Mathematics and Computer Science
2023; 8(4): 94-103
http://www.sciencepublishinggroup.com/j/mcs
doi: 10.11648/j.mcs.20230804.12
ISSN: 2575-6036 (Print); ISSN: 2575-6028 (Online)

On Computing the Metric Dimension of the Families of Alternate Snake Graphs

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To cite this article:

Basma Mohamed, Mohamed Amin. On Computing the Metric Dimension of the Families of Alternate Snake Graphs. *Mathematics and Computer Science*. Vol. 8, No. 4, 2023, pp. 94-103. doi: 10.11648/j.mcs.20230804.12

Received: September 13, 2023; **Accepted**: October 8, 2023; **Published**: October 30, 2023

Abstract: Consider a robot that is trying to determine its current location while navigating a graph-based environment. To know how distant it is from each group of fixed landmarks, it can send a signal. We handle the problem of precisely identifying the minimum number of landmarks needed and their ideal placement to guarantee the robot can always discover itself. The number of landmarks in the graph is its metric dimension, and the collection of nodes on which they are distributed is its metric basis. The smallest group of nodes required to uniquely identify each other node in a graph using shortest path distances is known as the metric dimension of the graph. We consider the NP-hard problem of finding the metric dimension of graphs. A set of vertices B of a connected graph G resolves G if every vertex of G is uniquely identified by its vector of distances to the vertices in B. The minimum resolving set is called the metric basis and the cardinality of the basis is called the metric dimension of G. Metric dimension has applications in a wide range of areas such as robot navigation, telecommunications, combinatorial optimization, and pharmacocatual chemistry. In this paper, we determine the metric dimension of the family of alternate snake graphs including alternate snake, alternate k-polygonal snake, double alternate triangular snake and triple alternate triangular snake graph.

Keywords: Metric Basis, Metric Dimension, Alternate Snake Graphs

1. Introduction

Let $G=(V, E)$ be a connected graph and $d(u,v)$ be the shortest path between two vertices $u,v \in V(G)$. An ordered vertex set $B=\{x_1,x_2,\ldots,x_k\} \subseteq V(G)$ is a metric basis of G if the following two conditions are satisfied:

i) $\forall\ v \in V(G)$, the representation $r(v|B) = (d(v, x_1), d(v,x_2),\ldots,d(v, x_k))$ is unique.

ii) B with minimum cardinality.

The cardinality number of B is the metric dimension of G and is denoted dim (G).

Slater [1, 2] introduced the notion of a metric basis as a locating set of G and uses the cardinality of B as a locating number to uniquely identify the location of an intruder in a network. Harary and Melter [3] introduced independently the notion of metric basis as a resolving set of G and the cardinality of B as a metric dimension. In [4], Khuller et al. have shown that determining the metric dimension of a graph is an NP-complete problem. Many scholars have improved an upper bound for the metric dimension of several graphs or determined their exact values. In [9], Saputro et al. obtained a sharp bound for the metric dimension of the lexicographic product of a connected graph G and an arbitrary graph H. In [6], Chartrand et al. showed that the path graph P_n, $n \geq 2$ has a constant metric dimension 1, cycle graph C_n, $n \geq 3$ has metric dimension 2, complete graph K_n, $n \geq 2$ has metric dimension n-1 and complete bipartite graph $K_{s,t}$, $s,t \geq n \geq 4$ has metric dimension n-2. T is a tree of metric dimension 1, if T is a path.

In [10], Nawaz et al. proved that the total graph of path power three $T(P_n^3)$ and four $T(P_n^4)$ has an unbounded metric dimension. In [11], Nazeer et al. found that the metric dimension of two-middle path graph $Two-Mid(p_q)$, $q \geq 3$ is 2, three-middle path graph $Three-Mid(P_q)$, $q \geq 3$ is

3, three-total P_q $Three{-}T(P_q)$, $q \geq 3$ is 3, reflection middle tower path graph $RL(Tower_s)$, $s \geq 3$ is 2, middle tower path graph *Middle Tower*$_s$, s=2 is 1, *Middle Tower*$_s$, $s \geq 3$ is 2, symmetrical planar pyramid graph *SPPs* is 2 and reflection symmetrical planar pyramid graph *RL*(*SPPs*) is 2. In [12], Ahmad et al. determined the metric dimension of kayak paddle graph *KP* (ℓ, *m, n*) and cycles C_n with chord and proved that both families possess a metric dimension 2. In [13], Borchert et al. computed the metric dimension of the circulant graphs C_n (±1,±2) and have shown that if $n \equiv 1$ (mod 4), then dim(C_n (±1,±2)) = 4. In [14], Imran et al. investigated the metric dimension of barycentric subdivision of Möbius ladders, the generalized Petersen multigraphs *P*(*2n, n*) and proved that they also have metric dimension 3 when *n* is even and 4 when *n* is odd. In [15], Jäger et al. proved that the metric dimension for $Z_n \times Z_n \times Z_n$, $n \geq 2$ is $\left\lfloor \frac{3n}{2} \right\rfloor$. In [16], Ahmad et al. proved that the metric dimension of $P(n,2)ʘK_1$ graph is 3, where $ʘ$ is corona product. The metric dimension of convex polytopes has been studied in [17-20]. In [21], Imran et al. proved that the metric dimension of the m-level gear graph $J_{2n,m}$ $n \geq 4$, $m \geq 3$ is $\left\lfloor \frac{2n}{3} \right\rfloor + (m-1)\left\lceil \frac{2n}{3} \right\rceil$ and the metric dimension of the generalized gear graph J_{3n} is $\left\lfloor \frac{n}{2} \right\rfloor$ for every $n \geq 6$. In [22], Pan et al. showed that the metric dimension of the splitting graphs of path *S* (P_n) and cycle *S* (C_n), $n > 8$ are $\left\lceil \frac{n}{3} \right\rceil$. In [23], Siddiqui et al. computed the metric dimension of antiweb-gear graphs AWJ_{2n}, $n \geq 15$ is $\left\lceil \frac{n+1}{3} \right\rceil$, m-level wheel $W_{n,m}$, $n \geq 7$, $m \geq 3$ is $\left\lfloor \frac{2n+2}{5} \right\rfloor + (m-1)\left\lfloor \frac{2n+4}{5} \right\rfloor$. The first attempt to heuristically compute the smallest connected dominant resolving set of graphs using a binary version of the equilibrium optimization algorithm was made by Mohamed et al. [24]. The first binary implementation of the Enhanced Harris Hawks Optimization was provided by Mohamed et al. [25] in an attempt to heuristically compute the minimal connected resolving set of graphs. Mohamed et al. [26] investigated the metric dimension of subdivisions of a number of graphs, including the Lilly graph, the Tadpole graph, and the special trees star tree, bistar tree, and coconut tree. Mohamed et al. [27] examined the specific value of the secure resolving set for a few networks, including the trapezoid network, *Z*-(P_n) network, open ladder network, tortoise network, and $P_{2n} \,\overline{\vee}\, P_n$ network. Additionally, they calculated the domination numbers for several networks, including the twig network, double fan network, bistar network and linear kc_4 - snake network. Mohamed [28] focused on the contraction and bijection of the metric dimension when a robot is moving across a network that is modelled by the (2, 1) C_4-snake graph, $2\Delta_2$-snake graph and $3C_4$-snake graph.

