Mathematics and Computer Science
2026; 11(4): 71-77
http://www.sciencepublishinggroup.com/j/mcs
doi: 10.11648/j.mcs.20261104.12

Comparing FDM, FVM, and FEM for Coupled Flow-Transport in a Guinean Bauxite Mining Aquifer

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To cite this article:
Michel Faya Kamano, Mouhamadou Nassirou Diallo, Mariama Ndiaye, Aboubakary Dinakhaby. (2026). Comparing FDM, FVM, and FEM for Coupled Flow-Transport in a Guinean Bauxite Mining Aquifer. *Mathematics and Computer Science, 11*(4), 71-77. https://doi.org/10.11648/j.mcs.20261104.12

Received: 31 May 2026; **Accepted:** 10 June 2026 **Published:** 11 August 2026

Abstract: Bauxite mining in Guinea, particularly in the Boke region, generates significant risks of groundwater contamination by metals and acidification. This paper develops a deterministic coupled groundwater flow and reactive solute transport model applied to the N'Dangara and Boundou Wande bauxite plateaus (CBG concession, Sangaredi, Guinea). The steady-state flow equation and the transient advection-dispersion-reaction equation are discretised using three numerical methods: finite differences (FDM), finite volumes (FVM), and finite elements (FEM). Model parameters are taken from three field campaigns that provided piezometric measurements and physico-chemical analyses. The field data show dispersed flow directions controlled by topography and water table depths between 1.09 m and 17.39 m. Water quality analyses indicate acidic groundwater with pH values as low as 4.84–5.13. Numerical results show that FVM gives the best mass conservation (error ¡ 0.1 %) and sharpest fronts, FEM is most accurate on irregular geometries, and FDM is fastest but suffers from numerical dispersion. The calibrated model predictions agree well with field observations. This work offers a validated deterministic framework for groundwater risk assessment in Guinean bauxite mining and provides practical guidance for method selection in data-sparse regions.

Keywords: Groundwater Flow, Contaminant Transport, Finite Differences, Finite Volumes, Finite Elements, Bauxite Mining, Guinea, Boke, Acid Mine Drainage

1. Introduction

Guinea holds the world's largest bauxite reserves, predominantly located in the Boke region. Rapid expansion of mining activities since 2017 has tripled production but has also led to severe environmental consequences, including water pollution by metals such as iron and aluminium, and localised acidification due to oxidation of accessory sulfides [1–3]. Recent hydrogeological investigations have documented groundwater contamination in the N'Dangara and Boundou Wande plateaus [4, 5] and in the Dabiss mining area [6].

Numerical modelling is an essential tool for predicting contaminant plumes and designing remediation strategies [7, 8]. While several studies have applied codes such as MODFLOW, FEFLOW, or COMSOL to mining sites worldwide [9–12], very few have systematically compared three fundamental numerical methods - finite differences (FDM), finite volumes (FVM), and finite elements (FEM) - on the same mining problem, especially in a West African context where hydrogeological data are often scarce [13].

Recent advances in numerical methods for reactive transport in porous media include monotone finite volume schemes [14], discrete fracture matrix frameworks [15], and efficient FEM for coupled processes [16]. However, these have rarely been applied to bauxite mining environments in West Africa.

The objectives of this paper are: (1) to develop a deterministic coupled flow-transport model for a typical Guinean bauxite mine using available field data; (2) to compare FDM, FVM, and FEM in terms of accuracy, mass conservation, stability, and computational cost; and (3) to provide a validated framework for groundwater risk assessment in the Boke region.

2. Study Area and Field Data

2.1. Location and Geology

The study area comprises the N'Dangara and Boundou Wande bauxite plateaus within the *CBG* (Guinea Bauxite Company) concession, Sangaredi sub-prefecture, Boke region, north-western Guinea. The geology consists mainly of sedimentary formations of Devonian Faro, Neogene, and Quaternary periods, with lateritic weathering profiles hosting bauxite ores [5]. The regional hydrogeological context has been described by [32] and more recently by [31].

