Petroleum Science and Engineering
2025, Vol. 9, No. 1, pp. 13-21
https://doi.org/10.11648/j.pse.20250901.12

Research Article

Development of Maximum Efficient Rate Model and Improvement of Erosional Velocity-Based Correlation for Vertical Oil Wells in the Niger Delta

Aniefiok Livinus[1, *], Moses Effiong Ekpenyong[2], Aniekan Morgan[1], Edet Ita Okon[3], Seyeneofon Emmanuel Ukpong[3]

[1]Petroleum Engineering Department, University of Uyo, Uyo, Nigeria
[2]Computer Science Department, University of Uyo, Uyo, Nigeria
[3]Chemical Engineering Department, TopFaith University, Mkpatak, Nigeria

Abstract

Maximum efficient rate (MER) and erosional velocity are known to be vital concepts in oil and gas production, and producing a well at a maximum efficient rate remains a critical concern to the oil and gas operators, the production engineer and the regulator. Well testing and equilibrium concept are commonly used by oil and gas players to determine the MER of a well. However, little adjustment of the plot axes of the production rate, choke size and tubing head pressure can affect the accuracy of the MER determination. Additionally, there are no known generalized correlations to compare results with that of the MER tests. Furthermore, oil regulatory bodies in Nigeria have no known published models for estimating the technical allowable rate, unlike other regulatory bodies in other countries. This work therefore presents the outcomes of the formulation of MER and the improved erosional velocity-based correlations for vertical oil wells, using MER test data from the Niger Delta region of Nigeria. Multiple linear regression (MLR) and probabilistic modeling approaches were considered. The predicted normalized MER results compared favorably well with the MER test data, with an absolute average error of 7.62%. For the case examples, de-normalization of the predicted MER results increases the absolute average error. Among the predicted P10, P50 and P90 MER results, the predicted P10 results are the nearest to the MER test results. Improvement in the predicted probabilistic results depends on the mean value of the predicted normalized MER considered. The combination of the MER model and the improved erosional velocity-based correlation can be a useful tool for MER test results verification and determination, and in overall for optimization of oil wells.

Keywords

Technical Allowable Rate, Well Optimization, Optimum Production Rate, Production System Models, Erosional Velocity

[*]Corresponding author: aniefioklivinus@uniuyo.edu.ng (Aniefiok Livinus)

Received: 10 March 2025; **Accepted:** 31 March 2025; **Published:** 27 April 2025

1. Introduction

Many production challenges such as; early water/gas coning and breakthrough, and erosion-corrosion, can emanate from a higher production rate. Even at sand free or clean services, where sand production rate is as low as a few pounds per day, erosion damage could be very severe at high production rate [1]. In "clean service," the wear, that is; the material wastage associated with erosion, is primarily governed by flow velocity, mixture density and two-phase flow regimes [2, 3].

Oil and gas operators apply different methods to limit erosion-corrosion of mild steel lines and equipment during the production of hydrocarbons from underground reservoirs. One of the frequently used methods is limiting the flow velocity to a so-called "erosional velocity," under which it is assumed that no erosion corrosion would occur [4]. Erosional velocity greatly influences tubing sizing in the design stage; oversizing of tubing unnecessarily increases construction costs whilst underestimating the required size of tubular can lead to catastrophic erosion/corrosion failures [5].

Owing to the controversies surrounding the formulation basis and sometimes the non-realistic C-values of the API RP 14E [6] equation, Equation (1), as reported in many published reports, Livinus and SeyeneOfon [7] recently developed new erosional velocity models that incorporate the approximate quantifiable effects of both tubing head pressure and tubing size, as presented in Equation (2). Equation (2) was based on the results of numerous production performance simulations using NODAL analysis approach.

$$V_e = \frac{C}{\sqrt{\rho_m}} \tag{1}$$

$$V_e = \frac{-0.03076THP + 3081.81ID}{\sqrt{\rho_m}} \tag{2}$$

where V_e is fluid erosional velocity, ft/s; C (100 for continuous service, 125 for intermittent service for both solid-free flowing fluid, and reduced value if solid particles are present) is an empirical constant; ρ_m is gas/liquid mixture density at flowing pressure and temperature, lb/ft^3; THP is tubing head pressures, psig; and ID is the tubing internal diameter, ft.

