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corrigendum

Corrigendum to: A Ford-Based Branch-and-Bound with Minimum Cut Lower Bounds for the Resource-constrained Project Scheduling Problem

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Nature of correction. Substantive. The original Lemmas 4.1–4.4, Lemmas 5.2–5.3, Theorems 5.2–5.4, the dominance rule, the branching rule, and several abstract and computational claims are withdrawn as stated and replaced below. The corrected results are valid only under the additional hypotheses (N), (D), and (B) stated in Sections 2–4.

1. Scope of the Corrigendum

This corrigendum corrects the following categories of error in the original article:

1) the minimum-cut network of Section 4.2 has infinite-capacity source-sink paths, so the minimum cut is not the claimed finite quantity;
2) the min-cut value Φ_k is a resource quantity, while $\sum_i r_{ik} d_i / R_k$ is a time quantity; the inequality stated in Lemma 4.2 is dimensionally invalid and false;
3) the lower bound $\mathrm{LB}_{\mathrm{cut}} = \max_k \Phi_k$ + current_time is not a time lower bound;
4) the dominance condition of Definition 5.1 is incompatible with the earlier-start condition and does not support the copying argument;
5) the single-eligible-activity branching rule of Section 5.3 is not exhaustive.

The corrected theory below is valid under the additional hypotheses (N), (D), and (B), stated where they are first needed.

2. Withdrawal of the Original min-cut Statements

2.1. Withdrawn Items

The following items of the original article are withdrawn in their entirety: the time-expanded network construction of Section 4.2 with infinite-capacity arcs (τ_j, τ_{j+1}), (s, τ_0), and (τ_T, t); Lemma 4.1 (Feasibility of the Network); the inequality of Lemma 4.2,

$$\Phi_k \geq \left\lceil \frac{\sum_{i \in \mathcal{U}} r_{ik} d_i}{R_k} \right\rceil;$$

Lemma 4.4 defining

$$\mathrm{LB}_{\mathrm{cut}}(S) = \max_k \Phi_k + \text{current_time};$$

and Theorem 5.4 and the claim that $\mathrm{LB}_{\mathrm{cut}}$ dominates $\mathrm{LB}_{\mathrm{workload}}$.

Remark 2.1. The reason for withdrawal is twofold. First, because the arcs listed in the original Section 4.2 all have capacity $+\infty$, the network contains a source-sink path of infinite capacity; the minimum source-sink cut is therefore not a meaningful finite bound on the required resource capacity. Second, even

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if the network were finite, a cut capacity would measure a resource amount, whereas the workload bound measures time; the inequality stated in Lemma 4.2 is not dimensionally consistent.

2.2. Additional Hypothesis (N): Finite Admissible Network

Hypothesis 1 (N). For each resource k and each candidate remaining horizon $H = T - t_0$, let $N_k(H)$ be a finite time-expanded network with source s, sink t, and integral capacities, satisfying:

(N1) $N_k(H)$ has no s-t path consisting only of infinite-capacity arcs;

(N2) for every feasible completion of the remaining activities within horizon H, there exists an s-t cut in $N_k(H)$ of capacity at most $R_k H$.

An admissible explicit choice of $N_k(H)$ is the finite time-expanded relaxation of Möhring et al., used under its stated hypotheses. The original infinite-capacity construction is withdrawn. Let $\Phi_k(H)$ denote the value of a minimum s-t cut in $N_k(H)$.

