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The Soil Slope Stability in Failure with the Use of the Random Process Based on the Kriging's Interpolation Model

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Abstract: The complexity of geotechnical engineerings such as slopes and Embankment is reflected not only in the change of various geotechnical parameters but also in the implicit, non-analytical, and even unverifiable nature of its functional models. Due to this feature, an example of a slope limit equilibrium model in algorithm development with an easy application for direct solution of slope engineering stability and reliability is investigated in this paper. First, the slope limit equilibrium model will be called to obtain a suitable example of the basic rock and soil parameters and the corresponding slope stability coefficient; Then, the anisotropic correlation mapping method of the Kriging model is used in ground statistics to express the value of slope performance function as a random process and process control variables are determined through samples, then Monte Carlo simulation and active learning methods together are combined. The test specimens are set according to the search rules and determine the most probable region of rupture on the slope where the slope function represented by the random process is obtained through an iterative loop, finally, the random process function is used to obtain the probability of slope rupture through simple and direct calculation in this field. Engineering analysis and calculation results show that the accuracy of this method is equivalent to the Monte Carlo simulation method, but the calculation process is simpler and has a lower and more economical calculation cost.

Keywords: Slope Stability, Random Process Kriging Model, Monte Carlo Method, Active Review

1. Introduction

Theory and methods of analysis and analysis of the stability and the reliability of the tunnel [1-3] and slope [4-9] have been one of the pillars of researchers in the field of geotechnical engineering in recent years. Researchers have introduced many new methods in their theoretical research and many new achievements have been made. The breakdown and analysis of the slope reliability coefficient on the Limit Equilibrium Method in most cases indicates an unclear performance and the traditional methods are not able to solve the confidence to use it. Using Monte Carlo simulation (MCS) to perform calculations directly can solve such problems and the calculation accuracy is high, but the high volume of computations and low computational efficiency limit its application in practical engineering. In recent years, some approximate model methods have been developed to express implicit performance functions such as Response Surface Methodology, Artificial Neural Network, Car Vector Support (SVM), Kriging model, and other methods [10]. Response Surface Methodology uses a small response to replace the implied function. It is simple and easy to run, but when dealing with complicated functions, nonlinearity errors occur [11]; Artificial Neural Network, on the other hand, will not have a good ability to amplify the problems of small samples of "excessive learning problems." [12]; (SVM) solves some of the Artificial Neural Network problems [13], but the selection of meta-parameters is time-consuming and its modeling cost is high [14]. The finite element method with the shear strength reduction technique is primarily employed to evaluate the slope stability of this profile. It is concluded that the inclination and strength of the weak zone and the water conditions are the most critical parameters and control the stability [15].

Integrates a new error rate-based adaptive Kriging method called REAK to analyze the failure probability of soil slopes. Under the weakly stationary spatial distribution assumption, mean and variance of soil properties are considered constant, and the autocorrelation function is defined using a spatial lag. The reliability method, REAK, employs Kriging surrogate modeling for adaptive and strategic sampling of random variables to generate most effective training points for model refinement this feature significantly reduces the high computational demand for failure probability analyses of slopes with spatially varying soil properties [16].

The proposed method efficient slope reliability analysis based on the Monte Carlo simulation (MCS) makes use of an active learning function and cross-validation techniques to select the most suitable training samples to update the SVM model. The proposed method can estimate the slope reliability with a small number of evaluations of the slope performance function, thus improving the efficiency significantly. Method performs better in terms of computational efficiency to obtain similar estimation accuracy of the failure probability for the investigated examples [17].

Reliability analyses to alleviate the computation work when employing the strength reduction method (SRM), an efficient analysis method based on active-learning radial basis function (ARBF) surrogate model is. This model uses the active-learning function to select trained samples near the limit state surface to update the surrogate model, which accelerates the convergence speed of the training process once a stable surrogate model is established, Monte Carlo simulation (MCS) is used to calculate the probability of system failure. Together with two typical soil slope cases, are tested to illustrate the computational efficiency and model stability of the introduced ARBF [18].

2. Kriging (Kriging Interpolation Method)

Interpolation is based on the estimation of spatial statistical models, a process in which the value of a quantity at points with known coordinates can be obtained using the same value of the same quantity at other points with known coordinates. One of the most important space statistic estimator methods is the Kriging model which was named after one of the pioneers of geostatistics, Danie G. Krige. Kriging is a method of estimation, which is based on the logic of the average moving weight-bearing. Kriging is a method of ground data with navigation data that are based on the variance of the space. Kriging variance space is known to be a function of the distance. Absolute estimation in interpolation is one of the main features of the Kriging model in the sense that the amount of quantitative estimation at the sampling points is equal to the measured value and the variance of the estimation is zero. This feature causes the Kriging estimator to draw lines. The equivalent value exceeds the maximum sampling points and does not tend to close and bypass and exceed the borders of the limited study. In other words, this model minimizes the amount of variance in estimating the unknown quantity of points or coordinates, and this estimation can be defined as:

$$\hat{Z}(s_0) = \sum_{i=1}^{N} \lambda_i Z(s_i) \tag{1}$$

$\hat{Z}(S_i)$= The measured value for the sample;

λ_i= Weight or importance attached to the sample;

S_0 = The forecast;

N = Number of the measured values.

