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The Star Domination Polynomial in Hypergraphs

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Abstract: This paper introduces the star domination polynomial of hypergraphs as an extension of the domination polynomial in graphs. Based on the concept of star domination, we define the star domination polynomial as the generating function that counts the star dominating sets of a hypergraph according to their cardinalities. We establish several fundamental properties of this polynomial and investigate its behavior under the disjoint union of hypergraphs. Explicit formulas are obtained for the star domination polynomial of important classes of hypergraphs, including r-uniform complete hypergraphs and sunflower hypergraphs, by characterizing their star dominating sets of different cardinalities. The proposed polynomial provides an algebraic representation of the distribution of star dominating sets and offers a new tool for studying domination in hypergraphs. These results extend the theory of domination polynomials to hypergraphs and provide a foundation for further research on domination parameters, hypergraph invariants, and related combinatorial polynomials.

Keywords: Hypergraph, Domination, Domination Polynomial

1. Introduction

Domination is a fundamental topic in graph theory and has attracted considerable attention due to its rich theoretical structure and numerous applications in areas such as communication networks, social network analysis, facility location, biological systems, and computer science. Over the years, several variants of domination have been developed to address different structural and practical considerations. Standard references on graph theory and domination in graphs include the works of Chartrand and Lesniak [5] and the comprehensive monographs by Haynes et al. [9, 10].

Many real-world systems involve interactions among more than two entities simultaneously, making graphs inadequate for modeling such relationships. Hypergraphs provide a natural and more expressive framework for representing these higher-order interactions. A hypergraph $H = (V, \mathcal{E})$ consists of a finite nonempty vertex set V and a family of nonempty subsets $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, called hyperedges, such that

$$\bigcup_{e \in \mathcal{E}} e = V.$$

A hypergraph is said to be *simple* if no hyperedge is properly contained in another hyperedge, and r*-uniform* if every hyperedge contains exactly r vertices. Graphs form a special class of hypergraphs in which every hyperedge has cardinality two. For hypergraph theoretic terminology we use [6–8].

Extending graph-theoretic concepts to hypergraphs has become an active area of research, and domination is no exception. Owing to the richer incidence structure of hypergraphs, several notions of domination have been proposed. The classical definition, introduced by Acharya [11], generalizes graph domination by declaring a vertex dominated whenever it is adjacent to a vertex in the dominating set.

Definition 1.1. [11] Let $H = (V, \mathcal{E})$ be a hypergraph. A subset D of V is called a dominating set of H if for every $v \in V - D$, there exists $u \in D$ such that u and v are adjacent. The minimum cardinality of a dominating set of H is called the domination number of H and is denoted by $\gamma(H)$.

More recently, Divya et al. [1, 2] proposed an edge-based formulation in which domination is defined through the hyperedge structure. This approach, referred to as the *star domination* model, naturally extends the classical notion of domination in graphs while capturing the intrinsic characteristics of hypergraphs. Throughout this paper, we adopt the star domination framework introduced in [1, 2].

Definition 1.2. [1, 2] Let $H = (V, \mathcal{E})$ be a hypergraph. A subset D of V is called a star dominating set of H if for every $v \in V - D$, there exists an edge $e \in \mathcal{E}$ such that $v \in e$ and $e - \{v\} \subseteq D$. The minimum cardinality of a dominating set of H is called the star domination number of H and is denoted by $\gamma^*(H)$.

2. The Star Domination Polynomial

The domination polynomial introduced by Alikhani and Peng [3] has received considerable attention as it encodes the distribution of dominating sets in a graph. Motivated by this concept and the star domination model for hypergraphs, we introduce the *star domination polynomial*, which enumerates the star dominating sets of a hypergraph according to their cardinalities. This polynomial provides an algebraic tool for investigating star domination in hypergraphs and naturally extends the domination polynomial from graphs to the hypergraph setting.

Definition 2.1. Let $SD_k(H)$ be the collection of star dominating sets of a finite hypergraph H with cardinality k and let $sd_k(H) = |SD_k(H)|$. Then the star domination polynomial $SD_k(H)$ of H is defined as

$$SD(H; x) = \sum_{k=\gamma^*(H)}^{|V(H)|} sd_k(H) x^k \tag{1}$$

where $\gamma^*(H)$ is the star domination number of H.

