to be pure and perfect to allow equilibrium in all markets under very specific sufficient conditions. So the competition must fulfill the following points [2]:

1. Introduction

1. Atomicity: no agent is capable of "weighing in" and a fortiori of setting the price alone;
2. Product homogeneity: all firms deliver products that buyers consider to be identical, homogeneous or substitutable;
3. Free entry and exit on the market: no opposition to the arrival or departure of the firms that make up the industry, which also means that returns to scale are assumed to be non-increasing;
4. The free circulation of factors of production: capital and labor are perfectly mobile and can move from one industry to another;
5. Transparency of information: absence of over-the-counter exchange and, on the contrary, the existence of an overall mechanism.

As everything is not perfect, the electricity market faces its own specific difficulties. In the theoretical study, the price of electricity balances supply and demand, whatever the hazard. On the other hand, in practice, taking into account the physical characteristics of the electricity and the transaction costs, the price becomes lower than the highest contingent supply costs. In summary, the presence of the inelasticity of demand in the short term and the absence of equilibrium prices in extreme peak situations attack the electricity market from the inside.

The following figure illustrates, in normal times, the level of consumption sufficient to calculate an equilibrium price p0:

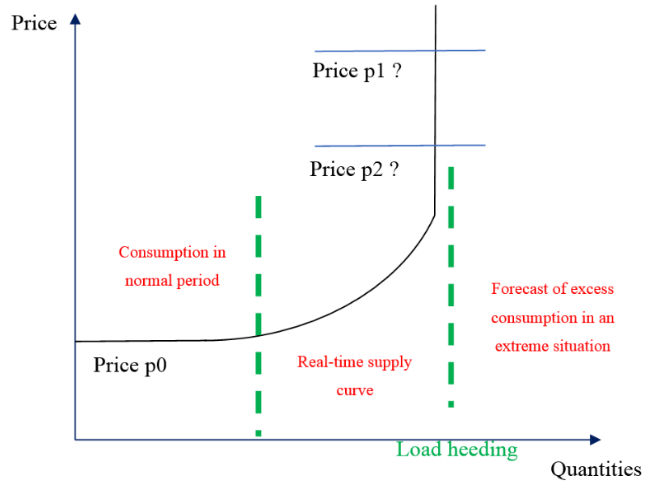

Figure 2. Principle of supply and demand in the event of an extreme peak situation [3].

2. Economic Modeling of the Electricity Market and Competition Study

By knowing the structure and the forms/reforms of the electricity market, let us model this market on the economic plan by taking into account its principles according to its actors.

Through the details of the possible strategies on the opening of a market, one identifies what corresponds to the electricity market with planning models.

2.1. Principle of the Electricity Market

A market evolves towards its equilibrium for the following reasons [13]:

(1) Prices regulate buying and selling intentions;

(2) When buying and selling intentions do not match, the price adjusts.

The equilibrium price is the only relevant price because the amount of desired supply is equal to the amount of desired demand at that price.

The competitive equilibrium must be at the point where the supply and demand curves intersect. [4]

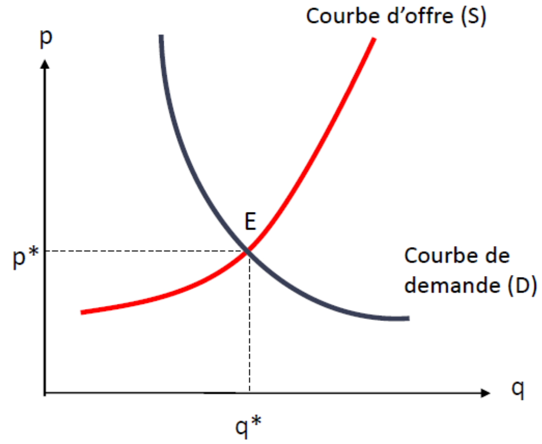

Figure 3. *Balance of supply and demand.*

It is at price $p<p^*$, demand is q_D and supply is q_S: not all demand is satisfied hence there is excess demand.

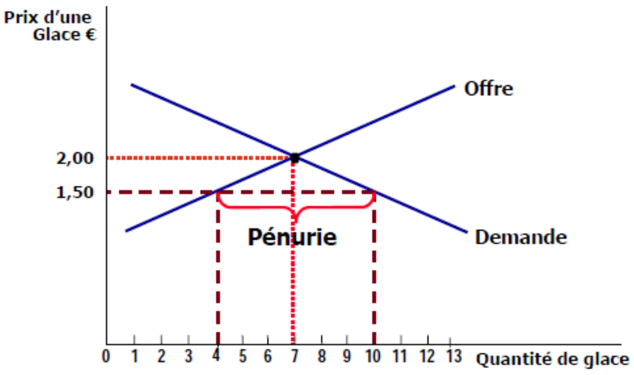

Figure 4. *Excess demand and price adjustment.*

Then at the price $p>p^*$, the demand is q_D and the supply is q_S: all the production is not sold and there is excess supply.

