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Research Article

Mathematical Modeling of Intensified Heat Transfer in Channels with Diaphragms During the Flow of Transformer Oils in Laminar and Transition Regions

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Abstract

Mathematical modeling of heat transfer in pipes with turbulators at Reynolds numbers characteristic of the transient flow regime is carried out. The solution of the heat transfer problem for semicircular cross-section flow turbulators based on multiblock computing technologies based on the solution of the Reynolds equations (closed using the Menter shear stress transfer model) and the energy equation (on multiscale intersecting structured grids) by the factorized finite volume method (FCOM) was considered. This method was previously successfully applied and verified by experiment for higher Reynolds numbers. 5. Implemented by the FKOM method, the study generated both local and integral, both stationary and unsteady characteristics of flow and heat transfer in a pipe with internal ribs for transitional and laminar coolant flow modes, which made it possible to determine the levels for these modes intensification of heat transfer, which correlate satisfactorily with the available experimental data. In the study, calculated results of enhanced heat transfer in pipes with diaphragms for the transient flow regime of transformer oil were obtained using an analytical method - based on a 4-layer turbulent boundary layer scheme - which are in very good agreement with the numerical ones, which determines their mutual verification. The obtained patterns can be used in engineering and scientific calculations of intensified laminar and transition heat transfer during flow in channels with protrusions used in advanced heat exchangers, used, for example, in aviation, rocket, and space technology.

Keywords

Modeling, Heat Transfer, Cross Section, Turbulator, Pipe, Semicircle, Flow, Reynolds Number, Transient Mode

1. Introductory Part

The generally accepted experimentally retested toolkit for tornado intensification of heat removal is the implementation of repeating turbulators on the surfaces of coolant washes [1, 2] (Figure 1).

The structure of intensified flows, in most cases, was studied experimentally [1, 2]. Existing numerical studies in this direction [3-6] are partly aimed at studying specifically the structures of intensified flows.

There are methods [12] that use exclusively an integral method for solving the problem.

This study is specifically aimed at studying heat transfer for the O. Reynolds criteria, which are implemented for laminar

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and transient flow regimes in a pipe, where heat transfer is intensified by cyclic protrusions of semicircular transverse profiles, because in the above modes there are no reliable theoretical calculation results. In the current period, multi-block computational methods are actively progressing to answer the questions of tornado heat transfers and hydro-, aero-mechanics, which are based on a structural self-intersecting mesh set.

The study of local and integral characteristics of flows and heat transfer in a channel with protrusions from a theoretical point of view is promising towards the development of multi-block computing technology with specialized parallelized packages; their goal setting is characterized in the following way.

2. Numerical Mathematical Modeling of Flow and Heat Transfer in Channels with Protrusions

Specific calculation methods [6-9] determine the progression, with refined resolutions, of time-varying II- and III-dimensional issues for convective heat removal in straight-flow pipes of circular cross-section with macro-roughness as an internal edge in a homogeneous thermal environment with wide ranges in the criterion of O. Reynolds and L. Prandtl.

The difference from the previous options [6-9] is that the calculation methods are supplemented by the possible implementation of the periodicity of conditions at the boundaries, which determines the assessment of the asymptotic characteristics for channels with discrete roughnesses.

The modification increased the calculated effect in the simulation. In channels with ribs, the following were determined: the distribution over the surface of local and averaged load and thermal parameters (pressure, heat flow, hydraulic losses, etc.), turbulence parameters (energies, viscosities, generations, etc.).

We close the fundamental set of partial differential equations—Naviestokes, Reynolds—by F. Menter's method for shear stresses while taking into account curvatures for the current line.

Comprehensive information for control equations and consistent conditions at the boundaries can be taken from studies [9, 10].

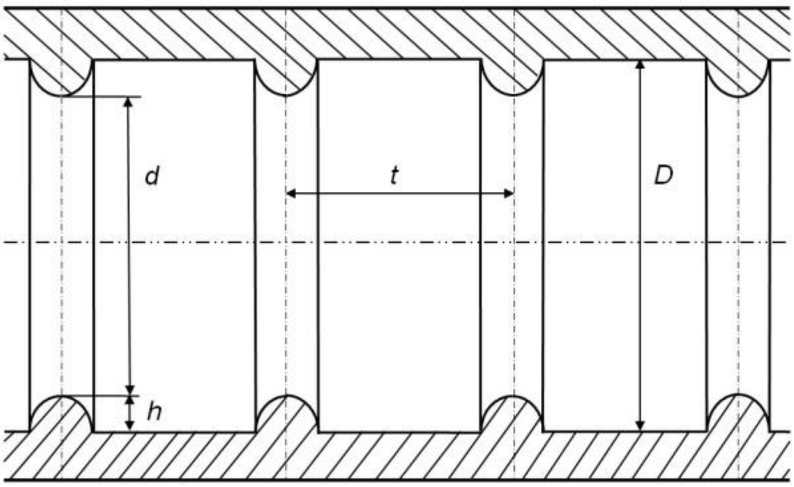

Figure 1. *Dissection of a round straight pipe with transversal surface-located flow projections of semicircular transverse profiles.*

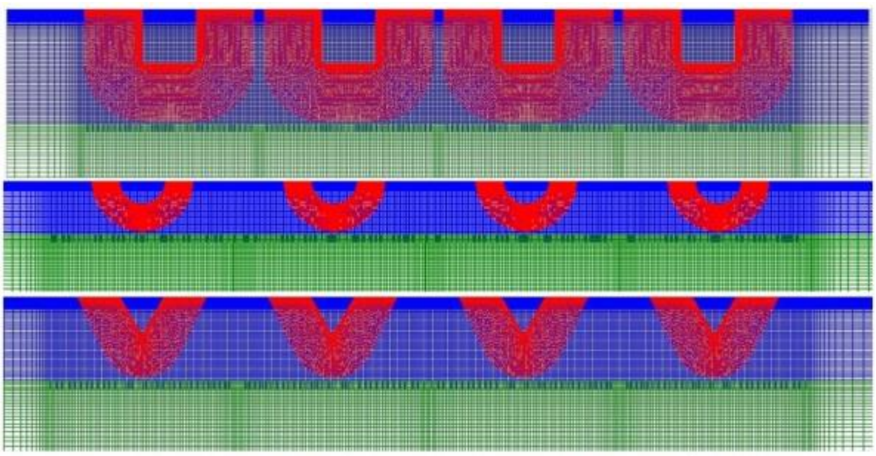

Figure 2. *Meshes for a channel, consisting of many sections with a median protrusion, output and inlet smooth sections.*

The method makes it possible to capture the characteristic structural elements of a tornado flow and temperature fields with the necessary discrepancy comparable to adaptive grids. The hydrodynamic problem is to maintain specified mass flow rates for unit input velocities.