2.3. Replacement for Lemmas 4.1–4.4

Lemma 2.2 (Necessary cut condition). *Suppose Hypothesis (N) holds. If the remaining activities can be completed within horizon H, then for every resource k,*

$$\Phi_k(H) \le R_k H.$$

Proof. By hypothesis (N2), any feasible completion admits an s-t cut of capacity at most $R_k H$. The minimum cut value is no larger. □

Lemma 2.3 (Cut-based time lower bound). *Define*

$$\mathrm{LB}_{\mathrm{cut}}(S) = t_0 + \min\{H \in \mathbb{Z}_{\ge 0} : \Phi_k(H) \le R_k H \text{ for all } k\}.$$

If the set in the minimum is empty, set $\mathrm{LB}_{\mathrm{cut}}(S) = +\infty$. *Then, for every feasible completion s of the partial schedule S,*

$$C_{\max}(s) \ge \mathrm{LB}_{\mathrm{cut}}(S).$$

Proof. Let $H_s = C_{\max}(s) - t_0$. By Lemma 2.2, $\Phi_k(H_s) \le R_k H_s$ for all k, so H_s belongs to the feasible set, and the minimum over that set is at most H_s. □

Lemma 2.4 (Integrality). *If all capacities in $N_k(H)$ are integers, then $\Phi_k(H)$ is an integer and can be computed by an integral max-flow algorithm, e.g. Dinic's algorithm.*

Withdrawn statement. Lemma 4.4 of the original article, defining

$$\mathrm{LB}_{\mathrm{cut}}(S) = \max_k \Phi_k + \mathrm{current_time},$$

is withdrawn. The correct time lower bound is $\mathrm{LB}_{\mathrm{cut}}$ of Lemma 2.3

2.4. Workload Bound and Corrected Comparison

The resource workload bound is retained as a separate bound:

$$\mathrm{LB}_{\mathrm{workload}}(S) = t_0 + \max_k \left\lceil \frac{W_k}{R_k} \right\rceil, \qquad W_k = \sum_{i \in \mathcal{U}} r_{ik} d_i,$$

where $\lceil \cdot \rceil$ denotes the ceiling function. No general inequality

$$\mathrm{LB}_{\mathrm{cut}} \ge \mathrm{LB}_{\mathrm{workload}}$$

is asserted. The following conditional replacement for Theorem 5.4 is valid.

Theorem 2.5 (Conditional comparison). *Suppose Hypothesis (N) holds. If for every resource k and every*

$$H < \left\lceil \frac{W_k}{R_k} \right\rceil$$

one has

$$\Phi_k(H) > R_k H,$$

then

$$\mathrm{LB}_{\mathrm{cut}}(S) \ge \mathrm{LB}_{\mathrm{workload}}(S).$$

Proof. Fix k. Let $H_k = \lceil W_k / R_k \rceil$. Suppose for a contradiction that

$$\mathrm{LB}_{\mathrm{cut}}(S) < +H_k.$$

Then the minimizing H in Lemma 2.3 satisfies $H < H_k$, and by the hypothesis $\Phi_k(H) > R_k H$. But the definition of the feasible set in Lemma 2.3 requires $\Phi_k(H) \le R_k H$ for all k, a contradiction. Hence

$$\mathrm{LB}_{\mathrm{cut}}(S) \ge t_0 + H_k,$$

and the conclusion follows by taking the maximum over k. □

Withdrawn statement. The unconditional Theorem 5.4 and its strictness claim are withdrawn. Under the hypotheses of Theorem 2.5 the comparison holds; no domination is asserted otherwise.

3. Replacement for the Dominance Rule

Withdrawn statement. Definition 5.1 and Lemma 5.2 of the original article are withdrawn. The cumulative-consumption condition is incompatible with shifting activities earlier, and the copying argument does not apply to the original condition.

Additional Hypothesis (D): Pointwise Dominance

Hypothesis 2 (D). For a partial schedule P and a decision time t_0, the resource profile of P after t_0 is the function

$$t \mapsto \sum_{i \in S(P)} \rho_i^k(t), \qquad t \geq t_0,$$

where

$$\rho_i^k(t) = \begin{cases} r_{ik}, & s_i(P) \leq t < s_i(P) + d_i, \\ 0, & \text{otherwise.} \end{cases}$$

Dominance is defined as in Definition 3.1 below.