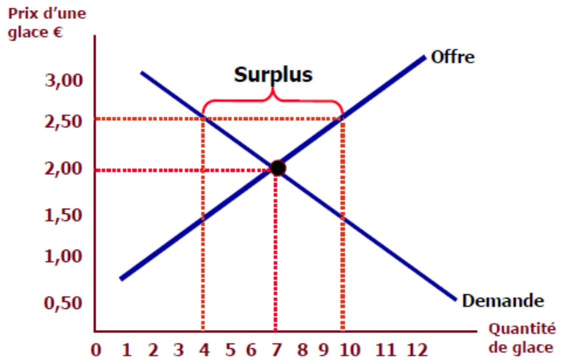

Figure 5. *Excess supply and price adjustment.*

2.2. Discrete Version of the Model for Investment Planning and Production of Interconnected Networks

One assumes that the objective of cooperation between players is the minimization of discounted total costs. Considering a finite date T of the game and keeping the cooperating networks face the following optimization problem [12]:

$$min \sum_{t=0}^{T} \rho^t \left\{\sum_i \left[C_p^i(K_i, q_i) + C_I^i(K_i, I_i) + C_u^i(k_i, u_i)\right]\right\}$$

One will define a cooperative game with n people in characteristic form as follows.

Let be the set of networks and denote by a coalition of s players. One minimizes the sum of discounted costs for all possible coalitions and defines the characteristic function v as follows:

$$v(\{i\}) = 0$$

$$v(S) = C(S) - \sum_{i \in S} C(\{i\})$$

3. Distribution of Production Based on the Principle of Hybridizing Methods

For a simplistic model of a competitive electricity market, taking into account [7, 11]:

1. Atomicity: a competitive price with the Bertrand model;
2. The homogeneity of the products: consider the active power produced with the Cournot model;
3. Free entry and exit on the market: rational producers with a Nash equilibrium;
4. Free movement of factors of production: no restrictions on exchanges with the Cournot-Nash equilibrium;
5. Transparency of information: production investment planning with the uncertainties of several producers.

3.1. Network Status Analysis

The power distribution calculation consists of determining all the power flows and voltages in the network for a given load case. Four quantities are associated with each node of the system: the active and reactive powers as well as the modulus and the phase of the voltage. Only two of these four variables are known in a node, the other two being determined during the calculation.