Kriging model has good flexibility, when it is processing data for higher dimensions it overcomes the limitations of non-parametric models and has stronger predictability than parametric models.

3. Monte Carlo Simulation (MCS)

Simulation is the creation of a fictitious environment and the use of a theoretical model to estimate the behavior of an existing system in the real world in which the analyst attempts to model an organization in the real world. Monte Carlo simulation was named because it was first used in casinos in the Monte Carlo neighborhood (a city in Monaco) and is a technique for calculating the uncertainty in predicting a possible event. It is also called random number simulation. The Monte Carlo method requires a mathematical-statistical model with two general determinable and random components for the variable under consideration.

To the characteristics of the perfect model of Kriging, scientists in the year of the last of them in the field of construction and engineering to work has been. Kaymaz [19] used the method Kriging For parsing and analysis capabilities ensure structural use it. Zhang Qi and colleagues [20] proposed a method of measuring important to base on the model Kriging to calculate the reliability model proposes that and pointed out that this method of performance calculated above. Xie Yanmin and colleagues [21] stated the use of algorithms Kriging to calculate the reliability can be non-linear, the equation key to reducing the reference to the amount calculated to no effect on the accuracy of cut off. To improve efficiency, Akard and colleagues [22] proposed the idea of improvement MCS and from the algorithm Kriging based on benefits Kriging provide that. This article has the objective complexity of the analysis, the ability to ensure the stability of engineering, geotechnical, the idea of Akard And colleagues [23]. To the theory of the balance of the engineering slope combination has and a method of searching for active and based on the analysis and the analysis of reliability, stability of the slope Kriging create the works.

The method to be activated from the repeat loop to follow the example of the optimization model is and where possible most points to that in the stable development of the river, identify with it. To determine the probability of failure is just a bit of calculation required is the MCS The stage of decomposition and analysis of empirical reached is that you can to comfort for the reliability engineering, dip used to be.

4. Nonlinear Features of Slope Mode

Many methods and theories have been developed to analyze and design slope stability, but the limit equilibrium method is currently the earliest, most mature, and one of the most common methods for rock-soil slope stability analysis based on resistance theory. The cut is the color of Columbus. First, the soil or rock mass is assumed and the body of the model with slip potential is divided into several vertical blocks and it is assumed that these blocks are with objects with high plasticity. Next, the balance between the external force of the soil and the resistance created by the internal resistance is used to calculate the stability of the soil slope under its load. Slope stability assurance coefficient is defined as the ratio of shear strength along the entire soil surface to actual shear stress, ie $F_s = {}^{\tau_f}/_{\tau_0}$ as shown in Figure 1, each of the n soil strips in the landslide body is defined as i is shown. The known values are: the weight of the tape itself W_i, the low angle of inclination α_i, the width of the tape b_i, the low water pressure pores U, the low length l_i and the resistance index c_i and $tan\varphi_i$ at the slip surface.

For the whole body slip, to achieve a balance of forces, the unknowns needed to solve are as follows:

1. Natural reaction force N_i at the bottom of each block, number n;
2. The coefficient of stability of the slope F_s (assuming F_s of each block is equal) is defined by the coefficient of stability the tangential force T_i at the bottom of the bar can be natural. The forces N_i and F_s are obtained, ie:

$$T_i = \tau_i l_i = {}^{\left(N_i \tan\varphi_i + c_i l_i\right)}\Big/_{F_s} \tag{2}$$

3. Count the force adjacent bars between ordinary E_i strips at the interface, $i-1$;
4. The tangential force X_i (between E_i and X_i or angle fromθ_i) $i-1$ will be measured.
5. The position of the resulting force Z_i is a combination of the force E_i and X_i, the number $i-1$. Thus, there are a total of $4i-2$ unknowns, and all we can get is the balance of force and torque horizontally and vertically of each block. There are a total of $3n$ equations. Compared to the number of $i-2$ equations, there are still unknowns that cannot be solved, so some assumptions must be made to obtain the stability coefficient F_s.