Observation 2.1. From the definition we can see that:

1) For any finite Hypergraph H, the degree of $SD(H; x)$ is $|V(H)|$ and the leading coefficient is one.
2) The smallest exponent of $SD(H; x)$ equals the star-domination number $\gamma^*(H)$.

This polynomial not only determines the star domination number $\gamma^\star(H)$ as its smallest exponent, but also provides a complete distribution of star-dominating sets across different sizes, offering a refined perspective on domination in hypergraphs

Example 2.1. Consider the 3-uniform complete hypergraph $H = (V, \mathcal{E})$ where $V = \{v_1, v_2, v_3, v_4\}$. We know that $\gamma^*(K_4^3) = 2$ and any 2-set of vertices serves as minimal star dominating set. Hence $sd_2(K_4^3) = \binom{4}{2} = 6$. similarly $sd_3(K_4^3) = \binom{4}{3} = 4$ and clearly $sd_4(K_4^3) = 1$. Consequently

$$SD(K_4^3; x) = x^4 + 4x^3 + 6x^2.$$

The above example illustrates how the star domination polynomial can be computed explicitly for a small complete hypergraph. Motivated by this, it is natural to investigate the general structure of star domination polynomials in the broader class of complete r-uniform hypergraphs. In what follows, we present the generalization to K_n^r.

Theorem 2.1. Let K_n^r be the complete r-uniform hypergraph on n vertices. Then the star domination polynomial of K_n^r is given by

$$\mathrm{SD}(K_n^r; x) = \sum_{k=r-1}^{n} \binom{n}{k} x^k = (1+x)^n - \sum_{k=0}^{r-2} \binom{n}{k} x^k.$$

Proof. The star-dominating sets of K_n^r are exactly the subsets of V of size k with $k \geq r-1$. The number of such sets is $\binom{n}{k}$ for each k, and hence

$$\mathrm{SD}(K_n^r; x) = \sum_{k=r-1}^{n} \binom{n}{k} x^k.$$

Finally, by the binomial theorem, this can be written as

$$(1+x)^n - \sum_{k=0}^{r-2} \binom{n}{k} x^k,$$

completing the proof.

Theorem 2.2. Suppose $H = (V, \mathcal{E})$ is a hypergraph of order n consisting of a single edge of cardinality t. Then

$$\mathrm{SD}(H; x) = t x^{n-1} + x^n.$$

Proof. Let e be the only edge in H where $|e| = t$, and let $S \subseteq V$ be a star-dominating set of H. By the definition of star domination, for each vertex $v \in V \setminus S$ there must exist an edge $e \in \mathcal{E}$ such that $e \setminus \{v\} \subseteq S$.

Since H has only one edge e, all isolated vertices must necessarily belong to S. Moreover, within e, all but at most one vertex must also belong to S to satisfy the condition. Hence, the minimum size of a star-dominating set is $n-1$, i.e., $\gamma^\star(H) = n-1$ and the only contributions to the polynomial are from sizes $n-1$ and n. Counting them gives

$$\mathrm{SD}(H; x) = \underbrace{t}_{\#\text{ of }(n-1)\text{-sets}} x^{n-1} + \underbrace{1}_{\#\text{ of }n\text{-sets}} x^n = t x^{n-1} + x^n,$$

as required.

Corollary 2.1. If H is a hypergraph consisting of a single edge on n vertices i.e., $t = n$ and there are no isolated vertices, then

$$\mathrm{SD}(H; x) = n x^{n-1} + x^n,$$

and the star domination number is $\gamma^\star(H) = n-1$.

Theorem 2.3. If $H = H_1 \cup H_2$ is the disjoint union of two hypergraphs H_1 and H_2, then

$$\mathrm{SD}(H; x) = \mathrm{SD}(H_1; x)\,\mathrm{SD}(H_2; x).$$

Proof. Every vertex of H lies either in H_1 or in H_2. A subset $S \subseteq V(H)$ is a star-dominating set of H if and only if $S_1 = S \cap V(H_1)$ is a star-dominating set of H_1 and $S_2 = S \cap V(H_2)$ is a star-dominating set of H_2. Conversely, any pair (S_1, S_2) of star-dominating sets in H_1 and H_2 combines to give a star-dominating set $S_1 \cup S_2$ in H. Thus, the star-dominating sets of H are in bijection with ordered pairs of star-dominating sets of the components, and the size of such a union is $|S_1 \cup S_2| = |S_1| + |S_2|$. Counting by size yields

$$sd_k(H) = \sum_{i+j=k} sd_i(H_1)\, sd_j(H_2),$$

where $sd_k(G)$ denotes the number of star-dominating sets of size k in G.