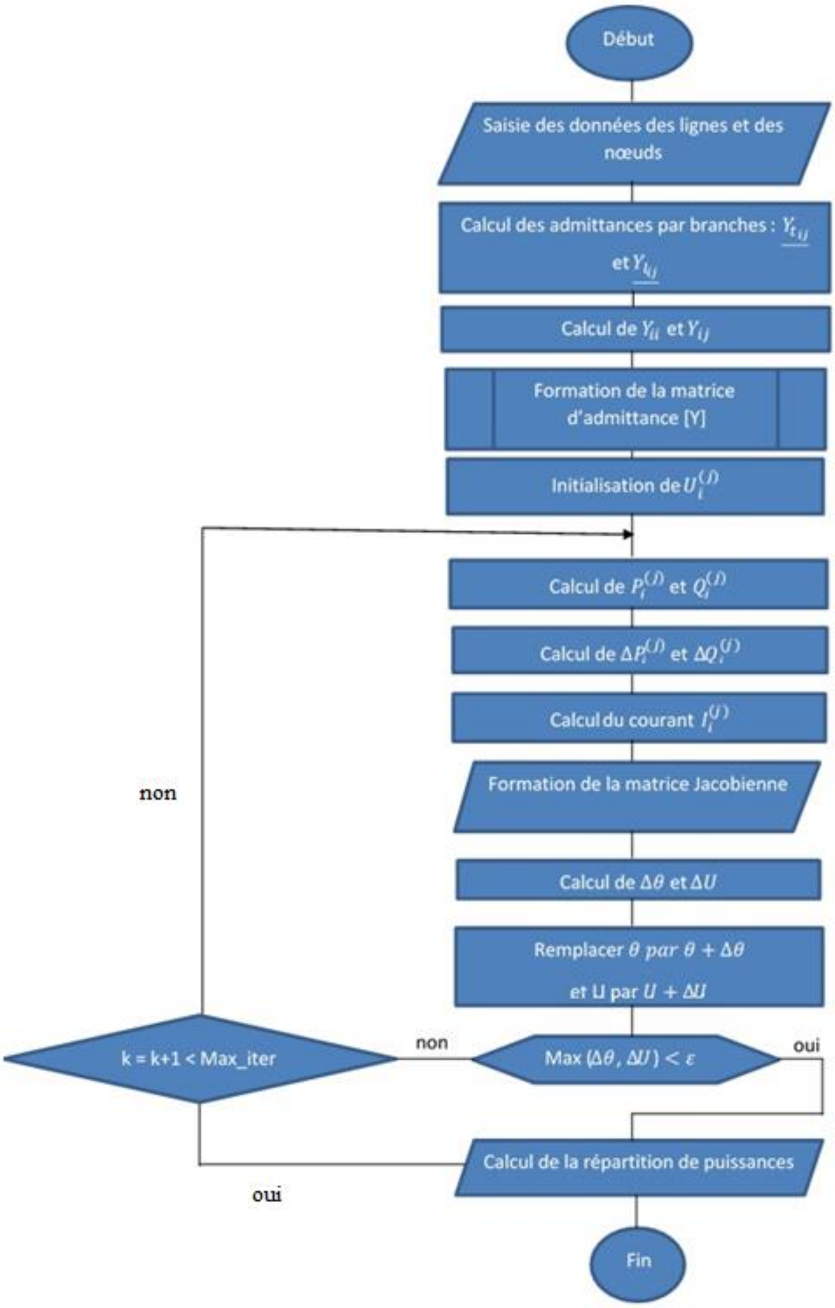

Figure 6. *Newton-Raphson flowchart.*

3.2. Optimal Production Distribution

The location of the power stations and the loads, as well as the connections of a network being known for the next time slot, the problem consists in determining at a given moment the distribution of the productions between these power stations, so as to satisfy the consumption given in minimizing a function representing the total production cost. [5, 6]

This distribution and this cost may include reactive powers or be limited to active powers only.

Moreover, to be realistic, it is necessary to take into account certain constraints of inequality reflecting physical or contractual limitations.

The cost of starting up the units will have been taken into account beforehand in an optimum management study for the reservoirs, also providing the daily operating plan for the hydraulic storage power stations, or their equivalent marginal cost.

Therefore, one has on the one hand power plants whose active power production is imposed, on the other hand power plants characterized by an hourly production cost depending on their regime.

If the characteristics of a unit *i* are known, you can deduce its hourly cost:

$$C_i = f(P_i, Q_i, U_i)$$

The physical limits of the generators and the limits of static stability force us to impose limits on the active and reactive powers produced P_i, Q_i.

$$P_i\min \leq P_i \leq P_i\max$$

$$Q_i\min \leq Q_i \leq Q_i\max$$

The necessary conditions for an extremum are obtained by constructing the function:

$$\Phi(x, u) = C(x, u) + (\lambda)_t.\left(g(x, u)\right)$$

Consider a system of algebraic equations, in general nonlinear. The elements of the vector (λ) are called Lagrange multipliers. They can be attributed an economic significance.

The last matrix equation is that of the load-flow.

By associating a Lagrange multiplier λi to each function Ci, you can see the association of our objective with the Lagrangian.

$$L(\lambda, P) = C(P) + \sum_{i=1}^{n} \lambda_i\left[\sum P_L + pertes - \sum P_i\right]$$

$$= \sum C_i P_{Gi} + \lambda\left[\sum P_L + pertes - \sum P_i\right]$$

3.3. Results of the Application of the Methods

One chooses the case of the interconnected network of Antananarivo also called RIA, since that is where the doctoral school is located, so it is easier to have the data necessary for the study to be done for this theme.

The RIA has during our study 50 nodes, including 16 lines and cables then 59 transformers.

With a network with N nodes, you have 2N data and 2N unknowns.

For the balance node, you impose U and θ then you obtain P and Q.

For consumer-nodes or PQ-nodes, you impose P and Q then you obtain U and θ.

Finally for the node-producer or node-PV, you impose P and U then you obtain Q and θ.

For this RIA, N=50: so you have 100 unknowns with 100 equations to find them.

This balance sheet node is node n°1: PIA (Ambohimanambola) with a voltage of 63kV.

