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Research Article

On Fuzzy Baire-Separated Spaces and Related Concepts

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Abstract

In this paper, a new class of fuzzy topological spaces, namely fuzzy Baire-separated spaces is introduced in terms of fuzzy Baire sets. Several characterizations of fuzzy Baire-separated spaces are established. It is shown that fuzzy Baire sets lie between disjoint fuzzy P-sets and fuzzy F_σ- sets in a fuzzy Baire-separated space. Conditions under which fuzzy topological spaces become fuzzy Baire-separated spaces are established. Fuzzy nowhere dense sets are fuzzy closed sets in fuzzy nodec spaces and subsequently a question will arise. Which fuzzy topological spaces [other than fuzzy hyperconnected spaces, fuzzy globally disconnected spaces] have fuzzy closed sets with fuzzy nowhere denseness? For this, fuzzy topological spaces having fuzzy closed sets with fuzzy nowhere denseness are identified in this paper. It is verified that fuzzy ultraconnected spaces are non fuzzy Baire -separated spaces. The means, by which fuzzy weakly Baire space become fuzzy Baire -separated spaces and in turn fuzzy Baire - separated spaces become fuzzy seminormal spaces, are obtained. There are scope in this paper for exploring the inter-relations between fuzzy Baire spaces and Baire -separated spaces.

Keywords

Fuzzy Nowhere Dense Set, Fuzzy Residual Set, Fuzzy Baire Set, Fuzzy Weakly Baire Space, Fuzzy Strongly Baire Space, Fuzzy Nodec Space, Fuzzy Seminormal Space, Fuzzy Globally Disconnected Space

1. Introduction

Any application of mathematical notions depends firmly how one introduces basic ideas that may yield various theories in various directions. If the basic idea is appropriately introduced, then not only the existing theories stand but also the possibility of emerging new theories increases. On these lines, the notion of fuzzy sets as a new approach for modelling uncertainties was introduced by L. A. Zadeh [26] in 1965.The concept of fuzzy topological space was introduced by C. L. Chang [5] in 1968. Based on this concept, many studies have been conducted in general theoretical areas and in various application sides.

The notion of Baire sets in classical topology was introduced and studied by Andrzej Szymanski [2].The concept of fuzzy Baire sets in fuzzy topological spaces was introduced and studied by G.Thangaraj and R.Palani [14] in terms of fuzzy open sets and fuzzy residual sets and further studied in [18]. In classical topology, W. Adamski [1] introduced the concept of Baire-separation in topological spaces. Motivated on these lines, the notion of fuzzy Baire - separation in fuzzy topological spaces is introduced in tems of fuzzy Baire sets. Conditions under which fuzzy topological spaces become fuzzy Baire-separated spaces, are obtained. The fuzzy topo-

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logical spaces having fuzzy closed sets with fuzzy nowhere denseness are identified. It is found that fuzzy ultraconnected spaces are non fuzzy Baire -separated spaces. Condition under which a fuzzy weakly Baire space becomes a fuzzy Baire-separated space is obtained. Also the means by which a fuzzy Baire -separated space becomes a fuzzy semi normal space is also obtained.

2. Preliminaries

Several basic notions and results used in the sequel, are given for making the exposition self - contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to C. L. Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$ for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [5]: A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions:

(a). $0_X \in T$ and $1_X \in T$,

(b). If A, B $\in$ T, then $A \wedge B \in T$,

(c). If $A_i \in T$ for each i $\in J$, then $\vee_i A_i \in T$.

T is called a fuzzy topology for X and the pair (X,T) is a fuzzy topological space, or fts for short. Members of T are called fuzzy open sets of X and their complements, are called fuzzy closed sets in X.

Definition 2.2 [5]: Let (X,T) be a fuzzy topological space and λ be any fuzzy set defined on X. The fuzzy interior, the fuzzy closure and the complement of λ are defined respectively as follows:

(i). $\text{int}(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$;

(ii). $\text{cl}(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1-\mu \in T\}$;

(iii). $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X,T), the ion $\psi = \vee_i (\lambda_i)$ and the intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as

(iv). $\psi(x) = \sup_{i \in I}\{\lambda_i(x) / x \in X\}$;

(v). $\delta(x) = \inf_{i \in I}\{\lambda_i(x) / x \in X\}$.

Lemma 2.1 [3]: For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \text{int}(\lambda) = \text{cl}(1-\lambda)$ and (ii). $1 - \text{cl}(\lambda) = \text{int}(1-\lambda)$.

Definition 2.3: A fuzzy set λ in a fuzzy topological space (X,T) is called a

(i). fuzzy G_δ-set in (X,T if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$; fuzzy F_σ-set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$ [4].

(ii). fuzzy regular-open set in (X,T) if $\lambda = \text{int cl}(\lambda)$; fuzzy regular-closed set in (X,T) if $\lambda = \text{cl int}(\lambda)$ [3].

(iii). fuzzy semi – open set in (X,T) if $\lambda \leq \text{cl int}(\lambda)$; fuzzy semi – closed set in (X,T) if $\text{int cl}(\lambda) \leq \lambda$ [3].

(iv). fuzzy dense set in (X,T) if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in (X,T) [9].

(v). fuzzy nowhere dense set in (X,T) if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) = 0$, in (X,T) [9].

(vi). fuzzy first category set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category [9].

(vii). fuzzy residual set in (X,T) if $1-\lambda$ is a fuzzy first category set in (X,T) [10].

(viii). fuzzy Baire set in (X,T) if $\lambda = \mu \wedge \delta$, where μ is a fuzzy open set and δ is a fuzzy residual set in (X,T) [14].

(ix). fuzzy σ-nowhere dense set in (X,T) if λ is a fuzzy F_σ-set in (X,T) such that $\text{int}(\lambda) = 0$ [12].

(x). fuzzy σ-boundary set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $\mu_i = \text{cl}(\lambda_i) \wedge (1-\lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X,T) [15].

(xi). fuzzy strongly nowhere dense set in (X,T) if $\lambda \wedge (1-\lambda)$ is a fuzzy nowhere dense set in (X,T) [16].

Definition 2.4 [22]: Let (X,T) be a fuzzy topological space. A fuzzy closed set λ in X is called a fuzzy P-set if $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ- set in (X,T) implies that $\lambda \leq 1 - \text{cl}(\mu)$ in X.

Definition 2.5: A fuzzy topological space (X,T) is called a

(i). fuzzy submaximal space if for each fuzzy set λ in X, such that $\text{cl}(\lambda) = 1$, $\lambda \in T$ [4].