Production well testing through a test separator is by far the most common practice to measure flow rates [8]. The maximum efficient rate (MER) is known to be a vital concept in oil and gas production that helps to achieve a critical balance between maximizing production and preserving reservoir integrity, ensuring that oil and gas resources are produced in a sustainable and environmentally responsible manner. It can be seen as the maximum daily rate at which oil and gas can be produced for a long period of time without adversely affecting the practical ultimate recovery from the existing development strategy of a reservoir, as defined by Savage et al. [9], Bruce [10], and Gallun [11]. Water free oil production rate from water drive oil reservoirs requires the determination of critical oil rate. Over the years, various critical oil rate models, such as; the models of Meyer and Garder [12]; Chierici et al. [13]; Høyland et al. [14]; Menouar and Hakim [15]; Zhang et al. [16]; Tabatabaei et al. [17]) developed from theoretical and empirical means predict an uneconomical production rate. Obe et al. [18] reported empirical technique that enables estimation of a critical choke size, and corresponding production rates beyond which there is a high risk of early water breakthrough. The method involves the plot of the log of the average of well test rates at various choke settings. Aleruchi et al. [19] used MER test to resolve early time well performance rate, with a cased study of Field X in the Niger Delta. He pointed out that the MER obtained may cone water depending on the standoff of the perforated interval from the oil-water contact.

Equilibrium concepts are also used by oil players in the Niger Delta, where plots of THP versus Rate and Choke size versus Rate are generated on the same graph and the point of intersection of the curves is considered the stable equilibrium and the corresponding rate, the MER. This is done after producing wells have been tested on at least three to four consecutive choke sizes of equal spacing during which the rates, pressures (wellhead and sometimes bottom-hole) and other production parameters are measured. Further bean up is stopped, if sand limit greater than 0.5% is produced and estimated drawdown exceeds 300 psi. Unfortunately, slight adjustment of the scaling of the vertical axis on the analysis plot gives different value of MER, which has been reported by Sukubo and Obi [20]. Georgeson et al. [21] presented the Least Square formulation approach to analyze MER test results for non-pool wells to address the scale error problem. Unfortunately, the approach is a conversion of the graphical representation into a mathematical expression.

Furthermore, there are no known correlations to compare results with that of the MER tests. Likewise, regulatory bodies in Nigeria have no known models for estimating the technical allowable rate, unlike other regulatory bodies in other countries. For instance, the government of Saskatchewan [22] put forward Equation (3) for the calculation of the maximum permissible rate of production for non-horizontal wells (MPR).

$$MPR = 0.5FA * FH * F\emptyset * Fsw * F1/B_{oi} \tag{3}$$

where, FA is the area factor, which is equal to the drainage unit expressed in legal subdivisions (LSDs) multiplied by 1.0188; FH is the thickness factor, and it is equal to the thickness of the pay expressed in metres to the nearest one tenth of a metre; $F\emptyset$ is the porosity factor, which is equal to the average porosity of the pay used to calculate FH, above, expressed in percent (%) and divided by 10; FSw is the interstitial water factor, which is equal to 1 minus the average interstitial water content of the pay used in FH, above, expressed as a decimal, divided by (1-0.25); and $F1/B_{oi}$ is the shrinkage factor, which is equal to the change in volume of oil from reservoir conditions to stock tank conditions, expressed as a decimal, divided by 0.75.

While for horizontal well, it is equal to the block MPR multiplied by the recovery multiplier (RM). The RM factor–which cannot exceed 2.0–is derived from the Equation (4):

$$RM = 1 + (L - 100)/500 \tag{4}$$

where, L is the length in metres of the productive portion of the horizontal wellbore of a horizontal well, or the sum of the productive horizontal wellbores of the horizontal well.

This work therefore uses data of MER tests from vertical oil wells in the Niger Delta to develop MER model, which when combined with the improved erosional velocity-based model, can be useful tools for MER test results verification and determination, and in overall for optimization of oil wells.

2. Gathered MER Test Data

As earlier mentioned, oil players in the Niger Delta use equilibrium concepts to determine MER, where plots of THP versus production rate and choke size versus rate are generated on the same graph and the point of intersection of the curves is considered the stable equilibrium and the corresponding rate, the MER. This is done after producing wells have been tested on at least three to four consecutive choke sizes of equal spacing during which the rates, pressures (wellhead and sometimes bottom-hole) and other production parameters are measured. Table 1 presents summary of range of MER test data collected from reports of some oil companies in Nigeria. The wells are mostly vertical and naturally flowing.

Table 1. Summary of the gathered MER test data.

Well type, geometry, artificial lift and strings	Tubing size (inch)	Tubing head pressure (psig)	Choke size (64th of an inch)	MER	GOR	Oil API	BSW (%)
Natural flowing and Gas Lift	2.375 - 4.5	108 - 3721	101 - 64	101 - 1850	57 - 27279	13.48 - 55.37	0.10 - 84

3. MER Model Formulations

3.1. Multiple Linear Regression (MLR)

Multiple linear regression (MLR) is the statistical approach considered for the model development in this work. The general multiple regression equation presented by Mustafar and Razali [23] is expressed in Equation (5).

$$y_i = \ \beta_o + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} \tag{5}$$

where, $i = n$ observations; y_i is the dependent variable (predicted value); x_i, the input variables; β_o, the intercept constant; β_p, the slope coefficients for each input variable.

