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Research Article

Modelling and Diagnosis of Hybrid Dynamic Systems by a Multi-Model Approach

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Abstract

Fault diagnosis is an essential task in ensuring the smooth operation of complex dynamic systems. The consequences of faults can be serious, leading to loss of life, harmful emissions to the environment, high repair costs and economic losses caused by unplanned production line stoppages. The work developed in this paper concerns the modeling and diagnosis of faults (sensor faults, system faults, actuator faults) in hybrid dynamic systems using our multi-model approach (which combines two sub-models, one continuous and the other discrete). The aim is to integrate three well-known tools in the literature: the Bond Graph, the Observer and the Timed Automata, to design a global diagnostic model. The hybrid dynamic system is modeled by connecting the tools for the continuous part, i.e. the bond graph and the observer, to the timed automata for the discrete part. The resulting model is used for fault diagnosis in two stages: The first is fault detection by analyzing the residuals generated by the system output and that of the observer. The second step involves fault localization, which results from analysis of the signature matrix and temporal identification of the system. The proposed method combines the advantages of these tools to obtain the best performance, particularly in the fault location phase. The simulation results prove the effectiveness of the proposed model for the hybrid dynamic system. Moreover, these results also evaluate the performance of the proposed diagnostic approach while reducing non-detections, detection delays and false alarms.

Keywords

Hybrid Dynamical System, Fault Diagnosis, Bond Graph, Observer, Timed Automata, Multi-Model Approach

1. Introduction

Industrial systems have become more and more complex due to an increasing automation of the production tools augmenting thus the risks of malfunctioning which can endanger the system itself and also its environment. The most important goal of automation today is to increase the operational safety of physical systems. It is for this reason that a monitoring system is implemented which is able to provide, at any time, the operating status of the various constituent parts of the system (sensor, actuator,...).

The objective of the diagnostic function is to increase safety in order to limit the consequences of faults that can be catastrophic for equipment and human lives, thus improving productivity and system performance. For the detection and localization of faults, several approaches have been developed by various research communities [1-4].

The work developed in this article focuses on modelling

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and diagnosis by a multi-model approach (which combines two sub-models, one continuous and the other discrete). The objective concerns the integration of three tools known in the literature: the Bond Graph, the Observer and the Timed Automata for the design of a global diagnostic model. The latter allows to detect and localize defects in order to minimize repair times, and thus, provide a reliable and easily interpretable diagnosis despite the complexity of the equipment. All these developments contribute to minimize the harmful consequences that can be catastrophic for the equipment and human safety. Thus, in order to properly study and diagnose Hybrid Dynamic Systems (HDS), our proposed approach will be illustrated with didactic examples.

2. Model of the Proposed Approach: Multi-Model Approach

The HDS are systems composed essentially of discrete and continuous dynamics interacting with each other. The modeling approach we are interested in for our work considers the HDS model, which is based on the combination of two submodels, one for the continuous aspects, formalized by state equations (often by differential equations), and the other, based on finite-state automata for the event-based aspects.

Generally, obtaining the model is a difficult and complex task, especially for processes due to their diversity and the coupling of energies that characterize them. Indeed, dynamic systems are composed of elements belonging to multi-energy domains (thermal, hydraulic, mechanical, electrical, etc.). The research themes, in the continuous domain of automation, are centered around the Bond Graph methodology. The choice of this methodology is explained by its ability to model with a unified and energetic approach the systems implementing several domains of physics, by its graphical aspects, and by its causal and structural properties for the analysis and the dynamic control of complex systems. The causal properties of the Bond graph methodology allow to deduce the model of the system in the form of state space. In this way, through the Bond Graph model, it becomes possible to obtain the knowledge of the system behavior.

Equation of state from the bond graph model

The Bond Graph methodology [5, 6] was chosen for its ability to model systems involving several fields of physics (thermal, hydraulic, mechanical, electrical, etc.) with a unified, energetic approach, for its graphical aspects, and for its causal and structural properties for the analysis and dynamic control of complex systems. Structurally, a bond graph model is state-observable if and only if there is a causal path between a detector Df or De and all dynamic elements C and I, as illustrated in Figure 1.

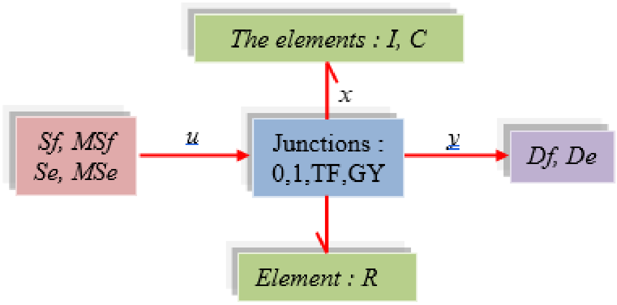

Figure 1. *Structure of a bond graph model.*

The equation of state from the bond graph model has the following form:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1}$$

In the bond graph model, the inputs "u" are the sources of flow (*Sf*, *MSf*) and effort (*Se*, *MSe*), the state variables "x" are the energy variables: the impulse (p) and the generalized displacement (q) associated with the C and I elements, and the measurements "y" are the effort (*De*) and flow (*Df*) detectors.

A dynamic system is in normal operation if it is capable of performing functions that are designed. A fault that occurs on one of the components of the system, can lead to a malfunction on the component and consequently on the global system.

3. Multi-Model Diagnosis

The industrial systems are generally of a hybrid nature, in other words, their behavior is based on the evolution and the interaction of discrete and continuous variables. For this type of system, few works have been devoted to the detection and diagnosis of faults. Moreover, the general principle of model-based fault detection methods is to compare the actual behavior of the system as it is known from measurements, with the behavior it should have under the assumption of proper operation given by a model.