(ii). fuzzy hyperconnected space if every non- null fuzzy open subset of (X,T) is fuzzy dense in (X,T) [6].

(iii). fuzzy extraresolvable space if er λ_i and $\lambda_j (i \neq j)$ arefuzzy dense sets in X,then $\lambda_i \wedge \lambda_j$ is-afuzzynowheredense set in X [25].

(iv). fuzzy nodec space if each fuzzy nowhere dense set is a fuzzy closed set in X [11].

(v). fuzzy ultraconnected space if whenever λ and μ are two non-zero fuzzy closed setsin (X,T), $\lambda \nleq 1-\mu$, in (X,T) [23].

(vi). fuzzy globally disconnected space if each fuzzy semi-open set is fuzzy open in X [20].

(vii). fuzzy Baire space if $\text{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in X [10].

(viii). fuzzy weakly Baire space if $\text{int}(\vee_{i=1}^{\infty}(\mu_i)) = 0$, where $\mu_i = \text{cl}(\lambda_i) \wedge (1-\lambda_i)$ and (λ_i)'s are fuzzy regular open sets in X [15].

(ix). fuzzy strongly Baire space if $\text{cl}(\vee_{i=1}^{\infty}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy strongly nowhere dense sets in X [16].

(x). fuzzy perfectly disconnected space if for any two non – zero fuzzy sets λ and μ defined on X with $\lambda \leq 1-\mu$, $\text{cl}(\lambda) \leq 1 - \text{cl}(\mu)$ in X [19].

(xi). fuzzy F′-space if $\lambda \leq 1-\mu$, where λ and μ are fuzzy F_σ-sets in (X,T), then $\text{cl}(\lambda) \leq 1 - \text{cl}(\mu)$, in X [21].

(xii). weak fuzzy Oz-space if for each fuzzy F_σ-set δ in (X,T), cl (δ) is a fuzzy G_δ-set in (X,T) [24].

(xiii). fuzzy semi normal space if given a fuzzy closedset λand a fuzzy open set μ such that $\lambda \leq \mu$, then there exists a fuzzy regular open set σ such that $\lambda \leq \sigma \leq \mu$ [7].

Theorem 2.1 [17]: If λ is a fuzzy Baire set in a fuzzy globally disconnected space (X,T), then λ is a fuzzy G_δ-set in (X,T).

Theorem 2.2 [17]: If λ is a fuzzy Baire set in a fuzzy submaximal space (X,T), then λ is a fuzzy G_δ -set in (X,T).

Theorem 2.3 [25]: If λ is a fuzzy residual set in a fuzzy extraresolvable space (X, T),then int $(\lambda) = 0$, in X.

Theorem2.4 [13]: Ifλ is a fuzzy residual set in a fuzzy topological space (X,T), then there exists a fuzzy G_δ- set μ in(X,T) such that $\mu \leq \lambda$.

Theorem2.5[23]: If λ and μ are fuzzy closed sets in a fuzzy hyperconnected space(X,T), thenλ and μ are fuzzy nowhere dense sets in (X,T).

Theorem2.6 [22]: If a fuzzy set λ is a fuzzy P-set in a fuzzy topological space(X,T) such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ- set in (X,T),then there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1-\mathrm{int}(\mu)$.

Theorem 2.7 [14]: If $\lambda = \mu \wedge (1-\alpha)$, where $\mu \in T$ and α is a fuzzy σ-nowhere dense set in a fuzzy topological space (X,T), then λ is a fuzzy Baire set in (X,T).

Theorem2.8 [15]: If λ is a fuzzy σ-boundary set in a fuzzy weakly Baire space (X,T), then λ is a fuzzy σ-nowhere dense set in (X,T).

Theorem 2.9 [16]: If λ is a fuzzy residual set in a fuzzy strongly Baire space,then int$(\lambda) = 0$, in X.

Theorem 2.10 [19]: If a fts (X,T) is a fuzzy perfectly disconnected space and $\lambda \leq 1-\mu$, for any two non-zero fuzzy sets $\lambda$and μ defined on X, then cl$(\lambda) \neq 1$ and cl$(\mu) \neq 1$, in (X,T).

Theorem 2.11 [10]:If λ is a fuzzy first category set in a fuzzy topological space (X,T), then there is a fuzzy F_σ -set η in (X,T) such that $\lambda \leq \eta$.

Theorem 2.12 [8]: If a fuzzy topological space (X,T) is a fuzzy second category(but not fuzzy Baire) and fuzzy hyperconnected space and λ is a fuzzy residual set in (X,T), then

(i).λ is a fuzzy nowhere dense set in (X,T);

(ii).$(1-\lambda)$ is a fuzzy dense set in (X,T).

3. Baire – Separated Spaces

Definition 3.1: A fuzzy topological space (X,T) is called a fuzzy Baire-separated space if for each pair of fuzzy closed sets μ_1 and $\mu_2$in(X,T) such that$\mu_1 \leq 1-\mu_2$, there exists a fuzzy Baire set η in (X,T) such that $\mu_1 \leq \eta \leq 1-\mu_2$.

Example3.1:Let X = {a,b,c}. Let I = [0,1] and α, β and γ are the fuzzy sets defined on X as follows:

α:X→Iis defined by $\alpha(a) = 0.8$; $\alpha(b) = 0.7$; $\alpha(c) = 0.9$,

β:X→Iis defined by $\beta(a) = 0.6$; $\beta(b) = 0.9$; $\beta(c)= 0.8$,

γ:X→I is defined by $\gamma(a) = 0.7$; $\gamma(b) = 0.6$; $\gamma(c) = 0.7$.

Then, T={$0, \alpha, \beta, \gamma, \alpha\vee\beta, \beta\vee\gamma, \alpha\wedge\beta, \beta\wedge\gamma, \gamma\vee[\alpha\wedge\beta]$, 1} is a fuzzy topology on X. By computation, one can find that int cl $(1-\alpha) = 0$;int cl$(1-\beta) = 0$;int cl$(1-\gamma) = 0$; int cl$(1-[\alpha\vee\beta]) = 0$; int cl$(1-[\beta\vee\gamma]) = 0$; int cl $(1-[\alpha\wedge\beta]) = 0$;int cl $(1-[\beta\wedge\gamma]) = 0$ and