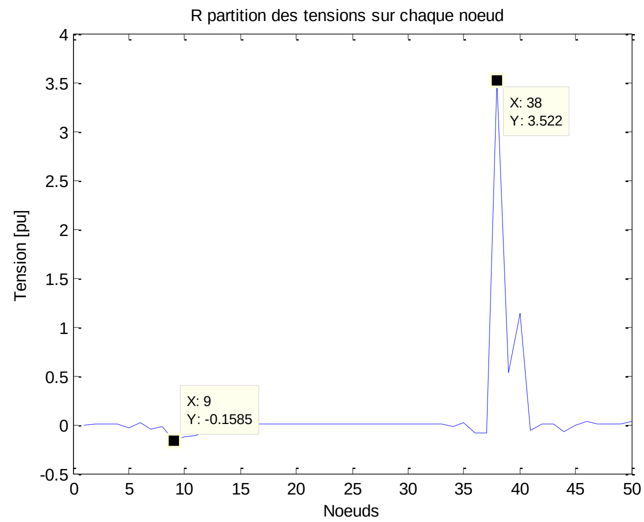

Figure 7. *The voltage on each node of the RIA.*

After the distribution, variations in voltage were observed at each node in order to detect undervoltage and overvoltage conditions. The furthest undervoltage was found at node 9 with (-0.16) pu, likely due to high load or demand in the vicinity of the affected node. The overvoltage at node 38 with 3.5 pu is attributed to high electrical energy production at this node, referred to as "Voltage Peak."

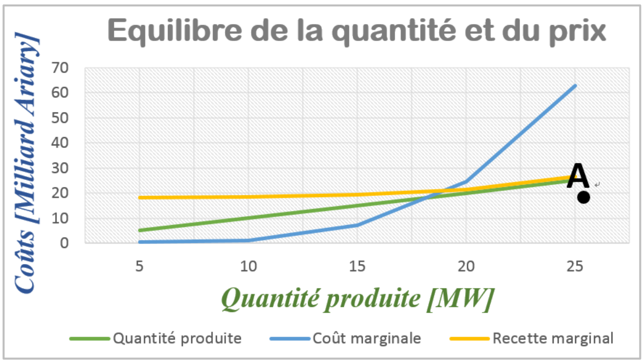

Figure 8. *Price-production balance.*

The marginal cost of this electrical energy is crucial for assessing the profitability of production and determining the equilibrium price that allows for maximum profit. This analysis is based on the price-production equilibrium curve established from data provided by Jirama and ORE, in collaboration with the Ecole Supérieure Polytechnique d'Antananarivo.

According to the previous figure, point A corresponds to the intersection where revenues equal costs, thus marking a minimum breakeven point where the company realizes neither profit nor loss.

To achieve maximum profit, when costs exceed revenues (Cost > Revenue), it is imperative that our production surpasses *point A*. In the context of the studied interconnected network, the principle of Nash equilibrium, according to game theory, is reached if and only if this production exceeds 18.75 MW. This ensures optimal profitability of this electrical production activity.

3.4. Cournot-Nash Equilibrium on the RIA

At time *t*, after calculating the distribution of loads in an interconnected network, you define: [8, 9]

1. The overall state of the power to be ensured ΔP_T^t;
2. The production of each incumbent operator with the installed initial capacity $P_{init,i}$;
3. The maximum allowed capacity for an operator at each site $P_{max,i}$.

For a player, one will set their new capacity to be installed on the network at time *t*, where it is for existing players p_i^t and for new operators p_j^t.

One considers *n* firms competing in a market. Each producer *i*:

$$\max \pi = \sum r_i q_i$$

With:

r_i: Cost price for a producer i; which is equal to (revenue - costs) [Ariary/MW].

q_i: Quantity produced by a producer i [MW].

Subject to the following constraints:

1) Sum of the powers to be distributed

$$\sum q_i = beq$$

2) Inequality:

$$\frac{a_i}{b_i} \leq \sum q_i \leq \frac{beq}{k}$$

With:

a_i: Common expenses for producer *i* [Ariary].

b_i: Selling price / Quantity of producer *i* [Ariary/MW].

k: Correlation coefficient with Supply-Demand equilibrium.

$$\frac{d\pi_i}{dq_i} = 0$$

The condition provides the best response from producer *i*, with quantities of other firms *j* fixed:

$$q_j^{BR}(q_i) = \frac{[a - b\sum_i q_i - c]}{2b}$$

The equilibrium is obtained by substitution:

$$q_i^* = \frac{[a + \sum_{k \neq i} c_k - nc_i]}{(n+1)b}$$

The power distribution will be done according to the Load Flow result; after obtaining negative active powers, you can find the Cournot-Nash equilibrium to stabilize the interconnected network under study.