Definition 3.1 (Dominance). Let P and Q be two partial schedules with a common decision time t_0. Partial schedule P dominates partial schedule Q if:

(D1) $S(P) = S(Q)$;

(D2) $s_i(P) \leq s_i(Q)$ for every $i \in S(P)$;

(D3) for every resource k and every $t \geq t_0$,

$$\sum_{i \in S(P)} \rho_i^k(t) \leq \sum_{i \in S(Q)} \rho_i^k(t).$$

Condition (D3) is a pointwise future resource-profile condition, not a cumulative condition.

Lemma 3.2 (Completeness of dominance). *Suppose Hypothesis (D) holds and P dominates Q. Then for every feasible completion s_Q of Q there is a feasible completion s_P of P with*

$$C_{\max}(s_P) \leq C_{\max}(s_Q).$$

Consequently, Q may be pruned if a dominating P has already been explored.

Proof. Let $\mathcal{U}$ be the common unscheduled set. Define s_P on $\mathcal{U}$ exactly as s_Q.

Precedence. Every scheduled predecessor i finishes in P no later than in Q, by condition (D2); hence it does not delay any successor in $\mathcal{U}$.

Resources. At every $t \geq t_0$ the scheduled contribution in P is pointwise no larger than in Q by condition (D3), and the unscheduled contribution is identical. Thus the capacity limit is respected.

Makespan. The unscheduled start times are copied unchanged; the makespan does not increase. □

4. Replacement for Branching and Completeness

Withdrawn statement. The single-eligible-activity branching rule of Section 5.3 is withdrawn: it is not exhaustive, since an optimal schedule may start an eligible activity other than the one with smallest ES, or may start several activities at the same decision time.

4.1. Additional Hypothesis (B): Exhaustive Branching

Hypothesis 3 (B). At decision time t, let E_t be the set of not-yet-scheduled activities whose predecessors are completed by t and which can be started at t without violating precedence or resource capacity. The algorithm branches over every nonempty feasible subset $A \subseteq E_t$ such that starting all activities in A simultaneously at t respects precedence and resource capacities. After scheduling A at t, it advances to the next event time determined by activity completions. The empty choice $A = \varnothing$ is also allowed and advances t to the next event time determined by activity completions.

4.2. Replacement for Lemma 5.3 and Theorem 5.2

Lemma 4.1 (Exhaustive branching). *Suppose Hypothesis (B) holds. Let S be a partial schedule at decision time t. For any feasible completion s of S, the set*

$$A_s = \{i \in E_t : s_i = t\}$$

is feasible and is enumerated by the branching rule. If $A_s = \varnothing$, the algorithm advances t without scheduling, which is also enumerated.

Proof. Since s is feasible, every $i \in A_s$ respects precedence and resource constraints at time t when started simultaneously; hence A_s is a feasible subset and is one of the subsets enumerated by Hypothesis (B). □

Theorem 4.2 (Completeness). *Suppose Hypothesis (B) holds. The search tree contains at least one leaf corresponding to an optimal schedule.*

Proof. Follow an optimal schedule s^*. At each decision time, the subset of activities started at that time is feasible and is enumerated by Lemma 4.1. Inductively, the algorithm follows a path to s^*. □

Theorem 4.3 (Termination). *The corrected algorithm terminates after a finite number of steps.*

Proof. The search tree has finite branching factor, since at each decision time it is bounded by the number of nonempty feasible subsets of E_t, which is finite. Along any path, each recursive call either schedules at least one activity or advances to a later event time. There are at most n scheduling calls and at most a finite number of event advances determined by the horizon. Hence every path has finite depth, and the total number of nodes is finite. □

Theorem 4.4 (Main correctness theorem). *Suppose Hypotheses (N), (D), and (B) hold, and that the lower bound used in pruning is* LB *of Lemma 4.5. Then the algorithm terminates and returns an optimal schedule for the RCPSP.*

Proof. Termination is Theorem 4.3. Completeness is Theorem 4.2: the search tree contains a leaf corresponding to an optimal schedule. The three pruning rules discard only nodes that cannot lead to a strictly better solution:

1) *lower-bound pruning:* if $\mathrm{LB} \geq UB$, no descendant can improve the incumbent, by admissibility of LB (Lemma 4.5);
2) *dominance pruning:* by Lemma 3.2, the discarded node cannot yield a strictly better solution than an explored dominating node;
3) *resource infeasibility:* if no feasible start time exists, the node has no feasible completions.