int cl $(1-[\gamma\vee(\alpha\wedge\beta)) = 0$. Thus $1-\alpha$, $1-\beta$, $1-\gamma$, $1-(\alpha\vee\beta)$, $1-(\beta\vee\gamma)$, $1-(\alpha\wedge\beta)$, $1-(\beta\wedge\gamma)$ and $1-(\gamma\vee[\alpha\wedge\beta])$ are fuzzy nowhere dense sets in (X,T). By computation, one can find that $1-(\beta\wedge\gamma) = (1-\alpha)\vee(1-\beta)\vee(1-\gamma)\vee(1-(\alpha\vee\beta))\vee(1-(\beta\vee\gamma))\vee(1-(\alpha\wedge\beta))$ and thus $1-(\beta\wedge\gamma)$ is a fuzzy first category set in (X,T). Then, $\beta\wedge\gamma$ is a fuzzy residual set in (X,T). For each fuzzy open setδ $(=\alpha, \beta, \gamma, \alpha\vee\beta, \beta\vee\gamma, \alpha\wedge\beta, \beta\wedge\gamma, \gamma\vee[\alpha\wedge\beta])$, $\delta\wedge[\beta\wedge\gamma] = \beta\wedge\gamma$is a fuzzy Baire set in(X,T). One can easily verify that for each pair of fuzzy closed sets μ_1 and μ_2{$= 1-\alpha$, $1-\beta$, $1-\gamma$,$1-(\alpha\vee\beta)$, $1-(\beta\vee\gamma)$, $1-(\alpha\wedge\beta)$, $1-(\beta\wedge\gamma)$, $1-(\gamma\vee[\alpha\wedge\beta])$} in (X,T) such that $\mu_1 \leq 1-\mu_2$, there exists a fuzzy Baire set $\beta\wedge\gamma$ in (X,T) such that$\mu_1 \leq \beta\wedge\gamma \leq 1-\mu_2$. Hence (X,T) is a fuzzy Baire – separated space.

Proposition 3.1: If λ is a fuzzy closed set and δ is a fuzzy open set in a fuzzy Baire - separated space (X,T) such that $\lambda \leq \delta$,then there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq \delta$.

Proof: Let λ be a fuzzy closed set and δbe a fuzzy open set in X, such that $\lambda \leq \delta$. Then, $\lambda \leq 1-(1-\delta)$, where λ and$(1-\delta)$are fuzzy closed sets in (X,T). Since (X,T) is a fuzzy Baire - separated space, there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq 1-(1-\delta)$ and it follows that $\lambda \leq \eta \leq \delta$, in (X,T).

Corollary 3.1: Ifλ is a fuzzy closed set and$\delta$is afuzzy opensetin a fuzzy Baire - separated space (X,T) such that $\lambda \leq \delta$, then there exists a fuzzy Bairesetη in (X,T)such that $\lambda \leq \eta \leq \delta$and$\eta$isnotafuzzydensesetinX.

Proof: By proposition 3.1, there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq \delta$, in (X,T). Now, cl$(\lambda) \leq$cl$(\eta) \leq$ cl(δ) and then, cl $(\eta) \leq$ cl(δ) and by Lemma 2.1,cl(δ) is a fuzzy regular closed set and thus cl$(\delta)$is a fuzzy closed set in (X,T). Thus, there is a fuzzy closed set cl(δ)in (X,T) such that cl $(\eta) \leq$ cl$(\delta) <1$. This implies that cl(η) is not a fuzzy dense set in (X,T). Nowcl (cl(η)) = cl(η) and cl (cl(η)) $\neq 1$, implies that cl$(\eta) \neq 1$ and thus η is not a fuzzy dense set in X.

Proposition 3.2: If $\lambda\vee\delta = 1$, where λ and δ are fuzzy open sets in a fuzzy Baire - separated space (X,T), then there exists a fuzzy Baire set η in (X,T) such that $1-\lambda \leq \eta \leq \delta$.

Proof: Suppose that $\lambda\vee\delta = 1$, whereλ and δ are fuzzy open setsin (X,T). Then, $1-(\lambda\vee\delta) = 0$. This implies that $(1-\lambda)\wedge(1-\delta) = 0$ and then $(1-\lambda) \leq 1-(1-\delta)$, where $(1-\lambda)$ and $(1-\delta)$are fuzzy closed sets in (X,T). Since (X,T) is a fuzzy Baire - separated space, there exists a fuzzy Baire set η in (X,T) such that $1-\lambda \leq \eta \leq 1-(1-\delta)$ and it follows that $1-\lambda \leq \eta \leq \delta$, in (X,T).

Proposition 3.3: If there exists a fuzzy Baire set η in a fuzzy topological space (X,T) such that $\delta_1 \leq \eta \leq 1-\delta_2$, for each pair of disjoint fuzzy F_σ-sets δ_1 and δ_2, then (X,T) is a fuzzy Baire - separated space.

Proof: Let δ_1 and $\delta_2$bea pair of disjoint fuzzyF_σ-sets in(X,T). Then, it follows that $\delta_1 \leq 1-\delta_2$ Since δ_1 and δ_2 are fuzzy F_σ -sets in (X,T), $\delta_1 = \bigvee_{i=1}^{\infty}(\alpha_i)$ and

$\delta_2 = \vee_{k=1}^{\infty}(\beta_k)$, where$(\alpha_i)$'sand$(\beta_k)$'sare fuzzy closedsets in (X,T). Now $\alpha_i \leq \delta_1$and $\beta_k \leq \delta_2$ and $\alpha_i \leq \delta_1 \leq 1-\delta_2 \leq 1-\beta_k$. By hypothesis, there exists a fuzzy Baire setηin X such that $\delta_1 \leq \eta \leq 1-\delta_2$. Thus, for a pair of fuzzy closed sets α_i and β_k in (X,T) such that $\alpha_i \leq 1-\beta_k$, there exists a fuzzy Baire set η in (X,T) such that $\alpha_i \leq \eta \leq 1-\beta_k$and hencethefts(X,T) is a fuzzy Baire – separated space.

Proposition 3.4: If λ is a fuzzy closed set and δ is a fuzzy open set in a fuzzy Baire - separated space (X,T) such that $\lambda \leq \delta$, then there exists a fuzzy residual setμin (X,T) such that $\lambda \leq \mu$.

Proof: Let λ be a fuzzy closed set andδbe afuzzy open set in (X,T) such that $\lambda \leq \delta$. Since(X,T) is a fuzzy Baire -separated space, by Proposition 3.1, there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq \delta$. Then,$\eta = \alpha \wedge \mu$, where α is a fuzzy open set and μ is a fuzzy residual set in X. Now$\alpha \wedge \mu \leq \mu$, impliesthat$\eta \leq \mu$ and $\lambda \leq \eta \leq \mu$. Thus, for a fuzzy closed set λin (X,T), there exists a fuzzy residual set μ in (X,T) such that $\lambda \leq \mu$, in (X,T).