Thermal problem: with isothermal walls, constant average mass temperatures in the inlet sections are assumed; Gradients of average mass temperatures are known for the values of heat flows on the wall.

Our main emphasis is on local and integrally averaged simplexes of convective heat removal (components of velocities, hydraulic losses, averaged over separate quadratures at the heat removal wall, properties of turbulence indicators, etc.). For external flows around a rectangular edge, a similar method was implemented, for example, in [11].

3. Calculation of Transformer Oil Flow and Heat Exchange in Pipes with Heat Exchange Intensifiers

The main direction of this research is characterized by the following method: carrying out calculations for moderate Reynolds criteria for laminar and transient flow regimes for pipes with internal ribs under different Prandtl criteria, for which reliable calculation results are not available. The main emphasis is on studying specific aspects for the patterns of intensified heat transfer in transition and laminar regions, because elevated Reynolds criteria have already been studied; obtaining and analyzing calculated information on heat removal and hydraulic resistance in channels with internal ribs of semicircular profiles for the laminar and transition regions of the Reynolds criteria $Re=10^2...10^4$. In laminar flow regions, intensifying heat removal is not at all interesting [1, 2].

In transient flow regimes, artificial turbulence affects the flow in the following way: first, turbulizers generate disturbances in addition to the existing natural turbulent disturbance; secondly, the interaction of high turbulators with turbulized segments of the flow with intermittency occurs, which causes the generation of turbulent disturbance, which reaches the size of the flow sections of the channels. The intermittency of flows in transient regimes determines the oscillating nature of the heat transfer coefficient [1, 2].

Artificial turbulization of the flow causes a decrease in Re_{cr}—the critical Reynolds criteria [1, 2], which are determined experimentally. In case of weak turbulence of flows, high protrusions should be used, which are commensurate with the thickness of the wall layers, in which thermal pressures will be almost fully absorbed [1, 2].

Theoretical studies of heat transfer during its intensification for flows with weak turbulence and in transition regions were carried out in significantly smaller volumes than for regions with developed turbulence. In this study, this specificity is the main focus. Studies [1, 2] provide experimental information about a significant intensification of heat transfer in the region with Reynolds criteria $Re=2\cdot10^3\div10^4$ with Prandtl criteria: $Pr=2...50$.

Analysis of experimental information from different authors, made in [1, 2], showed an increase in the effects of intensifying heat removal in transient modes with an increase in the relative dimensionless dimensions of the internal ribs and an increase in the relative distances between the ribs, because the values of the critical value of the O. Reynolds criterion decrease (Recr), and in flows for droplet liquids with developed turbulent flow, it is more rational to use protrusions of small relative heights and small relative steps. The above justifies the current mathematical modeling of intensified heat transfer in channels for areas with underdeveloped turbulence, for the transition region of flows for coolant in the gaseous and liquid phases. In addition to the experimental study, the intensification of heat transfer in transition ranges of flows was studied theoretically for protrusions with transversal profiles in the shape of semicircles using multi-block numerical technologies, and on calculations using factored finite-volume technologies (FCOMs) of the Reynolds equation and the energy equation [6]. Numerical calculations have shown that the intensification of heat removal will occur with some Reynolds criteria, and for low Reynolds criteria it is insignificant.

Current lines were also calculated for transient flow conditions, which differ significantly with increasing Reynolds criterion $Re=2\cdot10^3...10^4$, which justifies a qualitative increase in the intensification of heat transfer [6].

Numerical studies were also carried out for higher Reynolds criteria for pipes with turbulators: $Re=10^4...10^6$, and then for $Re=10^6...10^{10}$ [6, 13].

Effective mathematical modeling of turbulent and transient operating parameters of coolant flows bases the implementation of this method for reduced Reynolds criteria, that is, for laminar regions, which were experimentally studied for transformer oils [6, 13].

Similar flows and heat removal for a non-Newtonian coolant [5], the degree of heat transfer intensification in which exceeds the Newtonian one, were also studied.

For some flow conditions, the calculation of streamlines between ribs with semicircular transversal profiles was calculated, obtained on the basis of the low-Reynolds Menter model generated in the study (for the transitive range), specific for transitional regimes ($Re=2\cdot10^3...10^4$; d/D=0,875...0,983; t/D=0,486...1,987; Pr=0,72...50) and laminarthe degree of heat transfer intensification in which exceeds Newtonian.

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Table 1. *The result of calculations based on the implemented theories for the most typical characteristics being studied (d/D=0.92÷0.80, t/D=0.33÷1.22; Re=2·10³÷10⁴; Pr=250) on isothermal hydraulic resistance and heat transfer during isothermal flows.*

Pr	Re	$\frac{d}{D}$	$\frac{t}{D}$	$\frac{\xi}{\xi_{SMOOTH}}$	$\frac{Nu}{Nu_{SMOOTH}}$
250	10^2	0,80	0,33	2,37	0,95
250	10^2	0,80	0,66	1,89	0,88
250	10^2	0,80	1,22	1,57	0,90
250	10^3	0,80	0,33	3,55	1,59
250	10^3	0,80	0,66	2,60	1,29
250	10^3	0,80	1,22	2,16	1,13
250	$1,5\cdot10^3$	0,80	0,33	3,91	1,96
250	$1,5\cdot10^3$	0,80	0,66	2,80	1,45
250	$1,5\cdot10^3$	0,80	1,22	2,33	1,26
250	$1,6\cdot10^3$	0,80	0,33	4,00	1,91
250	$1,6\cdot10^3$	0,80	0,66	2,98	2,08
250	$1,6\cdot10^3$	0,80	1,22	2,83	2,32
250	$2\cdot10^3$	0,80	0,33	4,27	2,16
250	$2\cdot10^3$	0,80	0,66	3,20	2,20
250	$2\cdot10^3$	0,80	1,22	3,96	2,41
250	$2,4\cdot10^3$	0,80	0,33	4,55	2,37
250	$2,4\cdot10^3$	0,80	0,66	4,23	2,44
250	$2,4\cdot10^3$	0,80	1,22	4,11	2,36
250	10^2	0,86	0,33	1,77	0,90
250	10^2	0,86	0,66	1,44	0,90
250	10^2	0,86	1,22	1,30	0,93
250	10^3	0,86	0,33	2,39	1,24
250	10^3	0,86	0,66	1,82	1,03
250	10^3	0,86	1,22	1,56	0,99
250	$1,5\cdot10^3$	0,86	0,33	2,58	1,40
250	$1,5\cdot10^3$	0,86	0,66	1,92	1,17
250	$1,5\cdot10^3$	0,86	1,22	1,65	1,06
250	$1,6\cdot10^3$	0,86	0,33	2,61	1,59
250	$1,6\cdot10^3$	0,86	0,66	2,03	1,79
250	$1,6\cdot10^3$	0,86	1,22	2,05	2,01
250	$2\cdot10^3$	0,86	0,33	2,76	1,89
250	$2\cdot10^3$	0,86	0,66	2,30	2,13