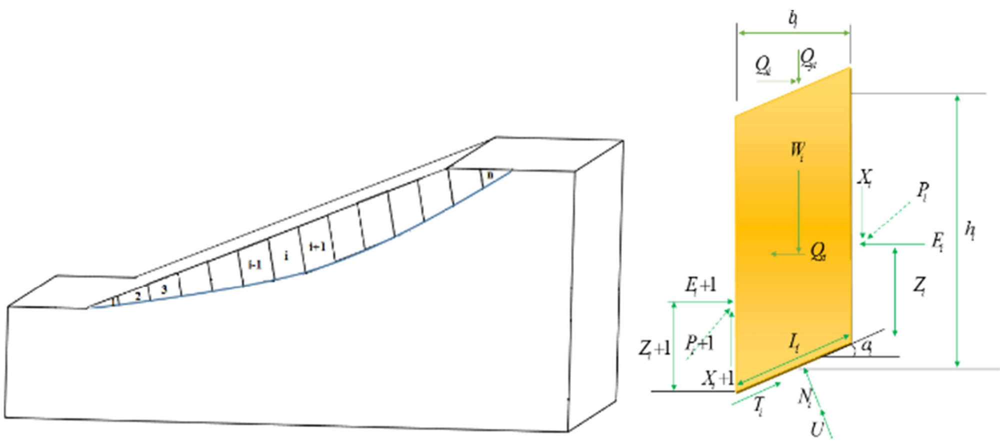

Figure 1. *Schematic view of ruptured surfaces.*

In this paper, the bishop method is applied and usable for rupture surface blocks, so it is selected. The Spencer method for rupture surface blocks is considered as an example, to obtain the rupture level reliability coefficient.

The Bishop method assumes that all X_i are zero, based on the equilibrium conditions, the formula for calculating the slope stability coefficient F_s can be obtained as follows:

And Spencer's law assumes that there is a fixed relationship between the force between the rupture surface blocks in the horizontal direction E_i and the perpendicular X_i:

Therefore, the directions of the force P are parallel between the rupture surface blocks. Given the limit equilibrium conditions, the Spencer method requires simultaneous equations (3) and (4) to solve the stability factor F_s.

$$\sum_{i=1}^{n} \frac{\dfrac{c_i b_i \sec\alpha_i}{F_s} + \dfrac{\tan\varphi_i}{F_s} W_i \cos\alpha_i - W_i \sin\alpha_i}{\cos\left(\alpha_i - \theta\right)\left[1 + \dfrac{\tan\varphi}{F_s}\tan\left(\alpha_i - \theta\right)\right]} = 0 \tag{3}$$

$$\sum_{i=1}^{n} \frac{\dfrac{c_i b_i \sec\alpha_i}{F_s} + \dfrac{\tan\varphi_i}{F_s} W_i \cos\alpha_i - W_i \sin\alpha_i}{\left[1 + \dfrac{\tan\varphi}{F_s}\tan\left(\alpha_i - \theta\right)\right]} = 0 \tag{4}$$

From Equation (1) and Equations (2) and (3) it can be seen that the Spencer and Bishop method of slope stability coefficient F_s is itself a function of C_i, φ_i, W_i and F_s, ie an implicit function, Etc is linear.

$$Z = F_s - 1 \tag{5}$$

It is noteworthy that the slope function of the function is also a complex nonlinear implicit function and it will be very difficult to use traditional methods to solve the slope stability reliability.

5. Stochastic Process Substitution Equation of the Performance Function Value

5.1. Kriging Random Process

The Kriging method was proposed by South African geologist Krige in 1951. It is a statistical forecasting method based on the stochastic process. It can obtain optimal, linear, and unbiased interpolation estimates for regionalized variables. It has a smoothing effect and statistics with the smallest estimated variance. Features [15]. The Kriging model assumes that the relationship between the response value of the system and the independent variables is expressed as follows:

$$y(x) = f^T(x)\xi + z(x) \tag{6}$$

It consists of a regression part and a random process, where ξ is the regression coefficient, $f\ (x)$ is the regression model, generally expressed as a polynomial $Z\ (x)$, is a random process with a mean value of 0 and a variance of σ_z, two interpolations The covariance of points $x^{(i)}, x^{(j)}$,

$$COV\left(z\left(x^{(i)}\right), z\left(x^{(j)}\right)\right) = \sigma_z^2 R\left(\rho; x^{(i)}, x^{(j)}\right) \tag{7}$$

In the formula, $x^{(i)}, x^{(j)}$ are the i, j components of the training sample, respectively, and $R = \left(\rho; x^i, x^{(j)}\right)$ is the correlation function with the parameterρ, which represents the spatial correlation between the training sample points. The most commonly used is Gaussian related functions (Gaussian Functions),

$$R\left(\rho; x^{(i)}, x^{(j)}\right) = \exp\left[-\sum_{k=1}^{n_{dv}} \rho_k \left(x^{(i)} - x^{(j)}\right)^2\right] \tag{8}$$

In the formula, n_{vd} is the number of known design variables, and p_k is the k element of the vector ρ.