It is observed that fuzzy nowhere dense sets are fuzzy closed sets in fuzzy nodec spaces [11] and under what conditionsdoesa fuzzy Baire – separated space has a fuzzy closed set with fuzzy nowhere denseness ?

Proposition 3.5: If λ is a fuzzy closed set and δ is afuzzy openset such that $\lambda \leq \delta$in a fuzzy Baire - separated space (X,T) in which fuzzy first category sets are fuzzy dense, then λ is a fuzzy nowhere densesetin X.

Proof: Let λ b ea fuzzy closed set and δ be a fuzzy open set in(X,T)such that $\lambda \leq \delta$. Since (X,T) is a fuzzy Baire - separated space, by Proposition 3.4, there exists a fuzzy residual set μ in (X,T)such that $\lambda \leq \mu$. Then, int $(\lambda) \leq$ int (μ). Now, $1-\mu$ is a fuzzy first category set in (X,T) and by hypothesis, cl $(1-\mu) = 1$. By Lemma2.1, $1-$ int$(\mu) =$ cl$(1-\mu) = 1$ and it follows that int$(\mu) = 0$ and then int$(\lambda) \leq 0$, in (X,T). That is, int$(\lambda) = 0$. Since λ is a fuzzy closedset in X, cl$(\lambda) = \lambda$and int cl$(\lambda) =$ int$(\lambda) = 0$. Hence the fuzzy closed set λ is a fuzzy nowhere dense set in X.

Proposition 3.6: If λ is a fuzzy closed set and δ is a fuzzy open set in a fuzzy Baire - separated space (X,T) such that $\lambda \leq \delta$,then there exists a fuzzy G_δ -set β in (X,T) such that $\beta \leq \delta$.

Proof:Let λ be a fuzzy closed set andδ be a fuzzy open set in (X,T)such that$\lambda \leq \delta$. Since (X,T) is a fuzzy Baire – separated space, by Proposition 3.1,there exists a fuzzy Baire set η in (X,T)such that $\lambda \leq \eta \leq \delta$. Now η beinga fuzzy Baireset in (X,T),$\eta = \alpha \wedge \mu$, where α is a fuzzy open set and μ is a fuzzy residual set in (X,T). Then, $\lambda \leq \alpha \wedge \mu \leq \delta$. By Theorem 2.4, for the fuzzy residual setμ in X there exists a fuzzy G_δ -set γ in (X,T) such that $\gamma \leq \mu$. Then,$\alpha \wedge \gamma \leq \alpha \wedge \mu = \eta$,in (X,T). Let$\beta = \alpha \wedge \gamma$. Then,$\beta$ is a fuzzy G_δ- set in (X,T)such that $\beta \leq \eta \leq \delta$. Hence,for the fuzzy open set δ, there exists a fuzzy G_δ –set β such that $\beta \leq \delta$, in (X,T).

Corollary 3.2 If μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy Baire -separated space (X,T), then there exists a fuzzy G_δ-set η in (X,T)such that $\eta \leq 1-\mu_2$ and $\eta \leq 1-\mu_1$.

Proof:Let μ_1 and μ_2 be disjoint fuzzy closed sets in X. Then, $\mu_1 \wedge \mu_2 = 0$. This implies that $\mu_1 \leq 1-\mu_2$ and $1-\mu_2$ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy Baire -separated space, by Proposition 3.6, there exists a fuzzy G_δ –set η in (X,T) such that $\eta \leq 1-\mu_2$. Also $\mu_1 \wedge \mu_2 = 0$, implies that $\mu_2 \leq 1-\mu_1$ and then it follows that there exists a fuzzy G_δ-set η in (X,T) such that $\eta \leq 1-\mu_1$.

Proposition3.7: If a fuzzy set λ is a fuzzy P-set such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ - set in a fuzzy Baire-separated space (X,T), then there exists a fuzzy Baireset η in (X,T) such that $\lambda \leq \eta \leq 1-int(\mu)$.

Proof: Let λ be a fuzzy P-set such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ- set in (X,T). Then, by Theorem 2.5, there exists a fuzzy open set δ in(X,T)such that $\lambda \leq \delta \leq 1-int(\mu)$. Since λ is a fuzzy P -set in X, λ is a fuzzy closed set and$\lambda \leq \delta$, in (X,T). Since (X,T) is a fuzzy Baire - separated space, by Proposition 3.1, there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq \delta$. This implies that $\lambda \leq \eta \leq \delta \leq 1-int(\mu)$ and hence $\lambda \leq \eta \leq 1-int(\mu)$, in (X,T).

The following proposition gives a condition for a fuzzy topological space to become a fuzzy Baire - separated space.

Proposition3.8: If $\mu_1 \leq (1-\mu_2) \wedge (1-\alpha)$, for each pair of fuzzy closed sets μ_1 and μ_2 and for a fuzzy σ-nowhere dense set α, in a fuzzy topological space (X,T), then (X,T) is a fuzzy Baire - separated space.

Proof: Let μ_1 and $\mu_2$be a pair of fuzzy closed sets such that $\mu_1 \leq (1-\mu_2) \wedge (1-\alpha)$, where α is a fuzzy σ-nowhere dense set in (X,T). Now$(1-\mu_2) \wedge (1-\alpha) \leq (1-\mu_2)$ implies that $\mu_1 \leq (1-\mu_2)$. Since$\mu_2$is a fuzzy closed set, $1-\mu_2$is a fuzzy open set in (X,T). Let $\eta = (1-\mu_2) \wedge (1-\alpha)$. Then, by Theorem 2.7, η is a fuzzy Baire set in (X,T). Thus, for a pair of fuzzyclosed sets μ_1 and μ_2 in (X,T) such that $\mu_1 \leq 1-\mu_2$,
the existence of a fuzzy Baire set η in (X,T) such that $\mu_1 \leq \eta \leq 1-\mu_2$, implies that (X,T) is a fuzzy Baire -separated space.

Corollary 3.3:If $\mu_1 \leq (1-\mu_2) \wedge \beta$, for each pair of fuzzy closed sets μ_1 and μ_2 and for a fuzzy G_δ -set set β such that cl$(\beta) = 1$, in a fuzzy topological space (X,T), then (X,T) is a fuzzy Baire -separated space.

Proof:Now $\beta = 1-(1-\beta)$,where$1-\beta$ is a fuzzy F_σ –set in (X,T) and int$(1-\beta) = 1-$cl$(\beta) = 1-1 = 0$. Then, $1-\beta$ is a fuzzy σ-nowhere dense set in (X,T) and $\mu_1 \leq (1-\mu_2) \wedge (1-(1-\beta))$. By Proposition 3.8, (X,T) is a fuzzy Baire - separated space.