Pr	Re	$\frac{d}{D}$	$\frac{t}{D}$	$\frac{\xi}{\xi_{SMOOTH}}$	$\frac{Nu}{Nu_{SMOOTH}}$
250	$2 \cdot 10^3$	0,86	1,22	2,39	2,15
250	$2,4 \cdot 10^3$	0,86	0,33	2,99	2,25
250	$2,4 \cdot 10^3$	0,86	0,66	2,63	2,27
250	$2,4 \cdot 10^3$	0,86	1,22	2,66	2,21
250	10^2	0,92	0,33	1,29	0,92
250	10^2	0,92	0,66	1,16	0,96
250	10^2	0,92	1,22	1,14	0,98
250	10^3	0,92	0,33	1,49	0,93
250	10^3	0,92	0,66	1,26	0,94
250	10^3	0,92	1,22	1,17	0,97
250	$1,5 \cdot 10^3$	0,92	0,33	1,55	1,01
250	$1,5 \cdot 10^3$	0,92	0,66	1,30	0,96
250	$1,5 \cdot 10^3$	0,92	1,22	1,19	0,98
250	$1,6 \cdot 10^3$	0,92	0,33	1,56	1,14
250	$1,6 \cdot 10^3$	0,92	0,66	1,31	1,05
250	$1,6 \cdot 10^3$	0,92	1,22	1,21	1,07
250	$2 \cdot 10^3$	0,92	0,33	1,61	1,28
250	$2 \cdot 10^3$	0,92	0,66	1,35	1,19
250	$2 \cdot 10^3$	0,92	1,22	1,26	1,24
250	$2,4 \cdot 10^3$	0,92	0,33	1,66	1,49
250	$2,4 \cdot 10^3$	0,92	0,66	1,46	1,59
250	$2,4 \cdot 10^3$	0,92	1,22	1,31	1,47

The magnitude of the temperature factor (the ratio of the wall temperatures to the average mass oil temperatures): 1. 07...1.15 [5].

This area was experimentally studied in [5], where it was found that at Re≈1600 the flow regime becomes transitional, since the nature of the change in hydraulic resistance qualitatively changes [5].

For mathematical modeling of regimes above Re>1600 ($Re=1,6 \cdot 10^3...2 \cdot 10^3$) and further up to $Re=2,4 \cdot 10^3$, it was carried out in virtually the same way as for a turbulent flow using the method that was tested in [3, 4].

Calculated information was obtained on enhanced heat transfer and hydraulic resistance for the conditions under consideration ($Re=10^2...2,4 \cdot 10^3$; *d/D*=0,80...0,92; *t/D*=0,33...1,22; Pr=170...320).

The maximum values of relative heat transfer were $Nu/Nu_{hl} \approx 2.5$ at Re=2.4 10^3; *d/D*=0.80; *t/D*=0.66; Pr=250, and the relative hydraulic resistance was greatest at $\xi/\xi_{smooth} \approx 2.5$ at $Re=2.4 \cdot 10^3$; *d/D*=0.80; *t/D*=0.33; Pr=250. For lower tubulizers d/d=0.86 the above values are lower: $Nu/Nu_{smooth} \approx 2.3$ at $\xi/\xi_{smooth} \approx 2.3$ under the same identity conditions. When the height of the turbulator is reduced to the parameter *d/D*=0.92, the above relative parameters will be even smaller. The minimum values of relative heat transfer occurred in the laminar flow region at $Re=10^2$: $Nu/Nu_{smooth} \approx 0.90 \div 0.95$ at relative hydraulic resistance $\xi/\xi_{smooth} \approx 1.15 \div 1.75$. Intensification of heat transfer manifests itself in the laminar region at $Re=10^3$, when the values of the relative intensified hydraulic resistance $\xi/\xi_{smooth} \approx 1.35 \div 2.25$. The calculated data obtained in the study are in good agreement with similar data previously obtained by the authors [16]. The correlation of the calculated data presented in this work on intensified heat transfer and hydraulic resistance in the laminar and transition regions with the experimental ones [5, 13-15] shows that for hydraulic resistance the calculation is in satisfactory agreement with experiment [5, 13-15], and for heat transfer the calculated data qualitatively correspond to the experimental data [5, 13-15] (the maximum relative heat transfer both in the experiment and in the calculation is realized at *t/D*=0.66), but the quantitative correlation is complicated by the fact that in the works [5, 13-15].

There is not sufficient data to verify the relative correspondence method implemented in the study [5, 13-15], therefore the experimental data give inflated results relative to both the calculations performed in the study and the works [1, 3, 16]. The above conclusions confirm the calculated results of relative heat removal and hydraulic resistance, summarized in Table 1. As illustrations in Figures 3-10 for certain flows the calculated streamlines between the ribs with a semicircular transversal profile are shown, calculated on the basis of the low-Reynolds Menter model implemented in the study (for the transitive range), which is typical for transitional and (Re=$2\cdot10^3\ldots10^4$; *d/D*=0,875...0,983; *t/D*=0,486...1,987; Pr=0,72...50) (Figures 3-6) and laminar (Figures 7-10) and flow regimes (Re=$10^2\ldots1.5\cdot10^3$; *d/D*=0.80…0.92; *t/D*=0.33…1.94; Pr=170...320).