Multiple regression analysis gives more meaningful outcomes, if the dataset used are accurate [24]. Therefore, the multiple linear regression tool available in Microsoft Excel package was used for the development of the MER model, using the MER test dataset collected from reports of some oil companies in Nigeria. Based on the knowledge of the parameters used in existing theoretical choke models (see, [25-27]) and the importance of other parameters in some published empirical choke models (see, [28-32]), five (5) input variables were considered, namely: choke size (64th of an inch), tubing head pressure, THP, psig; gas-oil ratio, GOR, scf/stb; basic sediment and water, BSW,%; and tubing internal diameter, ID, ft.

The input and output dataset were scaled using logarithmic function to prevent numerical instability and improve the fitting performance. Afterwards, the dataset was partitioned into test size of 30% and training data of 70% for the multilinear regression process. The regression statistics show very good fitting performance, with values of 0.996, 0.991, 0.977 and 0.249 for the multiple R, R-squared, adjusted R-squared and standard error, respectively.

From the multiple linear regression process, the expressions for estimating MER are given in Equations (6) and (7).

$$X = \ b_1 Log(Bean) + b_2 Log(THP) + b_3 Log(GOR) + b_4 Log(BSW) + b_5 Log(ID) \tag{6}$$

$$MER_{MLR} = 10^X \tag{7}$$

where, b_1 through b_5 denote the coefficients associated with the input variables ($Bean, THP, GOR, BSW, ID$). Thus, the coefficients of the developed model are presented in Table 2.

Table 2. *Coefficients of the developed MLR model.*

b_1	b_2	b_3	b_4	b_5
-0.012230978	0.410467617	0.317466263	-0.00452942	-0.92030048

Figure 1 shows the relationship between the percentage error and the predicted MER data. The percentage error bandwidth falls in the range of +24% and -15%, with an absolute average error of 7.62%.

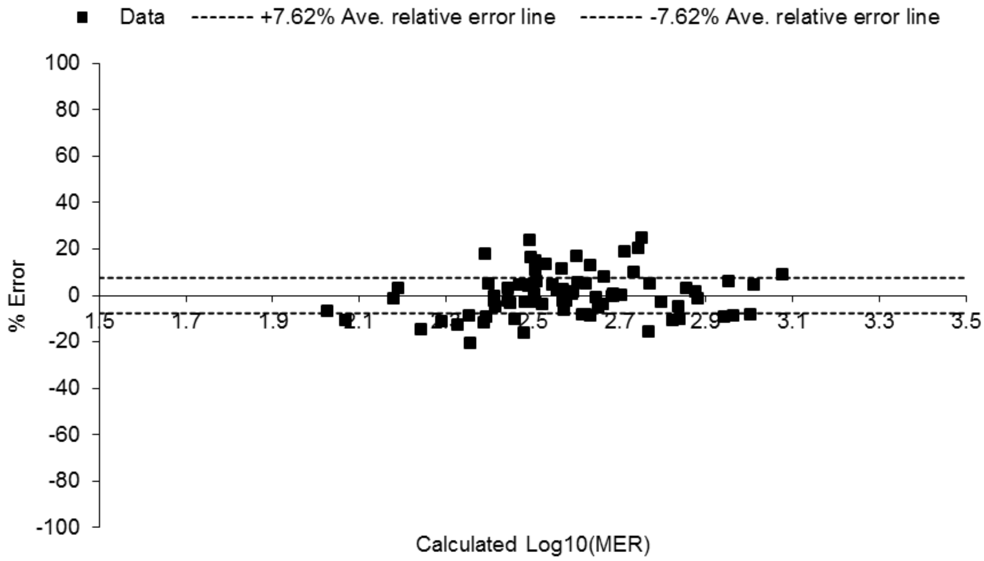


Figure 1. *Percentage error and the predicted log10 (MER) data.*

MER test data results from ten (10) wells were verified using Equations (6) and (7); the discrepancies of the predicted MER results from the well test values are quite high for more than eight (8) wells, as can be seen in Table 3. The percentage errors range from -103% to 53%; these error bandwidths are far away from the absolute average relative error of 7.62%, obtained from the multiple linear regression analysis; the error escalated from the de-normalization of the logarithmic function of X in Equation (7). To address this issue, probabilistic modeling approach was conceived.

Table 3. *Comparison of predicted and test MER results.*

WELL	MER, bopd	MER, bopd (Equations (6) & (7))	% ERROR
1	210	428	-103.81
2	290	461	-58.9655
3	180	320	-77.7778
4	370	319	13.78378
5	464	484	-4.31034
6	224.8	294	-30.7829
7	450	212	52.83648
8	188.5	248	-31.565
9	421	243	42.21165
10	536	297	44.58955

3.2. Probabilistic Modeling

Probabilistic model is a mathematical model that uses probability theory to make predictions about uncertain events. There are many different probabilistic models such as Bayes' theorem, Gaussian distribution, logistic regression, probability density function e.t.c., and the specific expressions to be used depend on the model and the problem to be solved. Details of several important common continuous univariate probabilistic models are often used in real life applications have been presented by Christensen [33], Wackerly et al. [34], and Nevzorov et al. [35]. In this work, the probability density function (PDF) has been considered, likewise the cumulative distribution function (CDF).