Proposition 3.9: If μ_1 and $\mu_2$are disjoint fuzzy closed sets in a fuzzy Baire - separated space (X,T), then there exists a fuzzy residual set β and a fuzzy first category set γ in (X,T) such that $\mu_1 \leq \beta$and $\mu_2 \leq \alpha \vee \gamma$, where α is a fuzzy closed set in (X,T).

Proof:Let μ_1 and $\mu_2$be disjoint fuzzy closed sets in (X,T). Then, $\mu_1 \wedge \mu_2 = 0$. This implies that $\mu_1 \leq 1 - \mu_2$ and $1 - \mu_2$is a fuzzy open set in (X,T). Then, by Proposition 3.1, there exists a fuzzy Baire set ηin (X,T) such that $\mu_1 \leq \eta \leq 1 - \mu_2$. Now $\eta \leq 1 - \mu_2$, implies that$\mu_2 \leq 1 - \eta$in (X,T). Since $\eta$is a fuzzy Baire set, $\eta = \lambda \wedge \delta$, where λ is a fuzzy open set and δ is a fuzzy residual set in (X,T) and thus $\mu_2 \leq 1 - (\lambda \wedge \delta) = (1 - \lambda) \vee (1 - \delta)$. Let $\alpha = 1 - \lambda$and $\gamma = 1 - \delta$. Then, $\alpha$is a fuzzy closed set and γ is a fuzzy first category set in (X,T). By Proposition 3.4, for the fuzzy closed set μ_1, there exists a fuzzy residual set β in (X,T) such that $\mu_1 \leq \beta$. Hence, for the disjoint fuzzy closed sets μ_1 and μ_2, there exists a fuzzy residual set β and a fuzzy first category set γin (X,T) such that $\mu_1 \leq \beta$and $\mu_2 \leq \alpha \vee \gamma$, where α is a fuzzy closed set in (X,T).

Proposition3.10: If μ_1 and $\mu_2$are disjoint fuzzy closed sets in a fuzzy Baire - separated space (X,T), then there exists a fuzz yresidual set β and a fuzzy F_σ -set δ such that $\mu_1 \leq \beta$and $\mu_2 \leq \delta$, in (X,T).

Proof: Let μ_1 and μ_2 be disjoint fuzzy closed sets in (X,T). Since (X,T) is a fuzzy Baire- separated space, by Proposition 3.9, there existsa fuzzy residual set$\beta$and a fuzzy first category set γ in (X,T) such that $\mu_1 \leq \beta$ and $\mu_2 \leq \alpha \vee \gamma$, where α is a fuzzy closed set in (X,T). By Theorem 2.11, for the fuzzy first category setγ, there is a fuzzy F_σ-set θ in (X,T) such that$\gamma \leq \theta$ and then $\alpha \vee \gamma \leq \alpha \vee \theta$. Since α is a fuzzy closed set, $\alpha \vee \theta$ is a fuzzyF_σ-set in (X,T). Let $\delta = \alpha \vee \theta$. Hence, for the disjoint fuzzy closed sets μ_1 and μ_2, there exists a fuzzy residual set β and a fuzzy F_σ –set δ such that$\mu_1 \leq \beta$and $\mu_2 \leq \delta$, in (X,T).

Proposition 3.11:Ifthe fuzzy sets (λ_i) 's (i = 1 to ∞)are fuzzy closed sets and(δ_i)'s (i = 1 to ∞) arefuzzyopen sets such that $\lambda_i \leq \delta_i$ina fuzzy Baire - separated space (X,T) in which fuzzy first category sets are fuzzy dense, then$\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open and fuzzy dense set in (X,T).

Proof:Let (λ_i)'s (i = 1 to ∞) be fuzzy closed sets and(δ_i)'s (i = 1 to ∞)be fuzzy open sets in a fuzzy Baire - separated space (X,T) such that $\lambda_i \leq \delta_i$. This implies that$\vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty}(\delta_i)$ and $cl[\vee_{i=1}^{\infty}(\lambda_i)] \leq cl[\vee_{i=1}^{\infty}(\delta_i)]$, in (X,T). Since (X,T) is a fuzzy Baire – separated space in which fuzzy first category sets are fuzzy dense, by Proposition 3.5, (λ_i) 's are fuzzy nowhere dense sets in (X,T). Then,$\vee_{i=1}^{\infty}(\lambda_i)$is a fuzzy first category set in (X,T). By hypothesis, $cl[\vee_{i=1}^{\infty}(\lambda_i)]=1$. This implies that $cl[\vee_{i=1}^{\infty}(\delta_i)] = 1$and thus$\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open and fuzzy dense set in (X,T).

Corollary 3.4: If the fuzzy sets (λ_i) 's (i = 1to ∞) are fuzzy closed sets and(δ_i)'s(i = 1 to ∞)are fuzzy open sets such that $\lambda_i \leq \delta_i$ina fuzzy Baire - separated space (X,T) in which fuzzy first category sets are fuzzy dense, then there exists a fuzzy first category and fuzzyF_σ –set $\lambda$and a fuzzy open and fuzzy dense set δin (X,T) such that $\lambda_i \leq \lambda$, $\delta_i \leq \delta$and $\lambda \leq \delta$.

Proof:Let (λ_i) 's(i = 1 to ∞) be fuzzy closed sets and (δ_i)'s (i = 1 to ∞)be fuzzy open sets in a fuzzy Baire - separated space (X,T) such that$\lambda_i \leq \delta_i$. Then,by Proposition 3.11,$\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open and fuzzy dense set in (X,T). Also $\vee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy first category set in (X,T). Since (λ_i) 'sare fuzzy closed sets, $\vee_{i=1}^{\infty}(\lambda_i)$is a fuzzy$F_\sigma$-set in (X,T). Let $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ and $\delta = \vee_{i=1}^{\infty}(\delta_i)$. Hence, there exists a fuzzy first category and fuzzy F_σ -set$\lambda$and a fuzzy open and fuzzy dense setδin (X,T) such that$\lambda_i \leq \lambda$, $\delta_i \leq \delta$and$\lambda \leq \delta$.

The following proposition gives a condition for a fuzzy topological space to become a fuzzy Baire - separated space.

Proposition 3.12: If $\lambda \leq \delta \wedge \mu$, foreach fuzzy closed set λ and each fuzzy open set δ, where μ is a fuzzy residual set in a fuzzy topological space (X,T), then (X,T) is a fuzzy Baire - separated space.

