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A Note on (i,j)-πgβ Closed Sets in Intuitionistic Fuzzy Bitopological Spaces

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Abstract: In this paper we introduce the concept of (i,j)–πgβ-closed set in intuitionistic fuzzy bitopological spaces as a generalization of πgβ-closed set in fuzzy bitopological space and study their related notions in bitopological spaces. Next, we introduce (i,j)–πgβ- open sets in intuitionistic fuzzy bitopological spaces, and investigate some of their basic properties. Using these concepts, the characterizations for the intuitionistic fuzzy pairwise (i,j)–πgβ continuous mappings are obtained. The relationships between intuitionistic fuzzy pairwise (i,j)–πgβ continuous mappings are discussed. Finally, we prove the irresoluteness in (i,j)–πgβ intuitionistic fuzzy bitopological spaces.

Keywords: IF Bitopological Spaces, IF (i, j)β-Open Sets, IF(i, j)β-Closed Sets, IF (I,j) πgβ-Open Sets, IF (i,j) πgβ-Closed Sets, IF (i,j) πgβ-Pairwise Continuous Function and IF (i,j) πgβ-Irresolute Function

1. Introduction

The notion of β-open set was introduced by Abd El-Monsef et al. [1] and Andrijevic [2]. Later on, as a generalization of the above mentioned set, πgβ sets have been introduced by Caldas and Jafari [4]. The concept of bitopological spaces (X,τ_i,τ_j) was introduced by Kelly J. C in 1963 [8] where X is a nonempty set and the bitopological spaces are equipped with two arbitrary topologies τ_i and τ_j, Where τ_i and τ_j are two topologies on X. After that several authors turned their attention towards generalizations of various concepts of topology for bitopological spaces. In 2005, the concept of (i,j)-β-open sets was defined and investigated by Raja Rajeswari and Lellis Thivagar [9]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [3]. Recently, Coker [5] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. On the other hand, Kandil [7] introduced the concept of fuzzy bitopological spaces as a natural generalization of Chang's fuzzy topological spaces.

In 2012, the notion of bitopological space was introduced in intuitionistic fuzzy topology by - Jin Tae Kim, Seok Jong Lee [6]. In this paper, the concepts of πgβ-closed set have been extended to the bitopological spaces in intuitionistic fuzzy topology and we introduce a new form of closed set called Intuitionistic fuzzy (IF) (i,j)-πgβ-closed set., The notion of IF (i,j)- πgβ -continuous function and irresolute function is introduced and studied.

2. Preliminaries

The interior and the closure of a subset A of an intuitionistic fuzzy bi topological space (IFBTS) (X,τ) are denoted by Int(A) and Cl(A), respectively.

In the following sections by X, Y and Z, we mean an intuitionistic fuzzy bi topological space (X,τ_i,τ_j), (Y, σ_i,σ_j) and (Z,η_i,η_j), respectively.

Throughout the paper the triplet (X,τ_1,τ_2) denotes an intuitionistic fuzzy bi topological space (IFBTS) and (X,τ_i,τ_j) be the intuitionistic fuzzy bi topological space, where $i,j \in \{1,2\}$, and $i \neq j$.

For, a subset A of a bi topological space (X,τ_i, τ_j), we denote the closure of A and the interior of A with respect to τ_i by τ_i- Cl(A) and τ_i -Int(A), respectively.

Definition 2.1. Let A be an intuitionistic fuzzy set in an IFBTS (X,τ_i,τ_j), Then A is said to be an

(i) IF (i, j)-semi open [6] if there exists an IF,τ_i -open set U in X such that $U \subseteq A \subseteq$ j- Cl(U) and IF (i, j)-semi closed [6] if there exists an IF,τ_i - closed set U in X such that j-Int(U) ⊆ A ⊆ U.

(ii) IF (i, j)-preopen [6], A ⊂ τi - Int(τ_j - Cl(A)), and IF (i, j)-pre closed if

τi - Cl((τ_j - Int(A)).

(iii) IF (i, j)-β-open [7]) if A ⊆ τ_j - Cl(τ_I - Int(τ_j - Cl(A))), where i, j = 1, 2 and i≠ j, and

(i, j)-β-closed [7]) if (τ_i - Int (τ_j - Cl(τ_i - Int (A)))) ⊆ A.

Definition 2.2. A subset A of an intuitionistic fuzzy bi topological space (X,τi, τj) is said to be IF (i, j)-g-closed if τ_j - Cl(A) ⊆U and U ∈ τ_i where i, j =1,2 and i≠j.

Definition 2.3. A subset A of an intuitionistic fuzzy bitopological space (X,τi, τj) is said to be IF (i, j)-gβ-closed if (i,j)- βCl(A) ⊆U whenever A⊆U and U∈ τ_i.

Definition 2.4. [6]. Let A be an intuitionistic fuzzy subset of X, Then A is said to be IF (i,j) regular-open if

A= (τi -Int(τ_j -Cl(A))). The union of all IF (i,j) regular - open set is known as IF (i,j) π-open.

The complement of IF (i,j) π-open set is IF (i,j)-π-closed.

3. Intuitionistic Fuzzy (i,j)- πgβ- Closed Set

Definition 3.1. A subset A of an intuitionistic fuzzy bitopological space (X,τi,τj) is said to be IF(i,j)-πgβ-closed [6] if IF(j,i)–βCl(A) ⊆ U whenever A⊆U and U is IF(i,j)-π-open.

Definition 3.2 A subset A of a bitopological space (X,τi, τj) is said to be

(i) IF(i,j)-πgβ-closure of A, is defined by the intersection of all IF(i,j)-πgβ-closed sets containing A. That is IF(i,j)-πgβ-Cl(A) = ∩{F⊆X:F∈ (i,j)-πgβ-Cl(X), A⊆ F} &

(ii) The IF(i,j)-πgβ-interior of A is defined by the union of all IF(i,j)-πgβ-open sets contained in A. Thus IF(i,j)-πgβ-int(A)=∪{F⊆X: F∈(i,j)-πgβO(X), F⊆ A}

Theorem 3.1 Let (X,τ_1,τ_2) be an intuitionistic fuzzy bitopological space and A be a subset of X. Then x∈IF(j,i)-πgβ-Cl(A) if and only if for every IF(i,j)-πgβ-open set U containing x, such that U∩A≠∅.