Given a known training sample $S = \left[x^{(1)}, x^{(2)}, x^{(3)} \ldots, x^{(n)}\right]$ and its true response value $Y = \left[y^{(1)}, y^{(2)}, y^{(3)} \ldots, y^{(n)}\right]$, m is the capacity of the training sample, the estimated value of any point to be measured x_{new}.

$$\hat{y}(x_{new}) = f^T(x_{new})\hat{\xi} + r(x_{new}) R^{-1}\left(Y - F\hat{\xi}\right) \tag{9}$$

Where R is a symmetric matrix composed of $R = (\rho; S)$with a diagonal element of 1 and a size of$m \times m$; F is an m-dimensional vector composed of regression models at m sample points; $f(x_{new})$ is the regression polynomial, Is determined by the actual situation of the specific project, generally a polynomial not higher than the second-order can be used; $r(x_{new})$is the correlation vector between the test point and the training sample, and its expression is,

$$r(x_{new}) = \left[R\left(\rho; x_{new}, x^{(1)}\right), \ldots, R\left(\rho; x_{new}, x^{(m)}\right)\right] \tag{10}$$

Maximum likelihood estimation factor.

$$\hat{\sigma}_z^2 = \frac{1}{m}(Y - F\xi)^T R^{-1}(Y - F\xi) \tag{11}$$

Under the assumption of the Gaussian process, the relevant model needs to solve the unknown quantity ρ to construct the optimal Kriging model. According to the maximum likelihood estimate, it can be obtained,

$$\hat{\sigma}_z^2 = \frac{1}{m}(Y - F\xi)^T R^{-1}(Y - F\xi) \tag{12}$$

Where ρ can be solved by,

$$Min.\phi(\rho) = \left|R(\rho; S)\right|^{\frac{1}{m}} .\hat{\sigma}_z^2 \tag{13}$$

The optimization problem is obtained.

5.2. Stochastic Process Alternative Equation Construction

According to the basic principle of the Kriging model, the approximate display expression of the performance function is taken as formula (9), and the random parameter affecting the slope stability is expressed as a random variable $x = [x_1, x_2, x_3, \ldots, x_n]$, then the slope function of the approximate expression of function Z is:

$$\hat{Z} = \hat{y}(x) = f^T(x)\hat{\xi} = R^{-1}\left(Y - F\hat{\xi}\right) \tag{14}$$

First, given a known slope parameter training sample $S = \left[x^{(1)}, x^{(2)}, x^{(3)} \ldots, x^{(n)}\right]S$ with a capacity of m, substitute the limit balance method to obtain the true response$Y = Z = \left[Z^{(1)}, Z^{(2)}, Z^{(3)} \ldots, Z^{(m)}\right]$ of the sample, then obtain the performance function approximate expression (14) according to equations (10) to (13) coefficients. Take the regression polynomial $f(x)$ as 0th order [19], then the slope work can be obtained. Therefore, the Kriging stochastic model of energy function can be expressed as the following:

$$\hat{Z} = \hat{\xi} + r(x) R^{-1}\left(Y - 1\hat{\xi}\right) \tag{15}$$

The minimum mean square error between the performance function Z and its Kriging predicted value $\hat{Z}$ can be expressed as [18]

$$\sigma_{\hat{Z}}^2(x) = \sigma_{\hat{Z}}^2\left[1 + u(x)^T\left(1^T R^{-1} 1\right)^{-1} u(x) - r(x)^T R^{-1} r(x)\right] \quad (16)$$

In the formula $u(x) = 1^T r(x) - 1$, where 1 represents a vector whose element is 1 and length is m.

6. Active Search for the Most Likely Failure Area

6.1. Monte Carlo Simulation (MCS)

The Monte Carlo method is also called the random sampling method or statistical test method. This method is to solve the failure probability from the perspective of frequency. First, a large number of variables that affect reliability are sampled, and then these sampled values are substituted into the performance function one by one, and the number of performance functions that are less than zero is accumulated, thereby determining the slope Probability of destruction. The Monte Carlo method has no restrictions on the problem to be solved. As long as the number of random sampling is large enough, a very high-precision solution can be obtained. The failure probability expressed by the Monte Carlo method can be written as:

$$p_f = p\{Z \leq 0\} = \frac{N_{Z\leq 0}}{N_{MC}} \quad (17)$$

where Z is the slope of the performance function, $N_{(MC)}$ is the total sampling times, and $N_{Z\leq 0}$ is the number of samples whose performance function is less than or equal to zero.

6.2. Active Learning Search Rules

The active learning method was first proposed by Lewis and Gale and was first used for text classification in the field of machine learning. It changes the traditional way of passive learning from known samples, but in the learning process, according to a certain strategy, the samples that are most conducive to the performance of the classifier are selected from the unidentified samples to further train the classifier. As few training samples as possible to achieve the highest possible classification accuracy. The difference between active learning and passive learning is that it needs to interact with external systems during the active learning process, while passive learning depends entirely on the learner itself. The difference between the two is shown in Figure 2.