Proof: Suppose that $\lambda \leq \delta \wedge \mu$, for each fuzzy closed set λ and each fuzzy open set δ, where μ is a fuzzy residual set in (X,T). Now $\delta \wedge \mu \leq \delta$, implies that $\lambda \leq \delta \wedge \mu \leq \delta$. Let $\eta = \delta \wedge \mu$. Then, η is a fuzzy Baire set in(X,T). Hence (X,T) is a fuzzy Baire – separated space.

Corollary 3.5: If $\lambda \vee \eta \leq \delta$, for each fuzzy closed set λ and each fuzzy open set δ,where η is a fuzzy first category set in a fuzzy topological space (X,T),then (X,T) is a fuzzy Baire – separated space.

4. Fuzzy Baire-Separated Spaces and Other Fuzzy Topological Spaces

Proposition4.1: If λ is a fuzzy closed set and δ is a fuzzy open set in a fuzzy Baire -separated and fuzzy globally disconnected space (X,T) with $\lambda \leq \delta$, then there exists a fuzzy G_δ –set η in (X,T) such that $\lambda \leq \eta \leq \delta$.

Proof: Let λ be a fuzzy closed set and δ be a fuzzy open set in (X,T) with $\lambda \leq \delta$. Since (X,T) is a fuzzy Baire -separated space, by Proposition 3.1, there exists a fuzzy Baire set η in (X,T) such that $\lambda \leq \eta \leq \delta$. Also since (X,T) is a fuzzy globally disconnected space, by Theorem 2.1, the fuzzy Baire set $\eta$is a fuzzy G_δ -set in X. Hence, for a fuzzy closed set λ and a fuzzy open set $\delta$with $\lambda \leq \delta$, there exists a fuzzy G_δ –set η in X, such that $\lambda \leq \eta \leq \delta$.

Corollary 4.1: If μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy Baire-separated and fuzzy globally disconnected space (X,T), then there exists a fuzzyG_δ -setηin (X,T) such that$\mu_1 \leq \eta \leq 1 - \mu_2$.

Proof:Let μ_1 and $\mu_2$be disjoint fuzzy closed sets in (X,T). Then, $\mu_1 \wedge \mu_2 = 0$. This implies that $\mu_1 \leq 1 - \mu_2$ and $1 - \mu_2$ is a fuzzy open set in(X,T). Since(X,T) is a fuzzy Baire – separated and fuzzy globally disconnected space, by Proposition 4.1, there exists a fuzzy G_δ –set η in (X,T) such that $\mu_1 \leq \eta \leq 1 - \mu_2$.

Proposition 4.2: If λ is a fuzzy closed set and δ is a fuzzy open set with $\lambda \leq \delta$ in a fuzzy Baire -separated and fuzzy submaximal space (X,T), then there exists a fuzzy G_δ – set η in (X,T) such that $\lambda \leq \eta \leq \delta$.

Proof:Let λ beafuzzyclosedsetand δ be a fuzzy open setin (X,T) with $\lambda \leq \delta$. Since(X,T) is a fuzzy Baire - separated space, by Proposition 3.1, there exists a fuzzy Bairesetηin (X,T) such that $\lambda \leq \eta \leq \delta$. Also since (X,T) is a fuzzy submaximal space, by Theorem 2.2, the fuzzy Baire set η is a fuzzy G_δ -set in (X,T). Hence, for a fuzzy closedset λ and a fuzzy openset δ with $\lambda \leq \delta$, thereexistsafuzzy G_δ -setηin X such that $\lambda \leq \eta \leq \delta$.

Corollary 4.2: If μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy Baire-separated and fuzzy submaximal space (X,T), then there exists a fuzzy G_δ -setη in (X,T) such that $\mu_1 \leq \eta \leq 1 - \mu_2$.

Corollary 4.3: If α is a fuzzy open set and β is a fuzzy closed set with $\beta \leq \alpha$ina fuzzy Baire-separated and fuzzy submaximal space (X,T), then there exists a fuzzy F_σ-set δ in (X,T) such that $\beta \leq \delta \leq \alpha$.

Proof:Let α be a fuzzy open set and β be a fuzzy closed set in (X,T) with $\beta \leq \alpha$. Then, $1-\alpha \leq 1-\beta$, where $1-\alpha$ is a fuzzy closed set and $1-\beta$ is a fuzzy open set in the fuzzy Baire - separated and fuzzy submaximal space (X,T). By Proposition 4.2, there exists a fuzzy G_δ –set η in (X,T) such that $1-\alpha \leq \eta \leq 1-\beta$. Then, it follows that $\beta \leq 1-\eta \leq \alpha$. Let $\delta = 1-\eta$. Then, δ is a fuzzy F_σ -set in (X,T) and $\beta \leq \delta \leq \alpha$, in (X,T).

It is observed that fuzzy nowhere dense sets are fuzzy closed sets in fuzzy nodec spaces [11] and subsequently a question will arise. Which fuzzy topological spaces [other than fuzzy hyperconnected spaces [23], fuzzy globally disconnected spaces] have fuzzy closed sets with fuzzy nowhere denseness? The following propositions give the answers for thi squestion.

Proposition 4.3:If λ is a fuzzy closed set and δ is a fuzzy open set with $\lambda \leq \delta$in a fuzzy Baire-separated and fuzzy extraresolvable space (X,T), then the fuzzy closed set λ is afuzzy nowhere dense set in (X,T).

Proof: Let λ be a fuzzy closed set and δbe a fuzzy open set in (X,T)with $\lambda \leq \delta$. Since (X,T) is a fuzzy Baire -separated space, by Proposition 3.4, there exists a fuzzy residual set μ in (X,T) such that $\lambda \leq \mu$. Then, int $(\lambda) \leq$ int (μ), in (X,T). Also since (X,T) is a fuzzy extra-resolvable space, by Theorem 2.3, int$(\mu)= 0$ and then int$(\lambda) \leq 0$. That is, int(λ)=0, in (X,T). Since λ is a fuzzy closed set,cl$(\lambda) = \lambda$ and int cl(λ) =int$(\lambda) = 0$. Hence the fuzzy closed set λ is a fuzzy nowhere dense set in (X,T).

Proposition 4.4: If λ is a fuzzy closed set and δ is a fuzzy open set with $\lambda \leq \delta$ in a fuzzy Baire - separated and fuzzy strongly Baire space (X,T), then the fuzzy closed set λ is a fuzzy nowhere dense set in (X,T).