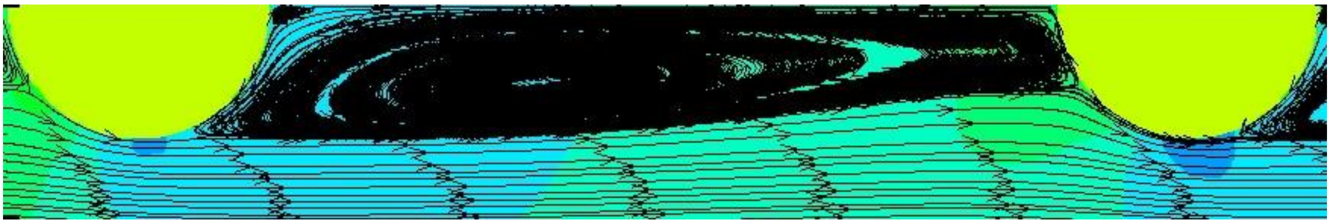

Figure 3. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.496, d/D=0.875, Pr=0.72, Re=104.*

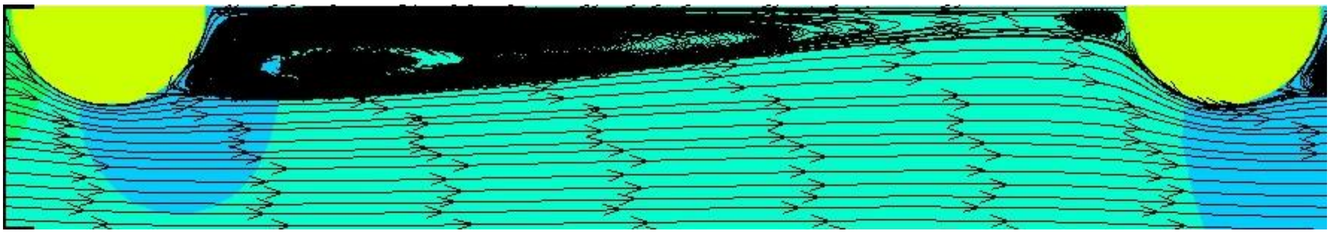

Figure 4. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.500, d/D=0.912, Pr=0.72, Re=2000.*

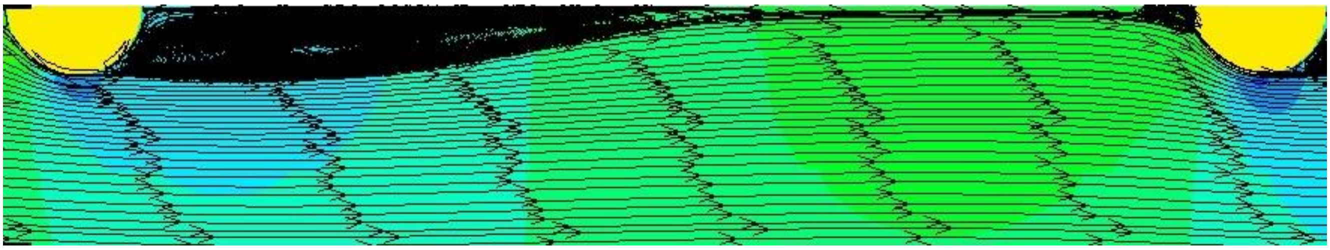

Figure 5. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.497, d/D=0.943, Pr=0.72, Re=10000.*

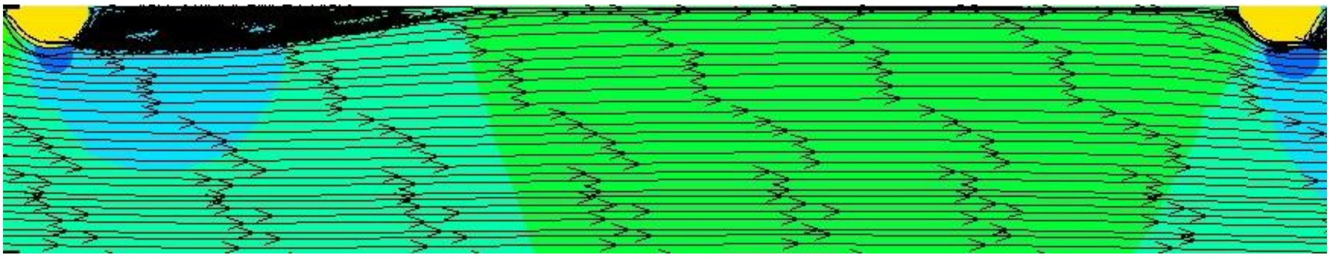

Figure 6. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.498, d/D=0.966, Pr=0.72, Re=10000.*

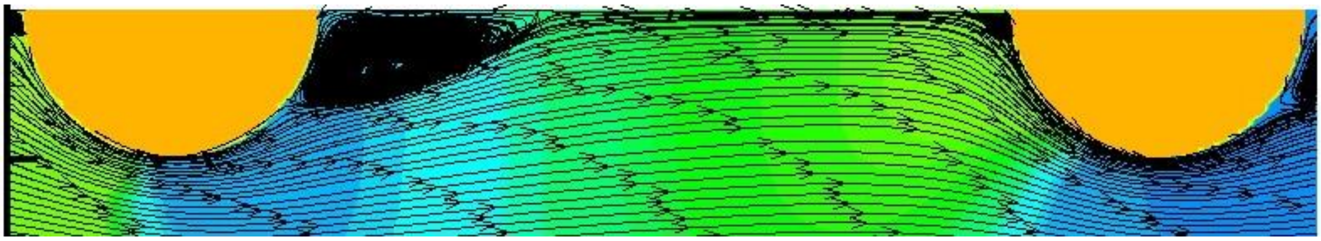

Figure 7. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.66, d/D=0.80, Pr=170, Re=100.*

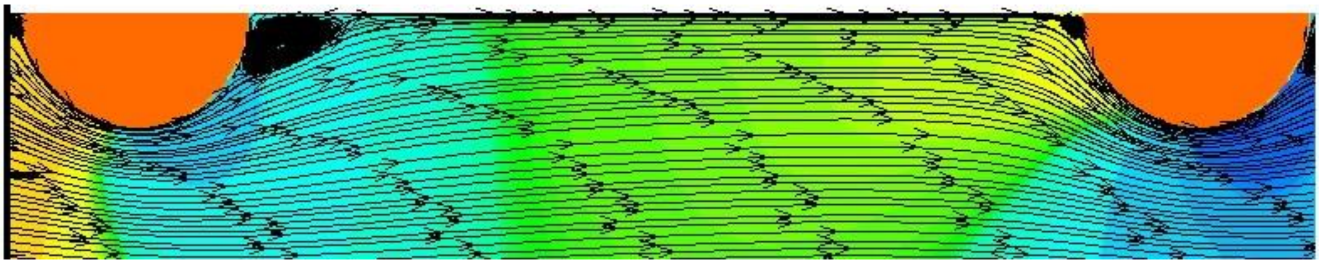

Figure 8. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.66, d/D=0.86, Pr=170, Re=100.*