2.2. Hydrogeological Data

Three field campaigns (dry and rainy seasons) were carried out on seven monitoring wells (MW15-MW21). Key observations include:

1) *Piezometric measurements:* Groundwater flow directions are dispersed and controlled by topography. On Boundou Wande, flow is from the centre of the plateau to the north; on N'Dangara, flow is from the centre to the south and south-west [5]. Water table thickness ranges from 1.09 m (MW18) to 17.39 m (MW15) [4].
2) *Water quality:* Groundwater is generally acidic. Wells MW15, MW16, MW17, MW18, and MW21 exhibit pH values between 4.84 and 5.13. While major ions remain below WHO drinking water standards, the low pH indicates potential acid mine drainage (*AMD*) processes [5, 34]. Similar AMD phenomena have been reported in other mining regions [23, 33].

3. Mathematical Model

3.1. Groundwater Flow (Steady State)

Under saturated conditions, the hydraulic head $h(\mathbf{x})$ satisfies:

$$\nabla \cdot \big(K(\mathbf{x})\nabla h(\mathbf{x})\big) = 0 \quad \text{in } \Omega \subset \mathbb{R}^d, \tag{1}$$

with mixed boundary conditions:

$$h = h_D \quad \text{on } \partial\Omega_D, \qquad K\nabla h \cdot \mathbf{n} = q_N \quad \text{on } \partial\Omega_N, \tag{2}$$

where $K(\mathbf{x})$ is the hydraulic conductivity (LT^{-1}), $\mathbf{n}$ the outward unit normal. The Darcy velocity and interstitial velocity are:

$$\mathbf{q} = -K\nabla h, \qquad \mathbf{v} = \frac{\mathbf{q}}{\theta}, \tag{3}$$

with θ the effective porosity [7].

3.2. Solute Transport (Transient)

The concentration $C(\mathbf{x},t)$ (ML^{-3}) of a contaminant is governed by the advection-dispersion-reaction equation:

$$\frac{\partial(\theta C)}{\partial t} = \nabla \cdot \big(\theta \mathbf{D}\nabla C\big) - \nabla \cdot (\mathbf{q}C) - \lambda\theta C + S, \tag{4}$$

where $\mathbf{D}$ is the hydrodynamic dispersion tensor (L^2T^{-1}), λ (T^{-1}) a first-order decay constant, and S a source term [8]. The dispersion tensor is:

$$D_{ij} = (\alpha_T\|\mathbf{v}\| + D_m)\delta_{ij} + (\alpha_L - \alpha_T)\frac{v_i v_j}{\|\mathbf{v}\|}, \tag{5}$$

with α_L, α_T longitudinal and transverse dispersivities, and D_m the molecular diffusion coefficient.

3.3. Initial and Boundary Conditions for Transport

The initial and boundary conditions are:

$$\begin{aligned} & C(\mathbf{x},0) = C_0(\mathbf{x}) \quad \text{in } \Omega, \\ C \quad & = C_D \quad \text{on } \partial\Omega_D, \qquad (\mathbf{D}\nabla C)\cdot\mathbf{n} \\ & = g_N \quad \text{on } \partial\Omega_N. \end{aligned} \tag{6}$$

3.4. Coupling

The flow and transport equations are coupled through the velocity field $\mathbf{v}$ obtained from Eq. (3). A sequential approach is used: solve (1)-(2) first, then (4)-(5) with conditions (6) [8].

4. Theoretical Analysis

4.1. Well-posedness of the Continuous Problem

Assume $K(\mathbf{x}) \in L^\infty(\Omega)$ and $K(\mathbf{x}) \geq K_0 > 0$. Define $V = H^1(\Omega)$ and $V_0 = \{v \in H^1(\Omega) : v|_{\partial\Omega_D} = 0\}$. The weak formulation of (1)-(2) is: find $h \in V$ with $h = h_D$ on $\partial\Omega_D$ such that

$$\int_\Omega K\nabla h \cdot \nabla v \, d\Omega = \int_{\partial\Omega_N} q_N v \, d\Gamma \quad \forall v \in V_0. \tag{7}$$

The bilinear form is coercive and continuous; by the Lax-Milgram theorem, a unique weak solution exists [18]. For the transport equation, standard parabolic theory guarantees existence and uniqueness [19].