Proof:

Suppose that x∈IF(j,i) πgβ-Cl(A),we shall show that U∩A≠∅ for every U∈IF (j,i) πgβO(X,x).

Suppose that there exists U∈IF(j,i) πgβO(X, x) such that U∩A ≠ ∅.

Then A⊆X\U and X\U is IF(i, j) πgβ-closed.

Since A⊆ X \U, IF(i, j) πgβ-Cl(A) ⊆IF(i, j) πgβ-Cl(X\U).

Since x ∈(j,i) πgβ-Cl(A), we have x∈(j,i) πgβ-Cl(X\U). Since X\U is IF(i,j)πgβ-closed,

we have x ∈X\U; hence x ∉U, which is a contradiction that x∈U. Therefore, U ∩A≠∅.

Conversely, suppose that U∩A≠∅ for every U∈IF(j,i) πgβO(X,x).

We can show that x ∈IF(j,i) πgβ-Cl(A). Suppose that x ∉(j,i) πgβ-Cl(A).

Then there exists U∈(j,i) πgβO(X, x) such that U∩A=∅.

This is a contradiction to U∩A ≠ ∅, Hence x∈IF(j,i) πgβ-Cl(A).

Lemma 3.1 Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. Then

(i) X\ IF(i,j)-πgβ-Int(A) = IF(i,j)-πgβ-Cl(X\A).

(ii) X\IF(i,j)-πgβ-Cl(A) = IF(i,j)-πgβ-Int(X\A).

Proof:

(i) Let x∈IF(i,j) πgβ-Cl(A). There exists V∈IF (i,j) πgβO(X, x) such that V∩A ≠ ∅.

Hence we obtain x∈(i, j) πgβ-Int(X\A). This shows that X\IF(i,j) πgβ-Cl(A)⊆IF(i,j)

πgβ-Int(X\A). Let x∈(i, j) πgβ-Int(X\A). Since IF(i,j) πgβ-Int(X\A)∩A=∅, we obtain

x∉IF(i, j) πgβ-Cl(A) hence x∈X\IF(i,j) πgβ-Cl(A).

Therefore, we obtain IF(i,j) πgβ Int(X\A)=X\IF(i,j) πgβ-Cl(A).

(ii) Similar to (i)

Remark 3.1: If A⊂B, then IF(i,j) β-Cl(A)⊂IF(i,j) β-Cl(B)&IF(i,j) β-Int(A)⊂IF(i,j)-β-Int(B) and hence IF(i,j) πgβ-Cl(A)⊂IF(i,j) πgβ-Cl(B) &IF(i,j) πgβ-Int(A)⊂IF(i,j) πgβ-Int(B).

Definition 3.3

A space X is said to be IF(i,j)-πgβT_1 if for any two distinct points x, y of X, there exists IF

(i, j)-πgβ open sets U, V such that x∈U but y∉U and y ∈V but x∉V.

Theorem 3.2 An intuitionistic fuzzy bitopological space X is IF (i,j) πgβT_1 if and only if {x} is IF(i,j)-πgβ closed in X for every x∈X.

Proof. If {x} is IF (i, j) πgβ closed in X for every x∈X, for x≠y, X\{x}, X\{y} are IF(i, j)-πgβ open sets such that y∈X\{x} and x∈X\{y}. Therefore, X is IF(i, j) πgβT_1.

Conversely, if X is IF (i,j)-πgβT_1, if and if y∈X\{x} then x≠y. Therefore, there exist IF(i,j) πgβ open sets Ui, Vj ∈X such that x∈U_i but y ≠Ui and y∈Vj but x∉Vj.

Let G be the union of all such Vj. Then G is an IF (i, j) πgβ- open set and G⊂X\{x}⊂X.

Therefore, X\{x} is an IF (i, j) πgβ open set in X.

Theorem 3.3: If A is IF(i,j) πgβ closed and A⊆B⊆(j,i) πgβ-Cl(A),then B is also IF(i,j) πgβ closed set.

Proof: Let U be an IF(i,j) π-open set in X such that B ⊆U, Since A is IF(i,j) πgβ closed, IF(j,i) πgβ-Cl(A)⊆U, B⊆IF(j,i) πgβ-Cl(A),implies IF(j,i)πgβ-Cl(B)⊆IF(j,i) πgβ-Cl(A), Hence IF(j,i) πgβ-Cl(B)⊆U. Hence B is also IF (i,j) πgβ closed.

Theorem 3.4 If A is IF(i, j) πgβ –closed set in X, then A∪(X\ IF(j,i) βCl(A) is also an

IF(i, j) πgβ closed set.

Proof: Let U be IF (i,j) π-open set in X such that A∪(X\ IF(j,i)-βCl(A)⊆U, then

(X\U)⊆ X\{A∪(X\IF(j,i)-βCl(A)}=X∩{A∪(X\i-Int(j-Cl(i-Int(A)}^{C}=X∩{A^C∩[IF(j,i) βCl(A)]=IF(j,i) πgβ-Cl(A)∩A^C = IF(j,i)- βCl(A)\A, since X\U is a IF(i,j) π–closed set and A is a IF(i, j) πgβ –closed set, by theorem X\U=ϕ and so X=U, Thus X is the only IF(i, j) πgβ -open set containing A∪(X\ IF(i, j) πgβ-Cl(A).

Hence A∪(X-IF(i, j)πgβ-Cl(A)) is a IF(i, j)-πgβclosed set.