By the definition of the star-domination polynomial, this relation is equivalent to $\mathrm{SD}(H;x) = \mathrm{SD}(H_1;x)\,\mathrm{SD}(H_2;x)$.

Corollary 2.1. If $H = \bigsqcup_{i=1}^{m} H_i$ is the disjoint union of m hypergraphs $H_1, H_2, \ldots, H_m$, then

$$\mathrm{SD}\left(\bigsqcup_{i=1}^{m} H_i; x\right) = \prod_{i=1}^{m} \mathrm{SD}(H_i;x).$$

Proof. A subset $S \subseteq V(H)$ is a star-dominating set in H if and only if its restriction $S_i = S \cap V(H_i)$ is a star-dominating set in each component H_i. Conversely, any choice of star-dominating sets $S_1, S_2, \ldots, S_m$ (one from each component) combines to form a star-dominating set $S = S_1 \cup S_2 \cup \cdots \cup S_m$ of H.

The size of S is the sum of the sizes of the individual S_i. Therefore, the number of star-dominating sets of size k in H is given by

$$sd_k(H) = \sum_{i_1+i_2+\cdots+i_m=k} \prod_{j=1}^{m} sd_{i_j}(H_j),$$

where $sd_\ell(G)$ denotes the number of star-dominating sets of size ℓ in G.

By the definition of the star-domination polynomial, this relation is equivalent to

$$\mathrm{SD}(H;x) = \prod_{i=1}^{m} \mathrm{SD}(H_i;x).$$

We now determine the star domination polynomial for sunflower hypergraphs, whose structure enables an explicit characterization of their star dominating sets. The definition of a sunflower hypergraph used here is adopted from [13].

Theorem 2.4. Let $H = \mathrm{SH}(n,p,h)$ be a sunflower hypergraph of order n and size k, with common core C and petals $P_1, \ldots, P_k$ such that

$$|C| = c = h - p, \qquad |P_i| = p \quad (1 \le i \le k),$$

and the petals P_i are pairwise disjoint. Thus $n = c + kp$.

Then the star-domination number is

$$\gamma^\star(H) = n - k = c + k(p-1),$$

and the star-domination polynomial is

$$\mathrm{SD}\big(\mathrm{SH}(n,p,h);x\big) = x^{\,n-k}(x+p)^k \;+\; c\,x^{\,n-1}.$$

Equivalently,

$$\mathrm{SD}\big(\mathrm{SH}(n,p,h);x\big) = \sum_{r=0}^{k} \binom{k}{r} p^{\,r}\, x^{\,n-r} \;+\; c\,x^{\,n-1}.$$

Proof. Let S be a star dominating set of H. At most one vertex of the core C can be omitted from S. If two distinct core vertices $u, w \in C$ were both omitted from a set S, then to star-dominate u we must have some edge e_i with $e_i \setminus \{u\} = (C \setminus \{u\}) \cup P_i \subseteq S$. But $(C \setminus \{u\})$ contains w, contradicting $w \notin S$. So at most one core vertex may be omitted. So every star dominating set falls into exactly one of two disjoint types.

Case 1: The whole core C is contained in S. If the core is included, then in each petal P_i we may either include all p petal vertices or omit exactly one vertex of that petal. (Omitting two or more vertices from the same petal would violate the star-domination condition for some omitted vertex of that petal.) Thus, for any fixed number r with $0 \le r \le k$, to obtain a set of size $n - r$ we choose exactly r petals from the k in which to omit one vertex, and from each chosen petal there are p choices which vertex to omit. This yields

$$\binom{k}{r} p^{\,r}$$

star-dominating sets of size $n - r$ in which the core is fully included. Summing r from 0 to k gives the contribution