The multi-model approach proposed in the present work is based on the interaction of two sub-models (discrete and continuous). The diagnostic method combines the advantages of the Observer and the Timed Automata to obtain the best performances, in particular in the fault location phase. Each step is described in a classical form. Figure 2 illustrates the multi-model structure of a hybrid dynamic system where the discrete part is described by a timed automata and the continuous part is given by a bond graph and an observer.

The principle of our diagnosis approach is based on the comparison between the actual behavior of the system and its expected behavior given by our multi-model approach proposed in the present work. Fault diagnosis is performed in two main stages:

The first is fault detection: if the residuals, generated by the observer model, are non-zero, and if an event does not occur at the desired time, then a fault is detected.

Once a fault has been detected, it needs to be localized.

The second step, therefore, concerns fault localization: based on the system's temporal identification and analysis of the signature matrix, a fault is then localized.

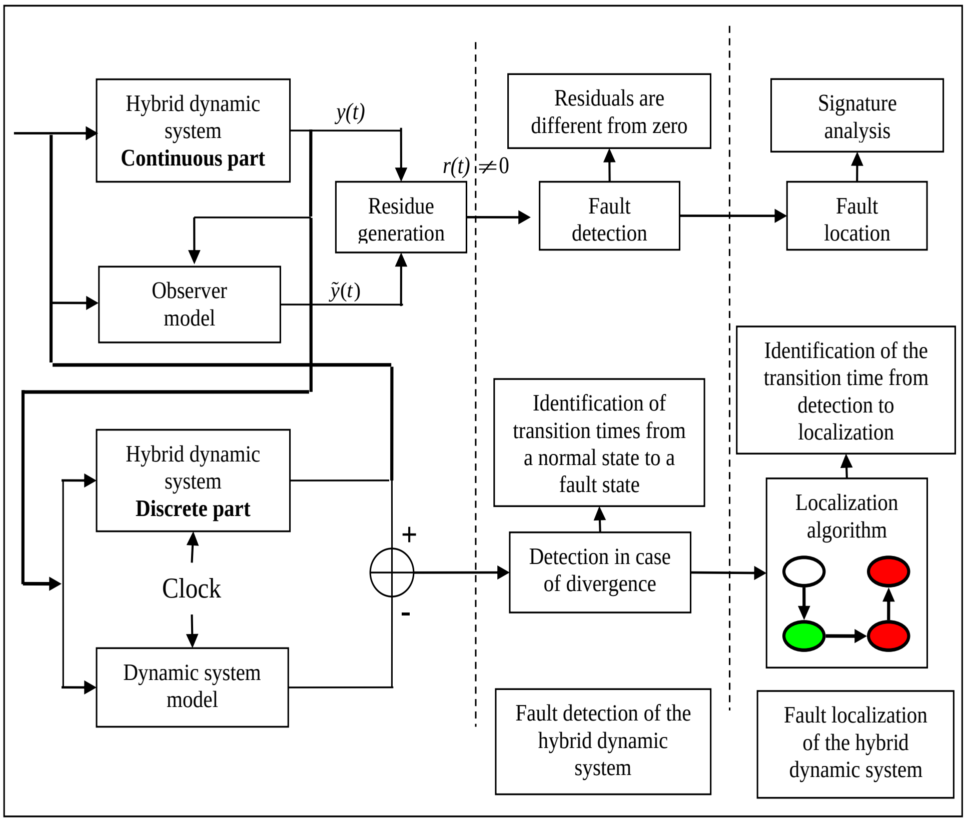

Figure 2. *Multi-model approach of hybrid dynamic system.*

3.1. Diagnosis of the Continuous Part

Residual fault sensitivity problems deserve further study. In the continuous part of the hybrid dynamic system, the first step is to generate fault sensitive residuals. The algorithm used to obtain the residuals is called residual generator. Three approaches (Parametric Estimation Approach [7-9], Analytic Redundancy Relations or Parity Space Approach [10, 11] and Observer Based Approach [12, 13]) are mainly used to construct this residue generator. A good comparison of these three approaches can be found in [14].

3.1.1. Diagnosis Based on State Observer

The residual is an indicator of the occurrence of a malfunction that affects a system and is often modeled by unknown additive signals.

In fact, the appearance of a significantly non-zero residual indicates an abnormal operation of the system, we speak of the detection of a fault. It is then interesting to identify the faulty component, it is the localization.

The diagnostic technique based on observers mainly allows the design of an observer structure that generates residuals allowing the detection and localization of the considered defects. Typically, observers produce estimates that can be subtracted from the available measurements to obtain the residuals. There are different approaches to observers for linear systems and for different classes of nonlinear systems. The main references can be found in [14-19].

3.1.2. The Observer Principle

It is assumed that the system is represented as a continuous linear dynamic model with p inputs, noted $u(t)$ and m measured outputs, noted $y(t)$. The set of n quantities describing the state of the system, noted $x(t)$, obeys the following differential system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \\ x(0) = x_0 \end{cases} \qquad (2)$$

Where the matrices $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{m\times n}$.

To diagnose a fault, the following observer is constructed:

$$\begin{cases} \dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + K(y(t) - \tilde{y}(t)) \\ \tilde{y}(t) = C\tilde{x}(t) \\ \tilde{x}(0) = \tilde{x}_0 \end{cases} \quad (3)$$

The observer has as input u(t) and y(t), and it is constructed to provide an estimate of the state, denoted $\tilde{x}(t)$. Since it has been assumed that the pair (A, C) is observable, the observer gain matrix K can be chosen such that (A - KC) is a stable matrix.

