Pure and Applied Mathematics Journal
2014; 3(1): 11-13
Published online March 20, 2014 (http://www.sciencepublishinggroup.com/j/pamj)
doi: 10.11648/j.pamj.20140301.13

Topological BCL$^+$- algebras

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To cite this article:
Yonghong Liu. Topological BCL$^+$- Algebras. *Pure and Applied Mathematics Journal.* Vol. 3, No. 1, 2014, pp. 11-13.
doi: 10.11648/j.pamj.20140301.13

Abstract: BCL/BCL$^+$-algebras [1, 2] are more extensive class than BCK/BCI/BCH-algebras [5-8], introduced by Yonghong Liu. In this paper, we aim to embed topological structure in BCL$^+$-algebras, thus we should be introduced to the notion of "good taste", which is both topological space and metric space. Thereupon, some fundamental properties of topological BCL$^+$-algebras are obtained.

Keywords: BCL-Algebra, BCL$^+$-Algebra, Topological BCL$^+$-Algebra

1. Introduction

In [1], Yonghong Liu introduced a new class of abstract algebra: BCL-algebra. Recently, Yonghong Liu introduced a wide class of abstract algebras: BCL$^+$-algebra (see [2, 4]). In [3], Deena Al-Kadi and Rodyna Hosny introduced the deformation of BCL-algebra. They continue to investigate the relation between BCL-algebra and d/BCH/BCI and BCK-algebra.

In this paper we are going to introduce topological spaces for the BCL$^+$-algebras, which is derived the fundamental properties, which we will need the following definitions and theorems:

Definition 1.1 [2]

An algebra $(X;*,1)$ is called a BCL$^+$-algebra if it satisfies the following laws hold: for any $x, y, z \in X$,

BCL$^+$*-1)* $x*x=1$.
BCL$^+$*-2)* $x*y=1$ and $y*x=1$ imply $x=y$.
BCL$^+$*-3)* $((x*y)*z)*((x*z)*y)=((z*y)*x)$.

Theorem 1.1 [2]

Assume that $(X;*,1)$ is any a BCL$^+$-algebra. Then the following hold: for any $x, y, z \in X$

i) $(x*(x*y))*y=1$.
ii) $x*1=x$ imply $x=1$.
iii) $((x*y)*(x*z))*(z*y)=1$.
iv) BCL$^+$*-2)* $x*y=1$ and $y*x=1$ imply $x=y$.

Theorem 1.2 [2]

An algebra $(X;*,1)$ is a BCL$^+$-algebra if and only if it satisfies the following conditions: for all $x, y, z \in X$

i) BCL$^+$*-1)* $x*x=1$.
ii) BCL$^+$*-2)* $x*y=1$ and $y*x=1$ imply $x=y$.
iii) $(((x*y)*z)*((x*z)*y))*((z*y)*x)=1$.
iv) $x*(1*y)=x$.

Definition 1.2 [2]

Suppose that $(X;*,1)$ is a BCL$^+$-algebra, the ordered relation if $x \le y$ if and only if $x*y=1$, for all $x, y \in X$, then $(X;\le)$ is partially ordered set and $(X;*,1)$ is an algebra of partially ordered relation.

Corollary 1.1 [2]

Let every $x \in X$. Then 1(one) is maximal element in a BCL$^+$-Algebra $(X;*,1)$ such that integral $1 \le x$ imply $x=1$.

2. Main Results

For a $g(x,y,z)=(x*y)*z$ -algebra, the topological space is just a basic concept in nature. We have the following definitions.

Definition 2.1

Let $(X;\circ,*,1)$ be a BCL_D^+ -algebra with two binary operations $\circ$ and $*$ that satisfies the following properties: for any $x, y, z \in X$.

BCL_D^+ *-1)* An algebra $D(X)=(X;\circ)$ is a distributive algebra.

BCL_D^+ *-2)* An algebra $P(X)=(X;*,1)$ is a $g(x,y,z)=(x*y)*z$ -algebra.

BCL_D^+ *-3)* (right weakly distribution) $x*(y\circ z)=(x*y)\circ(x*z)$.

BCL_D^+ *-4)* (left weakly distribution) $(y\circ z)*x=(y*x)\circ(z*x)$.

Definition 2.2

Let $(X;*,1)$ be a BCL$^+$-algebra and let $(X, \Im)$ be a topological space. If

$$\circ,*: X\times X \to X \text{ and } (x,y)\mapsto x*y \tag{2.1}$$

be a continuous mapping from the product space $(X\times X,\Re)$ to the topological space $(X,\Im)$, where $\Re$ is product topology of X, then we say that $(X;\Im,*,1)$ is a topological BCL$^+$-algebra.

Theorem 2.1

The product of any tow topological BCL$^+$-algebra is again a topological BCL$^+$-algebra.

