Pure and Applied Mathematics Journal
2021; 10(3): 84-88
http://www.sciencepublishinggroup.com/j/pamj
doi: 10.11648/j.pamj.20211003.13
ISSN: 2326-9790 (Print); ISSN: 2326-9812 (Online)

Minimum Degree Distance of Five Cyclic Graphs

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To cite this article:
Nadia Khan, Munazza Shamus, Fauzia Ghulam Hussain, Mansoor Iqbal. Minimum Degree Distance of Five Cyclic Graphs. *Pure and Applied Mathematics Journal.* Vol. 10, No. 3, 2021, pp. 84-88. doi: 10.11648/j.pamj.20211003.13

Received: May 26, 2021; **Accepted**: July 28, 2021; **Published**: August 4, 2021

Abstract: Let G be a connected graph with n vertices. Then the class of connected graphs having n vertices is denoted by G_n. The subclass of connected graphs with 5 cycles are denoted by G_n^5. The classification of graph $G \in G_n^5$ depends on the number of edges and the sum of the degrees of the vertices of the graph. Any graph in G_n^5 contains five linearly independent cycles having at least n+3 edges and the sum of degrees of vertices of 5-cyclic must be equal to twice of n+4. In this paper, minimum degree distance of class of five cyclic connected graph is investigated. To find minimum degree distance of a graph some transformations T have been defined. These transformation have been applied on the graph $G \in G_n^5$ in such a way that the resultant graph belongs to G_n^5 and also degree distance of *T(G)* is always must be less than G. For n=5, the five 5-cyclic graph has minimum degree distance 78 and the minimum degree distance of 5-cyclic graphs having six vertices is 124. In case of n greater than 6, a general formula for minimum degree distance is investigated. In this paper, we proved that the minimum degree distance of connected 5 cyclic graphs is $3n^2+13n-62$ by using transformations, for $n \geq 7$.

Keywords: Weiner Index, Graphical Sequence, Degree Distance, Five Cyclic Graphs

1. Introduction

For any graph $G \in G_n$, $d(x,y)$ represents the shortest distance between the vertices $x, y \in V(G)$ and the maximum of $d(x,y)$ for any vertices $x, y \in V(G)$ is defined to be the diameter of G, denoted by $giam(G)$. A well-known topological index called the Wiener index [8] is of a graph G is defined as

$$\sum_{x \in V(G)} \sum_{x \in V(G)} d(x,y).$$

A new graph invariant other than Wiener index was introduced by Dobrynin and Kotchetova in [1] and Gutman [2] whose results are reinvestigated in [10] and is defined as for any given graph $G \in G_n$ the degree distance of a vertex $v \in V(G)$ is defined by

$$D'(v) = d(v)D(v)$$

where $d(v)$ express the degree of a vertex v and $D(v) = \sum_{u,v \in V(G)} d(u,v)$. The degree distance of a graph G is defined as:

$$D'(G) = \sum_{v \in V(G)} D'(v) = \sum_{u,v \in V(G)} d(u,v)\big(d(x) + d(v)\big).$$

The two conjectures made by Dobrynin and Kochetova in [1] on the minimum and maximum values of the degree distance of a graph is discussed in [5]. In this paper, we determine all the extremal 5-cyclic graphs having the minimum degree distance.

In section 2, some known results are given for the proof of main result in this paper. In section 3, the characterization of extremal 5-cyclic graphs having minimum degree distance is discussed.

2. Some Lemmas

According to degree sequence trees have characterized by Moon [3] and Senior [4]. Connected unicyclic and bicyclic graphs has characterized according to degree sequence in [6]. Wei Zhu [7] has characterized connected tricyclic graphs via their degree sequence. N. Khan [9] has characterized connected four cyclic graphs according to degree sequence. The characterization of connected five cyclic graphs according to degree sequence is given in the following lemma.

Lemma 1. Let $n \geq 5$. The integers $n-1 \geq a_1 \geq a_2 \geq \cdots \geq a_n \geq 1$, are the degrees of the vertices of a graph $G \in G_n^5$ are, iff

(i) The degrees sum of n-vertices of 5-cyclic graph $\sum_{i=1}^{n} a_i = 2n+8$

(ii) $d_i \geq 2$, for at least five indices.

Proof: $\Longrightarrow$ Let $G \in G_n^5$. Then by the definition of 5-cyclic graph condition (i) and (ii) is satisfied. W_5 with an edge joining two alternate vertices of a cycle is the only 5-cyclic graph having the minimal order.

$\Longleftarrow$ For $n=5$, we have $a_1+a_2+a_3+a_4+a_5=18$. If $a_5 \geq 4$ then $a_1+a_2+a_3+a_4+a_5 \geq 20$ a contradiction. So $a_5 < 4$. If $a_5 = 3$ then $a_1+a_2+a_3+a_4=15$, which implies that $a_1=a_2=a_3=4, a_4=3$. This degree sequence has a unique G isomorphic to W_5 with an edge joining two alternate vertices of a cycle. If $a_5 < 3$, then $a_5 = 2$ and $a_1=a_2=a_3=a_4=4$ which is not a graphical sequence. Let $n \geq 6$ and assume that the result is true for all $k \leq n$.