Metric dimension has been used in several applications such as robot navigation in networks [4, 5, 29-31], application to pharmaceutical chemistry Chartrand et al. [[6]], application to pattern recognition Melter et al. [7], and application to wireless sensor network localization [8].

In this paper, the metric dimension of alternate snake graphs, double alternate triangular snake, and alternate triple triangular snake graph is investigated.

2. The Metric Dimension for Alternate Snake Graphs

Theorem 2.1 dim (*G*) =2 if *G* is

i) the alternate triangular snake *A*(T_n), $n > 3$.

ii) the alternate quadrilateral snake *A*(QS_n), $n \geq 8$.

iii) the alternate pentagonal snake *A*(PS_n), $n \geq 5$.

Proof. i) Consider the following cases of alternate triangular snake *A*(T_n).

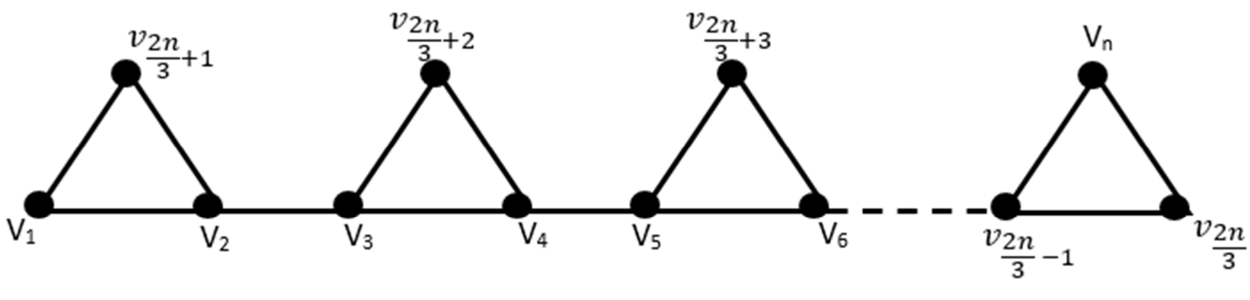

Figure 1. *Alternate triangular snake A(*T_n*).*

Case1. $n \equiv 0$(mod 3)

Let $n=3k$, $k > 1, k \in z^+$. The resolving set in general form is B = {v_1,v_{2k}} $V(A(T_n))$ as well as the representations of vertices $v_i \in V(A(T_n))$ in regard to *B* are

$$r(v_i|B) = \begin{cases} (i-1, 2k-i), & 1 \leq i \leq 2k; \\ (2i-n-5, 2n-2i+1), & 2k+1 \leq i \leq n. \end{cases}$$

Since all vertices have unique representations, we obtain dim($A(T_n)$) = 2.

B is also minimal, since Q = {v_1,v_2 } ⊂ *B* and *T*={ v_1,v_3 }⊂ *B* are not resolving sets:

Vertices v_{n-k} and v_n have equal distances to vertices of *Q* and *T*.

Case 2. $n \equiv 1$(mod 3)

Let $n=3k+1$, $k > 1, k \in z^+$. The resolving set in general form is B = {v_1,v_{2k}} $V(A(T_n))$ as well as the representations of vertices $v_i \in V(A(T_n))$ in regard to *B* are

$$r(v_i|B)=\begin{cases}(i-1,2k-i), & 1\le i\le 2k;\\ (i-1,1) & i=2k+1\\ (2i-n-5,2n-2i+1), & 2k+2\le i\le n.\end{cases}$$

Since all vertices have unique representations, we obtain dim($A(T_n)$) = 2.

B is also minimal, since $Q=\{v_1,v_2\}\subset B$ and $T=\{v_1,v_3\}\subset B$ are not resolving sets: vertices v_{n-k-1} and v_n have equal distances to vertices of Q and T.

Case 3. $n\equiv 2 (\mathrm{mod}\ 3)$

Let $n=3k+2$, $k>1, k\in z^+$. The resolving set is $B=\{v_1, v_{2k+1}\}$ $V(A(T_n))$ as well as the representations of vertices $v_i\in V(A(T_n))$ with regard to B are

$$r(v_i|B)=\begin{cases}(i-1,2k-i+1), & 1\le i\le 2k+1;\\ (i-1,1), & i=2k+2;\\ (2i-n-5,2n-2i+1), & 2k+3\le i\le n.\end{cases}$$

Since all vertices have unique representations, we obtain dim ($A(T_n)$) = 2.