The predicted MER in logarithmic form to the base of 10, denoted as X in Equation (6), resulted in an absolute average relative of 6% when compared to the MER well test results (also in logarithmic form) obtained from the multiple linear regression analysis performed.

Therefore, the continuous random variable, X, has a probability density function $f(x)$ which is integrated to find the probability that X falls in any interval:

$$P(b < X < c) = \int_{(X-(X*a)}^{(X+(X*a)} f(x)dx \quad (8)$$

where, $X \sim normal(\mu, \sigma)$, X (obtained from Equation (6)) is a normal random variable with mean, μ, and standard deviation, σ. a is considered to be 0.0762, being the absolute average relative error obtained from the multiple linear regression analysis performed.

$$b = X - (X * a) \quad (9)$$

$$c = (X + (X * a) \quad (10)$$

The probability density function of X is presented in Equation (11):

$$P(b < X < c) = \int_b^c \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx \quad (11)$$

Assuming the x-samples are; (X - (X * a)), X, and (X + (X * a)), the mean, μ, and the standard deviation, σ, can roughly be estimated by Equations (12) and (13), respectively.

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (12)$$

$$\sigma = \sqrt{(\frac{1}{n-1}\sum_{i=1}^{n}(x_i - x)^2)} \quad (13)$$

More number of x-samples of X could be generated within the upper limit and the lower limit regions and the mean, μ, and the standard deviation, σ, computed.

The cumulative distribution function, $F(x)$, gives to each real value of x the probability of X having values less than or equal to x, that is,

$$F(x) = \Pr(X \le x) = \int_{-\infty}^{x} f(t)dt, \quad (14)$$

Which implies that;

$$f(x) = \frac{dF(x)}{dx} \quad (15)$$

Therefore, the probability that the random variable, X, (the predicted MER in logarithmic form to the base of 10) takes values in the interval $(b, c]$, with $b \le c$, is given by;

$$\Pr(b < X \le c) = \int_b^c f(x)dx = F(b) - F(c) \quad (16)$$

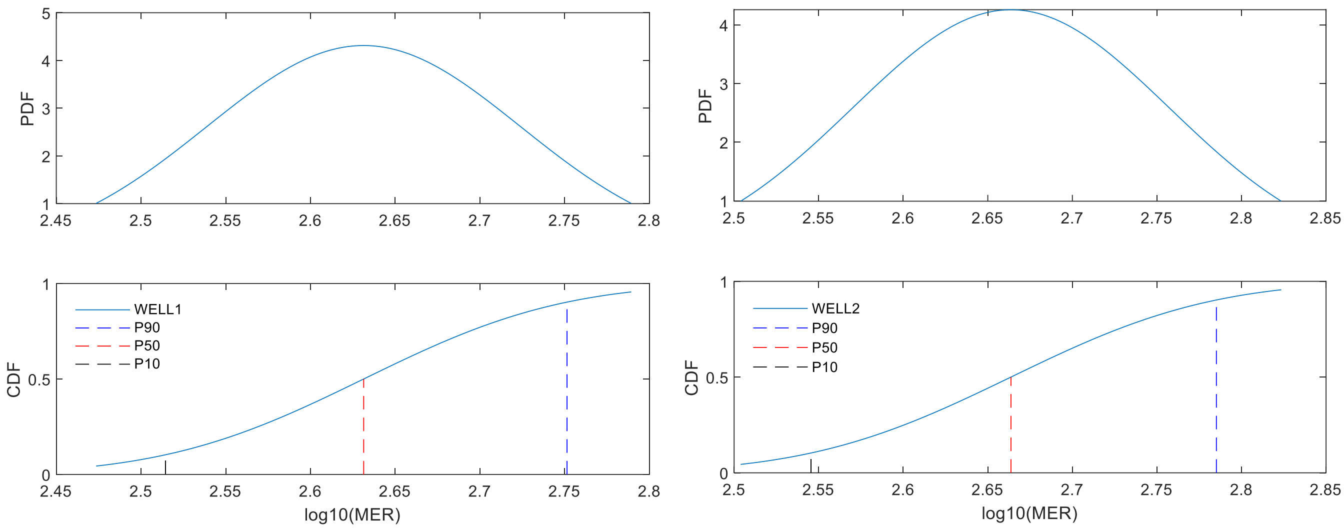
Finally, the pth percentile of the random variable X is the value x_p that separates the smallest p% of $X's$ values from the largest $(100 - p)$%. Probabilistically,

$$\mathrm{P}(X \le x_p) = p/100 \quad (17)$$

In this work, we considered the 10th, 50th and 90th percentiles as the probabilistic results for the predicted X, MER in logarithmic form to the base of 10, before the application Equation (6) to calculate the final MER results.