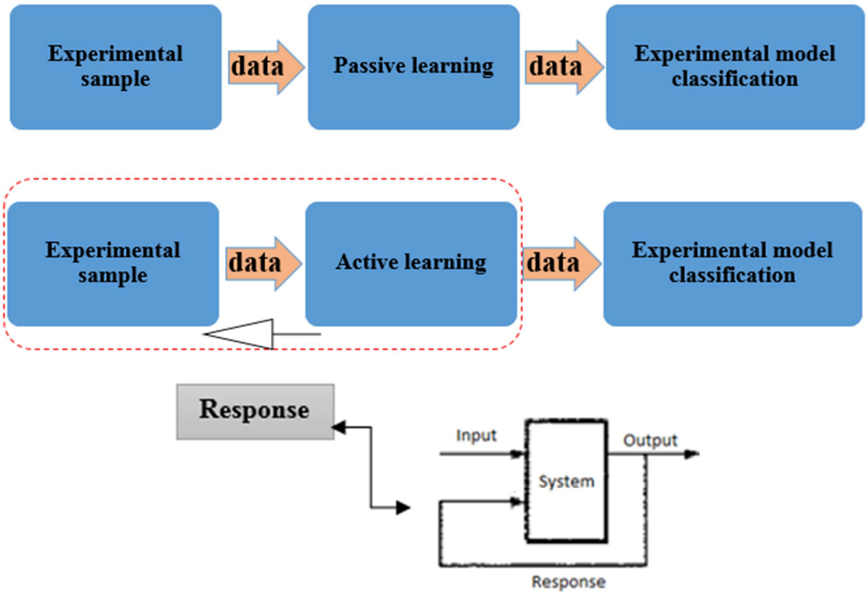

Figure 2. *Flow chart of active learning and passive learning.*

(5) Substitute x* into the limit balance method to obtain the true response value, update the training sample S, establish a new Kriging model, and repeat steps (4) ~ (5) until it meets min L(x) ≥2 [18] thinks that all the best training samples have been obtained and the learning has changed to be active.

The MCS method of slope reliability analysis is similar to the text classification in machine learning. It uses the surface formed by the performance function $Z = 0$ (ie the limit state surface) as the interface and divides the sample into two parts $Z \leq 0, Z > 0$ section. The key to the MCS method based on the Kriging fitting model lies in the fitting accuracy of the Kriging model to the sample points near the limit state surface and whether The Kriging model predicts the positive or negative function value of these points, which directly affects the failure of the final calculation. furthermore, is the probability accurate? Hence, these points are called dangerous points. The danger point has two characteristics:

1) Very close to the limit state surface, that is, the value of $|\hat{Z}(x)|$ is close to 0;
2) The Kriging variance $\sigma_{\hat{Z}}^2(x)$ corresponding to these points is relatively large.

Considering these two characteristics, construct the learning function $L(x)$ [18]

$$\left|\hat{Z}(x)\right| - L(x)\sigma_{\hat{z}}(x) = 0 \tag{18}$$

$L(x)$ Can measure the probability that the Kriging model predicts errors near the limit state surface.

Select a small number of training samples from x to establish the initial Kriging model, and then find the samples corresponding to $minL(x)$ to join the training samples, and repeat this process to achieve the purpose of improving the fitting accuracy of the Kriging model.

6.3. Execution Steps of Slope Reliability Search Method

The specific implementation steps of the slope reliability active learning algorithm with the combination of the Kriging model and MCS are as follows:

1) According to the actual problem of the slope, determination of the random variable $x = \{x_1, x_2, x_3, \dots, x_n\}$, that affects the stability of the slope, and draw a large-volume sample of x according to its statistical parameters and distribution, which is called the MC sample. The number isN_{MC}.
2) Randomly select a small number of samples from the MC sample as the initial training sample S, and substitute the limit balance method to obtain the response value of the sample (this paper uses the Slope/W module in Geo-Studio software to solve the problem, and the response value uses the minimum safety factor).
3) After obtaining the initial training sample and its response value, establish the initial Kriging model of the slope performance function according to sections 2.1 and 2.2 of this article, and predict the performance function value of the MC sample according to formula (15). The initial value of the failure probability of the slope can be obtained.
4) Active learning: The purpose is to search for the next best training sample point x^*, continuously update the training samples, and improve the prediction accuracy of the Kriging model for the performance function. The next best training sample point is the learning function (Eq. (18)) corresponds to the sample point at the minimum value.
5) Substitute x^* into the limit balance method to obtain the true response value, update the training sample S, establish a new Kriging model, and repeat steps (4) ~ (5) until it meets $minL(x) \geq 2$ [18] therefore, it thinks that all the best training samples have been obtained, which means it is at the active learning level.
6) Establish the optimal Kriging model from the best training samples, Using the formula (17), the slope failure probability P_f with higher accuracy can be obtained.
7) test whether the MC sample meets the requirements of the minimum number of samples required by the Monte Carlo method [24]

$$N_{MC} \geq \frac{(1-P_f)}{0.05^2 P_f} \tag{19}$$

If equation (19) is not satisfied, increase N_{MC} and restart learning until equation (19) is satisfied. The calculation process is shown in Figure 3.