3.1.3. Residual Generator by Observer

Our goal is to directly construct residual generators $r(t)$ from the observers:

$$\begin{cases} \dot{x}(t) = A.x(t) + B.u(t) \\ \dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + K(y(t) - \tilde{y}(t)) \\ r(t) = y(t) - \tilde{y}(t) = C(x(t) - \tilde{x}(t)) \end{cases} \quad (4)$$

Indeed, the residuals $r(t)$, in the case of observers, represent the difference between the real output and the estimated output. In other words, it is the estimation error.

The detection step is very important in diagnostic systems. It is used to determine whether or not a fault is present. If this step is not properly performed, faults may be detected incorrectly or not at all, or false alarms may appear. An alarm $A_i(t)$ assigned to the ith fault is obtained following the comparison of $r_i(t)$ to a threshold ε_i established during a measurement campaign made during the operation of the system in the absence of a fault.

$$A_i(t) = \begin{cases} 0, & \text{if } |r_i(t)| < \varepsilon_i \\ 1, & \text{if } |r_i(t)| \geq \varepsilon_i \end{cases} \quad (5)$$

When a fault is detected, it is necessary to locate it. Unlike detection, where only one residue is needed, the localization procedure requires a set (or vector) of residues. Consequently, this localization is performed from the signature matrix. Indeed, the residues are designed to be each affected by a subset of defects and insensitive to other defects. Thus, only one subset of residues reacts when a defect appears. Then, the signature matrix gathers the sensitivity information for the residues. It is defined as follows:

$$\begin{gathered} M_R : D \times R \mapsto \{0?,1\} \\ (d,r) \to M_R(i,j) \\ M_R(i,j) = \begin{cases} 1 \text{ if and only if } r_j \text{ is sensitive to fault } d_i \\ 0 \text{ if and only if } r_j \text{ insensitive to fault } d_i \end{cases} \end{gathered} \quad (6)$$

The dimensions of the signature matrix M_R are defined by the number of actuators and sensors, and the number of residuals obtained from the observer model. It is a binary matrix, with row i corresponding to the fault d_i and column j to the residual r_j.

$$M_R = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix} \quad (7)$$

3.2. Diagnosis of the Discrete Part

In some applications, temporal information is essential and must be taken into account explicitly by the model. The models that have this characteristic are called temporalized. In the description of discrete event systems, these are models where time is deterministic (temporal Petri nets, timed automata [20]) and models where time is random (Markov chain, queuing networks). We present, in the following, only one tool among the modelling possibilities in the discrete domain, the timed automata. In the context of our work on the discrete part of the hybrid system, our objective is to design a diagnostic system, called a diagnoser, which allows to analyze, detect and localize a fault in a system.

3.2.1. Timed Automata

A timed automata is a tool for modeling and diagnosing systems in real time [21, 22]. This tool allows to generate a model to be used for the analysis of a system and in particular for the verification of the system operation, the detection and the localization of faults. The main objective of the timed automata is to integrate time in the model. The integration of time is justified by the growing interest in real-time systems in recent years.

3.2.2. Temporal Analysis: Search for Characteristic Times

The diagnostic method detailed in our work of the discrete part of the hybrid system is based essentially on the characteristic times of the system. They often correspond to the beginning of the system operation, Figure 3. If these times are not respected, a fault is detected. This subsection explains how to collect these particular instants. The first step is to establish the dynamics of the system. The objective is to know how the system behaves over time. The simulation allows to collect the characteristic times.

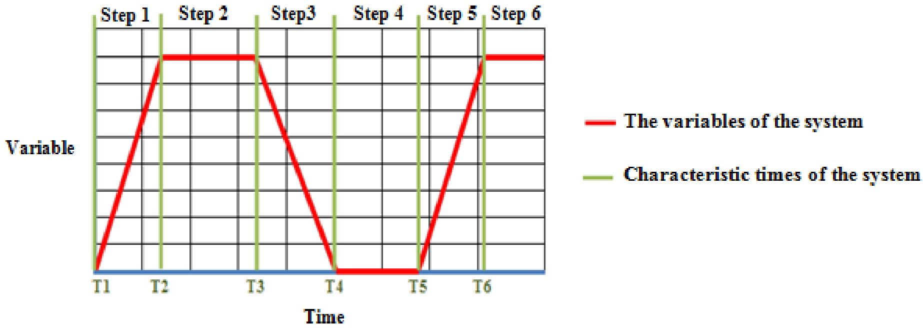

Figure 3. *Example of characteristic times (T1 to T6).*

3.2.3. Principle of Fault Detection

The temporal evolution of the system leads to a succession of states. A discrete transition Ti from a state q_i to another state q_{i+1} occurs when two conditions are satisfied. The first is related to some logical conditions that can be caused by discrete events generated by discrete actuators or discrete sensors. The second is related to the given time t that must elapse.

In fact, the detection principle in the discrete part of the hybrid system and as we have seen previously is based on the comparison, at each instant, between the real state of the system, given by the set of sensor states, and the one given by the normal behavior model of the system. Thus, if an event does not come at the desired time, a fault is detected, Figure 4.

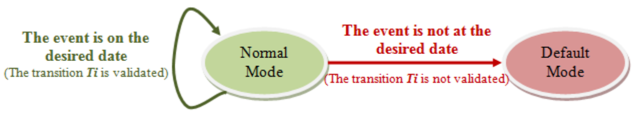

Figure 4. *Two operating modes used.*

3.2.4. Fault Location Principle

The second step of the diagnosis concerns the location of a fault. Following the detection of a fault, and the different fault modes identified in the Failure Mode and Effects Analysis (FMEA), we need to know how the fault will propagate in the system and how it will modify the occurrence of future events.