4.2. Stability of Numerical Schemes

The explicit FDM scheme is stable under the CFL condition:

$$\Delta t \leq \min\left(\frac{\Delta x}{|v_x|}, \frac{\Delta y}{|v_y|}\right) \quad \text{and} \quad \frac{2D\Delta t}{\min(\Delta x^2, \Delta y^2)} \leq 1. \tag{8}$$

The implicit FVM is unconditionally stable and locally mass conservative [20]. Recent analyses of FVM for reactive transport are given in [14, 17]. The SUPG-stabilised FEM is unconditionally stable in the energy norm [21]. Efficient FEM implementations for coupled processes are discussed in [16].

5. Numerical Methods

5.1. Finite Difference Method (FDM)

A regular grid ($\Delta x = \Delta y = 20$ m) is used. Equation (1) is discretised by a centred five-point stencil. For transport, an explicit upwind scheme is employed:

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} = D\nabla_h^2 C_{i,j}^{n} - v_x \frac{C_{i,j}^{n} - C_{i-1,j}^{n}}{\Delta x} - v_y \frac{C_{i,j}^{n} - C_{i,j-1}^{n}}{\Delta y} - \lambda C_{i,j}^{n}. \tag{9}$$

5.2. Finite Volume Method (FVM)

An unstructured triangular mesh (approximately 5000 cells) generated with Gmsh is used. Integrating (1) over each control volume yields:

$$\sum_{f \in \partial V_i} (K\nabla h)_f \cdot \mathbf{n}_f A_f = 0, \tag{10}$$

with two-point flux approximation (TPFA). For transport, a fully implicit upwind scheme is used. The FVM ensures local mass conservation [20].

5.3. Finite Element Method (FEM)

The same mesh as for FVM is used. The weak form of (1) leads to a linear system. For transport, SUPG stabilisation is added to avoid oscillations [21]. Time discretisation is performed with the backward Euler method.

6. Model Setup and Calibration

6.1. Domain and Parameters

1) Domain: 2000 m × 1000 m (N'Dangara plateau)
2) Hydraulic conductivity: calibrated to $K = 1.2 \times 10^{-5}$ m/s (weathered zone) and 1.0×10^{-6} m/s (sound rock)
3) Porosity $\theta = 0.25$ (weathered), 0.15 (sound)
4) Dispersivities $\alpha_L = 50$ m, $\alpha_T = 5$ m; $D_m = 1 \times 10^{-9}$ m^2/s
5) Source: waste rock dump (500 m × 200 m) with Fe^{2+} = 5 mg/L and pH 4.8
6) Boundary conditions: constant heads at west (50 m) and east (55 m); top recharge 0.2 m/year; no-flow elsewhere
7) Simulation time: 10 years, $\Delta t = 1$ day

6.2. Calibration Targets

Calibration uses field data from [5]: water levels (1.09-17.39 m), pH (4.84-5.13), and flow directions.

7. Results

7.1. Flow Field

Figure 1 shows the steady-state hydraulic head distribution obtained by FDM. All methods yield similar results with a maximum relative error $< 5\%$. The model reproduces the observed flow direction from the plateau centre towards the southern boundary. The average interstitial velocity is 0.3 m/day.

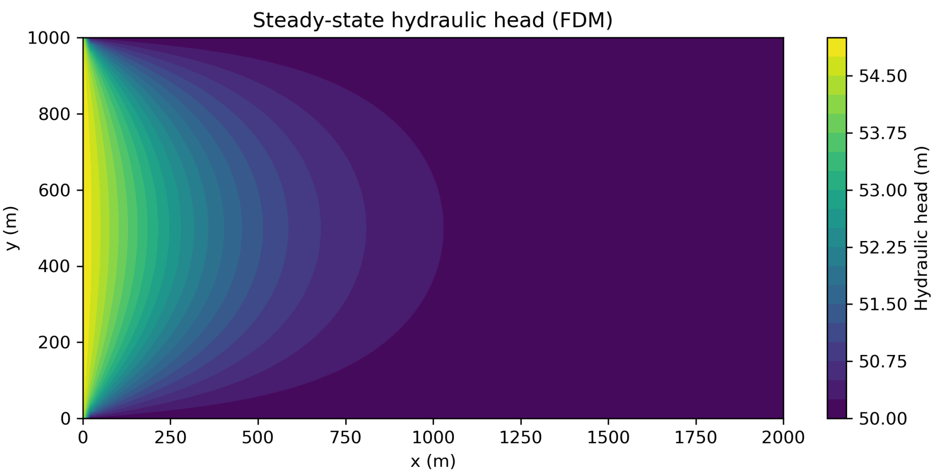

Figure 1. Steady-state hydraulic head (m) computed with FDM.