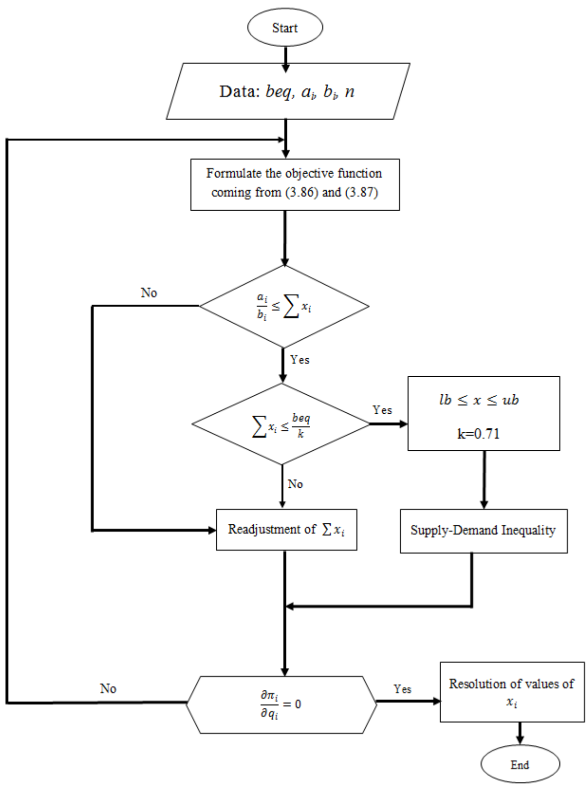

Figure 9. Operating principle of the model.

During this study, the interconnected network has 50 nodes, including 16 lines and cables, and 59 transformers.

The voltage levels on the HT and MT sides are: 138 kV, 63 kV, 35 kV, and 20 kV. After the Load Flow analysis of the interconnected network, one is able to determine the power to be shared among the producers: 102.833 MW.

It should be noted that profits are also dependent on the production shares for each chosen scenario. Taking into account the marginal cost and the number of selected producers n, one is able to derive the following scenarios [10, 15]:

***Table 1**. Private producers' shares on the RIA with their profits.*

For n=5		For n=10		For n=20	
Profits [Milliard ariary]	**Power produced [MW]**	**Profits [Milliard ariary]**	**Power produced [MW]**	**Profits [Milliard ariary]**	**Power produced [MW]**
20,056	26,737	0,709	0,945	0,709	0,945
0,684	0,912	0,684	0,912	0,684	0,912
0,921	1,228	0,921	1,228	0,921	1,228
54,767	73,012	16,581	22,105	0,837	1,116
0,709	0,945	0,732	0,976	0,732	0,976
		0,626	0,834	54,759	73,012
		0,716	0,954	0,716	0,954
		0,706	0,941	0,706	0,941
		0,695	0,927	0,695	0,927
		54,767	73,012	0,686	0,914
				0,675	0,900
				0,665	0,873
				0,655	0,945
				0,709	0,912
				0,684	0,900
				9,385	12,513
				0,837	1,116
				0,732	0,976
				0,626	0,834
				0,716	0,954

The results show remarkable stability in total production, maintained at *120.833 MW* regardless of the number of producers.

Profit strongly depends on the quantity of electricity produced. Increasing the number of producers leads to a more uniform distribution of power, which could have significant implications for the economic and organizational aspects of the system.

3.5. Interpretation of the Results

To distribute power on an interconnected network, technical and economic parameters must be combined. Therefore, hybridizing Newton-Raphson Load Flow and game theory in the electricity market with Cournot-Nash equilibrium improves competition in electricity production.

By observing the distribution of active powers alone and considering the objective function.