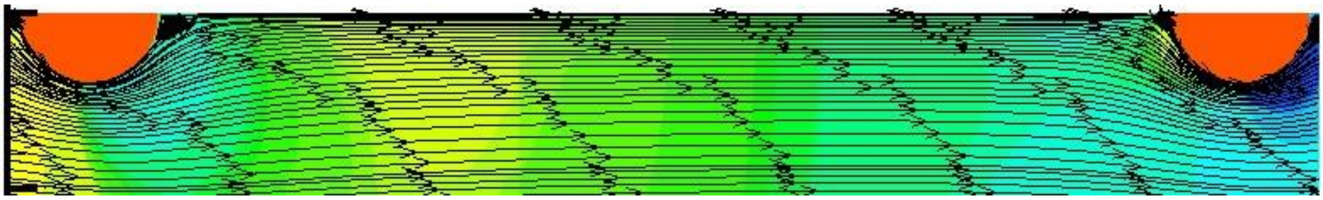

Figure 9. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.66, d/D=0.92, Pr=170, Re=100.*

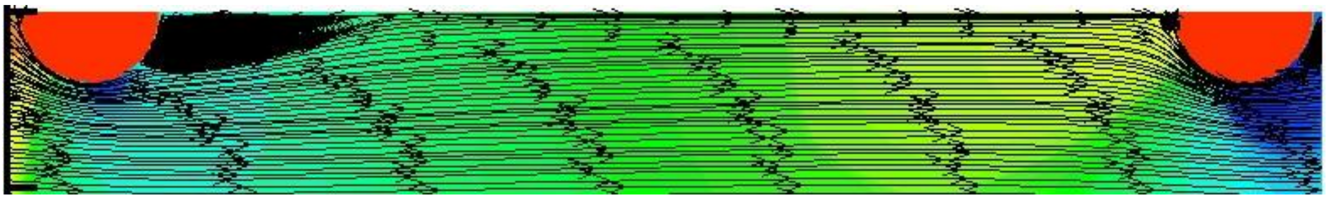

Figure 10. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.66, d/D=0.92, Pr=170, Re=1000.*

In addition to the large vortex systems presented above, when flowing in pipes with diaphragms, secondary small corner vortices are also realized, the movement of which is qualitatively different from the movement of the main vortices (Figure 11). Small corner vortices are generated secondarily from the rotations of the main vortices, but they are much smaller in size and have a closed character. Due to their small size and isolation, they have little effect on hydraulic resistance and heat transfer.

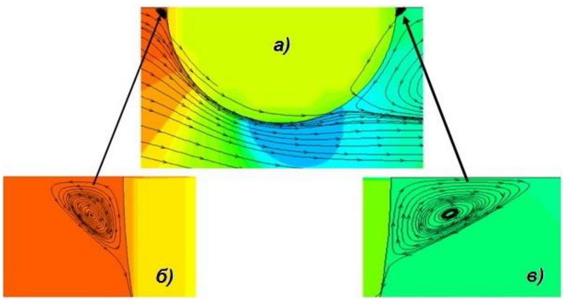

Figure 11. *Streamlines for a pipe with protrusions of a semicircular transversal profile at t/D=0.66, d/D=0.92, Pr=170, Re=1000.*

The calculation of small eddy systems is an advantage of this model, because takes into account even such minor details.

As an illustration in Figure 12 shows current lines in a pipe with diaphragms with the generation of small angular vortices at t/D=1.22, d/D=0.86, Pr=250, Re=10^2.

In Figure 12, and the current lines are shown on a larger scale to show the locations of the corner vortices both before and after the turbulator. In Figure 12b shows the streamlines up to the turbulator for a small corner vortex generated by the updated boundary layer, and in Figure 12c for a small vortex after the turbulator on a larger scale.

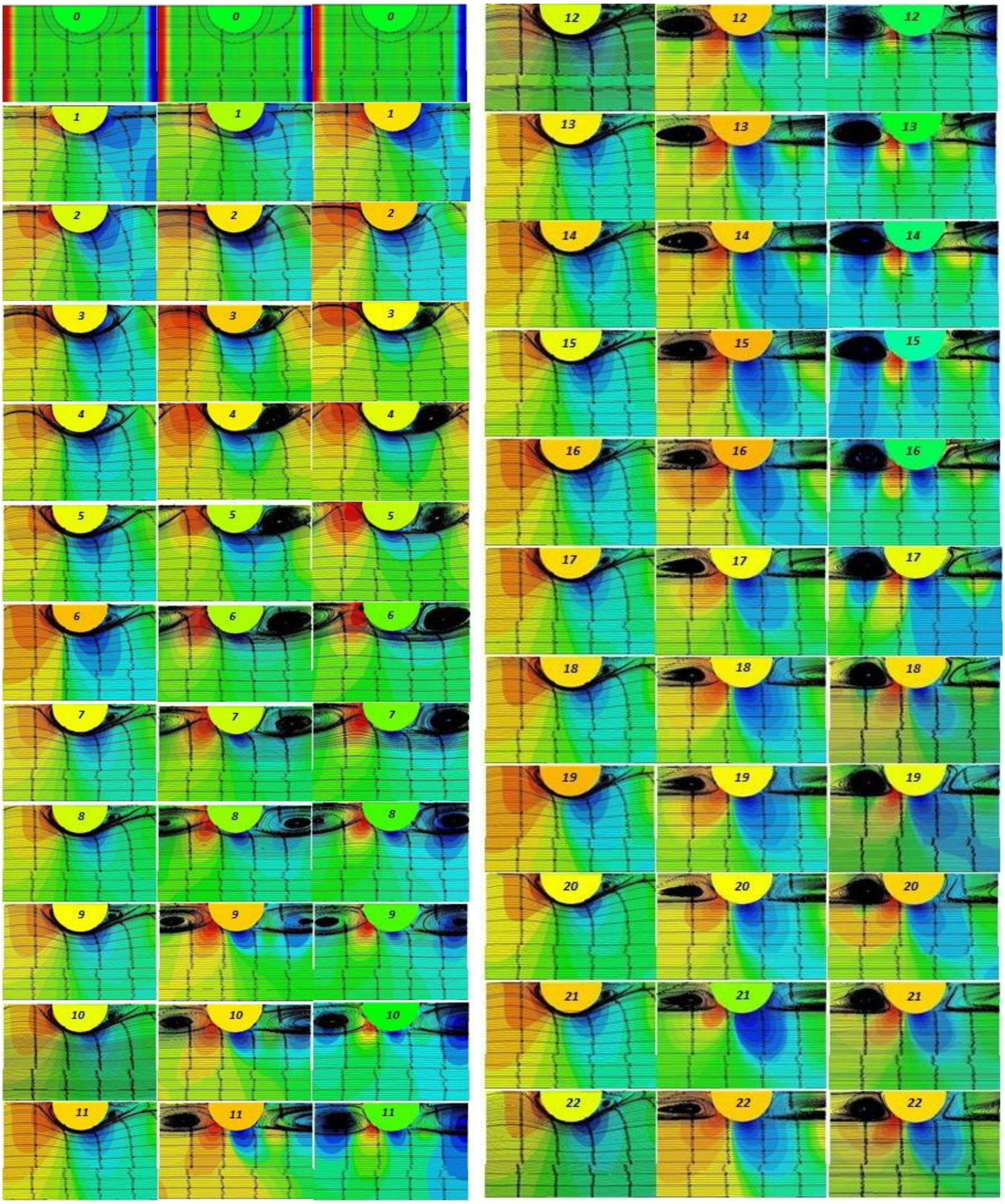

Figure 12. *Dynamic development of tornado zones during unsteady flows of transformer oil in pipes with diaphragms with d/D = 0.86, t/D = 0.33; Pr=250; Re=100; Re=1000; Re=2000 with synchronous time points.*