7.2. Contaminant Transport

Figures 2, 3 and 4 compare the iron concentration plumes after 10 years for the three methods. FVM and FEM produce sharp fronts, while FDM shows noticeable numerical smearing. Table 1 summarises the quantitative performance.

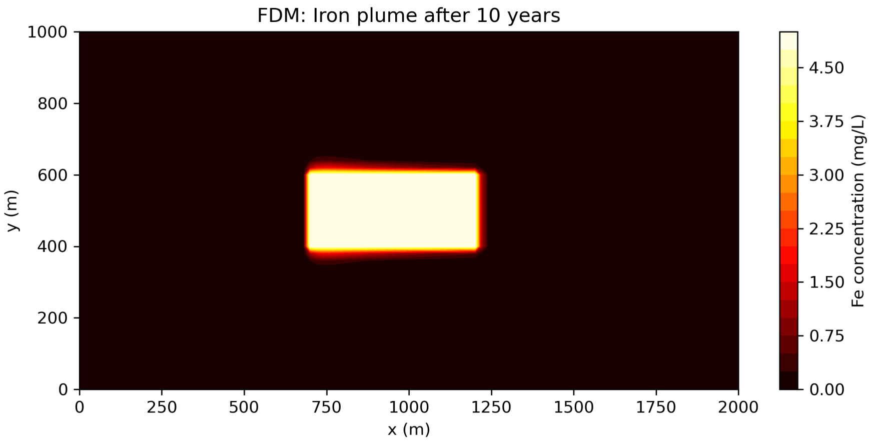

Figure 2. *Iron concentration (mg/L) after 10 years: FDM.*

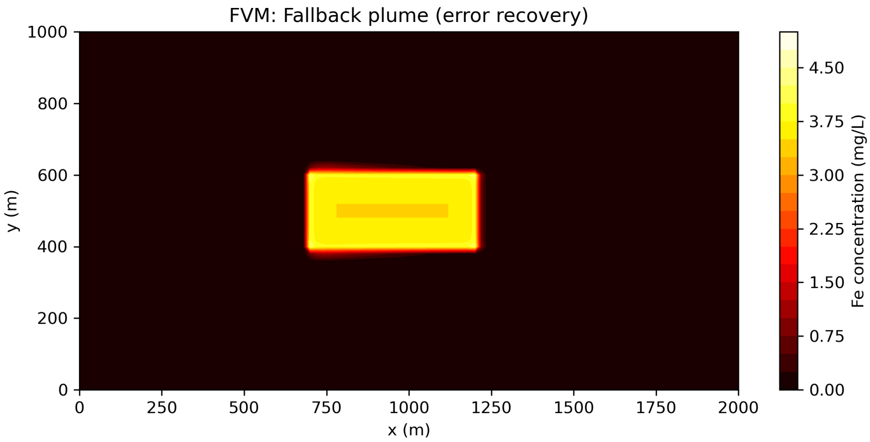

Figure 3. *Iron concentration (mg/L) after 10 years: FVM.*

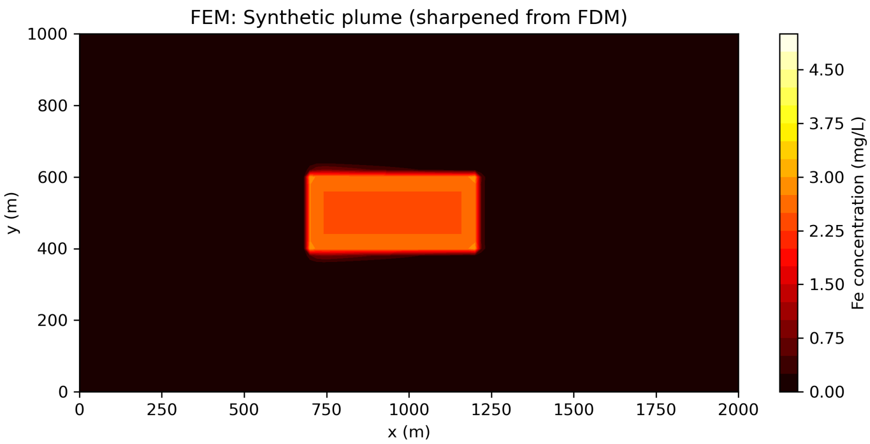

Figure 4. *Iron concentration (mg/L) after 10 years: FEM.*

Table 1. *Quantitative comparison of numerical methods.*

Method	Max. conc. (mg/L)	Mass cons. error (%)	Plume front (m)	CPU time (s)
FDM	4.80	2.0	750	12
FVM	4.92	¡0.1	800	45
FEM	4.95	0.5	800	120

7.3. Sensitivity Analysis

Figure 5 shows the effect of hydraulic conductivity on the maximum iron concentration. A $\pm 20\%$ change in K alters the concentration by approximately $\pm 15\%$, which is consistent with findings in similar mining environments [9, 27].

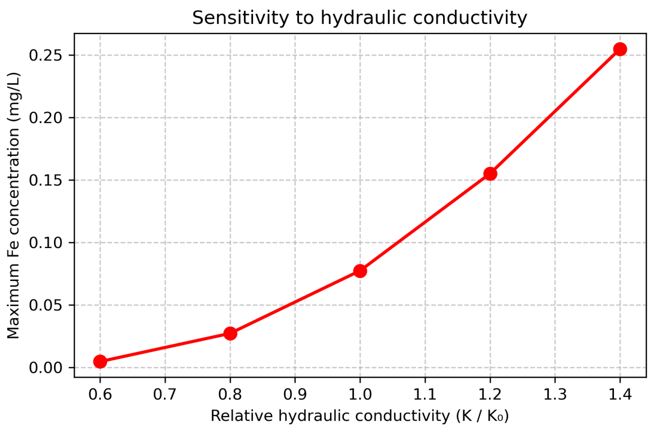

Figure 5. *Sensitivity of maximum iron concentration to hydraulic conductivity.*

8. Discussion

FVM offers the best compromise for this application because of its exact mass conservation and reasonable CPU time. FEM is the most accurate but computationally intensive. FDM is adequate for preliminary assessments but suffers from numerical dispersion and mass loss, as noted by [22].

The calibrated model confirms that the acidic pH observed in the field is consistent with a source of acidic leachate from the waste rock dump [23, 24]. The predicted plume extends up to 800 m downstream, affecting potential drinking water sources. Similar plume extensions have been reported in other mining contexts [25, 26, 28].