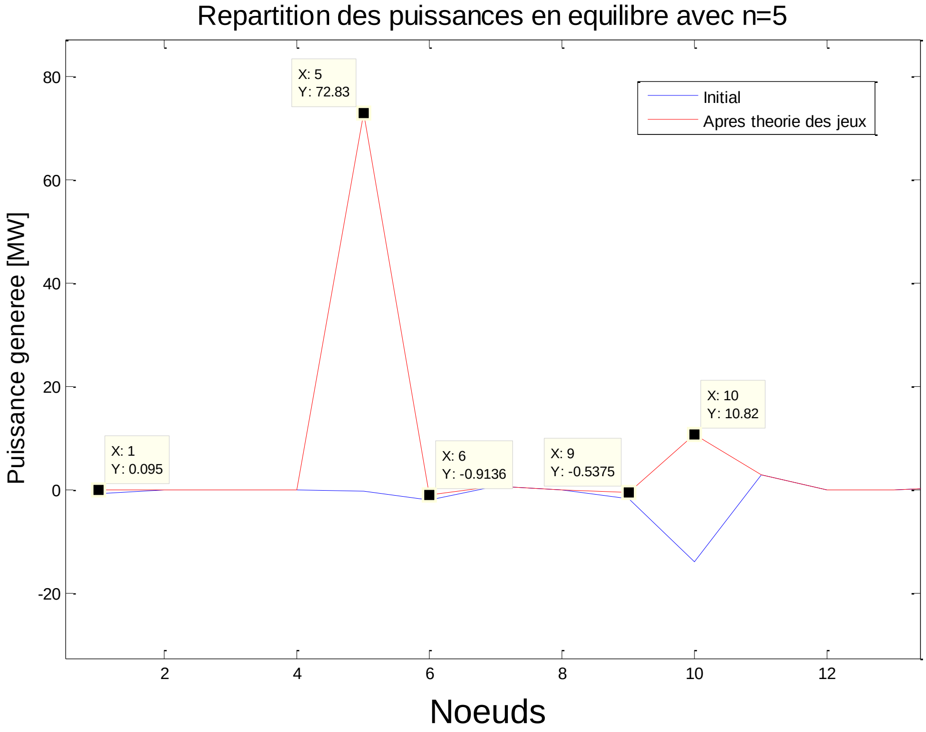

Figure 10. Distribution of powers at Cournot-Nash equilibrium with n=5.

Observation reveals a distribution of power among 5 nodes, in accordance with the prior constraint of power distribution among 5 producers, where underlying tensions are present. Disparities in generated powers stem from each producer's fixed and variable production costs, thus inducing distinct levels of production and profit. This diversity reflects the Cournot-Nash equilibrium, where each producer seeks to maximize their profit while considering the minimum selling price, within an oligopolistic competitive context.

Next, still considering active power only after Cournot-Nash equilibrium with n=10, one has:

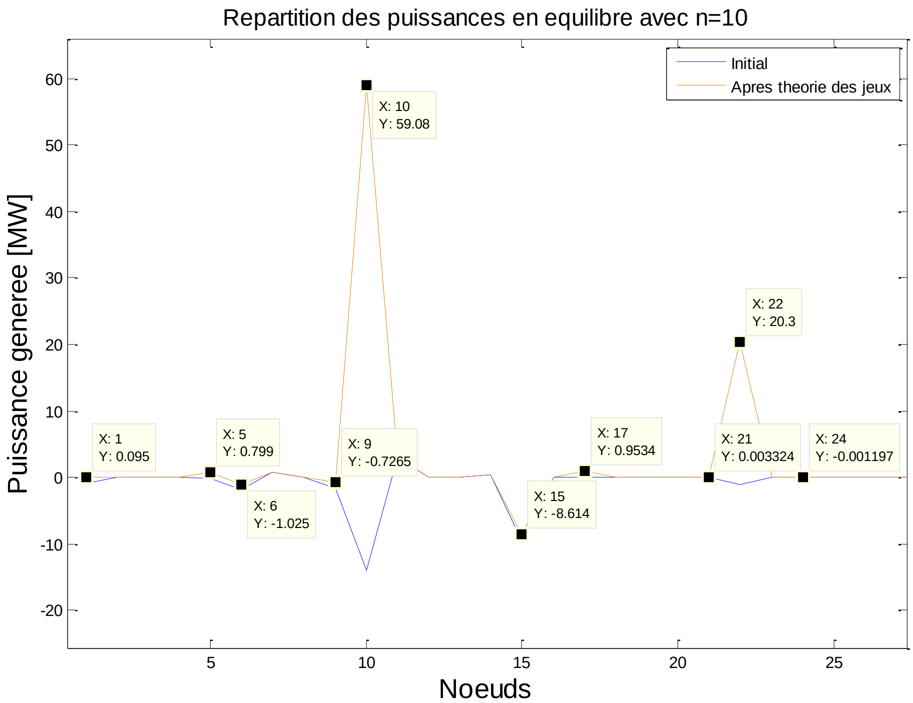

Figure 11. Power distribution after equilibrium with n=10.

According to the previous figure, it is noted that, despite the variation in the number of producers, the maximum power remains constant at 73.012 MW, in line with one of the pre-existing constraints. Furthermore, the next maximum power is reduced to 22.105 MW compared to its previous value of 26.737 MW when the number of producers is increased to 5. This observation indicates a redistribution of power among market producers.

Thus, the constraint of maximum power remains stable, but a decrease in the next maximum power is observed with an increase in the number of producers, suggesting adjustments in the competitive dynamics of the interconnected network.