$$\sum_{r=0}^{k} \binom{k}{r} p^{\,r} x^{\,n-r}.$$

Case 2: Exactly one core vertex is omitted. By the claim, we may omit at most one core vertex. If one core vertex is omitted then no petal vertex may be omitted: for any petal P_i and any leaf $v \in P_i$, the set $e_i \setminus \{v\} = C \cup (P_i \setminus \{v\})$ contains the missing core vertex, so $e_i \setminus \{v\} \nsubseteq S$. Thus when one core vertex is omitted every petal must be entirely included, and the size of such an S is

$$kp + (c-1) = n - 1.$$

There are exactly c choices (one for each omitted core vertex), so Case 2 contributes c star-dominating sets of size $n-1$, i.e. the term $c\,x^{\,n-1}$ in the star dominating polynomial.

Adding the contributions from Case 1 and Case 2 yields

$$\mathrm{SD}(H;x) = \sum_{r=0}^{k} \binom{k}{r} p^{\,r} x^{\,n-r} + c\,x^{\,n-1}.$$

Noting that

$$\sum_{r=0}^{k}\binom{k}{r}p^{\,r}x^{\,n-r}=x^{\,n-k}(x+p)^{k},$$

we obtain the compact form

$$\mathrm{SD}(H;x)=x^{\,n-k}(x+p)^{k}+c\,x^{\,n-1}.$$

This completes the proof.

Example 2.2. Let $k=3,\ p=2,\ h=5$, so $c=h-p=3$ and $n=c+kp=9$. By the theorem,

$$\mathrm{SD}\big(\mathrm{SH}(9,2,5);x\big)=x^{6}(x+2)^{3}+3x^{8}.$$

Since $(x+2)^3=x^3+6x^2+12x+8$, we get

$$\mathrm{SD}\big(\mathrm{SH}(9,2,5);x\big)=x^{9}+9x^{8}+12x^{7}+8x^{6}.$$

Hence $\gamma^{\star}=6$ and the coefficients are:

$$sd_9=1,\qquad sd_8=9,\qquad sd_7=12,\qquad sd_6=8.$$

Corollary 2.2. Let H be a star hypergraph with k edges of cardinality p. Then

$$\mathrm{SD}(H;x)=x^{\,n-k}(x+p)^{k}+x^{\,n-1},$$

where $n=1+kp$ is the order of H and each petal has size p. The star domination number of H is

$$\gamma^{\star}(H)=n-k=1+k(p-1).$$

Proof. The star hypergraph is a special case of the sunflower hypergraph $\mathrm{SH}(n,p,h)$ with core size $c=1$. Substituting $c=1$ into the general formula

$$\mathrm{SD}(\mathrm{SH}(n,p,h);x)=x^{\,n-k}(x+p)^{k}+c\,x^{\,n-1}$$

yields the claimed expression. The minimum degree follows as $\gamma^{\star}(H)=n-k$, and by construction the number of $\gamma^{\star}$-sets is p^k, obtained by omitting one vertex from each of the k petals.

Definition 2.2 (Join of two r-uniform hypergraphs). Let $H_1=(V_1,\mathcal{E}_1)$ and $H_2=(V_2,\mathcal{E}_2)$ be two disjoint r-uniform hypergraphs. The *join* of H_1 and H_2, denoted $H_1\vee H_2$, is the r-uniform hypergraph

$$H_1\vee H_2=\Big(V_1\cup V_2,\ \mathcal{E}_1\cup\mathcal{E}_2\cup\mathcal{E}\Big)$$

where

$$\mathcal{E}=\{e: e\subseteq V_1\cup V_2, e\cap V_1\neq\phi, e\cap V_2\neq\phi\}$$

That is, $H_1\vee H_2$ contains all the edges of H_1 and H_2, together with every r-edge that meets both V_1 and V_2.

Remark 2.1. When $r=2$, this construction coincides with the usual *graph join*: every vertex of H_1 is adjacent to every vertex of H_2.