Proof: Let $(X_1;\Im,*_1,1_1)$ and $(X_2;\Im,*_2,1_2)$ be topological BCL$^+$-algebra, let (x_1,y_1) and (x_2,y_2) is any tow element in X, and defines:

$$\begin{cases} X=X_1\times X_2 \\ \circ:(x_1,y_1)\circ(x_2,y_2)=(x_1*_1x_2,y_1*_2y_2). \\ 1=(1_1,1_2) \end{cases} \tag{2.2}$$

Now suppose that U is a open neighborhood of $\Re$. By Definition 2.2, $\Re$ be product topology, so we conclude that B_1 is an open neighborhood of $x_1 *_1 x_2$, and B_2 is an open neighborhood of $y_1 *_2 y_2$ in X, we have $V_1\times V_2\subseteq U$. Since B_1 is an open neighborhood of x_1, and B_2 is an open neighborhood of x_2. Also, B_3 is a open neighborhood of y_1, and B_4 is a open neighborhood of y_2, we have

$$B_1*B_2\subseteq V_1 \text{ and } B_3*B_4\subseteq V_2. \tag{2.3}$$

Then

$$(B_1*B_2)\circ(B_3*B_4)\subseteq V_1\times V_2\subseteq U, \tag{2.4}$$

and we claim that in fact

$$(B_1*B_2)\times(B_3*B_4)=(B_1\times B_3)\circ(B_2\times B_4)\subseteq U, \tag{2.5}$$

where $B_1\times B_3$ is a open neighborhood of (x_1,y_1) in X, and $B_2\times B_4$ is a open neighborhood of (x_2,y_2) in X. Thus $\circ$ is a continuous and $(X;\Im,\circ,1)$ is a topological BCL$^+$-algebra. ■

Corollary 2.1

The product of any finite topological BCL$^+$-algebra is again a topological BCL$^+$-algebra.

Corollary 2.2

Any finite product of a topological BCL$^+$-algebra is again a topological BCL$^+$-algebra.

Example 2.1

Let X be a topological BCL$^+$-algebra and let $f(x,y,z)$ and $g(x,y,z)$ be ternary polynomial. Assume that for all $x,y,z\in X$. Suppose the following conditions hold:

i) $f(x,y,z)=((x*y)*z)*((x*z)*y)$, and

ii) $g(x,y,z)=(z*y)*x$.

If $(f(x,y,z),g(x,y,z))=1$, then we say that $f(x,y,z)$ and $g(x,y,z)$ are coprime topological BCL$^+$-algebras.

Example 2.2

Let X be a topological BCL$^+$-algebra and let $f(x,y,z)$, $g(x,y,z)$ and $h(x,y,z)$ be ternary polynomial. Assume that for all $x,y,z\in X$. Suppose the following conditions hold:

i) $f(x,y,z)=(z*y)*x$.

ii) $g(x,y,z)=(x*y)*z$.

iii) $h(x,y,z)=(x*z)*y$.

If $(f(x,y,z),g(x,y,z))=1$ and $(f(x,y,z),h(x,y,z))=1$.

Then

$$(f(x,y,z),g(x,y,z)*h(x,y,z))=1. \tag{2.6}$$

Definition 2.3

If BCL_D^+ -algebra $(X;\circ,*,1)$ has the property of distribution, let $\Im$ be topology of X and let $(X;\Im,\circ,*,1)$ be a topological BCL_D^+ -algebra. Then topological BCL_D^+ -algebra $(X;\Im,\circ,*,1)$ has also the property of distribution.

Theorem 2.2

Let $(X;\circ,*,1)$ be a BCL_D^+ -algebra and let $\Im$ be topology of X, then $(X;\Im,\circ,*,1)$ be a topological BCL_D^+ - algebra.

Proof: Let the operations $\circ$

$$\circ:(x_1,y_1)\circ(x_2,y_2)=(x_1*x_2,y_1*y_2). \qquad (2.7)$$

identically, element $1=(1,1)$. By Definition 2.1 and Definition 1.3, we have continuous mapping

$$\circ,*:(X\times X,\Re)\to(X,\Im). \qquad (2.8)$$

where $\Re$ is product topology of X. We deduce that $(X;\Im,\circ,*,1)$ be a topological BCL_D^+-algebra. ■

Definition 2.4

Let $(X;\Im,\circ,*,1)$ be a topological BCL_D^+-algebra. Suppose the following conditions hold: for any $x,y,z\in X$.

i) $\Im(x,y)\geq 1$ and $\Im(x,y)=1$ if and only if $x=y$.

ii) $\Im(x,y)=\Im(y,x)$.

iii) $\Im(x,z)\leq\Im(x,y)+\Im(y,z)$.

Then $(X,\Im)$ be a metric space of identically element 1.

Example 2.3

Let X be a set and let $\Im: X\times X\to\mathbb{R}$, $x,y\in X$, by Definition 2.3, we now have

$$\Im(x,y)=\begin{cases}1 & \text{if } x=y,\\ 0 & \text{if } x\neq y.\end{cases} \qquad (2.9)$$

Then $\Im$ be a discrete metric and $(X,\Im)$ be a discrete metric space.

Theorem 2.3

Let $(X;\circ,*,1)$ be a BCL_D^+-algebra and let $\Im$ a discrete topology of X, then $(X,\Im)$ be a topological space.

Proof: By Theorem 2.1, $(X,\Im)$ be a discrete metric space by Example 2.1, where discrete metric of X is induced by $\Im$. Thus $(X,\Im)$ be a topological space. ■

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