B is also minimal, since $Q=\{v_1,v_2\}\subset B$ is not resolving set: vertices v_3 and v_{n-k+1} have equal distance to vertices of Q.

ii) The alternate quadrilateral snake $A(QS_n)$ has the following cases:

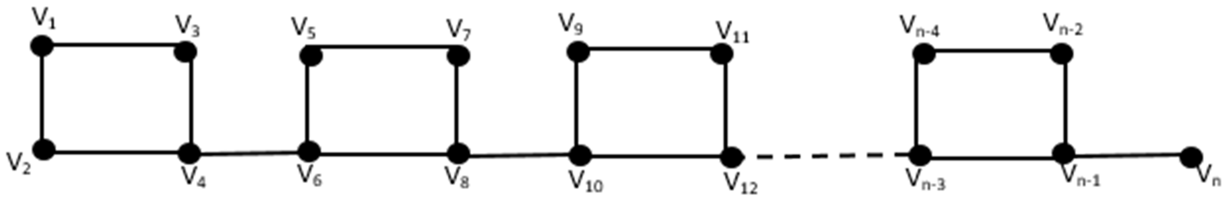

Figure 2. *Alternate quadrilateral snake A(QSn).*

Case 1: $n\equiv 0 (\mathrm{mod}\ 4)$

Let $n=4k$, $k\ge 2, k\in z^+$. The resolving set in general form is $B=\{v_2,v_{n-3}\}$ $V(A(QS_n))$ as well as the representations of vertices $v_i\in V(A(QS_n))$ in regard to B are as follow:

$$r(v_i|B)=\begin{cases}\left(\frac{i+1}{2},\frac{n-i+1}{2}\right), & 1\le i\le n-5;\ \text{for } i=1,3,\dots\\ \left(\frac{i-2}{2},\frac{n-i}{2}\right), & 2\le i\le n-4;\ \text{for } i=2,4,..\\ \left(\frac{i+1}{2},0\right), & i=n-3;\\ (2k-2,1), & i=n-2;\\ \left(\frac{i+1}{2},1\right), & i=n-1.\\ (2k-1,2), & i=n.\end{cases}$$

These representations are concluded to be distinct and dim ($A(QS_n)$) = 2.

B is also minimal, since $Q=\{v_2,v_3\}\subset B$ and $T=\{v_2,v_4\}\subset B$ are not resolving sets: vertices v_1 and v_4 have equal distance to vertices of Q *and* vertices v_1 and v_3 of T.

Case 2: $n\equiv 1 (\mathrm{mod}\ 4)$

Let $n=4k+1$, $k\ge 2, k\in z^+$. The resolving set in general form is $B=\{v_2,v_{n-4}\}$ $V(A(QS_n))$ as well as the representations of vertices $v_i\in V(A(QS_n))$ in regard to B are as follow:

$$r(v_i|B)=\begin{cases}\left(\frac{i+1}{2},\frac{n-i}{2}\right), & 1\le i\le n-6;\ \text{for } i=1,3,\dots\\ \left(\frac{i-2}{2},\frac{n-i-1}{2}\right), & 2\le i\le n-5;\ \text{for } i=2,4,..\\ (2k-1,0), & i=n-4;\\ \left(\frac{i-2}{2},1\right), & i=n-3;\\ (2k,1), & i=n-2;\\ \left(\frac{i-2}{2},2\right), & i=n-1.\\ (2k,3), & i=n.\end{cases}$$

These representations are concluded to be distinct and dim ($A(QS_n)$) = 2.

B is also minimal, since $Q=\{v_2,v_{n-5}\}\subset B$ and $T=\{v_2,v_{n-6}\}\subset B$ are not resolving sets:

vertices v_{n-1} and v_{n-4} have equal distance to vertices of Q and T.

Case 3: $n\equiv 2 (\mathrm{mod}\ 4)$

Let $n=4k+2$, $k\ge 2, k\in z^+$. The resolving set in general form is $B=\{v_1,v_{n-4}\}$ $V(A(QS_n))$ as well as the representations of vertices $v_i\in V(A(QS_n))$ in regard to B are as follow:

$$r(v_i|B) = \begin{cases} \left(\frac{i-1}{2}, \frac{n-i-1}{2}\right), & 1 \leq \text{i} \leq \text{n-3; for i=1,3,...} \\ \left(\frac{i+2}{2}, \frac{n-i}{2}\right), & 2 \leq \text{i} \leq \text{n-2; for i=2,4,..} \\ \left(\frac{i-1}{2}, 2\right), & \text{i = n-1.} \\ (\frac{i}{2}, 3), & \text{i = n.} \end{cases}$$

These representations are concluded to be distinct and dim $(A(QS_n)) = 2$.

B is also minimal, since $Q = \{v_1, v_{n-5}\} \subset B$ and $T = \{v_1, v_{n-7}\} \subset B$ are not resolving sets: vertices v_{n-1} and v_{n-4} have equal distance to vertices of Q and T.

iii) The alternate pentagonal snake $A(PS_n)$.

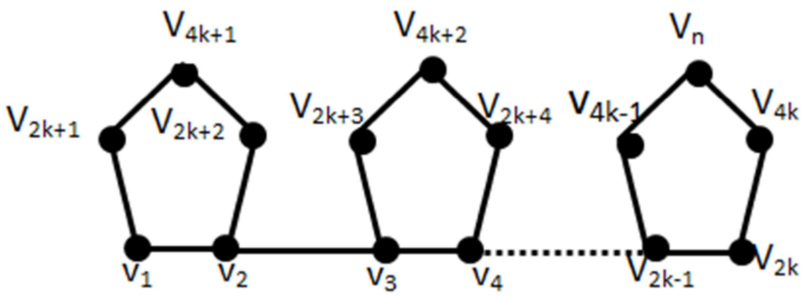

Figure 3. *Alternate pentagonal snake A(PSn).*

Case1. $n \equiv 0 \pmod 5$

Let $n = 5k$, $k \geq 1$, $k \in Z^+$. A minimal resolving set of $V(A(PS_n))$ is $B = \{v_1, v_{2k}\}$ $V(A(PS_n))$. The following representations of vertices $v_i \in V(A(PS_n))$ in regard to B are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i), & 1 \leq \text{i} \leq 2k; \\ (i-2k, n-i-2), & \text{2k+1} \leq \text{i} \leq 4k; \\ (2i-9k+3, 2n-2i+2), & \text{4k+1} \leq \text{i} \leq n. \end{cases}$$

Thus, dim $(A(PS_n)) = 2$.