Theorem 2.5. Let $H_1=(V_1,\mathcal{E}_1)$ and $H_2=(V_2,\mathcal{E}_2)$ be two disjoint r-uniform hypergraphs with $n_1=|V_1|$, $n_2=|V_2|$, and $n=n_1+n_2$. Let $H=H_1\vee H_2$ be their join (so H is r-uniform and contains all r-subsets that meet both V_1 and V_2). Then the star-domination polynomial of H is

$$\begin{aligned}\mathrm{SD}(H;x)&=\mathrm{SD}(H_1;x)+\mathrm{SD}(H_2;x)\\&\quad+\sum_{s=r-1}^{n}\left[\binom{n}{s}-\binom{n_1}{s}-\binom{n_2}{s}\right]x^{s}.\end{aligned}$$

Equivalently,

$$\begin{aligned}\mathrm{SD}(H;x)&=\mathrm{SD}(H_1;x)+\mathrm{SD}(H_2;x)\\&\quad+\Big((1+x)^{n}-\sum_{s=0}^{r-2}\binom{n}{s}x^{s}\Big)\\&\quad-\Big((1+x)^{n_1}-\sum_{s=0}^{r-2}\binom{n_1}{s}x^{s}\Big)\\&\quad-\Big((1+x)^{n_2}-\sum_{s=0}^{r-2}\binom{n_2}{s}x^{s}\Big).\end{aligned}$$

Proof. First observe that any set $S\subseteq V_1$ (respectively, $S\subseteq V_2$) is star-dominating in the join $H_1\vee H_2$ if and only if it is star-dominating in H_1 (respectively, H_2). Indeed, if $S\subseteq V_1$ is a star-dominating set of H_1, then all vertices of $V_1\setminus S$ are star dominated by S, so it remains to check vertices of V_2. By the definition of the join of two r-uniform hypergraphs, any $r-1$ vertices from V_1 together with a vertex from V_2 form an edge. Thus, for each $v\in V_2$, one can pick $r-1$ vertices from S so that S star dominates v. Hence every vertex of V_2 is star dominated by S, and the same reasoning applies when $S\subseteq V_2$. Therefore, the collections of star-dominating sets contained entirely within V_1 (or entirely within V_2) are exactly those counted by $\mathrm{SD}(H_1;x)$ and $\mathrm{SD}(H_2;x)$.

Next consider sets $S\subseteq V_1\cup V_2$ such that $S\cap V_1\neq\varnothing$ and $S\cap V_2\neq\varnothing$. We claim that S is star-dominating in H if and only if $|S|\geq r-1$, then . Since H is r-uniform, the condition $|S|\geq r-1$ is necessary. Let $v\in(V_1\cup V_2)\setminus S$, and without loss of generality assume $v\in V_1$. Because $|S|\geq r-1$ and S meets both V_1 and V_2, we can choose $r-1$ vertices from S with at least one from V_2. Then $\{v\}\cup(S'\subseteq S,\ |S'|=r-1,\ S\cap V_2\neq\phi)$ forms an edge in $\mathcal{E}$, so v is star dominated. Thus every such S is star-dominating. Conversely, it is clear that any star-dominating set meeting both V_1 and V_2 must have size at least $r-1$.

Therefore, the star-dominating sets of H fall into three classes:

1) star-dominating sets of H_1 (counted by $\mathrm{SD}^{\star}(H_1;x)$);
2) star-dominating sets of H_2 (counted by $\mathrm{SD}^{\star}(H_2;x)$);
3) mixed sets $S\subseteq V_1\cup V_2$ with $S\cap V_1\neq\varnothing, S\cap V_2\neq\varnothing$, and $|S|\geq r-1$.

For the third type, the number of s-subsets of $V_1\cup V_2$ meeting both parts is

$$\binom{n}{s}-\binom{n_1}{s}-\binom{n_2}{s},$$

since $\binom{n}{s}$ counts all s-subsets of $V_1 \cup V_2$, and we subtract those contained entirely in V_1 or entirely in V_2. Summing these quantities for $s \geq r-1$ gives the contribution of the mixed star-dominating sets.

Adding the three contributions yields the stated expression for $\mathrm{SD}(H;x)$, completing the proof.