The MER test data of the ten (10) wells reported in Table 3 were predicted by applying the probabilistic approach, with the generation of 100 x-samples, within the range of (X - (X * a)) and (X + (X * a)), and the mean, μ, and the standard deviation, σ, computed. MATLAB® was used to perform normal distributions of the PDF and the CDF results, and crude MATLAB codes were written to perform the calculations. Figure 2(a, b) show the graphs of the PDF and CDF for two of the wells studied.

Table 4 gives summary of all the 10th, 50th and 90th percentiles as the probabilistic results for the predicted X for the wells. Equation (7) was then used to estimate the MER results. The predicted P10 MER results have an error bandwidth of -54% to 55.73%, while the predicted P50 MER results show an error range of -42.28% to 103.79%. The predicted P90 MER results also show high discrepancies with an error range of -39.86% to 168.65%. The predicted P10 results are therefore nearer to the MER test results. In general, there was no significant improvement in the predicted MER, considering the P10, P50 and P90 percentiles of the predicted X. This is majorly as a result of taking the predicted X, as the mean of the random variables.



(a) PDF and CFD, with P10-P50-P90, for WELL 1

(b) PDF and CFD, with P10-P50-P90, for WELL 2

Figure 2. *PDFs and CDFs, with P10, P50, P90, for WELL1 and WELL 2.*

Table 4. *Comparison of predicted and test MER results for different percentiles of the calculated X.*

WELL	MER (bopd) (Test data)	Calculated X, log10 (MER)	MER (P10)	MER (P50)	MER (P90)	MER (bopd) (Equations (4) & (5))	% ERROR (P10)	% ERROR (P50)	% ERROR (P90)
1	210	2.6314	327.0393	427.9569	564.157	428	55.73	103.79	168.65
2	290	2.6637	351.0751	460.999	609.8177	461	21.06	58.97	110.28
3	180	2.5051	247.6852	319.9632	416.198	320	37.60	77.76	131.22
4	370	2.5038	246.9449	319.0068	414.954	319	-33.26	-13.78	12.15
5	464	2.6848	367.7901	483.9494	641.5049	484	-20.73	4.30	38.26
6	224.8	2.4683	228.1918	293.968	380.9781	294	1.51	30.77	69.47
7	450	2.3263	167.1091	211.9825	270.645	212	-62.86	-52.89	-39.86
8	188.5	2.3944	194.1333	247.9705	318.86	248	2.99	31.55	69.16
9	421	2.3856	190.4145	242.9965	312.1764	243	-54.77	-42.28	-25.85
10	536	2.4727	327.0393	427.9569	564.157	297	55.73	103.79	168.65

4. Improving the Erosional Velocity-Based Model Using MER Test Data

As earlier mentioned, the maximum efficient rate (MER) helps operators to achieve a critical balance between maximizing production and preserving reservoir integrity, ensuring that oil and gas resources are produced in a sustainable and environmentally responsible manner. Owing to the fact that oil and gas operators apply different methods to limit erosion-corrosion of mild steel lines and equipment during the production of hydrocarbons from underground reservoirs and couple with the fact various theories surrounding the formulation basis and sometimes the non-realistic C-values of the API RP 14E equation, given in Eq. (1), have been reported in many published reports, the idea of using MER test data to improve the robust erosional velocity model developed by Livinus and SeyeneOfon [7] from simulation data becomes expedient. To emphasize more, many studies have shown that various C-values have been adopted based on field and laboratory experiences. For instance, C-factors in the range of 145-195 could be considered for wells at their initial stage of completion [36, 37]. Castle and Teng [38] reported operational velocity up to three times the calculated value from the

API formula (3API) for various materials. Erichsen [39] and Salama [40] have reported a C-value of 726 for gas condensate wells and C-values above 300 for water injection wells, respectively.

The Multiple regression analysis was performed on the MER test data, after normalization, using regression tool available in Microsoft Excel package. The regression statistics show good fitting performance, with values of 0.878, 0.771, 0.753 and 0.322 for the multiple R, R-squared, adjusted R-squared and standard error, respectively.