Case 2. $n \equiv 1 \pmod 5$

Let $n = 5k+1$, $k \geq 1$, $k \in Z^+$. A minimal resolving set of $V(A(PS_n))$ is $B = \{v_1, v_{2k+1}\}$ $V(A(PS_n))$. The following representations of vertices $v_i \in V(A(PS_n))$ in regard to B are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i+1), & 1 \leq \text{i} \leq 2k+1; \\ (i-2k-1, n-i-1), & \text{2k+2} \leq \text{i} \leq 4k+1; \\ (2i-9k+1, 2n-2i+3), & \text{4k+2} \leq \text{i} \leq n. \end{cases}$$

Thus, dim $(A(PS_n)) = 2$.

Case 3. $n \equiv 2 \pmod 5$

Let $n = 5k+2$, $k \geq 1$, $k \in Z^+$. The resolving set in general form is $B = \{v_1, v_{2k+2}\}$ $V(A(PPS_n))$. The representations of vertices $v_i \in V(A(PPS_n))$ with regard to B are

$$r(v_i|B) = \begin{cases} (i-1, 2k+2-i), & 1 \leq \text{i} \leq 2k+2; \\ (i-2k-1, n-i), & \text{2k+3} \leq \text{i} \leq 4k+2; \\ (2i-9k-1, 2n-2i+3), & \text{4k+3} \leq \text{i} \leq n. \end{cases}$$

Since all vertices have different representations, we obtain dim $(A(PPS_n)) = 2$.

From case (1), case (2) and case (3), We conclude that metric dimension of alternate snake is 2.

Theorem 2.2. If $AS_n(C_k)$, $n \geq 3$, $k \geq 1$ is an alternate k-polygonal snake, then dim $(AS_n(C_k))=2$.

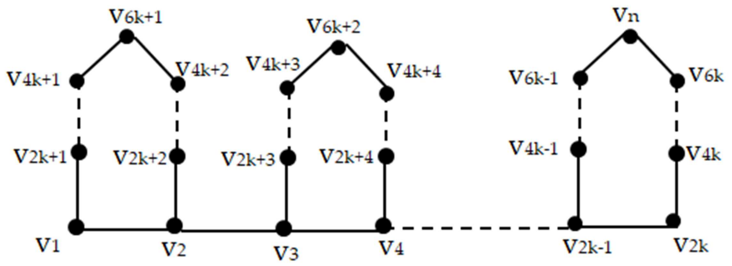

Figure 4. *Alternate k-pentagonal snake ASn(Ck).*

Proof. Consider the following cases of alternate k-pentagonal snake $AS_n(C_k)$.

Case 1. $n \equiv 0 (\text{mod } k)$

A minimal resolving set of $V(AS_n(C_k))$ is $B= \{v_1, v_{2k}\}$ $V(AS_n(C_k))$. The following representations of vertices $v_i \in V(AS_n(C_k))$ in regard to B are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i), & 1 \leq i \leq 2k; \\ (i-2k, 4k-i+1), & 2k+1 \leq i \leq 4k; \\ (i-4k+1, 6k-i+2), & 4k+1 \leq i \leq 6k; \\ (2i-12k+1, 2n-2i+3), & 6k+1 \leq i \leq n. \end{cases}$$

Thus, dim $(AS_n(C_k)) = 2$.

Case 2. $n \equiv 1 (\text{mod } k)$

A minimal resolving set of $V(AS_n(C_k))$ is $B= \{v_1, v_{2k+1}\}$ $V(AS_n(C_k))$. The following representations of vertices $v_i \in V(AS_n(C_k))$ in regard to B are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i+1), & 1 \leq i \leq 2k+1; \\ (i-2k-1, 4k-i+3), & 2k+2 \leq i \leq 4k+1; \\ (i-4k, 6k-i+4), & 4k+2 \leq i \leq 6k+1; \\ (2i-12k-1, 2n-2i+4), & 6k+2 \leq i \leq n. \end{cases}$$

Thus, dim $(AS_n(C_k)) = 2$.

Case 3. $n \equiv 2 (\text{mod } k)$

A minimal resolving set of $V(AS_n(C_k))$ is $B= \{v_1, v_{2k+2}\}$ $V(AS_n(C_k))$. The following representations of vertices $v_i \in V(AS_n(C_k))$ in regard to B are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i+2), & 1 \leq i \leq 2k+4; \\ (i-2k-1, 4k-i+4), & 2k+5 \leq i \leq 4k+2; \\ (i-4k-1, 6k-i+6), & 4k+3 \leq i \leq 6k+2; \\ (2i-12k+5, 2n-2i+3), & 6k+3 \leq i \leq n. \end{cases}$$

Thus, dim $(AS_n(C_k)) = 2$.

Theorem 2.3. If DA(Tn), n ≥ 8 is a double alternate triangular snake, then

$$\dim(DA(T_n)) = \begin{cases} \frac{n}{4} & if\ n = 8, 12, \ldots\ldots. \\ \frac{n-1}{4} & if\ n = 9, 13, \ldots\ldots. \\ \frac{n-2}{4} & if\ n = 10, 14, \ldots\ldots. \end{cases}$$

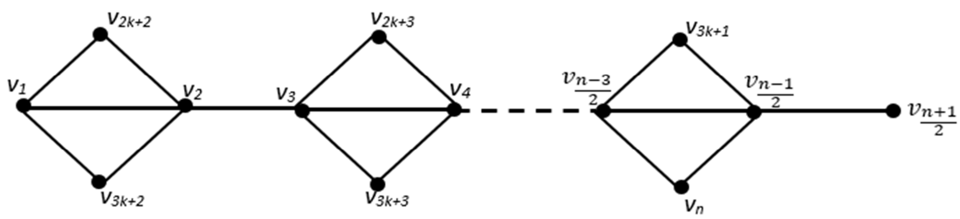

Figure 5. *Double alternate triangular snake $DA(T_n)$.*

Proof. The double alternate triangular snake has two cases.