Proof: Let λ be a fuzzy closed set and δ be a fuzzy open set in (X,T)with $\lambda \leq \delta$. Since (X,T) is a fuzzy Baire - separated space, by Proposition 3.4, there exists a fuzzy residual set μ in (X,T) such that $\lambda \leq \mu$. Then, int $(\lambda) \leq$ int (μ), in (X,T). Also since (X,T) is a fuzzy strongly Baire space, by Theorem 2.9, int$(\mu)= 0$ and then int $(\lambda) \leq 0$. That is, int$(\lambda) = 0$, in (X,T). Since$\lambda$is a fuzzy closed set in X, cl$(\lambda) = \lambda$ and int cl(λ) =int$(\lambda) = 0$. Hence the fuzzy closed set λ is a fuzzy nowhere dense set in (X,T).

Corollary 4.4: Ifμ_1 and $\mu_2$are disjoint fuzzy closed sets in a fuzzy Baire - separated and fuzzy strongly Baire space (X,T), then μ_1 and μ_2 are fuzzy nowhere dense sets in (X,T).

Proof: Let μ_1 and $\mu_2$be disjoint fuzzy closed sets in (X,T). Then, $\mu_1 \wedge \mu_2 = 0$. This implies that $\mu_1 \leq 1-\mu_2$ and $1-\mu_2$ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy Baire - separated space, by Proposition 3.4, there exists a fuzzy residual set δ in X such that $\mu_1 \leq \delta$. Then, int $(\mu_1) \leq$ int (δ), in (X,T). Also since (X,T) is a fuzzy strongly Baire space, by Theorem2.9, int$(\delta) = 0$ and then int$(\mu_1) \leq 0$. That is, int (μ_1) =0, in (X,T). Since $\mu_1$is a fuzzy closed set in X, cl$(\mu_1) = \mu_1$andintcl(μ_1) = int (μ_1) =0. Hence$\mu_1$is a fuzzy nowhere dense set in (X,T). Also $\mu_1 \wedge \mu_2 = 0$, implies that $\mu_2 \leq 1-\mu_1$and then it follows that μ_2 is also a fuzzy nowhere dense set in (X,T).

Corollary4.5: If μ_1 and $\mu_2$are disjointfuzzy closed setsin a fuzzy Baire -separated and fuzzy extraresolvable space (X,T), thenμ_1 and $\mu_2$arefuzzynowheredense setsin(X,T).

Proposition 4.5: If λ is a fuzzy closed set and δ is a fuzzy open set in a fuzzy Baire -separated and fuzzy second category (but not fuzzy Baire) and fuzzy hyper-connected space (X,T) such that $\lambda \leq \delta$, then the fuzzy closed set λ is a fuzzynowhere dense set in (X,T).

Proof: Let λ be a fuzzy closed set and δ be a fuzzy open set in (X,T) such that $\lambda \leq \delta$. Since (X,T) is a fuzzy Baire -separated space, by Proposition 3.4, there exists a fuzzy residual set μ in (X,T) such that $\lambda \leq \mu$. Then, int$(\lambda) \leq$ int(μ), in (X,T). Also since (X,T) is a fuzzy second category (but not fuzzy Baire) and fuzzy hyper-connected space, by Theorem 2.12,the fuzzy residual set μ is a fuzzy nowhere dense set in (X,T). Then, intcl$(\mu)= 0$ and int$(\mu) \leq$int cl(μ)implies that int$(\mu)= 0$ and thenint (λ)=0, in (X,T). Since λ is a fuzzy closed set, cl$(\lambda) = \lambda$ andint cl(λ) =int$(\lambda) = 0$. Hence the fuzzy closed set λ is a fuzzy nowhere dense set in (X,T).

The following proposition shows that fuzzy ultra-connected spaces, are not fuzzy Baire - separated spaces.

Proposition 4.6: If a fuzzy topological space(X,T) is a fuzzy ultraconnected space, then (X,T) is not a fuzzy Baire – separated space.

Proof: Let μ_1 and $\mu_2$be a pair of fuzzy closed sets in (X,T). Since (X,T)is a fuzzy ultraconnected space, $\mu_1 \nleq 1-\mu_2$ in (X, T). Then, it is not possible to find a fuzzy Baire set η in (X,T) such that $\mu_1 \leq \eta \leq 1-\mu_2$ Hence (X,T) is not a fuzzy Baire - separated space.

The following proposition gives a condition under which fuzzy weakly Baire spaces become fuzzy Baire - separated spaces.

Proposition 4.7:I f$\mu_1 \leq (1-\mu_2)\wedge(1-\alpha)$, where α is a fuzzy σ-boundary set and μ_1 and μ_2 are fuzzy closed sets in a fuzzy weakly Baire space (X,T),then (X,T) is a fuzzy Baire - separated space.

Proof: Let μ_1 and μ_2 be a pair of fuzzy closed sets such that $\mu_1 \leq (1-\mu_2)\wedge(1-\alpha)$, where α is a fuzzy σ-boundary set in (X,T). Since (X,T) is a fuzzy weakly Baire space, by Theorem 2.8, the fuzzy σ-boundary set α is a fuzzy σ-nowhere dense set in X. Thus, $\mu_1 \leq (1-\mu_2)\wedge(1-\alpha)$, where α is a fuzzy σ-nowhere dense set in (X,T). Then, by Proposition 3.8, (X,T) is a fuzzy Baire - separated space.

Proposition 4.8:If $\mu_1 \leq 1-\mu_2$, for any two non - zero fuzzy sets μ_1 and μ_2 in a fuzzy perfectly disconnected and fuzzy Baire - separated space (X,T), then there exists a fuzzy Baire set η in (X,T) such that$\mu_1 \leq \eta \leq 1-\mu_2$.

Proof: Let μ_1 and $\mu_2$be any two fuzzy sets defined on X such that $\mu_1 \leq 1-\mu_2$. Since (X,T) is a fuzzy perfectly disconnected space, $\mu_1 \leq 1-\mu_2$implies that $cl(\mu_1) \leq 1-cl(\mu_2)$. By Theorem 2.10, $cl(\mu_1) \neq 1$ and $cl(\mu_2) \neq 1$, in (X,T). Also since (X,T) is a fuzzy Baire -separated space, for the fuzzy closed sets $cl(\mu_1)$and $cl(\mu_2)$, there exists a fuzzyBaire set η in (X,T) such that $cl(\mu_1) \leq \eta \leq 1-cl(\mu_2)$. Now $\mu_1 \leq cl(\mu_1) \leq \eta \leq 1-cl(\mu_2) \leq 1-\mu_2$, implies that $\mu_1 \leq \eta \leq 1-\mu_2$, in X.