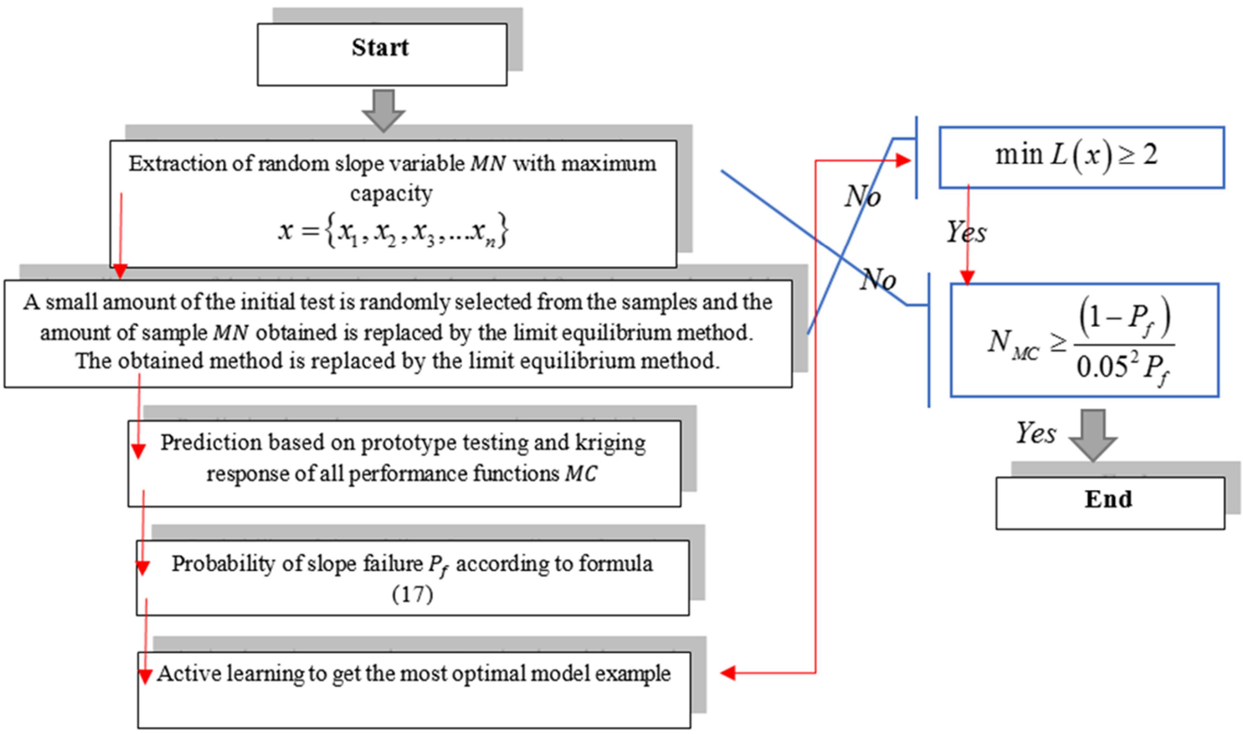

__Figure 3.__ Computational process flow chart.

7. Case Analysis

Two calculation examples are used to verify the feasibility of the method in this paper. Case 1 is a homogeneous slope, and Case 2 is a non-homogeneous slope.

7.1. Example 1

It is known that the cross-sectional geometry of a homogeneous soil slope is shown in Figure 5. Assume that the cohesion c, the internal friction angle φ, and the severity γ of each soil layer are all independent normal random variables, and their statistical characteristics are shown in Table 1.

The calculation is based on the Bishop method in the limit balance method, and the constructed 12 sets of initial training samples and their corresponding performance function values are shown in Table 2.

The initial Kriging model is established with these 12 sets of initial training samples, and active learning is carried out according to the description in Section 3.2.

When the training samples are updated to 28 groups, the active learning reaches convergence, that is, the accuracy of the Kriging model at this time is considered to be high enough, and the corresponding failure probability $P_f = 7.751 \times 10^{-4}$ is shown in Table 3.