The dynamic model contains all possible states (normal and faulty states) of the system, which allows us to follow its temporal evolution. From there, thanks to the trajectory followed to go from an initial state to a fault state, we are able to locate a fault by quantifying the times taken in the transitions.

To better understand the different phases of the diagnosis of the hybrid dynamic system by our multi-model approach (construction of the hybrid dynamic model, detection phase, localization phase), we will describe these ideas in more detail through a two-tank system.

4. Application to a Two-Tank System

4.1. Description of the System

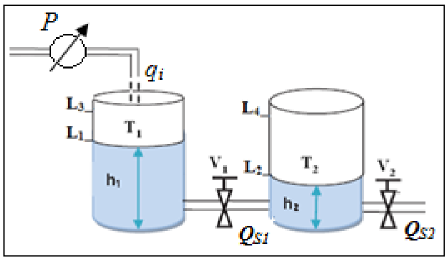

Figure 5. *Example of a two-tank system.*

The system, figure 5, consists of two cylindrical tanks: the first tank T_1 of section S_1 (S_1 = 0.0154 m^2) and height h_1 and the second tank T_2, of section S_2 (S_2 = 0.0154 m^2) and height h_2. The tanks communicate through a valve V_1 (of hydraulic resistance R_{V1}) with a flow rate Q_{S1}, always open. The system has a single inlet: volume flow qi (qi = 10^{-4} m^3/s) through a pump P. The output flow of the second tank Q_{S2} is allowed by a valve V_2 of hydraulic resistance R_{V2}. We put four sensors of Boolean nature: two level sensors $L_1$and L_2 and two overflow sensors L_3 and L_4.

The controls of the valves V_1, V_2 and the pump P are of AON (All Or Nothing) nature. When the pump P is stopped its flow qi is null, when it is in operation its flow qi is equal to 10^{-4} m^3/s. The variables V_1, V_2, P are of Boolean nature. This allows us to see two combinations for this example among the 8 possible combinations, grouped in Table 1.

Table 1. *Construction of the two modes.*

Modes	***P***	V_1	V_2
Mode 1: Filling	1	1	0
Mode 2: Draining	0	1	1

The overall objective of the system is to maintain the liquid in the two tanks at a defined level:

$$\begin{cases} h_1 \le 0.8m \Rightarrow if\ h_1 \ge 0.8m\ then\ L_1 = 1 \\ h_2 \le 0.5m \Rightarrow if\ h_2 \ge 0.5m\ then\ L_2 = 1 \end{cases} \quad (8)$$

4.2. Modelling of Hybrid Dynamic Systems

The bond graph model representing the system is shown in Figure 6.

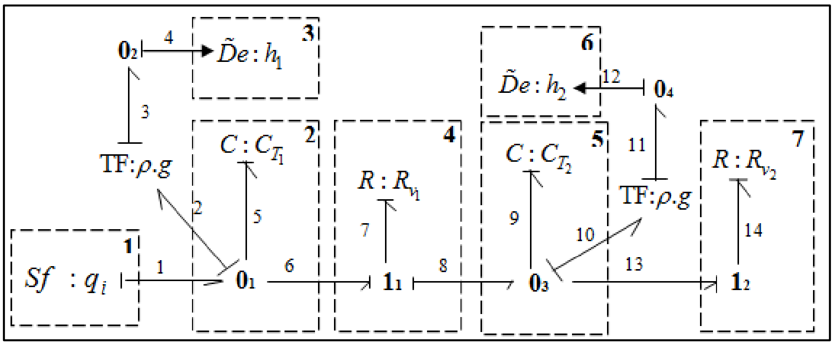

Figure 6. *Bond Graph model of the two-tank system.*

1: Flow source Sf models volume flow q_i; 2: Element C models storage tank T_1; 3: Effort detector De models level sensor L_1; 4: Element R models valve V_1; 5: Element C models storage tank T_2; 6: Effort detector De models level sensor L_2; 7: Element R models valve V_2.

Where g is the gravity constant and ρ the density.

As explained earlier, the equations of state of the system can be deduced directly from the bond graph model (Figure 6).

Let's choose as first equation the one corresponding to the junction 0_1, so we have:

$$S_1 \frac{dh_1}{dt} = q_i\ P - a\ S_c \sqrt{2g(h_1 - h_2)}\ V_1 \quad (9)$$

Where P is the control of a pump P of the On/Off type, a the flux coefficient between 0 and 1, S_c is the section of the conduit in m^2 and V_1 is the control of a valve V_1 of the On/Off type.

A second one can be generated from the junction equation 0_3. Thus, we can write:

$$S_2 \frac{dh_2}{dt} = a\ S_c \sqrt{2g(h_1 - h_2)}\ V_1 - a\ S_c \sqrt{2g\ h_2} V_2 \quad (10)$$

Where V_2 the control of the valve V_2 of the On/Off type.

After linearization around the stationary operating regime, the continuous part of the system, equation (9) and equation (10), can be modeled as follows:

$$\begin{cases} \begin{pmatrix} \dot{Q}_{S_1} \\ \dot{Q}_{S_2} \end{pmatrix} = \begin{pmatrix} -\frac{aS_c}{S_1}\sqrt{\frac{g}{2(h_{10}-h_{20})}}V_1 & 0 \\ \frac{aS_c}{S_2}\sqrt{\frac{g}{2h_{20}}}V_1 & -\frac{aS_c}{S_2}\sqrt{\frac{g}{2h_{20}}}V_2 \end{pmatrix} \begin{pmatrix} Q_{S_1} \\ Q_{S_2} \end{pmatrix} + \begin{pmatrix} \frac{aS_c}{S_1}\sqrt{\frac{g}{2(h_{10}-h_{20})}}P \\ 0 \end{pmatrix} (q_i) \\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} Q_{S_1} \\ Q_{S_2} \end{pmatrix} \end{cases} \quad (11)$$

Where

a = 0.095 and S_c = 3.7895 10^{-4} m^2 and g = 9.81 m/s^2 the gravity constant.

h_{10} is the level in tank T_1 in stationary operation.

h_{20} is the level in tank T_2 in stationary operation.