Theorem 3.5: If A is IF(i,j) π-open and IF(i,j) πgβ-closed, then A is IF (i,j) β-closed.

Proof. Since A is IF (i,j) π-open and IF(i,j) πgβ-closed, IF(j,i) β-Cl(A)⊆A, but A⊆IF(j,i) β-Cl(A), So A=IF(j,i) β-Cl(A). Hence, A is IF (i,j) β closed.

Theorem 3.6. Let A be an IF(i,j)-πgβ-closed in intuitionistic fuzzy bitoplogical space X. Then IF(j,i) β-Cl(A)\A does not contain any nonempty IF(i,j) π-closed set.

Proof. Let U be a nonempty IF (i,j) π-closed subset of IF(j,i)-βCl(A)\A. Then A ⊆ X\U, where A is IF (i, j)- πgβ-closed and X\U is IF (i, j)-π-open. Thus IF(j,i)-βCl(A)⊆X\U, or U⊆X\IF(j,i)-βCl(A). Since by assumption U⊆IF(j,i)-βCl(A), we get a contradiction.

Corollary 3.2. Let A be IF(i,j)- πgβ-closed in X. Then A is IF(i,j) β-closed if and only if IF(j,i)-βCl(A)\A is IF(i, j)- π-closed.

Proof. Necessity: Let A be an IF(i, j) πgβ-closed, By hypothesis IF(j,i) β-Cl(A)=A and so IF(j,i) βCl(A)\A=φ which is IF(i, j) π-closed.

Sufficiency. Suppose IF(i,j) βCl(A)\A is IF(i,j) π-closed Then by theorem 3.6,

IF (j,i) β-Cl(A)\A=ϕ, that is, IF(j,i) β-Cl(A)=A. Hence, A is IF(i,j) β-closed.

Definition 3.4

In a bitopological space (X,τ_1, τ_2),Let B ⊆ A ⊆X. Then we say that B is IF(i,j) πgβ -closed relative to A if IF (j,i) $\beta Cl_A(B)$⊆ U where B⊆ U and U is IF(i,j) π-open in A.

Theorem 3.7 Let B⊆A⊆X where A is a IF (i,j) πgβ -closed and IF (i,j) π-open set. Then B is IF (i,j) πgβ-closed relative to A if and only if B is IF(i,j)-πgβ -closed in X.

Proof. Here, B⊆A and A is both a IF(i,j) πgβ -closed and IF(i,j) open set, then IF(j,i) βCl(A)⊆A and thus IF (j,i) β-Cl(B)⊆IF (j,i) β-Cl(A)⊆A. Now from, A\IF(j,i) β-Cl(B) = IF (j,i) $\beta Cl_A(B)$, we have IF(j,i)-βCl(B) =IF(j,i) β-$Cl_A(B)$ ⊆A. If B is IF(i,j)-πgβ -closed relative to A and U is IF(i,j) -π-open subset of X such that B ⊆U, then B = B∩A ⊆U ∩ A where U∩A is IF(i,j) π-open in A. Hence as B is IF(i,j)-πgβ-closed relative to A, IF (j,i)-β-Cl(B)=IF(j,i) β-$Cl_A(B)$⊆U∩A⊆ U. Therefore B is IF(i,j) πgβ -closed in X.

Conversely if B is IF (i,j)πgβ-closed in X and U is an Intuitionistic fuzzy bitopological

π-open subset of A such that B⊂U, then U=V∩A for some open subset V of X. As B⊆V and B is IF(i,j) πgβ-closed in X, IF(j,i) β-Cl(B)⊆V. Thus IF (j,i) $\beta Cl_A(B)$ =IF(j,i) β-Cl(B)∩A⊆V∩A=U.

Therefore B is IF (i,j) πgβ -closed relative to A.

Corollary 3.3. Let A be IF(i,j) π- open IF(i,j) πgβ -closed set. Then A∩U is IF(i,j) πgβ closed whenever U∈IF (j,i)β-Cl(X).

Proof. Since A is IF(i,j) πgβ--closed and IF(i,j) π-open, then IF(j,i) β-Cl(A)⊆A and thus A is IF(i,j) β-closed. Again, as U ∈ IF(j,i) β-Cl(X) and A ∈ IF(j,i)βCl(X), So A⋂U ∈ IF(j,i)βCl(X).

Now, A⋂U⊆A ⟹IF(j,i)β-Cl(A⋂U) ⊆ A which means that A⋂U is IF (i,j)-πgβ-closed.

Theorem 3.8 Let A and B be any two Intuitionistic fuzzy subsets of (X, τ_1,τ_2). Then the following properties hold:

IF(i, j) πgβ-Int(A∩B) ⊂ IF (i, j) πgβ-Int(A) ∩IF (i, j) πgβ-Int(B).

IF(i, j) πgβ-Int(A)∪(i, j)πgβ-Int(B) ⊂IF(i, j) πgβ Int(A∪B).

Proof. (i). Since A∩B⊂A and A∩B⊂B, by Remark 3.1 we have IF(i, j) πgβ Int(A∩B)

⊂IF(i, j) πgβ-Int(A) and IF(i,j) πgβ Int(A∩B)⊂IF(i,j) πgβ-Int(B).

Therefore, IF (i, j) πgβ Int(A∩B) ⊂IF(i, j) πgβ-Int(A)∩IF (i, j)πgβ-Int(B).

(ii). We have IF(i, j) πgβ-Int(A)⊂IF (i, j) πgβ Int(A∪B) and

IF(i, j)πgβ-Int(B)⊂IF(i, j) πgβ-Int(A∪B).

Then IF(i,j)πgβ-Int(A)∪IF(i,j)πgβ-Int(B)⊂IF(i,j) πgβ Int(A∪B).

Theorem 3.9 A subset A of X is IF (i,j)-πgβ-open if and only if U⊂IF(j,i) β-Int(A) whenever U is IF (i,j) π closed in X and U⊂A.