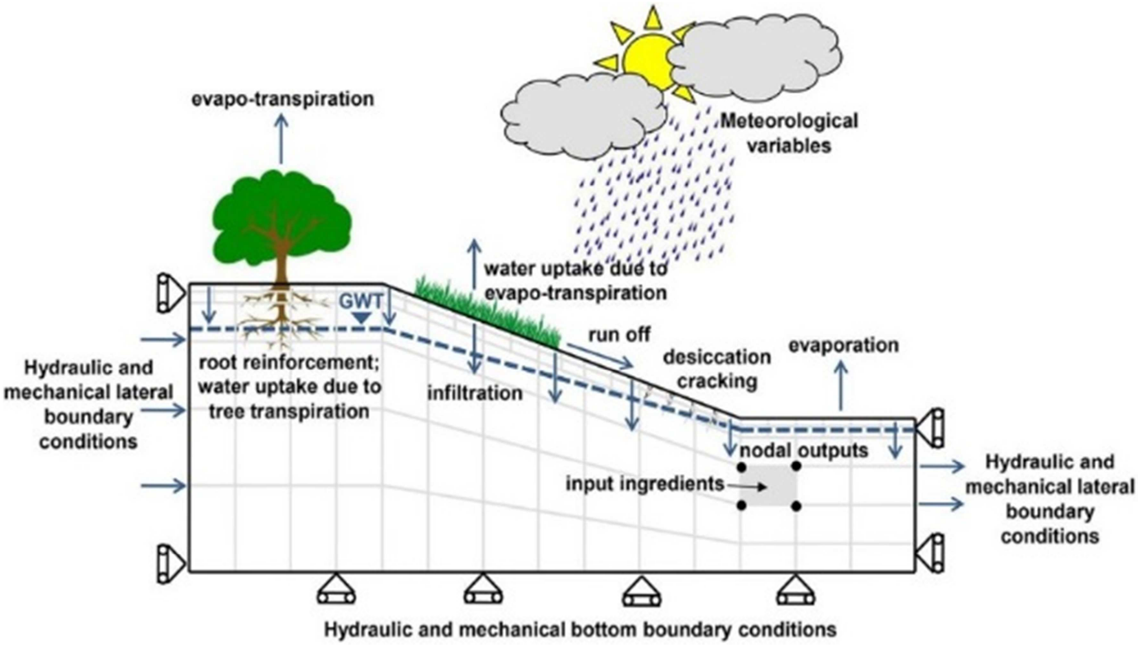

Figure 4. *Schematic overview of an embankment or natural slope of the ground.*

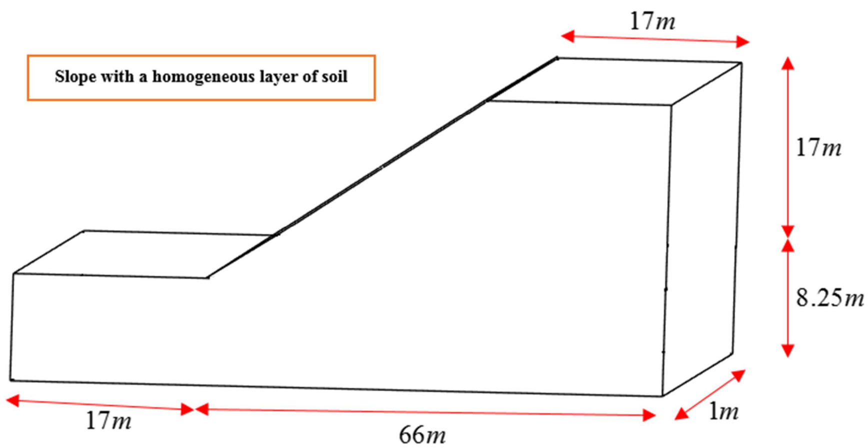

Figure 5. *Slope geometry of the first example.*

Table 1. *Statistical characteristics of soil parameters in the first example.*

Cohesion		Friction		Soil Specific Weight	
Total amount	Average standard deviation	Total amount	Average standard deviation	Total amount	Average standard deviation
27.75	4.95	29.7	5.94	33	6.6

***Table 2.** Experimental examples example 1.*

Sample	Cohesion	Friction	Soil Specific Weight	Factor of Safety
1	24.915	40.9035	33.462	2.3298
2	28.17	29.106	36.96	1.47015
3	20.4765	27.0105	29.8485	1.2111
4	20.3775	39.3855	18.4305	2.76705
5	25.1955	25.938	35.244	1.1781
6	31.3995	26.169	32.3895	1.5642
7	26.268	28.446	37.158	1.34475
8	29.733	27.357	35.772	1.45035
9	28.7265	34.452	14.5695	3.5838
10	29.436	27.7035	36.102	1.45365
11	32.373	27.093	32.3565	1.67805
12	18.7605	28.9245	31.1025	1.2177

***Table 3.** Calculation of results.*

Calculation Method	Calculated Value	Failure Probability Factor P_f	Factor of Safety β	Perecent Relative Error
Monte Carlo Method	16^5	10.1409×10^{-4}	5.332	Not
Response Level Method (RSM)	37.95	1.198×10^{-3}	5.016	9.801
The Method Used in this Article	46.2	7.7517×10^{-4}	5.458	3.795

It can be seen from Table 3 that when the calculation result of direct Monte Carlo simulation 16^5 times are an approximate accurate solution, the relative error of the slope stability reliability index calculated by the method in this paper is 3.795%, the error is small. Moreover, the slope stability limit equilibrium analysis and the calculation workload is only 28 times. Furthermore, the analysis of the stability equilibrium on the slope is only 46.2 times, which is far less than the Monte Carlo method.

7.2. Example 2

The cross-sectional geometry of a heterogeneous soil slope [14] is shown in Figure 6. The soil slope contains two soil layers and the soil gravity is γ= 21 $^{kN}/_{m^3}$. Set the cohesive force c of each soil layer and the internal friction angle φ as the mutual statistical characteristics of independent normal random variables are shown in Table 4.