Proposition 4.9: If the fuzzy sets μ_1 and μ_2 are disjointfuzzy F_σ -sets in a fuzzy F'- space and fuzzy Baire - separated space (X,T), then there exists a fuzzy Baire set η in (X,T) such that $\mu_1 \leq \eta \leq 1-\mu_2$.

Proof: Let μ_1 and $\mu_2$be disjoint fuzzy F_σ-sets in (X,T). Then, $\mu_1\wedge\mu_2 = 0$. This implies that $\mu_1 \leq 1-\mu_2$. Since (X,T) is a fuzzy F'- space,$\mu_1 \leq 1-\mu_2$implies that $cl(\mu_1) \leq 1-cl(\mu_2)$. Also since (X,T) is a fuzzy Baire-separated space, for the fuzzy closed sets $cl(\mu_1)$ and $cl(\mu_2)$, there exists a fuzzy Baire set η in (X,T) such that $cl(\mu_1) \leq \eta \leq 1-cl(\mu_2)$. Now $\mu_1 \leq cl(\mu_1) \leq \eta \leq 1-cl(\mu_2) \leq 1-\mu_2$, implies that $\mu_1 \leq \eta \leq 1-\mu_2$,in (X,T).

Proposition 4.10: If the fuzzy sets μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy Baire - separated and fuzzy globally disconnected space (X,T), then there exists a fuzzy G_δ – set η and a fuzzy F_σ-set δ in (X,T) such that μ_1 η and$\mu_2 \leq \delta$.

Proof: Let μ_1 and $\mu_2$be disjoint fuzzy closed sets in (X,T). Since (X,T) is a fuzzy Baire – separated space, by Corollary 4.1, there exists a fuzzy G_δ –set η in (X,T) such that $\mu_1 \leq \eta \leq 1-\mu_2$. Now$\eta \leq 1-\mu_2$, implies that$\mu_2 \leq 1-\eta$, in (X,T). Since η is a fuzzyG_δ -set,$1-\eta$ is a fuzzy F_σ-set in (X,T). Let$\delta = 1-\eta$. Hence, for the disjoint fuzzy closed sets μ_1 and μ_2, there exists a fuzzyG_δ –set η and a fuzzy F_σ –set δ in (X,T) such that $\mu_1 \leq \eta$and $\mu_2 \leq \delta$.

Proposition 4.11: If the fuzzy sets μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy Baire -separated and fuzzy submaximal space (X,T), then there exists a fuzzy G_δ –set η and a fuzzy F_σ –set δ in (X,T) such that $\mu_1 \leq \eta$ and $\mu_2 \leq \delta$.

Proof: The proof follows from Corollary 4.2.

Proposition 4.12: If the fuzzy sets μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy Baire -separated, fuzzy globally disconnected and weak fuzzy Oz-space (X,T),then there exist fuzzyG_δ –sets $\eta$and θ in (X,T) such that $\mu_1 \leq \eta$ and $\mu_2 \leq \theta$.

Proof: Let μ_1 and $\mu_2$be disjoint fuzzy closed sets in (X,T). Since(X,T) is a fuzzy Baire - separated, fuzzy globally disconnected space, by Proposition 4.10, there exists a fuzzyG_δ –set η and a fuzzy F_σ –set δ in (X,T) such that $\mu_1 \leq \eta$and$\mu_2 \leq \delta$. Now $\mu_2 \leq \delta$ implies that $cl(\mu_2) \leq cl(\delta)$, and then since μ_2 is a fuzzy closed set, $\mu_2 = cl(\mu_2) \leq cl(\delta)$, in(X,T). Also since (X,T) is a weak fuzzy Oz-space, for the fuzzy F_σ-setδ, $cl(\delta)$ is a fuzzy G_δ -set in (X,T). Let $\theta = cl(\delta)$. Hence, for the disjoint fuzzy closed sets μ_1 and μ_2, there exist fuzzy G_δ –sets $\eta$and θin (X,T) such that $\mu_1 \leq \eta$ and $\mu_2 \leq \theta$.

The following proposition gives a condition under which fuzzy Baire – separated spaces become fuzzy semi normal spaces.

Proposition 4.13:If a fts (X,T)is a fuzzy Baire -separated space in which fuzzy Baire sets are fuzzy regular open sets, then (X,T) is a fuzzy seminormal space.

Proof: Let λ be a fuzzy closed set and δ be a fuzzy open set in (X,T)such that$\lambda \leq \delta$.Since (X,T) is a fuzzy Baire – separated space, by Proposition 3.1, there exists a fuzzy Baire set η in (X,T)such that $\lambda \leq \eta \leq \delta$. By hypothesis, the fuzzy Baire set η is a fuzzy regular open set in (X,T) and thus for the fuzzy closed set λ and the fuzzy open set δ such that $\lambda \leq \delta$, there exists a fuzzy regular open set η in (X,T) such that $\lambda \leq \eta \leq \delta$. Hence it follows that (X,T) is a fuzzysemi-normal space.

Proposition 4.14: If a fts (X,T) isa fuzzy Baire - separated space in which for each fuzzy Baire set η, $\eta = int(\gamma)$,where $(1-\gamma) \in T$, then (X,T) is a fuzzy semi normal space.

Proof:The proof follows from Lemma 2.1 and Proposition 4.13.

5. Conclusion

In this paper, the notion of fuzzy Baire-separated spaces is introduced in terms of fuzzy Baire sets. It is shown that fuzzy Baire sets lie between disjoint fuzzy P-sets and fuzzy F_σ-sets in a fuzzy Baire-separated space. Also it is obtained that fuzzy Baire sets which lie between fuzzy closed sets and fuzzy open sets in a fuzzy Baire-separated space are not fuzzy dense sets. The conditions under which fuzzy topological spaces become fuzzy Baire-separated spaces are identified. Fuzzy nowhere dense sets are fuzzy closed sets in fuzzy nodec spaces and subsequently the fuzzy topological spaces having fuzzy closed sets with fuzzy nowhere denseness are identified and it is shown that fuzzy ultraconnected spaces are not fuzzy Baire - separated spaces. The means, by which fuzzy weakly Baire spaces become fuzzy Baire -separated spaces and the fuzzy Baire-separated spaces become fuzzy seminormal spaces are obtained. In the future work, the inter-relations between fuzzy Baire spaces and Baire -separated spaces have to be explored.

Conflicts of Interest

The authors declare no conflicts of interest.

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