Case 1. When k is even (k is the number of blocks)

Subcase 1. $n \equiv 0 (\text{mod } 4)$

We write $n=4k$, $k \geq 2, k \in z^+$. The resolving set in general form is $B = \{v_{2k+1}, v_{2k+2,\ldots,} v_{3k}\}$ $V(DA(T_n))$ as well as the representations of vertices $v_i \in V(DA(T_n))$ in regard to B are

$$r(v_i|B) = \begin{cases} (1,3,5,\ldots\ldots\ldots,2k-1), & i=1; \\ (1,2,4,\ldots\ldots\ldots,2k-2), & i=2; \\ (2,1,3\ldots\ldots,2k-3), & i=3; \\ (3,1,2,\ldots\ldots,2k-4), & i=4; \\ (i-1,i-3,\ldots,1,3,5,\ldots\ldots.,2k-i), & 5 \le i \le 2k-1, \text{ where } i \text{ is odd}; \\ (i-1,i-3,\ldots,1,2,4,\ldots\ldots.,2k-i), & 6 \le i \le 2k-2, \text{ where } i \text{ is even}; \\ (i-1,i-3,\ldots\ldots\ldots\ldots\ldots\ldots..,1), & i=2k; \\ (0,3,5,\ldots\ldots\ldots\ldots\ldots..,2k-1), & i=2k+1; \\ (3,0,3,\ldots\ldots\ldots\ldots\ldots\ldots..,2k-3), & i=2k+2; \\ (2i-5k+5,2i-5k+3,\ldots\ldots\ldots..,0,3,5,\ldots\ldots,7k-2i-1), & 2k+3 \le i \le 3k-1; \\ (2i-5k+5,2i-5k+3,\ldots\ldots\ldots..,0), & i=3k; \\ (2,3,5,\ldots\ldots.,2k-1), & i=3k+1; \\ (3,2,3\ldots\ldots\ldots\ldots\ldots\ldots,2k-3), & i=3k+2; \\ (2i-7k+5,\ 2i-7k+3,\ldots\ldots.,2,3,5,\ldots\ldots..,9k-2i-5), & 3k+3 \le i \le n-1; \\ (2k-1,\ 2k-3,\ldots\ldots\ldots\ldots.3,2), & i=n. \end{cases}$$

Assume that $B = \{v_{2k+1}, v_{2k+2,\ldots,} v_{3k}\}$ is the basis for $DA(T_n)$. Every element in $V((DA(T_n))$ has unique representation. So B is resolving set for $DA(T_n)$.

If we remove any vertex from B then it is not resolving set for $DA(T_n)$. Let $B' = \{v_{2k+1}, v_{2k+2,\ldots,} v_{3k-1}\}$ be subset of $V((DA(T_n))$ then there is at least one pair of vertices which gives same representation. So B' is not resolving set for $DA(T_n)$. Hence $\dim(DA(T_n)) = \frac{n}{4}$.

Subcase 2. $n \equiv 1 (\mathrm{mod}\ 4)$

Let $n=4k+1$, $k \ge 2$, $k \in z^+$. The resolving set in general form is $B = \{v_{2k+2}, v_{2k+3,\ldots,} v_{3k+1}\}$ $V(DA(T_n))$ as well as the representations of vertices $v_i \in V(DA(T_n))$ in regard to B are

$$r(v_i|B) = \begin{cases} (1,3,5,\ldots\ldots\ldots,2k-1), & i=1; \\ (1,2,4,\ldots\ldots\ldots,2k-2), & i=2; \\ (i-1,i-3,\ldots,1,3,5,\ldots\ldots.,2k-i), & 3 \le i \le 2k-1, \text{ where } i \text{ is odd}; \\ (i-1,i-3,\ldots,1,2,4,\ldots\ldots.,2k-i), & 4 \le i \le 2k-2, \text{ where } i \text{ is even}; \\ (i-1,i-3,\ldots\ldots\ldots\ldots\ldots\ldots..,1), & i=2k; \\ (i-1,i-3,\ldots\ldots\ldots\ldots\ldots\ldots..,2), & i=2k+1; \\ (0,3,5,\ldots\ldots\ldots\ldots\ldots\ldots..,2k-1), & i=2k+2; \\ (2i-5k+5,2i-5k+3,\ldots\ldots\ldots..,0,3,5,\ldots\ldots,7k-2i-5), & 2k+3 \le i \le 3k; \\ (i-k-2,i-k-4,\ldots\ldots\ldots.,0), & i=3k+1; \\ (2,3,5,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,2k-1), & i=3k+2; \\ (2i-7k+5,\ 2i-7k+3,\ldots\ldots.,2,3,5,\ldots\ldots..,9k-2i-5), & 3k+3 \le i \le n-1; \\ (2i-7k+5,\ 2i-7k+3,\ldots\ldots\ldots\ldots\ldots.,2), & i=n. \end{cases}$$

Since all vertices have different representations, we obtain dim $(DA(T_n)) = \frac{n-1}{4}$.