The improved erosional velocity-based model for vertical oil wells in the Niger Delta is therefore presented in Equation (18);

$$V_e = \frac{THP^{-0.069} ID^{-1.24}}{\sqrt{\rho_{oil}}} \tag{18}$$

Comparing Equation (18) to Equation (1) shows that the C-values of the classical API RP 14E equation can be determined by Equation (19). This therefore will better represent vertical oil wells in the Niger Delta.

$$C = THP^{-0.069} ID^{-1.24} \tag{19}$$

Equation (18), together with Equations (6) and (7), could be used in the optimization of an oil well, and to perform MER estimation without test data for vertical oil wells in the Niger Delta. Either the equilibrium concept used for MER test data analysis, where plots of THP versus production rate and choke size versus rate are generated on the same graph and the point of intersection of the curves is considered the stable equilibrium and the corresponding rate, or the traditional iteration processes can be used for the MER estimation task.

5. Conclusion

The maximum efficient rate (MER) and erosional velocity are known to be vital concepts in oil and gas production that helps to achieve a critical balance between maximizing production and preserving reservoir integrity, ensuring that oil and gas resources are produced in a sustainable and environmentally responsible manner. This work used data of MER tests from vertical oil wells in the Niger Delta to develop MER model, and to improve an existing erosional velocity model.

The multiple linear regression tool available in Microsoft Excel package was used for the development of the MER model, using the MER test dataset collected from reports of some oil companies in Nigeria. Based on the knowledge of the parameters in existing choke models and the importance of other parameters, five (5) input variables were considered, namely: choke size (64th of an inch), tubing head pressure (psig), gas-oil ratio (scf/stb), basic sediment and water (%), and tubing internal diameter (ft). The regression statistics showed very good fitting performance to the normalized MER data. The predicted normalized MER results compared favorably well with the MER test data, with an absolute average error of 7.62%. For the case examples, MER test data results from ten (10) wells were verified using Equations (6) and (7); the discrepancies of the predicted MER results from the well test values were quite high. The percentage errors range from -103% to 53%; these error bandwidths are far away from the absolute average relative error of 7.62%, obtained from the multiple linear regression analysis; the error escalated from the de-normalization of the logarithmic function of X. However, due to increase of discrepancies between the predicted and the test MER results, observed in the de-normalization MER values, probabilistic modeling approach was carried out; the probability density and the cumulative distribution functions. Also, 10th, 50th and 90th percentiles were considered. For the case examples considered in the work, the predicted P10 MER results have an error bandwidth of -54% to 55.73%, while the predicted P50 MER results show an error range of -42.28% to 103.79%. The predicted P90 MER results also show high discrepancies with an error range of -39.86% to 168.65%. Improvement in the predicted probabilistic results depend on the mean value considered.

Lastly, improved erosional velocity-based model for vertical oil wells in the Niger Delta that could be used in the optimization of an oil well and to perform MER estimation without test data, either through the equilibrium concept or the traditional iteration processes, was developed. The combination of the MER model and the improved erosional velocity-based correlation can be a useful tool for MER test results verification and determination, and in overall for optimization of oil wells.

Abbreviations

MER	Maximum Efficient Rate
MLR	Multiple Linear Regression
API	American Petroleum Institute
THP	Tubing Head Pressure
C	Empirical Constant in Erosional Velocity Model
V_e	Erosional Velocity
ID	Tubing Internal Diameter
MPR	Maximum Permissible Rate
FA	Area Factor
FH	Thickness Factor
$F\emptyset$	Porosity Factor
FSw	Interstitial Water Factor
RM	Recovery Multiplier
L	Length in Metres of the Productive Portion of the Horizontal Wellbore of a Horizontal Well
GOR	Gas-Oil Ratio
BSW	Basic Sediment & Water
$Bean$	Choke Size
PDF	Probability Density Function

CDF	Cumulative Distribution Function
MATLAB®	Matrix Laboratory
P10	10^{th} Percentile
P50	50^{th} Percentile
P90	90^{th} Percentile

Acknowledgments

The authors would like to thank the management of Tertiary Education Trust Fund (TETFUND) Projects for the award of an institutional based research (IBR) grant, with reference number: TETF/DR&D/CE/UNI/UYO/IBR/2024/VOL.1. Also, the authors express their gratitude to the management of the University of Uyo and Topfaith University for providing enabling environments for the research.

Author Contributions

Aniefiok Livinus: Conceptualization, Funding Acquisition, Supervision, Methodology, Writing – original draft, Writing – review & editing

Moses Effiong Ekpenyong: Funding Acquisition, Supervision

Aniekan Morgan: Data curation, Formal analysis, Writing – review & editing

SeyeneOfon Emmanuel Ukpong: Data curation, Formal analysis, Writing – review & editing

Edet Ita Okon: Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Cardoso Filho, C., and Orazem, M. E. (2001). Application of a Submerged Impinging Jet to Investigate the Influence of Temperature, dissolved CO_2, and Fluid Velocity on Corrosion of Pipeline Grade Steel in Brine, CORROSION 2001, paper 01058. Paper No: C2001-01058, pp. 1-19; https://doi.org/10.5006/C2001-01058

[2] Papavinasam, S., and Revie, R. W. (2004). Review of Testing Methods and Standards for oilfield corrosion inhibitor., Proceedings of EUROCORR 2004, Nice, paper 10.