In particular, when restricted to $r = 2$, star domination in hypergraphs reduces to the usual domination in graphs. Consequently, in this special case the formula simplifies to the domination polynomial of the join of two graphs which is already established in [3]

Corollary 2.3. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two disjoint graphs, with $n_1 = |V_1|$, $n_2 = |V_2|$ and $n = n_1 + n_2$. Then the star-domination polynomial of the graph join $G = G_1 \vee G_2$ is

$$\begin{aligned}\mathrm{SD}(G_1 \vee G_2; x) = {} & \mathrm{SD}(G_1; x) + \mathrm{SD}(G_2; x) \\ & + \big((1+x)^{n_1} - 1\big)\big((1+x)^{n_2} - 1\big).\end{aligned}$$

Proof. For $r = 2$, from above theorem we get

$$\begin{aligned}\mathrm{SD}^\star(G;x) &= \mathrm{SD}^\star(G_1;x) + \mathrm{SD}^\star(G_2;x) \\ &\quad + \sum_{s=1}^{n} \left[\binom{n}{s} - \binom{n_1}{s} - \binom{n_2}{s}\right] x^s \\ &= \mathrm{SD}^\star(G_1;x) + \mathrm{SD}^\star(G_2;x) \\ &\quad + \big((1+x)^n - 1\big) - \big((1+x)^{n_1} - 1\big) \\ &\quad - \big((1+x)^{n_2} - 1\big). \\ &= \mathrm{SD}^\star(G_1;x) + \mathrm{SD}^\star(G_2;x) \\ &\quad + \big((1+x)^{n_1} - 1\big)\big((1+x)^{n_2} - 1\big).\end{aligned}$$

Example 2.3. Let $r = 3$ and consider sunflowers with core size $c = 1$ (hence petal size $p = r - c = 2$). Take H_1 to be a sunflower with $k_1 = 2$ petals, so $n_1 = 1 + 2 \cdot 2 = 5$, and H_2 a sunflower with $k_2 = 3$ petals, so $n_2 = 1 + 3 \cdot 2 = 7$. Form the join $H = H_1 \vee H_2$, hence $n = n_1 + n_2 = 12$.

Using the formula for sunflower hypergraphs,

$$\mathrm{SD}(\mathrm{SH}; x) = x^{\,n-k}(x+p)^k + c\,x^{\,n-1},$$

we obtain for the components

$$\mathrm{SD}(H_1; x) = x^{5-2}(x+2)^2 + x^4 = x^5 + 5x^4 + 4x^3,$$

$$\mathrm{SD}(H_2; x) = x^{7-3}(x+2)^3 + x^6 = x^7 + 7x^6 + 12x^5 + 8x^4.$$

All mixed subsets meeting both parts with size $s \geq r-1 = 2$ are star-dominating in the join. The mixed contribution is

$$\begin{aligned}M(x) &= \sum_{s=2}^{12} \left(\binom{12}{s} - \binom{5}{s} - \binom{7}{s}\right) x^s \\ &= 35x^2 + 175x^3 + 455x^4 + 770x^5 + 917x^6 + 791x^7 \\ &\qquad + 495x^8 + 220x^9 + 66x^{10} + 12x^{11} + x^{12}.\end{aligned}$$

Therefore the star-domination polynomial of the join is

$$\begin{aligned}\mathrm{SD}(H_1 \vee H_2; x) &= \big(x^5 + 5x^4 + 4x^3\big) \\ &\qquad + \big(x^7 + 7x^6 + 12x^5 + 8x^4\big) + M(x) \\ &= x^{12} + 12x^{11} + 66x^{10} + 220x^9 \\ &\qquad + 495x^8 + 792x^7 + 924x^6 + 783x^5 \\ &\qquad + 468x^4 + 179x^3 + 35x^2.\end{aligned}$$

In particular, $\gamma^\star(H_1 \vee H_2) = r - 1 = 2$ and the coefficient of x^2 equals the number of mixed 2-subsets meeting both parts, namely 35.

3. Conclusion

In this article, we introduced the concept of the *star domination polynomial* for hypergraphs as an extension of the domination polynomial in graph theory. This polynomial acts as a generating function that enumerates star dominating sets of different cardinalities in a hypergraph. Through this algebraic representation, structural information related to star domination can be captured in a compact form.

The concept of the star domination polynomial opens several possibilities for further study. In particular, it would be interesting to determine the polynomial for other families of hypergraphs and to explore additional structural and algebraic properties associated with it. Such investigations may contribute to a deeper understanding of domination-related parameters in hypergraphs and their applications.

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Conflicts of Interest

The authors declare no conflicts of interest.

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