Proof: Necessity. Let A be IF(i,j)-πgβ-open and U⊂A, where U is IF (i, j) π -closed.

Then (X\A)⊂(X\U) and (X\U) is IF(i, j) π-open.

Therefore, IF (j,i)- β-Cl(X\A)⊂(X\U).

Hence IF (j,i) β-Cl(X\A) = X\IF(j,i) β-Int(A)⊂(X\U).

Thus we have U ⊂IF (j, i) β-Int(A).

Sufficiency. If U is IF (i, j) π-closed and U⊂IF (j,i) β-Int(A) whenever U⊂A, then (X\A)⊂(X\U) and (X\⊂IF(j,i) β-Int(A)⊂(X\U). That is, IF(j,i) πgβ-Cl((X\A) ⊂(X\U).

Therefore, (X\A) is IF (i, j)- πgβ -closed and hence A is IF (i,j) πgβ--open.

Definition 3.5

The IF (i,j) π-kernel of A is the intersection of all IF(i,j) π-open sets containing A and the intersection of all the IF(i,j) πgβ-open sets containing A which is by the usual notation, IF(i,j) πgβ-ker(A).

Lemma 3.2 Let X be an intuitionistic fuzzy bi topological space and x∈X. The following are equivalent.

(i) x ∈IF (i, j)-πgβker{y}

(ii) y ∈ IF(i, j)-πgβker{x}

Proof. (i) ⇒ (ii): If y ∉IF(i, j)-πgβker{x}, then there exists an IF(i, j)-πgβ open set U containing x such that x ∉IF (i, j)-πgβ ker{y}

(ii) ⇒ (i): *Proof* is similar.

Lemma 3.3 The following statements are equivalent for any two points x, y in an intuitionistic fuzzy bi topological space X.

(i) IF(i, j)-πgβker{x} ≠ IF(i, j)-πgβker{y}

(ii) IF(i, j)-πgβ-Cl{x} ≠IF(i, j)-πgβ-Cl{y}

Proof. (i) ⇒ (ii): Let IF(i, j)-πgβker{x} ≠ IF(i, j)-πgβker{y}

Then there exists a point z in X such that z ∈ IF(i, j)-πgβker{x}&z ∉IF (i, j)-πgβker{y} z∈IF (i,j)-πgβker{x}⇒{x}∩IF (i,j)-πgβker{z}≠ϕ. Then we have x∈IF (i, j)πgβ ker{z}. From

z∉IF (i, j)-πgβker{y} it follows that {y}∩IF(i,j)-πgβker{z}=ϕ. Since x∈IF(i, j)-πgβker{z},

IF(i, j)-πgβker{x} ⊂IF(i, j)-πgβker{z} and {y} ∩IF(i, j)-πgβker{x}=ϕ,

Hence IF (i, j)-πgβker{x} ≠ IF(i, j)-πgβker{y}.

(ii)⇒ (i): Let IF (i, j) πgβ-Cl{x} ≠ IF (i, j) πgβ-Cl{y}, Then there exists a point

z in X such that z ∈IF (i, j)-πgβ-Cl{x} and z ≠IF (i, j)-

πgβCl-{y}

Hence there exists an IF(i, j) πgβ -open set containing z and, x but not y. Therefore, y ∉ IF (i, j) πgβker{x} and IF (i, j) πgβker{x} ≠ IF(i, j) πgβker{y}

Definition 3.6 A space X is said to be IF (i, j)-πgβR_0 if every IF(i, j)-πgβ-open set contains the IF(i, j)-πgβ-closure of each of its singletons.

Lemma 3.4 A space X is IF(i, j)-πgβ R_0 if and only if for any x and y in X and IF(i,j)-πgβcl{x} ≠IF (i, j)-πgβcl{y} implies and IF(i, j)-πgβcl{x} ∩ IF(i, j)-πgβcl{y}=φ.

Proof. Necessity. If X is and (i, j)-πgβR_0 and x, y in X such that and (i, j)-πgβcl{x} ≠ (i, j)-πgβcl{y}then there exists z∈ and (i, j)-πgβcl{x} such that z ∉ (i, j)-πgβcl{y}, Therefore, there exists V∈ and (i, j) πgβ open set (X) such that y ∉ V and z ∈ V and hence x ∈V. Thus we get x ∉ (j,i) πgβ-Cl{y} and therefore, x ∈X\(j,i) πgβ-Cl{y}. This implies that (i,j) πgβ-Cl{x}⊂X\(i,j) πgβ-Cl{y}, and therefore, (j,i)-πgβ-Cl{x}∩(j,i)-πgβ-Cl{y}=φ

Sufficiency. Let V be (i, j)-πgβ open and x∈V. If y∈X\V, then x ≠y and x ∉(j,i)-πgβ-Cl{y}. This shows that (j,i) πgβ-Cl{x} ≠ (j,i) πgβ-Cl{y} and hence by our assumption, (j,i)-πgβ-Cl{x}∩(j,i)-πgβ-Cl{y}=φ Hence y ∉(j,i)-πgβ-Cl{x}. Therefore, (j,i)-πgβ-Cl{x} ⊂V.

Theorem 3.10 A space X is (i, j)-πgβR_0 if and only if for any x and y in X,

(i, j)-πgβker{x} ≠ (i, j)-πgβker{y} implies (j,i)-πgβ-Cl{x}∩(j,i)-πgβ-Cl{y}=φ.