[3] Madani, S. F., Huizingab, S., Esaklul K. A., Nesic, S. (2019) Review of the API RP 14 Erosional Velocity Equation: Origin, Applications, Misuses, Limitations and Alternatives. Wear 426-427 (Part A): 620-636. https://doi.org/10.1016/j.wear.2019.01.119

[4] Shirazi, S., McLaury, B., Shadley, J., and Rybiki, E., (1995). Generalization of API TP 14E Guideline for Erosive Services. Journal of Petroleum Technology -J PETROL TECHNOL. 47: 693-698. https://doi.org/10.2118/28518-PA.

[5] Ariana, M. A., Esmaeilzadeh F. and Mowla, D. (2018). Beyond the Limitations of API RP-14E Erosional Velocity A Field Study for Gas Condensate Wells *Phys. Chem. Res.*, Vol. 6, No. 1, 193-207, https://doi.org/10.22036/pcr.2017.93963.1403

[6] American Petroleum Institute, (1984). Recommended Practice (1984) Design and Installation of Offshore Production Platform Piping Systems, API Recommended Practice 14E (RP 14E), Fourth Edition.

[7] Livinus, A. and Ukpong, S. E. (2023). How Oil and Gas Companies can derive C-Factors in the API RP 14E Erosional Velocity Models for their 'Clean Service' Pipelines. Pet Petro Chem Eng J 2023, 7(4): 000364. https://doi.org/10.23880/ppej-16000364

[8] Zangl, G., Hermann, R., Schweiger, C., (2014). Comparison of methods for stochastic multiphase flow rate estimation. In: Paper SPE 170866 MS Presented at the SPE Annual Technical Conference and Exhibition Held in Amsterdam. The Netherlands, 27 – 29 October 2014. https://doi.org/10.2118/170866-MS

[9] Savage W. K., Thachuk A. R., Hassan Beykpour and Lohrenz J. H., (1974). MER: Ultimate Recovery and Production Rate, SPE 4933 presented at the Rocky Mountain Regional Meeting of the Society of Petroleum Engineers of AIME, to be held in Billings, Mont., May 15-16, 1974. https://doi.org/10.2118/4933-MS

[10] Dan A. Bruce, (1976). Maximum Efficiency Rate – Its Use and Misuse in Production Regulation. Journal Article in Natural Resources Lawyer, Vol. 9, No. 3 (1976), pp 442.

[11] Rebecca A. Gallun, (2001). Fundamentals of Oil and Gas Accounting. Chapter 1, pp 24, 2001.

[12] Meyer H. I., Garder A. O. (1954) Mechanics of two immiscible fluids in porous media. J Appl Phys 25: 1400–1406. https://doi.org/10.1063/1.1721576

[13] Chierici GL, Ciucci GM, Pizzi G (1964) A systematic study of gas and water coning by potentiometric models. J Pet Technol 16: 923 929. https://doi.org/10.2118/871-PA

[14] Høyland LA, Papatzacos P, Skjaeveland SM (1989) Critical rate for water coning: correlation and analytical solution. Society of Petro leum Engineers, London. https://doi.org/10.2118/15855-PA

[15] Menouar HK, Hakim AA (1995) Water coning and critical rates in vertical and horizontal wells. Society of Petroleum Engineers, London. https://doi.org/10.2118/29877-MS

[16] Zhang L, Zhao Y, Liu Z (2011) A novel steady-state productivity equa tion for horizontal wells in bottom water drive gas reservoirs. Petrol Sci 8: 63–69. https://doi.org/10.1007/s12182-011-0116-2

[17] Tabatabaei M, Ghalambor A, Guo B (2012) An analytical solution for water coning in vertical wells. Society of Petroleum Engineers, London. https://doi.org/10.2118/113106-PA

[18] Obe, E., Davis, R., Reynolds, K., Ibrahim, J., Oragwu, A., Chukwu, F. & Inegbenose, A. (2011). Optimum Rate Estimation Guide in Mature Water-Prone Reservoirs. SPE 150792, the Nigeria Annual International Conference and Exhibition held in Abuja, Nigeria, 2011. https://doi.org/10.2118/150792-MS

[19] Aleruchi, W. M., Obinna, E., Sonny, T., (2019) Resolving Early time well performance rate issues using MER test data. NAPE Bulletin, V. 28 No. 2 (Nov. 2019), P. 84-88.