$Q_{S_1} = aS_c\sqrt{2g(h_1 - h_2)}$ (Where Q_{S1} is the flow through valve V_1)

$Q_{S_2} = aS_c\sqrt{2gh_2}$ (Where Q_{S2} is the flow through valve V_2)

Therefore, according to V_1, V_2 and P and from Table 1, two state representations from the bond graph model of the two-tank system are obtained as follows:

Mode 1: $P = 1$; $V_1 = 1$; $V_2 = 0$

$$\begin{cases}\begin{pmatrix}\dot{Q}_{S_1}\\ \dot{Q}_{S_2}\end{pmatrix} = \begin{pmatrix} -\frac{aS_c}{S_1}\sqrt{\frac{g}{2(h_{10}-h_{20})}} & 0 \\ \frac{aS_c}{S_2}\sqrt{\frac{g}{2h_{20}}} & 0\end{pmatrix}\begin{pmatrix}Q_{S_1}\\ Q_{S_2}\end{pmatrix} + \begin{pmatrix}\frac{aS_c}{S_1}\sqrt{\frac{g}{2(h_{10}-h_{20})}}\\ 0\end{pmatrix}(q_i)\\ \begin{pmatrix}y_1\\ y_2\end{pmatrix}=\begin{pmatrix}1&0\\ 0&1\end{pmatrix}\begin{pmatrix}Q_{S_1}\\ Q_{S_2}\end{pmatrix}\end{cases} \qquad (12)$$

Mode 2: $P = 0$; $V_1 = 1$; $V_2 = 1$

$$\begin{cases}\begin{pmatrix}\dot{Q}_{S_1}\\ \dot{Q}_{S_2}\end{pmatrix} = \begin{pmatrix} -\frac{aS_c}{S_1}\sqrt{\frac{g}{2(h_{10}-h_{20})}} & 0 \\ \frac{aS_c}{S_2}\sqrt{\frac{g}{2h_{20}}} & -\frac{aS_c}{S_2}\sqrt{\frac{g}{2h_{20}}}\end{pmatrix}\begin{pmatrix}Q_{S_1}\\ Q_{S_2}\end{pmatrix}\\ \begin{pmatrix}y_1\\ y_2\end{pmatrix}=\begin{pmatrix}1&0\\ 0&1\end{pmatrix}\begin{pmatrix}Q_{S_1}\\ Q_{S_2}\end{pmatrix}\end{cases} \qquad (13)$$

The Timed Automata representing the system in normal operation is given in Figure 7.

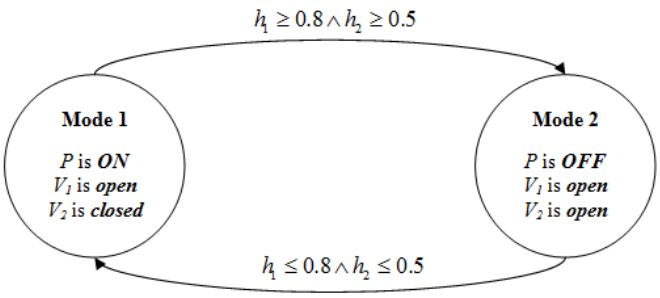

Figure 7. *Timed Automata of the two-tank system (« ∧ » = The logical function AND).*

4.3. Simulation of the Evolution of the System

The simulation is realized during a total simulation time equal to 700s, with the following initial conditions: $h_1(0) = h_2(0) = 0$.

The evolution of the liquid levels h_1 and h_2 are given in Figure 8.

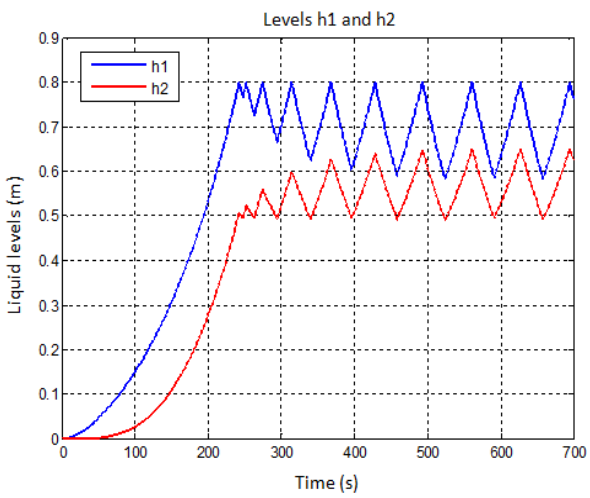

Figure 8. *The liquid levels in the two tanks (continuous evolution).*

The state of the valves and the state of the pump are given in Figure 9.