Proof. Suppose that X is (i, j) πgβ -R_0 and if for any x and y in X, (i, j)-πgβker{x} ≠ (i, j)-πgβker{y} then by Lemma 3.3 (j,i)-πgβ-Cl{x} ≠ (j,i)-πgβ-Cl{y}. If z ∈(i, j)-πgβker{x} ∩(i, j)-πgβker{y} then from z ∈ (i,j)-πgβker{x} and by Lemma 3.2, it follows that x ∈ (i,j)-πgβker{z}. Since x∈(i,j)-πgβker{x}, by Lemma 3.4, (i,j)-πgβker{x}=(i,j)-πgβker{z}. Similarly, we have (i,j)-πgβker{y}= (i,j)-πgβker{z} is a contradiction. Therefore, (i,j) πgβker {x}∩ (i,j) πgβker {y} = φ

Conversely, let x, y be any two points in X such that (i, j)-πgβker{x} ≠ (i, j)-πgβker {y} implies

(i, j)-πgβker{x}∩(i, j)-πgβker{y}=ϕ; If (i, j) πgβ ker{x} ≠ (i, j) πgβ ker{y}, then by Lemma 3.3, (i, j)-πgβker{x} ≠ (i, j)-πgβker{y}, Hence (i,j)-πgβker{x} ∩ (i, j)-πgβker{y}=φ. (i,j)-πgβCl{x}∩ (i, j)-πgβCl{y}=φ. For, if z ∈ ((i, j)-πgβker{x} then x ∈(i, j)-πgβker{z} ≠ (i, j)-πgβker{y}) and therefore, (i, j)-πgβker{x}\(i,j)-πgβker{y}≠φ. Therefore, by hypothesis, ((i, j)-πgβker{x} ≠ (i, j)-πgβker{y} Then z ∈ ((i, j)-πgβ-Cl{x} ≠ (i, j)-πgβ-Cl{y} implies that (i, j)-πgβker{x} = (i, j)-πgβker{z} ≠(i, j)-πgβker{y} a contradiction. Therefore, by Lemma 3.4, X is (i, j)-πgβ R_0.

Theorem 3.11. Let (X,τ_1,τ_2) be an intuitionistic fuzzy bitopological space. If A and B are two IF(i,j)-πgβ-open sets in X such that IF(j,i)-πgβ-Cl(A)∩B=∅ and A∩IF(j,i)-πgβ-Cl(B)=∅, then A ∪ B is IF(i,j) πgβ-open.

Proof. Let A and B be two IF (i,j) πgβ-open sets in X such that IF(j,i) πgβ-Cl(A)∩B=∅. and A∩IF (j,i) πgβ-Cl(B)=∅. Suppose V is an IF π-closed and V ⊂ A∪B. Clearly

V⊂A and V⊂B. Then V ∩ IF(j,i) πgβ-Cl(A) ⊂ A∩IF(j,i)-πgβ-Cl(A) = A and V∩IF(j,i) πgβ-Cl(B) ⊂ B ∩IF(j,i) πgβ-Cl(B) = B. By hypothesis we have V∩IF(j,i) πgβ-Cl(A) ⊂ IF(j,i) πgβ-Int(A) and (V∩IF(j,i) πgβ-Cl(B) ⊂ IF(j,i) πgβ-Int(B).

This implies V∩(j,i) πgβ-Cl(A)⊂IF(j,i) πgβ-Int(A) and V ∩IF(j,i) πgβ-Cl(B)⊂IF (j,i) πgβ-Int(B). Then V∩IF(j,i) πgβ-Cl(A)⊂IF (j,i) πgβ-Int(A)∪(V ∩(j,i) πgβ-Cl(B)⊂IF (j,i) πgβ-Int(B), Which implies (V∩(j,i) πgβ-Cl(A)∪(j,i) πgβ-Cl(B)⊂IF (j,i) πgβ-Int(A)∪ IF (j,i) πgβ-Int(B).

Thus (V ∩IF (j,i) πgβ-Cl(A∪B)⊂IF (j,i) πgβ-Int(A) ∪IF (j,i) πgβ-Int(B). Further,

V= V∩(A∩B)⊂V∩IF (j,i)πgβ-Cl(A∪B), we have V⊂(j,i) πgβ-Int(A∪B)⊂(V∩(j,i) πgβ-Cl(A∪B)

⊂IF (j,i) πgβ-Int(A∪B)⊂(V∩IF(j,i) πgβ-Cl(A∪B)⊂(j,i) πgβ-Int(A)∪IF(j,i) πgβ-Int(B).

This shows that V ⊂IF(j,i) πgβ-Int(A∪B). Hence A∪B is (i,j) πgβ-open.

Theorem 3.12 Let (X,τ_1,τ_2) be an intuitionistic fuzzy bitopological space. If A is (i,j)− πgβ-open set relative to B such that A ⊂ B ⊂ X and B is (i,j)− πgβ-open relative to X, then A is (i,j) − πgβ-open relative to X.

Proof. Let U ⊂ A and U be IF (i,j) π-closed. Suppose A is (i,j) πgβ-open relative to B. Then we have U ⊂IF (j,i) β-Int(A), Which implies that there exists an IF (j,i) β-open set V_1 such that

U ⊂ V_1 ∩ B ⊂ A. Let U ⊂ B and U be IF π-closed. Suppose B is IF(i,j) − πgβ-open relative to X. Then we have U ⊂ IF (j,i) πgβ-Int(B) Which implies that there exists an IF (j,i) πgβ-open set V_2 such that U ⊂ V_2 ⊂ B. Further U ⊂ (V_1 ∩B)∩V_2) ⊂ (V_1 ∩B)∩B = V_1 ∩B ⊂ A. This shows that U⊂ IF (j,i) πgβ-Int(A). Hence A is IF (i,j) πgβ -open relative to X.

4. IF (i,j)- πgβ -Continuous Functions and Irresoluteness

Definition 4.1 Let f: $(X,\tau_1,\tau_2,) \to (Y,\sigma_1,\sigma_2)$ be a mapping from an IFBTS X to an IFBTS Y. Then f is said to be IF pairwise continuous if the inverse image of every,σi–open set of Y is IF (i,j)- open in X, where i ≠j, i, j = 1,2.