The calculation is based on the Spencer method in the limit balance method, and the constructed 12 sets of initial training samples and their corresponding performance function values are shown in Table 5.

The initial Kriging model is established with these 12 sets of initial training samples, and active learning is carried out according to the description in Section 3.2. When the training samples are updated to 48 groups, convergence is achieved. It is considered that the accuracy of the Kriging model at this time is sufficiently high, and the corresponding failure probability is $P_f = 2.19 \times 10^{-2}$. The calculation results are shown in Table 6.

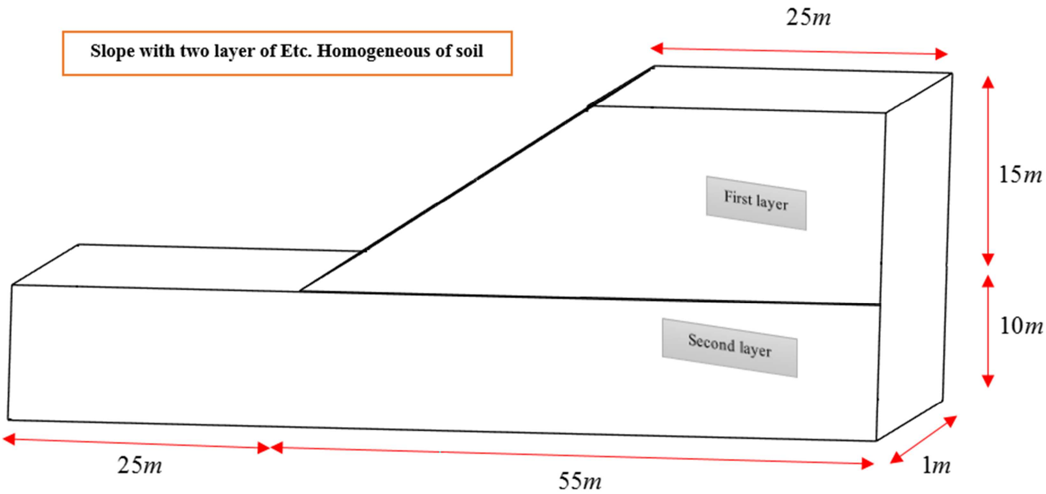

***Figure 6.** Slope geometry of the second example.*

***Table 4.** Statistical properties of soil parameters in the second example.*

Layers of Soil	Cohesion		Friction	
	Total Amount	Average Standard Devation	Total Amount	Average Standard Devation
1	63.2115	12.639	0	0
2	39.501	7.9035	19.8	19.8

Table 5. *Experimental examples of the second example.*

Sample	Cohesion	Friction	Soil Specific Weight	Factor of Safety
1	71.709	36.2505	18.084	0.9999
2	89.0505	46.4805	19.041	1.51305
3	512325	41.4645	21.4665	0.7194
4	47.784	44.22	18.2985	0.55935
5	63.5745	39.072	18.051	0.95865
6	74.415	38.742	20.2785	1.20615
7	89.001	31.944	20.724	1.21935
8	57.948	37.6035	21.9945	0.9438
9	77.6985	60.918	18.7935	1.7193
10	37.686	33.792	21.252	0.0924
11	54.549	43.7415	18.612	0.87285
12	60192	34.1055	19.833	0.85305

Table 6. *Calculates the results obtained.*

Calculation Method	Calculated Value	Failure Probability Factor P_f	Factor of Safety β	Perecent Relative Error
Monte Carlo Method	165^3	0.022935	3.63	Not
Response Level Method (RSM)	37.95	1.198×10^{-3}	5.016	9.801
The Method Used in this Article	79.2	0.021945	3.65	1.27

It can be seen from Table 6 that when the calculation result of the direct Monte Carlo simulation 165^3 times is the approximate accurate solution, the slope calculated by the method in this paper has the relative error of the stability reliability index of 1.27%. The calculation accuracy can meet the actual requirements of the project, and the calculation workload of the slope stability limit equilibrium analysis is only 48 times. The workload of calculating the limit equilibrium analysis is only 79.2 times the slope stability, which is less than one-thousandth of the Monte Carlo method.

8. Conclusion

Aiming at the complexity and uncertainty of slope stability reliability analysis, this paper takes slope as an example to study an easy-to-implement stability reliability calculation method, and the following results have been specifically achieved:

1) Using the Kriging geostatistical anisotropy correlation interpolation method, taking the Bishop model and Spencer model of limit equilibrium theory as examples, the stochastic process alternative expression equation of the slope function value is derived.
2) Introducing active learning rules, the iterative procedure for updating the replacement equation of slope performance function, and the search and determination method of the most likely failure area of stability are proposed.
3) A Monte Carlo simulation method that replaces the stochastic process equation in the most likely failure region is established; this method does not directly call the original performance function in the failure probability calculation, and the calculation process is straightforward.
4) Based on summarizing the detailed execution process of the above method, the application analysis of engineering examples shows that the process of the method is clear.

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