

An Application of the New Integral “Aboodh Transform” in Cryptography

Abdelilah K. Hassan Sedeeg^{1,2}, Mohand M. Abdelrahim Mahgoub^{1,3}, Muneer A. Saif Saeed⁴

¹Mathematics Department Faculty of Sciences and Arts, Almikwah-Albaha University, Albaha, Saudi Arabia

²Mathematics Department Faculty of Education, Holy Quran and Islamic Sciences University, Khartoum, Sudan

³Mathematics Department Faculty of Sciences, Omdurman Islamic University, Khartoum, Sudan

⁴Computer Department Faculty of Sciences and Arts, Almikwah-Albaha University, Albaha, Saudi Arabia

Email address:

aelilah63@hotmail.com (A. K. H. Sedeeg), mahgob10@hotmail.com (M. M. A. Mahgoub), moneer5000@yahoo.com (M. A. S. Saeed)

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Abstract: Cryptography is the science of providing security for information, It has been used historically as a means of providing secure communication between individuals. Message encryption has become very essential to avoid the threat against possible attacks by hackers during transmission process of the message. In this paper authors have proposed a method of cryptography, in which authors have used Aboodh transform for encrypting the plain text and corresponding inverse Aboodh transform for decryption.

Keywords: Cryptography, Encryption, Decryption, Aboodh Transform

1. Introduction

Cryptography, the mathematics of encryption, plays an indispensable part in numerous fields, and a vast range of daily activities, such as electronic commerce, bank card payments and electronic building and so on. Cryptography is the only most important tool that avoids the threat against possible attacks by hackers during transmission process of the message. It is one of the cornerstones of Internet security. Cryptography is the only most important tool that avoids the threat against possible attacks by hackers during transmission process of the message.

Cryptography [5-10] referred almost exclusively to encryption, which is the process of converting ordinary information (called plaintext) into unintelligible text (called cipher text). Decryption is the reverse, in other words, moving from the unintelligible cipher text back to plaintext. A cipher (or cipher) is a pair of algorithms that create the encryption and the reversing decryption. The detailed operation of a cipher is controlled both by the algorithm and in each instance by a "key". The key is a secret (ideally known only to the communicants), usually a short string of characters, which is needed to decrypt the cipher text. (Fig. 1) [11-15].



Fig. 1. Basic encryption and Decryption.

2. Proposed Technique

In the present paper a new cryptographic scheme is proposed using Aboodh Transform [1-4]. Aboodh transform is used for encrypting the plain text and corresponding inverse Aboodh transform is used for decryption. Aboodh transform was introduced by Khalid Aboodh in 2013. Aboodh transform is a widely used integral transform in mathematics and electrical engineering that transforms a function of time into a function of complex frequency. The inverse Aboodh transform takes a complex frequency domain function and yields a function defined in the time domain. Proposed algorithm provides as many transformations as per the requirements which are the most useful factor for changing key. Therefore it is very difficult for an eyedropper to trace the key by any attack. The implementation has been done in visual basic (VB).

3. Aboodh Transform

Definition: Consider functions in the set A defined by

$$A = \{f(t): \exists M, k_1, k_2 > 0, |f(t)| < Me^{-vt}\}$$

For a given function in the set M must be finite number, k_1, k_2 may be finite or infinite. Aboodh transform denoted by the operator $A[.]$ is defined by the integral equation

$$A[f(t)] = K(v) = \frac{1}{v} \int_0^{\infty} f(t) e^{-vt} dt \geq 0, k_1 \leq v \leq k_2$$

Properties of Aboodh transform:-

Linearity:- Aboodh transform is a linear transformation which means that the transform of a sum of waveforms is the sum of their transforms. Stated formally the linearity property is

$$A[a.f(t) + b.g(t)] = a.A[f(t)] + b.A[g(t)]$$

Where a, b are constants.

The above result can easily be generalized to more than two functions.

Aboodh transformation & Inverse Aboodh Transform of some elementary functions:-

Elementary functions include algebraic and transcendental functions.