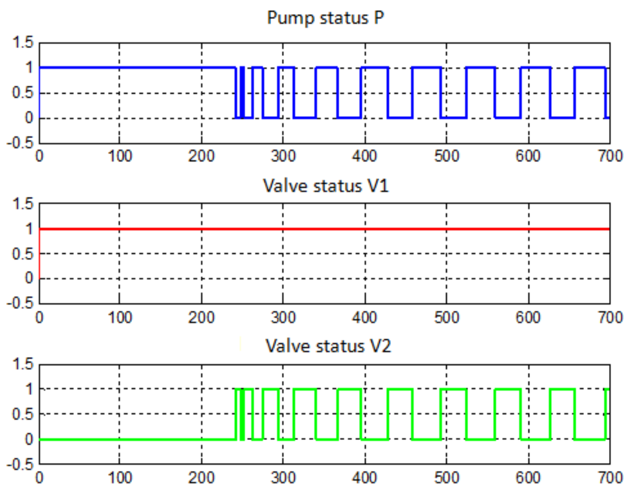

Figure 9. *The state of the valves and the pump (the discrete evolution).*

4.4. Diagnosis of Hybrid Dynamic Systems

4.4.1. Analysis of the Observer

As it has been assumed that the couple (A, C) is observable and the matrix (A - KC) is stable, the observer is constructed by assuming that there are no uncertainties:

$$\text{System}:\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{cases}\quad\Rightarrow\quad\text{Observer}:\begin{cases}\dot{\tilde{x}}(t)=A\tilde{x}(t)+Bu(t)+K(y-\tilde{y})\\ \tilde{y}(t)=C\tilde{x}(t)\end{cases}\tag{14}$$

With

Mode 1: P = 1; V_1 = 1; V_2 = 0

$$A=\begin{pmatrix}-\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}} & 0\\ \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}} & 0\end{pmatrix};\quad B=\begin{pmatrix}\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}\\ 0\end{pmatrix};\quad C=\begin{pmatrix}1&0\\0&1\end{pmatrix};\quad K=\begin{pmatrix}0.0717&0\\0.0083&0.0800\end{pmatrix}$$

Mode 2: P = 0; V_1 = 1; V_2 = 1

$$A=\begin{pmatrix}-\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}} & 0\\ \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}} & -\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}\end{pmatrix};\quad B=0;\quad C=\begin{pmatrix}1&0\\0&1\end{pmatrix};\quad K=\begin{pmatrix}0.0717&0\\0.0083&0.0717\end{pmatrix}$$

4.4.2. Defects Considered

Sensor faults

We consider two faults d_1, d_2 respectively for the two sensors L_1, L_2. That is to say that the two flows Q_{S1} and Q_{S2} change according to d_1, d_2. As a result, the measurement equations become:

$$\begin{cases}y_1=Q_{S1}=aS_c\sqrt{2g\left(h_1+d_1-(h_2+d_2)\right)}\\ y_2=Q_{S2}=aS_c\sqrt{2g\left(h_2+d_2\right)}\end{cases}\tag{15}$$

In fact, these defects d_1, d_2 consist in simulating a different operation than the one expected.

System faults

These faults are presented as leaks in the two tanks T_1 and T_2. The effect of these leaks on the differential equations is similar to the effect of the different flows, but these leaks are considered as outflows, and this leads us to implement these flows with minus signs (-): (d_6 and d_7 respectively for the first and second tank).

$$\begin{cases}\dot{Q}_{S_1}=-\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}V_1Q_{S_1}+\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}Pq_i-\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}d_6\\ \dot{Q}_{S_2}=\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_1Q_{S_1}-\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_2Q_{S_2}-\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}d_7\end{cases}\tag{16}$$

Actuator faults

We consider two faults d_3, d_4 respectively for the two valves V_1, V_2 and a fault d_5 for the pump P. Therefore, the differential equations become:

$$\begin{cases}\dot{Q}_{S_1}=-\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}V_1\bar{d}_3Q_{S_1}+\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}P\bar{d}_5q_i\\ \dot{Q}_{S_2}=\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_1\bar{d}_3Q_{S_1}-\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_2\bar{d}_4Q_{S_2}\end{cases}\tag{17}$$

These faults d_3, d_4, d_5 thus consist in simulating a different operation than the one expected.

4.4.3. Generation of Residuals

As we explained in the previous sections, the deviation signal or residual between the measurements and the estimation of the outputs (estimation error on the outputs) is given by this equation:

$$r(t) = y(t) - \tilde{y}(t) = C(x(t) - \tilde{x}(t)) \tag{18}$$

We associate the relation (18) and the system (15), (16), (17) and (14), we obtain the following systems:

$$\begin{cases} r_1 = y_1 - \tilde{y}_1 = x_1 - \tilde{x}_1 \Rightarrow \dot{r}_1 = \dot{x}_1 - \dot{\tilde{x}}_1 \\ \Rightarrow \dot{r}_1 = -\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}V_1\bar{d}_3Q_{S_1} + \dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}P\bar{d}_5q_i \\ -\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}d_6 + \dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}V_1Q_{S_1} \\ -\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}Pq_i + 0.0183(y_1 - \tilde{y}_1) \\ \Rightarrow \dot{r}_1 = -\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}V_1\bar{d}_3Q_{S_1} + \dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}P\bar{d}_5q_i \\ -\dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}d_6 + \dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}V_1Q_{S_1} - \dfrac{aS_c}{S_1}\sqrt{\dfrac{g}{2(h_{10}-h_{20})}}Pq_i \\ +0.0183\left(aS_c\sqrt{2g(h_1+d_1-(h_2+d_2))} - aS_c\sqrt{2g(h_1-h_2)}\right) \end{cases} \tag{19}$$