Definition 4.2 Let f: $(X,\tau_1,\tau_2,) \to (Y,\sigma_1,\sigma_2)$ be a mapping from an IFBTS X to an IFBTS Y. Then f is said to be IF pairwise πgβ continuous if $f^{-1}(A)$ is IF(i,j)πgβ -open in X for each IFσ_1–open set A in Y and $f^{-1}(B)$ is IF(j,i) πgβ -open in X for each IFσ_2–open set B in Y.

Theorem 4.1 For a function f: $(X,\tau_1,\tau_2) \to (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:

(i) f is IF pairwise πgβ -continuous.

(ii) For each point x in X and each σ_i -open set F in Y such that f (x) ∈ U, there exists an IF (i, j) -πgβ -open set A in X such that x ∈ A, f (A) ⊂ U.

(iii) The inverse image of each σ_i -closed set in Y is IF (i, j)- πgβ -closed in X.

(iv) For each subset A of X, f (IF(j,i) - πgβ -Cl(A))⊂σi-Cl(f(A)).

(v) For each subset B of Y, IF(j,i) πgβ-Cl(f^{-1}(B)) ⊂f^{-1} (σ_i -Cl(B)).

(vi) For each subset C of Y, f^{-1}(σ_i Int(C)) ⊂ (j,i) πgβ-Int(f^{-1}C)).

Proof:(i) $\Rightarrow$(ii): Let $x\in X$ and U be a σ_i –open set of Y containing f(x). By (i),$f^{-1}(U)$ is an IF(i, j)-$\pi g\beta$-open in X, Let $A= f^{-1}(U)$,Then $x\in A$ and $f(A) \subset U$.

(ii)$\Rightarrow$(i) Let U be σi – open in Y and let $x\in f^{-1}(U)$. Then $f(x)\in U$, By (ii) there is an IF (i, j)$\pi g\beta$ open set Ux in x such that $x\in Ux$ and $f(Ux) \subset U$, Then $x\in Ux \subset f^{-1}(U)$,

Hence $f^{-1}(U)$ is IF(i, j) $\pi g\beta$- open in X.

(i)$\Rightarrow$(iii) For any subset B of Y, $f^{-1}(Y\backslash B) = X\backslash f^{-1}(B)$

(iii)$\Rightarrow$(iv) Let A be a subset of X. Since $A\subset f^{-1}(f(A))$, we have $A\subset f^{-1}(\sigma i\text{-}Cl(f(A))$,Now σi-Cl(f(A) is σi- closed in Y. Hence IF (i,j)-$\pi g\beta$-Cl(A) $\subset f^{1}(\sigma i\text{-}Cl(f(A))$ for IF (i,j)- $\pi g\beta$-Cl(A) is the smallest IF(i,j) $\pi g\beta$- closed set containing A.

Then f(IF(i,j)- $\pi g\beta$-Cl(A)$\subset$ (σi-Cl(f(A)).

(iv)$\Rightarrow$(iii) Let U be any IF(i, j)- $\pi g\beta$-closed subset of Y.

Then f(IF(j,i) -$\pi g\beta$-Cl(f^{-1}(U))$\subset$ (σi-Cl(f(f $^{-1}$(U)))=σi-Cl(U)=U,

hence IF (j,i)-$\pi g\beta$-Cl(A)$\subset f^{-1}$(U) which implies f^{1}(F) is IF(i,j)-$\pi g\beta$ closed in X.

(iv)$\Rightarrow$(v) Let B be any subset of Y.

Now, f (IF(j,i)-$\pi g\beta$-Cl(f^{-1}(B))) $\subset\sigma_i$-$\pi g\beta$-Cl(A(f(f^{1}(B)))

$\subset\sigma_i$ -Cl(B).then, IF(j,i)-$\pi g\beta$-Cl(f^{1}(B))$\subset f^{1}(\sigma_i$ -Cl(B)).

(v)$\Rightarrow$(iv) Let B = f (A), where A is a subset of X.

Then, IF(i, j) $\pi g\beta$-Cl(A) $\subset$ IF(i, j)-$\pi g\beta$-Cl(f^{1}(B))

$\subset f^{1}(\sigma_i$ -Cl(B)) = $f^{1}(\sigma i$ Cl(f(A)),

Then f (IF(i,j) $\pi g\beta$-Cl(A))$\subset$(σi-Cl(f(A)).

(i)$\Rightarrow$(vi) Let B be any σi – open set in Y, then σi int(B)= B and $f^{1}(B)\backslash f^{-1}(\sigma_i$ -Int(B)) $\subset$IF(i, j)-$\pi g\beta$-Int(f^{1}(B)). Hence, f^{-1}(B)=IF(i,j)-$\pi g\beta$-Int(f^{1}(B)), Hence f^{-1}(B) is IF(i, j)-$\pi g\beta$-open in X.

Theorem 4.2. Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is ij-$\pi g\beta$-continuous and g: $(Y,\sigma_1,\sigma_2)\rightarrow (Z, \eta_1,\eta_2)$ is ij--$\pi g\beta$-irresolute function, then $g_0f:(X,\tau_1,\tau_2)\rightarrow (Z, \eta_1,\eta_2)$ is ij---$\pi g\beta$--continuous.

Proof. Let V $\in$ ij---$\pi g\beta$-C(Z), since g is ij---$\pi g\beta$-irresolute, then $g^{1}(V) \in$ ij---$\pi g\beta$-C(Y). Since f is ij---$\pi g\beta$--continuous, then $f^{1}(g^{1}(V)) = (g_0f)^{-1}$ (V) $\in$ji---$\pi g\beta$-C(X). Consequently, g_0f ij---$\pi g\beta$-continuous.

Theorem 4.3. Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be a function. If $g:(X,\tau_1,\tau_2)\rightarrow(X\times Y,\sigma_{1\times}\sigma_2)$ defined by g(x)= (x, f(x)) is an IF(i,j) $\pi g\beta$-continuous function, then f is IF(i,j) $\pi g\beta$-continuous.