1. $A(l) = \frac{l}{v^2}$, where l is constant.
2. $A(t^n) = \frac{n!}{v^{n+2}}$
3. $A^{-1}\left(\frac{1}{v^2}\right) = 1, A^{-1}\left(\frac{n!}{v^{n+2}}\right) = t^n$

4. Proposed Methodology

The following algorithm provides an insight into the proposed cryptographic scheme. The sender converts the original message or plain text into cipher text using the following steps.

4.1. Encryption Algorithm

- I) Treat every letter in the plain text message as a number, so that $A = 1, B = 2, C = 3, \dots, Z = 26, [space] = 0$.
- II) The plain text message is organized as finite sequence of numbers based on the above conversion. For example our text is “TEACHERS”. Based on the above step; we know that,

$$T = 20, E = 5, A = 1, C = 3, H = 8, R = 18, S = 19.$$

Therefore our plain text finite sequence is 20, 5, 1, 3, 8, 5, 18, 19.

- III) If $n + 1$ is the number of term in the sequence; consider a polynomial of degree n with coefficient as the term of the given finite sequence. Above finite sequence contains $7 + 1$ terms. Hence consider a polynomial $p(t)$ of degree 7.

$$p(t) = 20 + 5t + t^2 + 3t^3 + 8t^4 + 5t^5 + 19t^6 + 19t^7$$

Take Aboodh transform of polynomial $p(t)$.

$$\begin{aligned} A[p(t)] &= A[20 + 5t + t^2 + 3t^3 + 8t^4 + 5t^5 + 19t^6 + 19t^7] \\ &= \frac{20}{v^2} + \frac{5}{v^3} + \frac{2!}{v^4} + 3\frac{3!}{v^5} + 8\frac{4!}{v^6} + 5\frac{5!}{v^7} + 19\frac{6!}{v^8} + 19\frac{7!}{v^9} \\ &= \frac{20}{v^2} + \frac{5}{v^3} + \frac{2}{v^4} + \frac{18}{v^5} + \frac{192}{v^6} + \frac{600}{v^7} + \frac{12960}{v^8} + \frac{95760}{v^9} \\ &= \sum_{i=1}^{7+1} \frac{q_i}{v^{i+1}} \end{aligned}$$

Next find r_i such that $q_i \equiv r_i \pmod{26}$ for each $i, 1 \leq i \leq n + 1$. Therefore

$$q_1 = 20 \equiv 20 \pmod{26}, q_2 = 5 \equiv 5 \pmod{26}, q_3 = 2 \equiv 2 \pmod{26}$$

$$q_4 = 18 \equiv 18 \pmod{26}, q_5 = 192 \equiv 10 \pmod{26}, q_6 = 600 \equiv 2 \pmod{26}$$

$$q_7 = 12960 \equiv 12 \pmod{26}, q_8 = 95760 \equiv 2 \pmod{26}.$$

IV) Hence $q_i = 26k_i + r_i$. Thus we get a key k_i for $i = 1, 2, 3, \dots, n + 1$.

$$\begin{aligned} \therefore k_1 &= 0, k_2 = 0, k_3 = 0, k_4 = 0, k_5 = 7, k_6 = 23, k_7 \\ &= 498, k_8 = 3683 \end{aligned}$$

V) Now consider a new finite sequence $r_1, r_1, r_1, \dots, r_{n+1}$ i.e. 20, 5, 2, 18, 10, 2, 12, 2.

4.2. Decryption Algorithm

- I) Consider the cipher text and key received from sender. In the above example cipher text is “TEBRJBLB” and key is 0, 0, 0, 0, 7, 23, 498, 3683.
- II) Convert the given cipher text to corresponding finite sequence of numbers $r_1, r_1, r_1, \dots, r_{n+1}$, 20, 5, 2, 18, 10, 2, 12, 2.
- III) Let $q_i = 26k_i + r_i, \forall i = 1, 2, 3, \dots, n + 1$.