Subcase 3. $n \equiv 2 \pmod 4$

Let $n=4k+2$, $k \geq 2, k \in z^{+}$. The resolving set in general form is $B = \{v_{2k+3}, v_{2k+4,\dots,} v_{3k+2}\}$ $V(DA(T_n))$ as well as the representations of vertices $v_i \in V(DA(T_n))$ in regard to B are

$$r(v_i|B) = \begin{cases} (2,4,6,\dots\dots\dots,2k), & i=1; \\ (1,3,5,\dots\dots\dots,2k-1), & i=2; \\ (1,2,4,\dots\dots\dots,2k-2), & i=3; \\ (2,1,3,\dots\dots\dots,2k-3), & i=4; \\ (i-2,i-4,\dots,1,2,\dots\dots.,2k-i+1), & 5 \leq i \leq 2k-1, \text{ where } i \text{ is odd}; \\ (i-2,i-4,\dots,2,1,\dots\dots.,2k-i+1), & 6 \leq i \leq 2k, \text{ where } i \text{ is even}; \\ (i-2,i-4,\dots\dots,\dots\dots\dots.,3,1), & i=2k+1; \\ (i-2,i-4,\dots\dots\dots\dots\dots\dots..,4,2), & i=2k+2; \\ (0,3,5,\dots\dots\dots\dots\dots\dots..,2k-1), & i=2k+3; \\ (2i-5k+1,2i-5k-1,\dots\dots\dots..,0,3,5,\dots\dots,7k-2i-1), & 2k+4 \leq i \leq 3k+1; \\ (i-k-3,i-k-5,\dots\dots\dots.,0), & i=3k+2; \\ (2i-7k+1,2i-7k-1,\dots\dots.,2,3,5,\dots\dots..,9k-2i-1), & 3k+3 \leq i \leq n-1; \\ (2k-1,2k-3,\dots\dots\dots\dots\dots.,3,2), & i=n. \end{cases}$$

Since all vertices have different representations, we obtain dim $(DA(T_n)) = \frac{n-2}{4}$.

Case 2. When k is odd

Subcase 1. $n \equiv 0 \pmod 4$

Let $n=4k$, $k \geq 3, k \in z^{+}$. The resolving set in general form is $B = \{v_{2k+1}, v_{2k+2,\dots,} v_{3k}\}$ $V(DA(T_n))$ as well as the representations of vertices $v_i \in V(DA(T_n))$ in regard to B are

$$r(v_i|B) = \begin{cases} (1,3,5,\dots\dots\dots,2k-1), & i=1; \\ (1,2,4,\dots\dots\dots,2k-2), & i=2; \\ (i-1,i-3,\dots,1,3,5,\dots\dots.,2k-i), & 3 \leq i \leq 2k-1, \text{ where } i \text{ is odd}; \\ (i-1,i-3,\dots,1,2,4,\dots\dots.,2k-i), & 4 \leq i \leq 2k-2, \text{ where } i \text{ is even}; \\ (i-1,i-3,\dots\dots\dots\dots\dots\dots,1), & i=2k; \\ (0,3,5,\dots\dots\dots\dots\dots\dots,2k-1), & i=2k+1; \\ (2i-5k+4,2i-5k+2,\dots\dots\dots..,0,3,5,\dots\dots,7k-2i-4), & 2k+2 \leq i \leq 3k-1; \\ (i-k-1,i-k-3,\dots\dots\dots.,0), & i=3k; \\ (2,3,5,\dots\dots\dots\dots\dots\dots,2k-1), & i=3k+1; \\ (2i-6k-1,2i-6k-3,\dots\dots.,2,3,5,\dots\dots.,8k-2i+1), & 3k+2 \leq i \leq n-1; \\ (2k-1,2k-3,\dots\dots\dots\dots\dots,3,2), & i=n. \end{cases}$$

We assume that $B = \{v_{2k+1}, v_{2k+2,\dots,} v_{3k}\}$ is the basis for $DA(T_n)$. Every element in $V((DA(T_n))$ has unique representation. So B is resolving set for $DA(T_n)$.

If we remove any vertex from B then it is not resolving set for $DA(T_n)$. Let $B' = \{v_{2k+1}, v_{2k+2,\dots,} v_{3k-1}\}$ be subset of $V((DA(T_n))$ then there is at least one pair of vertices which gives same representation. So B' is not resolving set for $DA(T_n)$. Hence $\dim(DA(T_n)) = \frac{n-2}{4}$.

Subcase 2. $n \equiv 1 (\text{mod } 4)$

Let $n=4k+1$, $k \geq 3, k \in z^+$. The resolving set in general form is $B = \{v_{2k+3}, v_{2k+4,\dots,} v_{3k+2}\}$ $V(DA(T_n))$ as well as the representations of vertices $v_i \in V(DA(T_n))$ in regard to B are

$$r(v_i|B) = \begin{cases} (1,3,5,\dots\dots\dots,2k-1), & i = 1; \\ (1,2,4,\dots\dots\dots,2k-2), & i = 2; \\ (i-1, i-3,\dots,1,3,5,\dots\dots\dots,2k-i), & 3 \leq i \leq 2k-1, \text{ where } i \text{ is odd}; \\ (i-1, i-3,\dots,1,2,4,\dots\dots\dots,2k-i), & 4 \leq i \leq 2k-2, \text{ where } i \text{ is even}; \\ (i-1, i-3,\dots\dots\dots\dots\dots\dots,1), & i=2k; \\ (i-1, i-3,\dots\dots\dots\dots\dots\dots,2), & i=2k+1; \\ (0,3,5,\dots\dots\dots\dots\dots\dots,2k-1), & i=2k+2; \\ (2i-5k+2, 2i-5k,\dots\dots\dots,0,3,5,\dots\dots,7k-2i-2), & 2k+3 \leq i \leq 3k; \\ (i-k-2, i-k-4,\dots\dots\dots,0), & i=3k+1; \\ (2,3,5,\dots\dots\dots\dots\dots\dots,2k-1), & i=3k+2; \\ (2i-7k+2, 2i-7k,\dots\dots,2,3,5,\dots\dots,9k-2i-2), & 3k+3 \leq i \leq n-1; \\ (2i-7k+2, 2i-7k,\dots\dots\dots\dots,2), & i=n. \end{cases}$$

As all representations are distinct, we conclude that $\dim(DA(T_n)) = \frac{n-1}{4}$.