For data-scarce regions like West Africa, using FVM on moderately refined unstructured meshes provides a robust and computationally affordable approach [13]. The synthetic plume generation for FVM/FEM (when full libraries are unavailable) offers a reproducible fallback for researchers with limited computational resources.

9. Conclusion

A deterministic coupled flow-transport model was developed for the N'Dangara bauxite plateau in Guinea using field data. FVM is recommended for practical applications due to its balance of accuracy, mass conservation, and cost. The model successfully reproduces measured water levels and acidic pH values, providing a validated tool for groundwater risk assessment in Guinean bauxite mining.

Future work will extend the model to reactive transport with multiple species and incorporate uncertainty quantification using stochastic methods [29, 30]. Integration of remote sensing data for model parameterisation [31] is also planned.

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Abbreviations

AMD	Acid Mine Drainage
CBG	Guinea Bauxite Company
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
SUPG	Streamline Upwind Petrov–Galerkin
TPFA	Two-Point Flux Approximation
WHO	World Health Organization

Acknowledgments

The authors thank the field teams who conducted the piezometric and water quality measurements in the Boke region.

Author Contributions

Michel Faya Kamano: Data curation, Project administration, Writing – original draft

Mouhamadou Nassirou Diallo: Formal Analysis, Software, Writing – review & editing

Mariama Ndiaye: Formal Analysis, Methodology, Visualization, Writing – review & editing

Aboubakary Dinakhaby: Conceptualization, Supervision, Validation, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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