$$q_1 = 26(0) + 20 = 20, q_2 = 26(0) + 5 = 5, q_3 = 26(0) + 2 = 2,$$

$$q_4 = 26(0) + 18 = 18, q_5 = 26(7) + 10 = 192, q_6 = 26(23) + 2 = 600,$$

$$q_7 = 26(498) + 12 = 12960, q_8 = 26(3683) + 2 = 95760.$$

$$\begin{aligned} \text{IV) Let } p(v) &= \sum_{i=1}^{n+1} \frac{q_i}{v^{i+1}} \\ &= \frac{20}{v^2} + \frac{5}{v^3} + \frac{2}{v^4} + \frac{18}{v^5} + \frac{192}{v^6} + \frac{600}{v^7} + \frac{12960}{v^8} + \frac{95760}{v^9} \\ &= 20\frac{0!}{v^2} + 5\frac{1!}{v^3} + \frac{2!}{v^4} + 3\frac{3!}{v^5} + 8\frac{4!}{v^6} + 5\frac{5!}{v^7} + 18\frac{6!}{v^8} + 19\frac{7!}{v^9} \end{aligned}$$

V) Now take the Inverse Aboodh transform of $p(v)$.

$$\begin{aligned} \therefore A^{-1}[p(v), t] &= A^{-1}\left[20\frac{0!}{v^2} + 5\frac{1!}{v^3} + \frac{2!}{v^4} + 3\frac{3!}{v^5} + 8\frac{4!}{v^6} \right. \\ &\quad \left. + 5\frac{5!}{v^7} + 18\frac{6!}{v^8} + 19\frac{7!}{v^9}\right] \end{aligned}$$

$$p(t) = 20 + 5t + t^2 + 3t^3 + 8t^4 + 5t^5 + 19t^6 + 19t^7$$

VI) Consider the coefficient of a polynomial $p(t)$ as a finite sequence.

20, 5, 1, 3, 8, 5, 18, 19.

VII) Now translating the number of above finite sequence to alphabets. We get the original plain text as "TEACHERS".

5. Implementation of the Efficient Algorithm

Visual basic programming language is one of the most widely use high level language today because of its advantages [16]. In this parta program has been written in visual basic language (VB), forthe implementation of the Encryption Algorithms and Implementation Decryption Algorithms in section 4.

5.1. Implementation Encryption Algorithms

```
Private Sub Enc_btn_Click()
    Dim no_of_ltr As Integer
    Dim ltrs, e(26) As String
    no_of_ltr = Len(Me.text1)
    Dim p(26), ris(1000), g, qis(26) As Long
    Me.enc = "" Me.dcr = "" Me.mdd = "" Me.py = "" Me.dcr =
    "" Me.keys = "" Me.enc_txt = "" e(1) = "A"e(2) = "B"e(3) =
    "C"e(4) = "D"e(5) = "E"e(6) = "F"e(7) = "G"e(8) = "H"e(9) =
    "I"e(10) = "J"e(11) = "K"e(12) = "L"e(13) = "M"e(14) =
    "N"e(15) = "O"e(16) = "P"e(17) = "Q"e(18) = "R"e(19) =
    "S"e(20) = "T"e(21) = "U"e(22) = "V"e(23) = "W"e(24) =
    "X"e(25) = "Y"e(26) = "Z"
    For i = 1 Tono_of_ltr
        ltrs = Mid(Me.text1, (i), 1)
        MsgBox (ltrs)
        For j = 1 To 26
            If ltrs = e(j) Then
                MsgBox (i)
                MsgBox (j)
                MsgBox e(j)
                qis(i) = j * Factorial(i - 1)
                ris(i) = qis(i) Mod 26
                Me.enc = Me.enc & qis(i) & "-"
                Me.mdd = Me.mdd & ris(i) & "-"
                ks = ((qis(i) - ris(i)) / 26)
                Me.keys = Me.keys & ks & "-"
                Me.enc_txt = Me.enc_txt & e(ris(i))
            End If
        Next
        MsgBox (qis(i))
        LArray(i) = ltrs
        Me.enc = Me.enc & g & "-"
        Me.mdd = Me.mdd & ris(i) & "-"
        Me.dcr = Me.dcr & Chr(g)
    Next
End Sub
```

	Encryption
Original Text	TEACHERS
Coefficients of $p(t)[q_i]$	20-5-2-18-192-600-12960-95760-
Keys	0-0-0-0-7-23-498-3683-
Encryption No	20-5-2-18-10-2-12-2-
Cipher Text	TEBRJBLB

Fig. 2. Implementation Encryption.