Finally, still considering active power only after Cournot-Nash equilibrium with n=20, one has:

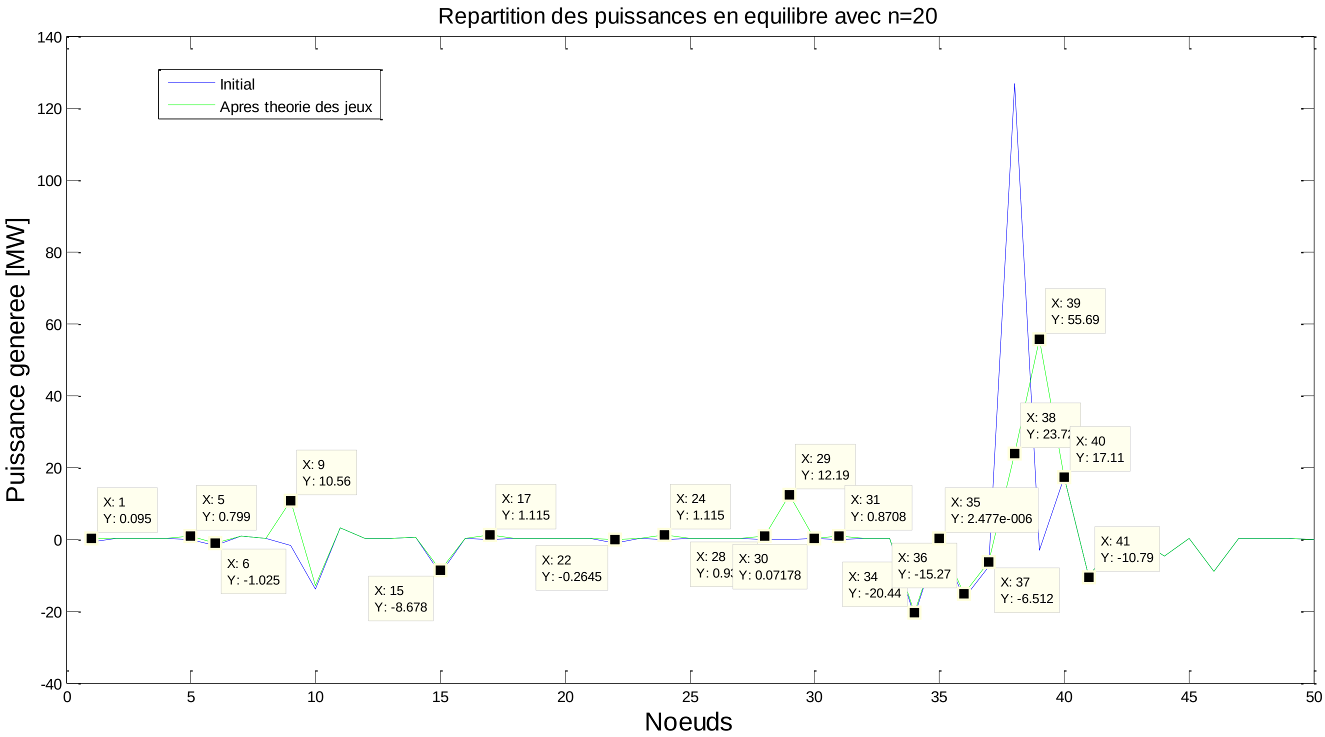

Figure 12. *Power distribution after equilibrium with n=20.*

The previous observations highlight the persistence of a constant maximum power, stabilizing successively at 73.102 MW and then at 12.513 MW. This consistency in maximum power values indicates a notable regularity in the behavior of the studied electrical network.

Furthermore, it is pertinent to note that this power distribution appears to be closely associated with substantial voltage differences across the network. Divergent voltages in different segments of the electrical network can influence the distribution of generated power. This observation is consistent with the principles of electrical network theory, where voltage differences can lead to power flows along preferred paths.

By analyzing the distribution of maximum power more closely as a function of the number of producers *(n)*, revealing trends emerge. For n=5, the maximum power is localized at node 05. When the number of producers increases to n=10, the maximum power concentrates at node 10. Finally, for n=20, the maximum power is observed at node 39.

These observations indicate a direct correlation between the location of maximum power and fluctuations in the number of producers in the network. This relationship between network topology and the distribution of maximum power highlights the crucial importance of voltage management and network configuration in the dynamics of electricity production and distribution.