Subcase 3. $n \equiv 2 (\text{mod } 4)$

Let $n=4k+2$, $k \geq 3, k \in z^+$. The resolving set in general form is $B = \{v_{2k+3}, v_{2k+4},\dots,v_{3k+2}\}$ $V(DA(T_n))$ as well as the representations of vertices $v_i \in V(DA(T_n))$ in regard to B are

$$r(v_i|B) = \begin{cases} (2,4,\dots\dots\dots,2k), & i = 1; \\ (1,3,\dots\dots\dots,2k-1), & i = 2; \\ (i-2, i-4,\dots,1,2,4,\dots\dots\dots,2k-i+1), & 3 \leq i \leq 2k-1, \text{ where } i \text{ is odd}; \\ (i-2, i-4,\dots,2,1,3,\dots\dots\dots,2k-i+1), & 4 \leq i \leq 2k-2, \text{ where } i \text{ is even}; \\ (i-2, i-4,\dots\dots\dots\dots\dots\dots,1), & i=2k, 2k+1; \\ (i-2, i-4,\dots\dots\dots\dots\dots\dots,2), & i=2k+2; \\ (0,3,5,\dots\dots\dots\dots\dots\dots,2k-1), & i=2k+3; \\ (2i-5k+2, 2i-5k,\dots\dots\dots,0,3,5,\dots\dots,7k-2i), & 2k+4 \leq i \leq 3k+2; \\ (2,3,5,\dots\dots\dots\dots\dots\dots,2k-1), & i=3k+3; \\ (2i-7k, 2i-7k-2,\dots\dots,2,3,5,\dots\dots,9k-2i), & 3k+4 \leq i \leq n-1; \\ (2k-1, 2k-3,\dots\dots\dots\dots,3,2), & i=n. \end{cases}$$

As all representations are distinct, we conclude that dim ($DA(T_n)$) = $\frac{n-2}{4}$.

Theorem 2.4. If $AT(T_n)$, $n \geq 10$ is alternate triple triangular snake graph, then dim ($AT(T_n)$) = $\frac{n-k}{2}$.

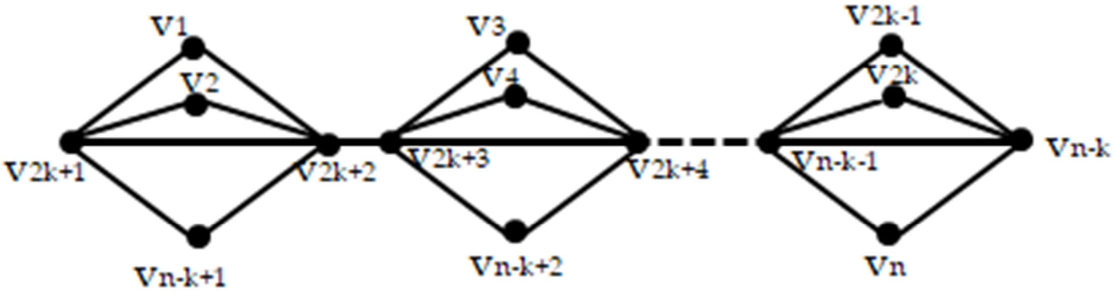

Figure 6. *Alternate triple triangular Snake graph AT(T$_n$).*

Proof. Let n=5k, $k \geq 2, k \in z^{+}$. The resolving set in general form is B = {$v_1, v_2, \ldots, v_{2k}$} $V(AT(T_n))$ as well as the representations of vertices $v_i \in V(AT(T_n))$ in regard to B are

$$r(v_i|B) = \begin{cases} (0,2,3,3,5,5,\ldots\ldots\ldots,2k-1,2k-1), & i=1; \\ (2,0,3,3,5,5,\ldots\ldots\ldots,2k-1,2k-1), & i=2; \\ (i,i,i-2,i-2,\ldots,3,3,0,2,\ldots\ldots.,2k-i,2k-i), & 3 \leq i \leq 2k, \text{ where } i \text{ is odd;} \\ (i-1,i-1,i-3,i-3,\ldots,2,0,3,3\ldots,2k+1-i,2k+1-i), & 4 \leq i \leq 2k, \text{ where } i \text{ is even;} \\ (i-1,i-1,i-3,i-3,\ldots\ldots\ldots\ldots\ldots\ldots..2,0), & i=2k; \\ (1,1,3,3,\ldots\ldots\ldots\ldots\ldots..,2k-1,2k-1), & i=2k+1; \\ (i-2k-1,i-2k-1,i-2k-3,i-2k-3,\ldots..1,1,2,2,\ldots\ldots,4k-i,4k-i), & 2k+2 \leq i \leq 4k-1; \\ (i-2k-1,i-2k-1,i-2k-3,i-2k-3,\ldots..1,1), & i=4k; \\ (2,2,3,3,5,5,\ldots\ldots\ldots..,2k-1,2k-1), & i=4k+1; \\ (3,3,2,2,3,3,\ldots\ldots.,2k-3,2k-3), & i=4k+2; \\ (2i-8k-1,2i-8k-1,2i-8k-3,2i-8k-3,\ldots,3,3,2,2,\ldots,10k-2i+1,10k-2i+1), & i=4k+3 \leq i \leq n-1; \\ (2k-1,2k-1,2k-3,2k-3,\ldots\ldots\ldots\ldots\ldots\ldots.,2,2), & i=n. \end{cases}$$

We conclude that these representations are distinct and dim ($AT(T_n)$) = $\frac{n-k}{2}$.

3. Conclusion

The metric dimension of alternate snake graphs have a constant metric dimension 2. The double alternate triangular snake and alternate triple triangular snake graph have an unbounded metric dimension as $n \rightarrow \infty$.

Open problem: Compute metric dimension of subdivision of alternate k-polygonal snake.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] Slater, P. J. Leaves of trees. *Congr Numer*. 1975; *14*, 549–559.

[2] Slater, PJ. Dominating and reference sets in a graph. *J. Math Phys Sci*. 1998; *22*: 445-455.

[3] Harary, F.; Melter, R. A. On the metric dimension of graph. *Ars Comb*. 1976, *24*, 191–195.