5.2. Implementation Decryption Algorithms

```
Private Sub Dec_Btn_Click()
    Dim pos, intCount As Integer
    Dim LArray(1000), ky(26), e(26), strTest, d, strArray(),
    ee(1000) As String
    Dim p(26), qis(1000), no(1000), ris(26), key(26) As Long
    LArray = Split(Me.text1)
    pos = Len(Me.text1)
    Me.key = "" Me.dcr = "" Me.mdd = "" Me.py = "" Me.dcr =
    "" Me.encrypted = "" e(1) = "A"e(2) = "B"e(3) = "C"e(4) =
    "D"e(5) = "E"e(6) = "F"e(7) = "G"e(8) = "H"e(9) = "I"e(10) =
    "J"e(11) = "K"e(12) = "L"e(13) = "M"e(14) = "N"e(15) =
    "O"e(16) = "P"e(17) = "Q"e(18) = "R"e(19) = "S"e(20) =
    "T"e(21) = "U"e(22) = "V"e(23) = "W"e(24) = "X"e(25) =
    "Y"e(26) = "Z"
    strTest = Me.keys
    strArray = Split(strTest, "-")
    For intCount = LBound(strArray) To UBound(strArray) - 1
        key(intCount) = (Trim(strArray(intCount)))
        MsgBox (intCount)
    Next
    For i = 1 Topos
        d = Mid(Me.text1, (i), 1)
        For j = 1 To 26
            If d = e(j) Then
                ris(i) = j * Factorial(i - 1)
                qis(i) = ris(i) Mod 26
                Me.enc = Me.enc & ris(i) & "-"
                Me.mdd = Me.mdd & qis(i) & "-"
                Me.py = Me.py & ris(i) & "-"
                qis(i) = ris(i) + (key(i - 1) * 26)
                Me.mdd = Me.mdd & qis(i) & "-"
                no(i) = (qis(i) / Factorial(i - 1))
                org_txt = org_txt & e(no(i))
                Me.encrypted = Me.encrypted & no(i) & "-"
                MsgBox ((ris(i))) - qis(i) / 26
            End If
        Next
        MsgBox (ris(i))
        LArray(i) = d
        g = Asc(d)
        p(i) = g
        qis(i) = p(i) Mod 26
        Me.py = Me.py & p(i) & "-"
        Me.enc = Me.enc & g & "-"
    Next
```

```

'Me.mdd = Me.mdd&qis(i) & "-"
'ee(i) = Chr(p(i))
'Me.encrypted = Me.encrypted&ee(i) & "-"
'Me.dcr = Me.dcr&Chr(g)
Next
End Sub
Public Function Factorial(ByVal X As Long) As Long
If X <= 1 Then
Factorial = 1
Else
Factorial = X * Factorial(X - 1)
End If
End Function

```

Decryption	
Cipher Text	TEBRJBLB
Keys	0-0-0-0-7-23-498-3683-
Cipher No	20-5-2-18-10-2-12-2-
qis	20-5-2-18-192-600-12960-95760-
Original No(ri)	20-5-1-3-8-5-18-19-
Original Text	TEACHERS

Fig. 3. Implementation Decryption.

6. Conclusion

Computers are good at doing mathematics. When we create a system to translate some piece of information into numbers (such as we do with text and ordinals or with space and coordinate systems), computer programs can process these numbers quickly and efficiently. Cryptography is one of the first lines of defense against hackers and crackers in today's world. Thus, it will stay important for a long time to come. In the proposed work a new cryptographic scheme is introduced using Aboodh Transform and the private key is the number of multiples of mod n. Visual Basic is more convenient to use in the Cryptography and is less prone to errors.

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