Proof. Let V be a σ_i -open set of Y. Then $f^{-1}(V) = X \cap f^{-1}(V) = g^{-1}(X\times V)$. Since g is an IF(i,j)-$\pi g\beta$ -continuous function and X$\times$V is a $\sigma_{i\times}\tau_i$ -open set of X$\times$ Y, f^{1}(V) is an IF(i,j)-$\pi g\beta$ -open set of X. Hence f is an IF (i, j)-$\pi g\beta$-continuous.

5. IF (i,j)-πgβ–Irresolute Functions

Definition 5.1 A map $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is called IF(i,j) -$\pi g\beta$ -irresolute if for each $x\in X$ and each V$\in$IF(i,j)-$\pi g\beta$O(Y,f(x)), there exists U$\in$IF(i,j)-$\pi g\beta$O(X; x) such that f(IF(j,i)) $\pi g\beta$-Cl(U)) $\subseteq$(IF(j,i) $\pi g\beta$-Cl(V).

Theorem 5.1 For a function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ the following are equivalent.

(i) f is IF(i,j) -$\pi g\beta$-irresolute.

(ii) IF(i, j)-$\pi g\beta$-Cl(f^{1}(B) $\subseteq f^{-1}$(IF(i,j)-$\pi g\beta$-Cl(B)) for every subset B of Y.

(iii) f (IF(i, j)-$\pi g\beta$-Cl(A)) $\subseteq$ IF(i, j)-$\pi g\beta$-Cl (f(A) for every subset A of X.

Proof:

(i)$\Rightarrow$ (ii): Let B be any subset of Y. Suppose that $x \notin f^{-1}$(IF(i,j)-$\pi g\beta$-Cl(B)). Then f(x)$\notin$IF(i,j)-$\pi g\beta$-Cl(B) and there exists V$\in$IF(i,j)-$\pi g\beta$O(X,f(x)) such that IF(i,j)-$\pi g\beta$-Cl (V)$\cap$B =ϕ; Since f is IF(i,j)-$\pi g\beta$ irresolute, there exists U$\in$ IF(i,j)-$\pi g\beta$-O(X, x) such that f IF(i,j)-$\pi g\beta$-Cl(U))$\subseteq$IF(i,j)-$\pi g\beta$-Cl(V). Therefore, f(IF(i,j)-$\pi g\beta$-Cl)$\cap$B =ϕ; and IF(i,j)-$\pi g\beta$-Cl)$\cap f^{-1}$ (B) =ϕ.

Hence, x$\notin$ IF(i,j)-$\pi g\beta$-Cl(f^{-1} (B)),Therefore, IF(i,j)-$\pi g\beta$-Cl(f^{-1}(B)$\subseteq f^{-1}$(IF(i,j)-$\pi g\beta$-Cl (B).

(ii)$\Rightarrow$ (iii): Let A be any subset of X.

Then IF(i,j)-$\pi g\beta$-Cl(A)$\subseteq$IF(i,j)-$\pi g\beta$-Cl(f^{1}(f(A)) $\subseteq f^{1}$(IF(i,j)-$\pi g\beta$-Cl(f(A)) and hence f (IF(i,j)-$\pi g\beta$-Cl(A))$\subseteq$ IF(i,j)-$\pi g\beta$-Cl(f(A)).

(iii)$\Rightarrow$(ii): Let B be a subset of Y By (iii), f(IF(i,j)-$\pi g\beta$-Cl(f^{1}(B)))$\subseteq$IF(i,j)-$\pi g\beta$-Cl(f(f^{1}(B))) $\subseteq$IF(i,j)-$\pi g\beta$-Cl(B) and IF(i,j)-$\pi g\beta$-Cl(f^{1}(B))$\subseteq f^{1}$(IF(i,j)-$\pi g\beta$-Cl(B)).

(ii)$\Rightarrow$(i): Let $x\in X$ and V$\in$(IF(i,j)-$\pi g\beta$O(Y, f(x)). Then IF(i,j)-$\pi g\beta$-Cl((V) and

Y\(IF(i,j)-$\pi g\beta$-Cl(V) are disjoint and f(x)$\notin$IF(i,j)-$\pi g\beta$-Cl(Y\IF(i,j)-$\pi g\beta$-Cl((V). Hence x$\notin f^{1}$ (IF(i,j)-$\pi g\beta$-Cl(Y \ IF(i,j)-$\pi g\beta$-Cl((V)) and by (ii), x$\notin$ IF(i,j)-$\pi g\beta$-Cl(Y\IF(i,j)-$\pi g\beta$-Cl((V). Then there exists U $\in$ IF(i,j)-$\pi g\beta$O(X; x) such that IF(i,j)-$\pi g\beta$-Cl(U)$\cap$ f^{-1}(Y \ IF(i,j)-$\pi g\beta$-Cl(V) = ϕ and then f(IF(i,j)-$\pi g\beta$-Cl(U) $\subseteq$ IF(i,j)-$\pi g\beta$-Cl(V). Hence, f is IF(i,j)-$\pi g\beta$-irresolute.

Theorem 5.2. For a function f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ the following properties are equivalent.

(i) f is IF(i,j)-$\pi g\beta$-irresolute.

(ii) f^{1}(V)$\subseteq$IF(i,j)-$\pi g\beta$-Int(f^{-1}(IF(i,j)-$\pi g\beta$-Cl(V)) for every V$\in$IF(i,j)-$\pi g\beta$O(Y).

(ii) IF(i,j)-$\pi g\beta$-(f $^{-1}$(V)$\subseteq$f ^{1}(IF(i,j)-$\pi g\beta$-Cl(V) for every V$\in$IF(i,j)-$\pi g\beta$O(Y).