[20] Sukubo I. and Obi P. (2018). A New Approach to Maximum Efficiency Rate Estimation in the Niger Delta. SPE-193491-MS, Nigeria Annual International Conference and Exhibition held in Lagos, Nigeria, 6–8 August 2018. https://doi.org/10.2118/193491-MS

[21] Georgeson, V. U., Wumi, I., Ajienka, J. A. (2019). A New Framework for the National Petroleum Production Capacity Determination. International Journal of Scientific and Technical Research in Engineering (IJSTRE) Volume 4 Issue 2, March 2019. www.ijstre.com

[22] Directive PNG012 (2015). Allowable Rate of Production: Oil Wells. The Oil and Gas Conservation Act, Government of Saskatchewan, Canada.

[23] Mustafar, I. B. and Razali, R. (2011). A Study on Prediction of Output in Oilfield Using Multiple Linear Regression. International Journal of Applied Science and Technology, 1(4): 107-113.

[24] Gulstad, R. L. (1995). The Determination of Hydrocarbon Reservoir Recovery Factors by Using Modern Multiple Linear Regression Techniques, MSc Dissertation, TexasTech University, Lubbock Texas, USA.

[25] Gilbert, W. E., (1954). Flowing and gas-lift well performance. API Drilling and Production Practice 126–157, 1954.

[26] Ros, N. C. J. (1960). An analysis of critical simultaneous gas/liquid flow through a restriction and its application to flow metering. Applied Science Research 9, 374–388. https://doi.org/10.1007/BF00382215

[27] Al-Rumah, M., Bizanti, M., (2007). New Choke Correlation for Sabriyah Field Kuwait, Paper SPE 105103 Presented at the 2007 Middle East Oil and Gas Show and Conference, Bahrain, 11-14 March. Society of Petroleum Engineers. https://doi.org/10.2523/105103-MS

[28] Abdul-Majeed, G. H., (1988). Correlations developed to predict two-phase flow through wellhead chokes. June 12 - 16. In: Paper PETSOC-88-39-26 Presented at the 39th Annual Technical Meeting of the Petroleum Society of CIM Held in Calgary, pp. 1–14. https://doi.org/10.2118/88-39-26

[29] Beiranvand, M. S., Khorzoughi, M. B., (2012). Introducing a new correlation for multiphase f low through surface chokes with newly incorporated parameters. SPE Prod. Oper. 27(4), 422–428. https://doi.org/10.2118/158649-PA

[30] Choubineh, A., Ghorbani, H., Wood, D. A., Moosavi, S. R., Khalafi, E., Sadatshojaei, E., (2017). Improved predictions of wellhead choke liquid critical-flow rates: modelling based on hybrid neural network training learning-based optimization. Fuel 207, 547–560. https://doi.org/10.1016/j.fuel.2017.06.131

[31] Ganat, T. A., Hrairi, M., (2018). A new choke correlation to predict flow rate of artificially f lowing wells. J. Petrol. Sci. Eng. 171, 1378–1389. https://doi.org/10.1016/j.petrol.2018.08.004

[32] Jumaah, H. A., (2019). Modify chock performance equation for tertiary reservoir wells in khabaz oil field. 1st international conference on petroleum technology and petrochemicals. IOP Conf. Ser. Mater. Sci. Eng. 579, 012023, 2019. https://doi.org/10.1088/1757-899X/579/1/012023

[33] Christensen U. (1984). Convection with pressure-and temperature-dependent non-Newtonian rheology. Geophysical Journal International, Vol. 77, Issue 2, May 1984, Pages 343-384. https://doi.org/10.1111/j.1365-246X.1984.tb01939.x

[34] Wackerly, D. D., Mendenhall III, W., Scheaffer, R. L. (2002). Mathematical Statistics with Applications. 7th Edition, Thomson Higher Education, Belmont, CA, USA.

[35] Nevzorov, V. B., Balakrishnan, N., Ahsanullah, M. (2003). Simple characterizations of Student's t_2 distribution. Journal of the Royal Statistical Society: Series D (The Statistician), Vol. 52, Issue 3, p. 395-400. https://doi.org/10.1111/1467-9884.00367

[36] Esmaeilzadeh, F. (2004). Future South Pars Development may include 9 5/8-in. Tubing, Oil Gas J., 2004, 102, 53-57.

[37] Mansoori, H., Esmaeilzadeh, F., Mowla, D. et al. (2013) Case study: production benefits from increasing C-values, Oil Gas J., 2013. 111, 64-73.

[38] Castle, M. J. and Teng, D. T. (1991). Extending Gas Well Velocity Limits: Problems and Solutions. SPE-22958-MS, Paper presented at the SPE Asia-Pacific Conference, Perth, Australia, November, 1991. https://doi.org/10.2118/22958-MS

[39] Erichsen, H. (1988). Nipple, Lock, and Flow Coupling Recommendations and Sub-Assembly Description for North Sea wells, Priv. Commun. Nor. Conoco, 1988.

[40] Salama, M. M. (2000). An alternative to API 14E Erosional Velocity Limits for Sand-laden Fluids, J. Energy Resour. Technol., 2000, 122, 71-77. https://doi.org/10.1115/1.483167