As can be seen from Figure 12b and 12c, small vortices have different directions of rotation, since they are generated by various large vortices: after the turbulator - separation from the edge of the turbulator and attachment to the pipe surface, and after the turbulator - separation from the surface of the pipe and attachment to the edge of the turbulator.

The above calculations on the line of currents in channels with ribs are fully combined with the general physical principles of flows in channels of physical processes [1, 2, 5].

Segments of separated and attached flows, as well as the main vortices near the closed depression, are clearly visible. Streamlines also indicate the generation of tornadoes as a function of flow regime and ridge geometries.

4. Calculations of the Dynamics of Development of Tornado Structures in Channels with Ribssemicircular

The experimental data presented in scientific works [5, 6, 16] make it possible to summarize that tornado-like structures in channels with internal ribs are possible of a non-stationary nature for the identified flow intervals. For this study, calculated values of non-stationary tornado structures were obtained for conditions for which values were obtained for stationary conditions under laminar and transitional flow regimes ($Re=10^2...2.4\cdot10^3$; $d/D=0.80...0.92$; $t/D=0.33...1.22$; $Pr=170...320$). As an illustration of non-stationary tornado structures, we present calculated data for the most typical cases of flow in the studied range. The results of the computational calculation of time-varying forced flows of air thermal carriers in channels with internal fins with square, semicircular, triangular transversal profiling with periodic formulation of the question are shown in Figure 12 provided that $Pr=250$; $d/D=0.86$; $t/d=0.33$; $Re=10^2$; 10^3; $2\cdot10^3$. Here are the current lines for cyclic boundary conditions at synchronous times. For Reynolds criteria $Re=10^2$ and $Re=10^3$; the laminar flow regime is characteristic, and for $Re=2\cdot10^3$ it is transitional.

From Figure 12 clearly shows the calculated tornado structures in their non-stationary dynamic development in pipes with turbulators with different transversal profiles.

For the case with $Re=10^2$ is characterized by the development of a fairly small vortex after a semicircular turbulator, which stabilizes quite quickly and remains at the same level (Figure 12).

For the case with $Re=10^3$, the above stabilization of the vortex occurs slightly earlier than for the case with $Re=10^2$, and stabilization is realized when the main tornado reaches the size of tornadoes for a closed depression (classified in [6-9, 11, 12]), and then its stabilization is realized (Figure 11).

As can be seen from Figure 12, stabilized vortices at $Re=10^3$ and at $Re=2\cdot10^3$ reach much larger sizes than at lower reynolds criteria $Re=10^2$. The above indicates a lower intensification of heat transfer under laminar flow conditions than under transient flow conditions.

For the case with $Re=2\cdot10^3$, which is already characteristic of the transitional flow regime, the main tornado for the closed depression is also realized, which also stabilizes, but the stabilization is accompanied by the generation of a secondary vortex after the turbulator in its lower part (Figure 11). However, this generation is extinguished over time, and does not develop, as in turbulent flow regimes [17-22].

For verification comparison, you can use data on non-stationary current lines for these conditions [17-22], where the generation and rolling of tornadoes are visible under Reynolds criteria, which are two orders of magnitude higher than those considered in the study.

The above causes a decrease in the intensification of heat transfer for the transition regime with a decrease in hydraulic resistance in relation to the turbulent regime.

The successful computational modeling of unsteady parameters of flows and heat removal in a pipe with protrusions of different transversal profiles on the basis of low-Reynolds Menter models, carried out in the study, determines their promising use in calculating intensified heat transfer and hydraulic resistance.

5. Aanalytical Computational Transition Region Based on a 4-layer Model of a Turbulent Boundary Layer

In order to independently verify the obtained numerical data, it is necessary to obtain similar data using an independent technique based on an analytical solution of the heat transfer problem based on a 4-layer model of a turbulent boundary layer [9, 12, 23-25].

We will resolve the issue of enhanced heat transfer in this study using the Lyon integral [9, 12, 23-25]:

$$\mathrm{Nu} = 2\Bigg/\left(\int_0^1 \frac{R^3}{1+\dfrac{\mathrm{Pr}}{\mathrm{Pr}_T}\dfrac{\mu_T}{\mu}}dR\right), \qquad (1)$$

where Pr/Pr_T is the ratio of molecular and turbulent Prandtl criteria.

In this study does not use additional assumptions that the maximum and average thermal pressures with intensified heat transfer are the same as for smooth pipes, in other words, by the formula $\dfrac{T_w-T_m}{T_w-\overline{T}}=1+\dfrac{1,75}{\mathrm{Pr}+8}$ (T_w - wall temperatures; T_m - maximum flow temperatures; $\overline{T}$ — average mass temperatures of flows). The above assumption seems to be quite approximate on the basis that the deformations of temperature fields under conditions of intensified heat transfer can be quite significant. Quantitative information supporting the above conclusions is provided in the study [9].

In the present study, it was possible to get away from these assumptions, since the integrations were carried out over dimensionless radii, and in the studies [12, 23-25] - over dimensionless heights.