Thus, the analysis of power distribution in the electrical network through game theory according to Cournot-Nash reveals a complex and interdependent dynamics. The observations highlight how electricity producers adjust their production based on the strategy adopted by other market players, thus influencing the location of maximum power. This approach provides a valuable framework for understanding competitive interactions in the electricity sector and underscores the importance of cooperation and coordination to ensure efficient operation of the electrical network.

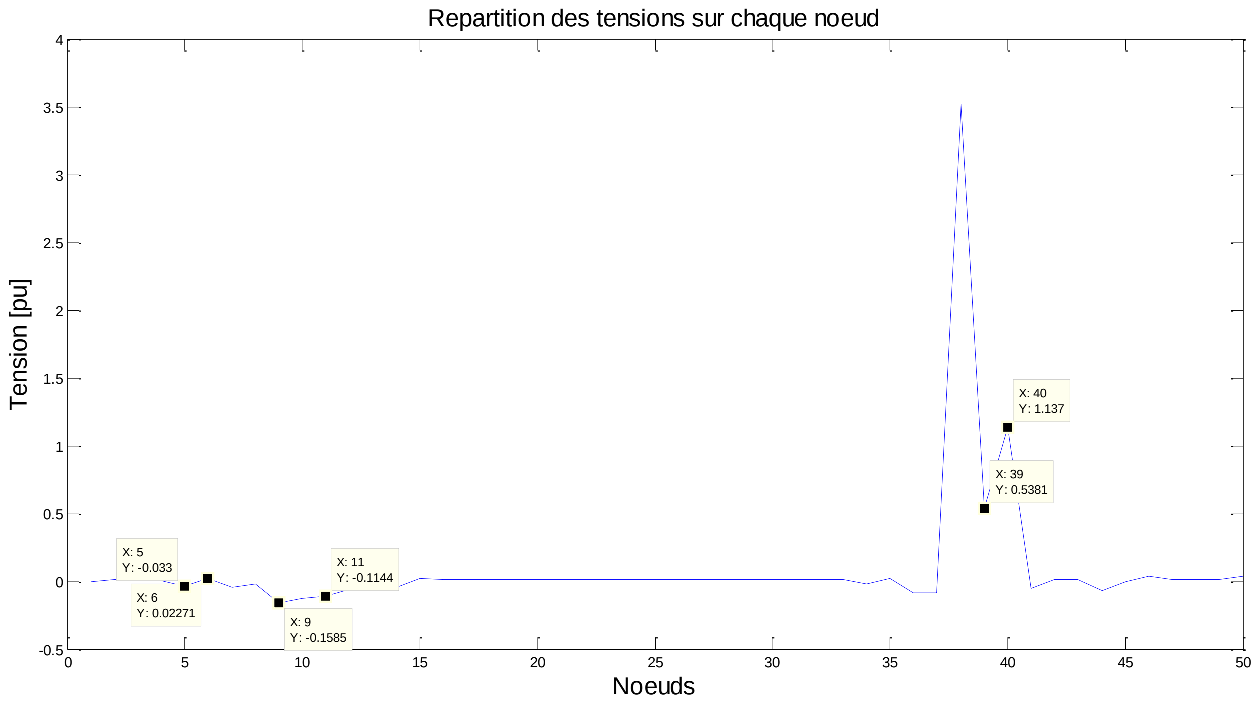

Figure 13. *The highest voltage variations between nodes.*

4. Conclusions

The electric power network has experienced monopolization in production, transportation, and even distribution. However, it has been liberalized for some time now, leading to the existence of competition.

Game theory, commonly used in economics, has been introduced to address this competition in the electricity market. Nash equilibrium implements a game with *n* players with repetition, and one has solved our market, considered as an oligopoly, using a dynamic game algorithm.

Planning production through the hybridization of Load Flow methods and game theory was the choice to effectively allocate the powers that need to be produced for a balance of supply and demand, as well as to manage competition and the stability of the interconnected network.

Distribution based on Cournot-Nash equilibrium establishes a competitive framework for the interconnected network. By setting the objective function and constraints to minimize production costs and maximize profits for each producer.

Abbreviations

HT	Haute Tension
Jirama	Jiro sy Rano Malagasy
MT	Moyenne Tension
ORE	Office de Régulation de l'Electricité
PIA	Poste d'Interconnexion d'Antananarivo
RIA	Réseaux Interconnecté d'Antananarivo

Author Contributions

Onja Voalintsoa: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Resources, Software, Visualization, Writing – original draft, Writing – review & editing

Andry August Randriamitantsoa: Project administration, Supervision, Validation

Solofo Hery Rakotoniaina: Supervision, Validation

Conflicts of Interest

The authors declare no conflicts of interest.

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