And

$$\begin{cases} r_2 = y_2 - \tilde{y}_2 = x_2 - \tilde{x}_2 \Rightarrow \dot{r}_2 = \dot{x}_2 - \dot{\tilde{x}}_2 \\ \Rightarrow \dot{r}_2 = \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_1\bar{d}_3Q_{S_1} - \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_2\bar{d}_4Q_{S_2} \\ -\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}d_7 - \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_1Q_{S_1} + \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_2Q_{S_2} \\ -0.0083(y_1 - \tilde{y}_1) + 0.02(y_2 - \tilde{y}_2) \\ \Rightarrow \dot{r}_2 = \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_1\bar{d}_3Q_{S_1} - \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_2\bar{d}_4Q_{S_2} \\ -\dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}d_7 - \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_1Q_{S_1} + \dfrac{aS_c}{S_2}\sqrt{\dfrac{g}{2h_{20}}}V_2Q_{S_2} \\ -0.0083\left(aS_c\sqrt{2g(h_1+d_1-(h_2+d_2))} - aS_c\sqrt{2g(h_1-h_2)}\right) \\ +0.02\left(aS_c\sqrt{2g(h_2+d_2)} - aS_c\sqrt{2gh_2}\right) \end{cases} \tag{20}$$

The system (19) shows that the residue r_1 is sensitive to faults d_1, d_2, d_3, d_5 and d_6. Thus, system (20) shows that residue r_2 is sensitive to faults d_1, d_2, d_3, d_4 and d_7. Therefore, when a fault occurs in the system, the residual r_1 and/or the residual r_2 becomes different from zero.

We notice the presence of the Boolean variables V_1, V_2 and P in the residuals r_1 and r_2, equation (19) and (20), therefore, we design two residuals for the two modes mentioned in the previous section, by substituting the values of V_1, V_2 and P according to Table 1. The different fault signatures are grouped in Table 2.

Table 2. *Signature matrix MR.*

Faults	Possible faults	Fault mode	Components	r_1	r_2
d_1	d_{11}	L_1 Stuck_Up	Level sensor L_1	1	1
	d_{12}	L_1 Stuck_Down			
d_2	d_{21}	L_2 Stuck_Up	Level sensor L_2	1	1
	d_{22}	L_2 Stuck_Down			
d_3	d_3	V_1 Stuck_Close	Valve V_1	1	1
d_4	d_{41}	V_2 Stuck_Close	Valve V_2	0	1
	d_{42}	V_2 Stuck_Open			
d_5	d_{51}	P Stuck_Off	Pump P	1	0
	d_{52}	P Stuck_On			
d_6	d_6	Leak in the tank 1	Tank T_1	1	0
d_7	d_7	Leak in the tank 2	Tank T_2	0	1

L_i Stuck_ Down means that the sensor L_i always remains in state 0 (the sensor does not detect high level); L_i Stuck_Up means that the sensor L_i always remains in state 1 (the sensor does not detect low level); V_i Stuck_Close means that the valve V_i remains closed on an open request; V_i Stuck_Open means that the valve V_i remains open on a close request; P_i Stuck_Off means that the pump P_i remains OFF on an ON request;; P_i Stuck_On means that the pump P_i remains ON on a OFF request;

All faults are detectable because no line is completely null, but are not locatable because the lines are not all different. Indeed, the three signatures of L_1, L_2 and V_1 are identical and equal to [1 1]. Similarly for the signatures of V_2 and T_2 equal to [0 1] and the signatures of P and T_1 equal to [1 0].

4.4.4. Temporal Identification of the System

The construction of the diagnoser by the Timed Automata being based on the temporal knowledge of the system, we need to know the times of the system as for example the time of opening or closing of the valves or the time of change of state of the sensors. From Figure 8 and Figure 9, the transition times determined for each phase of the system in normal operation are given in Table 3.

***Table 3**. Temporal identification of the system.*

Actions	Time in seconds	Time interval
Pump *P* ON	0	[0, 242.12]
Pump *P* OFF	242.12	
Opening of the valve V_2	242.12	[242.12, 247]
Closing of the valve V_2	247	
Activation of sensor L_1	242.12	[242.12, 242.12]
Deactivation of sensor L_1	242.12	
Activation of sensor L_2	242	[242, 246]
Deactivation of sensor L_2	246	

4.4.5. Simulation Results

The simulation was realized on Matlab - Simulink - State-flow: Simulink for the evaluation of the residuals of the continuous part and Stateflow for the modeling of the discrete model. The normal evolution of the residuals are presented in Figure 10. The simulation time is fixed at 700s. There are perturbations in the residuals that lead us to choose a detection threshold $\pm 10^{-7}$ ($\varepsilon_1 = \varepsilon_2 = 10^{-7}$).

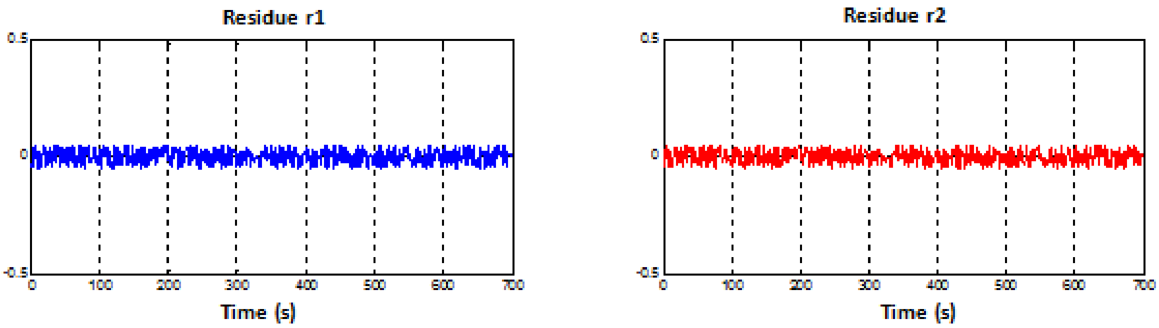

***Figure 10**. Residuals in normal operation (without faults).*

A fault is simulated at pump *P* (*P* Stuck_On) in the time interval [350s, 500s]. Figure 11 shows that r_l is sensitive to the introduced fault. This is confirmed by the matrix M_R presented in Table 2.