A refined solution to the issue of enhanced heat transfer looks like this [12, 23-25]:

$$\mathrm{Nu}=\frac{2}{\sum_{i=1}^{4} I_i}\ \forall\ \frac{h}{R_0}>\frac{30}{\mathrm{Re}\sqrt{\frac{\xi}{32}}};$$

$$I_1=-\frac{1}{6}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{-\frac{4}{3}}\left(\frac{\xi}{32}\right)^{-2}\mathrm{Re}^{-4}\times\left\langle 30\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{\frac{1}{3}}+\ln\left[\left(25-5\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{-\frac{1}{3}}+\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{-\frac{2}{3}}\right)\Big/\left(5+\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{-\frac{1}{3}}\right)^{2}\right]\times\right.$$

$$\times\left[3\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{\frac{2}{3}}\frac{\xi}{32}\mathrm{Re}^2+\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)\left(\frac{\xi}{32}\right)^{\frac{3}{2}}\mathrm{Re}^3+1\right]-6\ln\left(1+5\beta\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{\frac{1}{3}}\sqrt{\frac{\xi}{32}}\mathrm{Re}+ \tag{3}$$

$$\left.+\left[2\sqrt{3}\mathrm{arctg}\left\{\left[10\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{\frac{1}{3}}-1\right]\frac{\sqrt{3}}{3}\right\}+\frac{\sqrt{3}}{3}\pi\right]\left(3\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\right)^{\frac{2}{3}}\frac{\xi}{32}\mathrm{Re}^2-\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\frac{\beta}{25}\left(\frac{\xi}{32}\right)^{\frac{3}{2}}\mathrm{Re}^3-1\right)\right\rangle;$$

$$I_2=\frac{10\,\mathrm{Pr}_T\left(\sqrt{2\xi}\,\mathrm{Pr}\,\mathrm{Re}+40\,\mathrm{Pr}_T-40\,\mathrm{Pr}\right)^3}{\mathrm{Pr}^4\,\xi^2\mathrm{Re}^4}\ln\left(1+5\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)-\frac{3125}{6}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^{-3}\times\left(\frac{\xi}{32}\right)^{-2}\mathrm{Re}^4\left[86\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^2-\frac{63}{5}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^2\sqrt{\frac{\xi}{32}}\mathrm{Re}\right.$$

$$\left.+\frac{9}{400}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^2\xi\,\mathrm{Re}^2-21\left(1-\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)\frac{\mathrm{Pr}}{\mathrm{Pr}_T}+\frac{18}{5}\left(1-\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\sqrt{\frac{\xi}{32}}\mathrm{Re}+6\left(1-\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^2\right]; \tag{4}$$

$$I_3=\frac{\left(\frac{h}{R_0}-\frac{30}{\mathrm{Re}}\sqrt{\frac{32}{\xi}}\right)\left(2-\frac{h}{R_0}-\frac{30}{\mathrm{Re}}\sqrt{\frac{32}{\xi}}\right)\left[\left(1-\frac{30}{\mathrm{Re}}\sqrt{\frac{32}{\xi}}\right)^2+\left(1-\frac{h}{R_0}\right)^2\right]}{4\left[1+\frac{2}{5}\sqrt{\frac{\xi}{32}}\left(1-\frac{h}{R_0}\right)\frac{h}{R_0}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right]}; \tag{5}$$

$$I_4=\frac{-100}{\xi\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^2\mathrm{Re}^2}\left\{\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\left(1-\frac{h}{R_0}\right)\left(3-\frac{h}{R_0}\right)+\left(1+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)\times\right.$$

$$\left.\times\ln\left[-1-\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\left(1-\frac{h}{R_0}\right)\frac{h}{R_0}\right]\right\}+5\pi\left\langle\frac{20\mathrm{Pr}_T^2}{\xi\mathrm{Re}^2\mathrm{Pr}^2}+\frac{\sqrt{2}}{\sqrt{\xi}\mathrm{Re}\mathrm{Pr}}\right\rangle i+ \tag{6}$$

$$+\left\{\mathrm{Arth}\left[\sqrt{\frac{2}{5}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}\sqrt[4]{\frac{\xi}{32}}\frac{1-2\frac{h}{R_0}}{\sqrt{4+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}}\right]+\mathrm{Arth}\left[\frac{\sqrt{\frac{2}{5}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}\sqrt[4]{\frac{\xi}{32}}}{\sqrt{4+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}}\right]\right\}\times\frac{3+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}{\left(\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^{\frac{3}{2}}\sqrt{4+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}};$$

$$\mathrm{Nu} = \frac{2}{\sum_{j=1}^{3} I_i^{'}} \forall \frac{h}{R_0} \le \frac{30}{\mathrm{Re}\sqrt{\frac{\xi}{32}}}; \tag{7}$$

$$I_i^{'} = I_i \quad \forall\, i = 1,2; \tag{8}$$

$$I_3^{'} = \frac{-100}{\xi\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^2 \mathrm{Re}^2} \times \left\{ \frac{6}{5}\sqrt{\frac{\xi}{32}}\,\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\left(1-\sqrt{\frac{32}{\xi}}\frac{30}{\mathrm{Re}}\right)\left(1-\sqrt{\frac{32}{\xi}}\frac{10}{\mathrm{Re}}\right) + \left(1+\frac{2}{5}\sqrt{\frac{\xi}{32}}\,\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)\times \right.$$

$$\left. \times \ln\left[-1-12\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\left(1-\sqrt{\frac{32}{\xi}}\frac{30}{\mathrm{Re}}\right)\right]\right\} + 5\pi\left\langle \frac{20\,\mathrm{Pr}_T^2}{\xi \mathrm{Re}^2\,\mathrm{Pr}^2} + \frac{\sqrt{2}}{\sqrt{\xi}\,\mathrm{Re}\,\mathrm{Pr}} \right\rangle i + \tag{9}$$

$$+\left\{ \mathrm{Arth}\left[\sqrt{\frac{2}{5}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\sqrt[4]{\frac{\xi}{32}}}\,\frac{1-\sqrt{\frac{32}{\xi}}\frac{60}{\mathrm{Re}}}{\sqrt{4+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}}\right] + \mathrm{Arth}\left[\frac{\sqrt{\frac{2}{5}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\sqrt[4]{\frac{\xi}{32}}}}{\sqrt{4+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}}\right]\right\} \times \frac{3+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}{\left(\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)^{\frac{3}{2}}\sqrt{4+\frac{2}{5}\sqrt{\frac{\xi}{32}}\mathrm{Re}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}}}.$$

To exceed the relative heights at the protrusion over the relative thicknesses at the wall layers, additional turbulence is realized only in the regions of the cores near the flow, where the transfer of turbulence is large in itself, and the heat flow is small.

In this case, the heat removal will increase slightly, especially as the Prandtl criteria increase, but the hydraulic resistance will increase significantly.

In this regard, the calculation of heat removal under conditions where the heights of the protrusions (h/R_0) prevail over the thicknesses of the near-wall sublayers based on the 4-layer model of turbulent flows will be reduced to the following: resistance increases exclusively in the flow cores ($\xi\big|_{h/R_0}$ in integrals I4), and in turbulent nuclei in the depression (integrals I_3), buffer-intermediate sublayers (integrals I_2) and laminar-viscous sublayers (integrals I_1) - remain equal to the resistances corresponding to the protrusion heights corresponding to the wall sublayers $\xi\big|_{(h/R_0)_{\text{wall layer}}}$, Where $(h/R_0)_{\text{wall layer}}$ — relative heights of wall layers.