[4] Khuller, S.; Raghavachari, B.; Rosenfeld, A. Localization in Graphs; Technical Report CS-TR-3326; University of Maryland at College Park: College Park, MD, USA, 1994.

[5] Manjusha, R.; Sunny Kuriakose, A. Metric dimension and uncertainty of traversing robots in a network. *Int J Appl Graph Theory Wirel Ad Hoc Networks Sens Networks*. 2015, 7, 1-9.

[6] Chartrand, G.; Eroh, L.; Johnson, M. A.; Oellermann, O. R. Resolvability in graphs and the metric dimension of a graph. *Discret Appl Math*. 2000, *105*, 99–113.

[7] Melter, R. A.; Tomescu, I. Metric bases in digital geometry, Computer vision. Gr. *Image Process*. 1984, *25*, 113–121.

[8] Idrees, M.; Ma, H.; Wu, M.; Nizami, A. R.; Munir, M.; Ali, S. Metric Dimension of Generalized Möbius Ladder and its Application to WSN Localization. *J. Adv. Comput. Intell. Intell. Inform.* 2020, *24,* 3-11.

[9] Saputro, S. W., Simanjuntak, R..; Uttunggadewa, S. The metric dimension of the lexicographic product of graphs. *Discrete Math*. 2013, *313*, 1045-1051.

[10] Nawaz, S.; Ali, M.; Khan, M. A.; Khan, S. Computing Metric Dimension of Power of Total Graph. *IEEE Access*. 2021, *9*, 74550-74561.

[11] Nazeer S.; Hussain, M.; Alrawajeh F. A.; Almotairi, S. Metric Dimension on Path-Related Graphs. *Math. Probl. Eng*. 2021.

[12] Ahmad, A.; Bača, M.; Sultan, S. Computing the metric dimension of Kayak Paddles graph and Cycles with chord. *Proyecciones*. 2020, *39*, 287-300.

[13] Borchert, A.; Gosselin, S. The metric dimension of circulant graphs and Cayley hypergraphs. *Util. Math*. 2018, *106*, 125-147.

[14] Imran, M.; Siddiqui, M. K.; Naeem, R. On the metric dimension of generalized petersen multigraphs. *IEEE Access*. 2018, *6*, 74328-74338.

[15] Jäger, G.; Drewes, F. The metric dimension of $Z_n \times Z_n \times Z_n$ is 3n/2. *Theor. Comput. Sci*. 2020, *806*, 344-362.

[16] Ahmad, Z.; Chaudhary, M. A.; Baig, A. Q.; Zahid, M. A. On metric dimension of $P(n, 2) \odot K_1$ graph. *J. Discrete Math. Sci. Cryptogr.* 2021, *24,* 629-645.

[17] Imran, M.; Baig, A. Q.; Shafiq, M. K. Classes of convex polytopes with constant. *Util. Math.* 2013, *90*, 85-99.

[18] Imran, M.; AHMAD, A.; SEMANIČOVÁ-FEŇOVČÍKOVÁ, A. On classes of regular graphs with constant metric dimension. *Acta Math. Sci.* 2013, *33*, 187-206.

[19] Imran, M.; Bashir, F.; Baig, A. Q.; Bokhary, A. U. H.; Riasat, A.; Tomescu, I. On metric dimension of flower graphs $f_{n\times m}$ and convex polytopes. *Util. Math*. 2013, *92*, 389–409.

[20] Zuo, X.; Ali, A.; Ali, G.; Siddiqui, M. K.; Rahim, M. T.; Asare-Tuah, A. On Constant Metric Dimension of Some Generalized Convex Polytopes. *J. Math*. 2021.

[21] Imran, S., Siddiqui, M. K., Imran, M., Hussain, M., Bilal, H. M., Cheema, I. Z.,. & Saleem, Z. Computing the metric dimension of gear graphs. *Symmetry*, 2018, *10* (6), 209.

[22] Pan, H., Ali, M., Ali, G., Rahim, M. T., & Yang, X. On the families of graphs with unbounded metric dimension, IEEE Access, 2019.

[23] Siddiqui, H. M. A. & Imran, M. Computing the metric dimension of wheel related graphs. *Applied mathematics and computation*, 2014, *242*, 624-632.

[24] B. Mohamed, L. Mohaisen and M. Amin,"Binary Equilibrium Optimization Algorithm for Computing Connected Domination Metric Dimension Problem," *Scientific Programming*, vol. 2022, 2022.

[25] B. Mohamed, L. Mohaisen and M. Amin," Computing Connected Resolvability of Graphs Using Binary Enhanced Harris Hawks Optimization," *Intelligent Automation & Soft Computing*, vol. 36, no. 2, 2023.

[26] B. Mohamed and M. Amin, "The Metric Dimension of Subdivisions of Lilly Graph, Tadpole Graph and Special Trees," *Applied and Computational Mathematics*, vol. 12, no. 1, pp. 9-14, 2023.

[27] B. Mohamed and M. Amin, "Domination Number and Secure Resolving Sets in Cyclic Networks," *Applied and Computational Mathematics*, vol. 12, no. 2, pp. 42-45, 2023.

[28] B. Mohamed, Metric Dimension of Graphs and its Application to Robotic Navigation, *International Journal of Computer Applications*, vol. 184, no. 15, 2022.

[29] I. H. Jebril and N. M. Abdelqader,"Hyers-Ulam Stability of Quantum Logic Fuzzy Implication", *WSEAS Transactions on Information Science and Applications*, vol. 20, pp. 131-135, 2023.

[30] B. Mohamed and M. Amin, "A hybrid optimization algorithms for solving metric dimension problem," *International Journal on Applications of Graph Theory in Wireless Ad hoc Networks and Sensor Networks* (*GRAPH-HOC*), vol. 15, no. 1, 2023.

[31] B. Mohamed, "A Comprehensive Survey on the Metric Dimension Problem of Graphs and Its Types, *International Journal of Theoretical and Applied Mathematics*, vol. 9, no. 1, 2023.