None of these discards the node on the optimal path.

Optimality. Whenever a complete schedule is found, its makespan C is compared to the incumbent upper bound UB. Initially UB is any feasible upper bound (e.g. the sum of all durations). On termination, UB equals the minimum makespan over all explored complete schedules; by completeness, this equals OPT. □

4.3. Combined Lower Bound (Restated)

Lemma 4.5 (Combined lower bound). *Define*

$$\mathrm{LB}_{\mathrm{critical}}(S) = t_0 + \ell_{\mathrm{rem}}(S),$$
$$\mathrm{LB}(S) = \max\bigl(\mathrm{LB}_{\mathrm{critical}}(S), \mathrm{LB}_{\mathrm{cut}}(S)\bigr).$$

where $\ell_{\mathrm{rem}}(S)$ is the longest path length in the remaining precedence subgraph and $\mathrm{LB}_{\mathrm{cut}}$ *is as in Lemma 2.3. Then for every feasible completion s of S,*

$$C_{\max}(s) \geq \mathrm{LB}(S),$$

so $\mathrm{LB}(S)$ *is admissible.*

Proof. The critical-path term is a valid precedence lower bound; the cut term is valid by Lemma 2.3. The maximum of admissible lower bounds is admissible. □

5. Corrections to Abstract, Introduction, and Data Statement

Abstract

Replace the sentence

> "We provide complete rigorous proofs of correctness, including lemmas establishing admissibility of the Ford bound, the min-cut bound, dominance, termination, and optimality."

by

> "Under the finite-network hypothesis (N), the pointwise dominance condition (D), and the exhaustive branching condition (B), we provide proofs of admissibility, dominance, termination, and optimality."

Replace the sentence

> "This bound dominates the simple resource workload bound and significantly reduces the search tree size."

by

> "Under the additional condition of Theorem 2.5, the corrected cut bound can dominate the simple resource workload bound; this domination is not asserted unconditionally."

Computational Claims

The computational claims of Section 6 must be restated. If the corrected algorithm is used, all numerical entries in Tables 1–3 must be recomputed.

6. Summary of Errata

Original statement	Status	Replacement
Section 4.2 infinite-capacity network	withdrawn	finite network $N_k(H)$ under Hypothesis (N)
Lemma 4.1	withdrawn	Lemma 2.2
Lemma 4.2 and inequality $\Phi_k \geq \lceil W_k/R_k \rceil$	withdrawn	Lemma 2.3; no unconditional workload domination
Lemma 4.3	retained in corrected form	Lemma 2.4 (integrality under finite integral capacities)
Lemma 4.4, $\mathrm{LB}_{\mathrm{cut}} = \max_k \Phi_k + \mathrm{current_time}$	withdrawn	$\mathrm{LB}_{\mathrm{cut}} = t_0 + \min\{H : \Phi_k(H) \leq R_k H\}$
Theorem 5.4	withdrawn	Theorem 2.5 (conditional comparison)
Definition 5.1, cumulative dominance	withdrawn	Definition 3.1 (pointwise future profile)
Lemma 5.2	withdrawn	Lemma 3.2
Section 5.3 single-activity branching	withdrawn	chronological subset branching, Hypothesis (B)
Lemma 5.3, Theorem 5.2	withdrawn	Lemma 4.1, Theorem 4.2