The instant when the residual r_l exceeds its threshold ε_1 ($\varepsilon_1 = 10^{-7}$) represents the instant of defect detection. This instant, called $T_{detection}$, is equal to 367.9s.

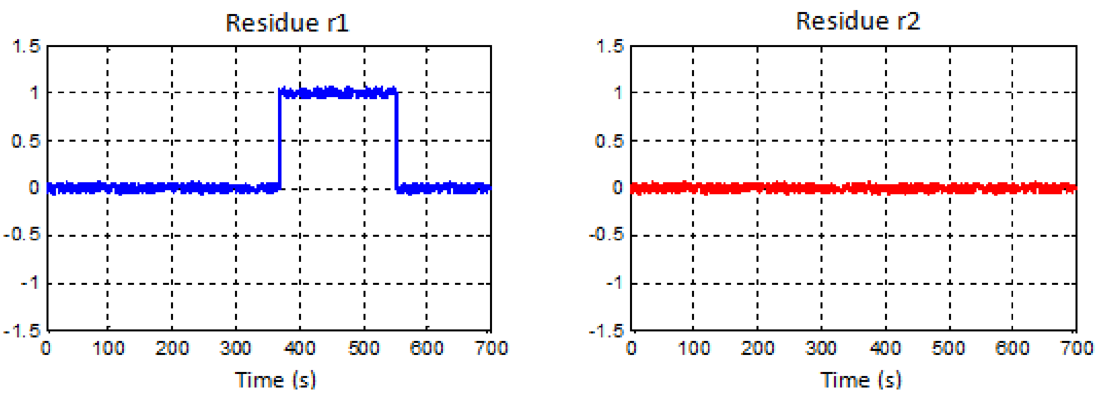

***Figure 11**. Residuals in fault mode (Pump fault P Stuck_On).*

The liquid levels, h_1 in pink and h_2 in blue, in the presence of a *P* Stuck_On fault are given in Figure 12.

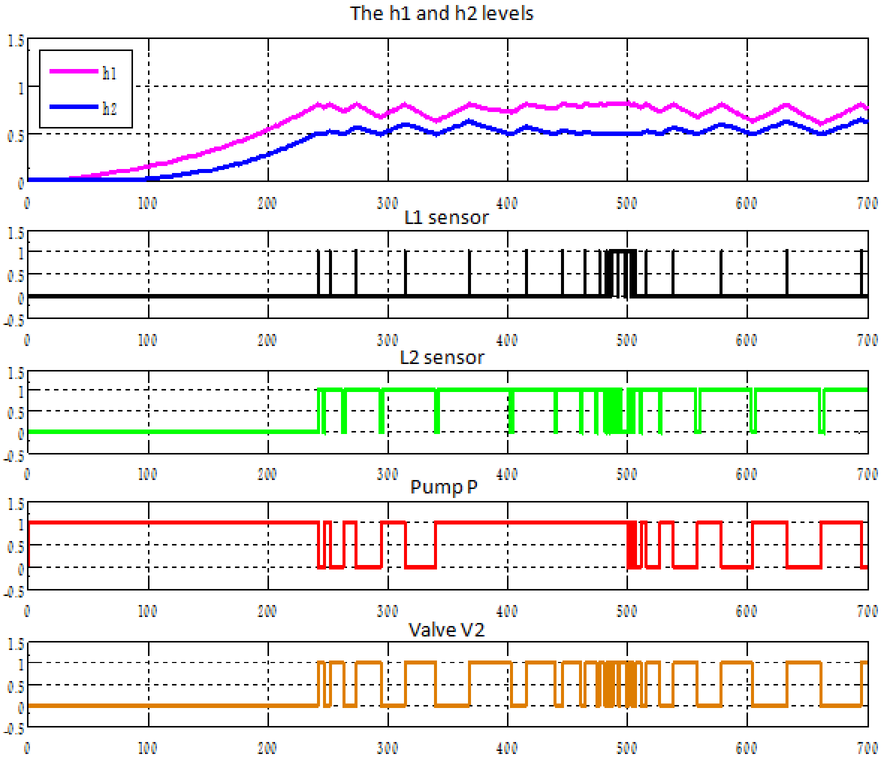

***Figure 12**. The states of the sensors and the pump in fault mode (Pump fault P Stuck_On).*

Thus, the black and green signals, Figure 12, show the state of the L_1 and L_2 sensors and their sensitivity to this fault (*P* Stuck_On). The states of the pump *P* and the valve V_2 are given by the red and brown signals.

In fact, in Figure 12, we can see that the sensor L_2 (green signal) remains in state 1; 4.88 sec after the opening of the valve V_2 and thus the residual r_1 at this instant is non-zero. So this instant, called $T_{localization}$, corresponds to the location of the fault on the pump *P* Stuck_On ($T_{localization}$ = 402s).

5. Conclusion

In this article, we have opted for a multi-model diagnosis method based on Bond Graph, Observer and Timed Automata. The choice of this method is justified by its ability to model and diagnose faults (sensors, actuator or system) and thus by its efficiency in the fault location phase.

The performances of the proposed multi-model diagnosis method have been validated on the hydraulic system with two tanks. The simulation of some faults (component faults, sensor faults and actuator faults) to provide a validity to our proposed approach has been realized.

Abbreviations

HDS	Hybrid Dynamic Systems
FMEA	Failure Mode and Effects Analysis

Conflicts of Interest

The authors declare no conflicts of interest.

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