Proof. (i)$\Rightarrow$(ii): Let V$\in$ IF(i,j)-$\pi g\beta$O(Y) and x $\in f^{1}$(V). Then f(x) $\in$ V and there exists U $\in$ (IF(i,j)-$\pi g\beta$O(X, x) such that f (IF(i,j)-$\pi g\beta$-Cl(U)) $\subseteq$ (IF(i,j)-$\pi g\beta$-Cl(V). Thus x $\in$ U $\subseteq$ (IF(i,j)-$\pi g\beta$-Cl(U) $\subseteq$ f^{1} IF(i,j)-$\pi g\beta$-Cl(V)) and x $\in$ IF(i,j)-$\pi g\beta$-Int(f^{1}IF(i,j)-$\pi g\beta$-Cl(V). Hence f^{-1}(V)$\subseteq$IF(i,j)-$\pi g\beta$-Int(f^{1}IF(i,j)-$\pi g\beta$-Cl(V).

(ii)$\Rightarrow$(iii): Let V$\in$ IF ji-$\pi g\beta$O(Y) and x $\notin f^{-1}$(IF ji -$\pi g\beta$-Cl(V).

Then f(x) $\notin$ IF(i,j)-$\pi g\beta$-Cl(V) and there exists W$\in$IF(i,j)-$\pi g\beta$O(Y,f(x)) such that W $\cap$ V=ϕ. and IF(i,j)-$\pi g\beta$-Cl(W)$\cap$V=ϕ. Then f^{-1}(IF(i,j)-$\pi g\beta$-Cl(W)$\cap f^{1}$(V)=ϕ.

Now x $\in f^{1}$(W) and by (ii), x$\in$ IF(i,j)-$\pi g\beta$-Int(f^{-1} (IF(i,j)-$\pi g\beta$-Cl(W)).

There exists U $\in$ IF(i,j)-$\pi g\beta$O(X; x) such that F(i,j)-$\pi g\beta$-Cl(U)

$\subseteq$ f^{-1}(IF(i,j)-$\pi g\beta$- Cl(W)). Thus IF(i,j)-$\pi g\beta$-Cl(U) $\cap f^{1}$(V)=ϕ

and hence x $\notin$ IF(i,j)-$\pi g\beta$-Cl(f^{1}(V)).

Thus we get IF(i,j)-$\pi g\beta$-Cl(f^{1}(V))$\subseteq$ f^{-1} (IF(i,j)-$\pi g\beta$-Cl (V)).

(iii (i) Let x $\notin$X and V$\notin$ IF(i,j)-$\pi g\beta$O(Y,f(x)) Then

V∩(Y\IF(i,j)-πgβ-Cl(V)=ϕ

and f(x)∉ IF(i,j)-πgβ-Cl(Y\IF(i,j)-πgβ-Cl(V).

Therefore, x ∉ f^{-1} (IF(i,j)-πgβ -Cl(Y\(IF(i,j)-πgβ-Cl(V))) and by (iii),

x ∉ ((IF(i,j)-πgβ-Cl(f^{1}Y\(IF(i,j)-πgβ-Cl(V))).

There exists U∈ (IF(i,j)-πgβO(X, x) such that

(IF(i,j)-πgβ -Cl(U)∩f^{1}(Y \(IF(i,j)-πgβ-Cl(V)) =ϕ.

Hence f(IF(i,j)-πgβ-Cl(U)) ⊆ IF(i,j)-πgβ-Cl(V) and hence f is IF(i,j)-πgβ-irresolute.

Theorem 5.3 Let f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g: $Y,\sigma_1,\sigma_2) \rightarrow$ (Z, η_1, η_2) be any two functions. Then

(i) $g_o f$ is IF πgβ-continuous, if g is IF-continuous and f is IF-πgβ-continuous,

(ii) g_0 f is IF-πgβ-continuous, if g is IF-πgβ -continuous and f is IF-πgβ -irresolute,

(iii) $g_0 f$ is IF-πgβ irresolute, if g is IF-πgβ-irresolute and f is IF-πgβ-irresolute.

Proof.(i) Let V be IF closed in Z. Then g^{-1}(V) is closed in Y, since g is IF- continuous. IF (i,j) πgβ-continuity of f implies that f^{-1}(g^{-1} (V)) is IF πgβ -closed in X. Hence $g_0 f$ is IF- πgβ-continuous.

(ii) Let V be closed in Z. Since g is IF-πgβ -continuous, g^{-1} (V) is IF πgβ -closed inY. As f is IF πgβ -irresolute, f^{-1}(g^{-1}(V)) is πgβ -closed in X. Hence gof is IF-πgβ - continuous.

(iii) Let V be IF(i,j) πgβ-closed in Z. Then g^{-1} (V) is IF πgβ-closed in Y, since g is IF (i,j) πgβ-irresolute. Because f is IF πgβ -irresolute, f^{-1} (g^{-1} (V)) is IF πgβ -closed in X. Hence $g_0 f$ is IF πgβ -irresolute.

Definition 5.2: A function Let f:X → Y is called a IF (i,j)πgβ -homeomorphism if

(i) f is bijective.

(ii) f is IF (i,j)πgβ -irresolute.

(iii) f^{1} is IF (i,j)πgβ - irresolute.

We denote the collection of all the IF (i,j)πgβ - homeomorphisms f: X → Y by IF (i,j)πgβ h(X).

Theorem. 5.4: The collection IF (i,j) πgβ h(X) is a group.

Proof: Define a binary operation *: IF (i,j)πgβh(X) → IF (i,j)πgβ h(X) by *(f, g)= $g_0 f$.

Then * is well-defined and it is easily proved that under this binary operation IF(i,j)πgβh(X) is a group.

6. Conclusion

In this paper, we introduce the concept of πgβ closed set in intuitionistic fuzzy bitopological spaces and study some of their properties. We also introduce the concept of πgβ continuous functions in bitopological spaces and some of their properties have been established. We hope that the findings in this paper are just the beginning of a new structure and not only will form the theoretical basis for further applications of topology on Intuitionistic fuzzy bi topological sets but also will lead to the development of information system and various fields in engineering.

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