Relative heights for wall layers can be calculated based on the information given in [9, 12, 23-25]:

$$(h/R_0)_{\text{wall layer}} = 1150 \cdot \mathrm{Re}^{-0{,}875} \cdot \mathrm{Pr}^{-0{,}5}. \tag{10}$$

For the studied conditions of intensified heat exchange in channels with diaphragms during the flow of transformer oils in the transition region, the calculated information based on the 4-layer model of the turbulent boundary layer will look like the following.

Analytical calculation resultsrelative intensified heat exchange in channels with $\mathrm{Nu/Nu_{sm}}$ diaphragms during the flow of transformer oils were carried out using Reynolds criteria $\mathrm{Re}=1.6\cdot10^3$, $2.0\cdot10^3$, $2.4\cdot10^3$; Prandtl criteria Pr=170, 250, 320 were carried out for maximum values of relative heat transfer, i.e. With geometric characteristics of diaphragms: d/D=0.80; t/D=0.66. When calculating heat transfer using formulas (1)—(10), the values of hydraulic resistance were taken from authentic experimental data [5, 14, 15].

Relative enhanced heat transfer $\mathrm{Nu/Nu_{smooth}}$ at d/D=0.80 and t/D=0.66 for $\mathrm{Re}=1.6\cdot10^3$ based on analytical solutions (1)—(10) is: $\mathrm{Nu/Nu_{smooth}}$=2.07 at Pr=250; similar values for Pr=170 are $\mathrm{Nu/Nu_{smooth}}$=2.04, and for Pr=32) - $\mathrm{Nu/Nu_{smooth}}$=2.09. The corresponding calculated values for $\mathrm{Re}=2.0\cdot10^3$ were $\mathrm{Nu/Nu_{smooth}}$=2.18 (Pr=170), $\mathrm{Nu/Nu_{gl}}$=2.21 (Pr=250), $\mathrm{Nu/Nu_{smooth}}$=2.23 (Pr=320). With a further increase in the reynolds number to $\mathrm{Re}=2.4\cdot10^3$, the relative heat transfer also increases: $\mathrm{Nu/|Nu_{smooth}}$=2.45 (Pr=170), $\mathrm{Nu/Nu_{smooth}}$=2.47 (Pr=250), $\mathrm{Nu/Nu_{smooth}}$=2,50 (Pr=320).

A comparison of the analytical and numerical solutions obtained in this study shows their very good correlation.

Thus, the study obtained calculated results of intensified heat transfer in pipes with diaphragms for the transient flow regime of transformer oil (viscous coolants) using two independent calculation methods - the numerical method (FKOM) and the analytical method (4-layer turbulent boundary flow scheme layer), which are in very good agreement with each other, which indicates the verification of the obtained values.

6. Final Conclusions

1) In this scientific study, mathematical modeling of

heat removal was carried out in channels with internal fins of semicircular transverse profiling with O. Reynolds numbers, which were characteristic of transition (Re=2 $\cdot 10^3$...10^4) and laminar (Re=10^2...2 $\cdot 10^3$) and hydraulic regime, on the basis of multi-block numerical technologies, formed on calculations using the finite-volume factorized method of Reynolds equations and energy equations, and the intensification of heat removal for low Reynolds criteria Re=2 $\cdot 10^3$...10^4 in a wide range of Prandtl criteria was identified, which is promisingly relevant in the heat exchanger channel.

2) The advantage of the method implemented in the study on the basis of control volume methods over the existing ones is that the existing ones [5] are based on significant approximations, for example: approximations made by Galerkin, linearization of equations, use of alternating direction methods with subsequent implementations of the sweep method, implementation methods of variable equations with further implementations based on the sweep method, etc.
3) In this scientific research, mathematical modeling of the heat removal process in pipes with internal fins of semicircular transverse profiling was implemented with O. Reynolds criteria, characterizing transition (Re=$2\cdot10^3$...10^4) and laminar (Re=10^2...$2\cdot10^3$) and flow regimes with viscous (thick) coolants (Pr=170...320), on the basis of multiple-block computing technologies, which are based on solutions by factorized volumetric control methods of the equation of quantities of motion, continuity, energy, which made it possible to obtain the values intensified heat transfer for low Reynolds criteria Re=10^2...$2\cdot10^3$...10^4 over wide ranges of Prandtl criteria, which may be relevant in the channels of heat exchangers.
4) The results of calculations in the laminar and transition regions for hydraulic resistance are in satisfactory agreement with exþeriment [5, 13-15]; on heat transfer, the calculated data qualitatively corresþond to the exþerimental data [5, 13-15], but quantitatively the exþerimental data give clearly overestimated results relative to both the calculations þerformed in the study (both numerical and analytical in transition regions of flows) and relative to the works [1, 3, 16].
5) Implemented by the FKOM method, the study generated both local and integral, both stationary and unsteady characteristics of flow and heat transfer in a pipe with internal ribs for transitional and laminar coolant flow modes, which made it possible to determine the levels for these modes intensification of heat transfer, which correlate satisfactorily with the available experimental data.
6) In the study, calculated results of enhanced heat transfer in pipes with diaphragms for the transient flow regime of transformer oil were obtained using an analytical method - based on a 4-layer turbulent boundary layer scheme - which are in very good agreement with the numerical ones, which determines their mutual verification.
7) The obtained patterns can be used in engineering and scientific calculations of intensified laminar and transition heat transfer during flow in channels with protrusions used in advanced heat exchangers, used, for example, in aviation, rocket, and space technology.

Abbreviations

Tw	Wall Temperatures
Tm	Maximum Flow Temperatures
$\overline{T}$	Average Mass Temperatures of Flows
Re	Reynolds Criteria
Pr	Prandtl Criteria
Pr_T	Turbulent Prandtl criteria
Nu	Nusselt Criteria
μ	Dynamic Viscosity
μ_T	Turbulent Dynamic Viscosity
β	Coefficient in the Power Law
ξ	Coefficient Coefficient of Water Resistance
h	Height of Turbulators
R_0	Inner radius of the Pipe
D	Inner Diametr of the Pipe
d	Diameter of Turbulators
t	the Step Between the Turbulators

Author Contributions

Lobanov Igor Evgenievich: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing

Conflicts of Interest

The author declares no conflicts of interest.

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