Case 1. If $a_n > 1$, then $a_n = 2$. Otherwise, $a_1+a_2+\cdots+a_n \geq 3n > 2n+8$. For $a_n = 2$ then for the equation $a_1+a_2+\cdots+a_{n-1} = 2n+6$, the following possibilities holds.

Subcase 1.1. If $a_1 = 10, a_2 = a_3 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_1 in figure 1.

Subcase 1.2. If $a_1 = 9, a_2 = 3, a_3 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_2 or G_3 in figure 1.

Subcase 1.3. If $a_1 = 8, a_2 = a_3 = 3, a_4 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_4, G_5, G_6 or G_7 in figure 1.

Subcase 1.4. If $a_1 = 8, a_2 = 4, a_3 = a_4 = \cdots = a_n = 2$, this degree sequence represents a unique graph is isomorphic to G_{35} in figure 1.

Subcase 1.5. If $a_1 = 7, a_2 = a_3 = a_4 = 3, a_5 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_8, G_9 or G_{10} in figure 1.

Subcase 1.6. If $a_1 = 7, a_2 = 5, a_3 = a_4 = a_5 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_6 or G_{34} in figure 1.

Subcase 1.7. If $a_1 = 7, a_2 = 4, a_3 = 3, a_4 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{33} in figure 1.

Subcase 1.8. If $a_1 = 6, a_2 = a_3 = a_4 = a_5 = 3, a_6 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{15} in figure 1.

Subcase 1.9. If $a_1 = 6, a_2 = a_3 = 4, a_4 = a_5 = a_6 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{11}, G_{12}, G_{13} or G_{14} in figure 1.

Subcase 1.10. If $a_1 = a_2 = 6, a_3 = a_4 = 3, a_5 = a_6 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{16} in figure 1.

Subcase 1.11. If $a_1 = 6, a_2 = 4, a_3 = a_4 = 3, a_5 = a_6 = \cdots = a_n = 2$, this degree sequence represents a unique graph is isomorphic to G_{32} in figure 1.

Subcase 1.12. If $a_1 = 6, a_2 = 5, a_3 = 3, a_4 = a_5 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{17} in figure 1.

Subcase 1.13. If $a_1 = 5, a_2 = a_3 = a_4 = a_5 = a_6 = 3, a_7 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{18} in figure 1.

Subcase 1.14. If $a_1 = 4, a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 3, a_8 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{31} in figure 1.

Subcase 1.15. If $a_1 = a_2 = a_3 = 4, a_4 = a_5 = 3, a_6 = a_7 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{19}or G_{30} in figure 1.

Subcase 1.16. If $a_1 = 6, a_2 = a_3 = 4, a_4 = a_5 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{36} in figure 1.

Subcase 1.17. If $a_1 = a_2 = 5, a_3 = 4, a_4 = a_5 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{37} in figure 1.

Subcase 1.18. If $a_1 = a_2 = 4, a_3 = a_4 = a_5 = a_6 = 3, a_7 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{20} or G_{21} in figure 1.

Subcase 1.19. If $a_1 = 4, a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 3, a_8 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{38} in figure 1.

Subcase 1.20. If $a_1 = a_2 = a_3 = a_4 = 4, a_5 = a_6 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{22}, G_{23} or G_{24} in figure 1.

Subcase 1.21. If $a_1 = a_2 = 5, a_3 = a_4 = 3, a_5 = a_6 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{25} in figure 1.

Subcase 1.22. If $a_1 = 5, a_2 = a_3 = 4, a_4 = 3, a_5 = a_6 = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{26} in figure 1.

Subcase 1.23. If $a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = 3, a_9 = a_{10} = \cdots = a_n = 2$, this degree sequence represents a unique graph isomorphic to G_{27}, G_{28} or G_{29} in figure 1.

Case 2. If $a_n = 1$, we consider the following two subcases:

Subcase 2.1. If $a_1 = n-1$, and for sufficiently large n, we have the following possibilities,

Subcase 2.1.1. If $a_1 = n-1, a_2 = \cdots = a_{11} = 2, a_{12} = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_1 in figure 2.

Subcase 2.1.2. If $a_1 = n-1, a_2 = a_3 = 3, a_4 = \cdots = a_{10} = 2, a_{11} = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_2 in figure 2.

Subcase 2.1.3. If $a_1 = n-1, a_2 = a_3 = a_4 = a_5 = 3, a_6 = a_7 = 2, a_8 = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_3 in figure 2.

Subcase 2.1.4. If $a_1 = n-1, a_2 = a_3 = 4, a_4 = 3, a_5 = a_6 = 2, d_7 = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_4 in figure 2.

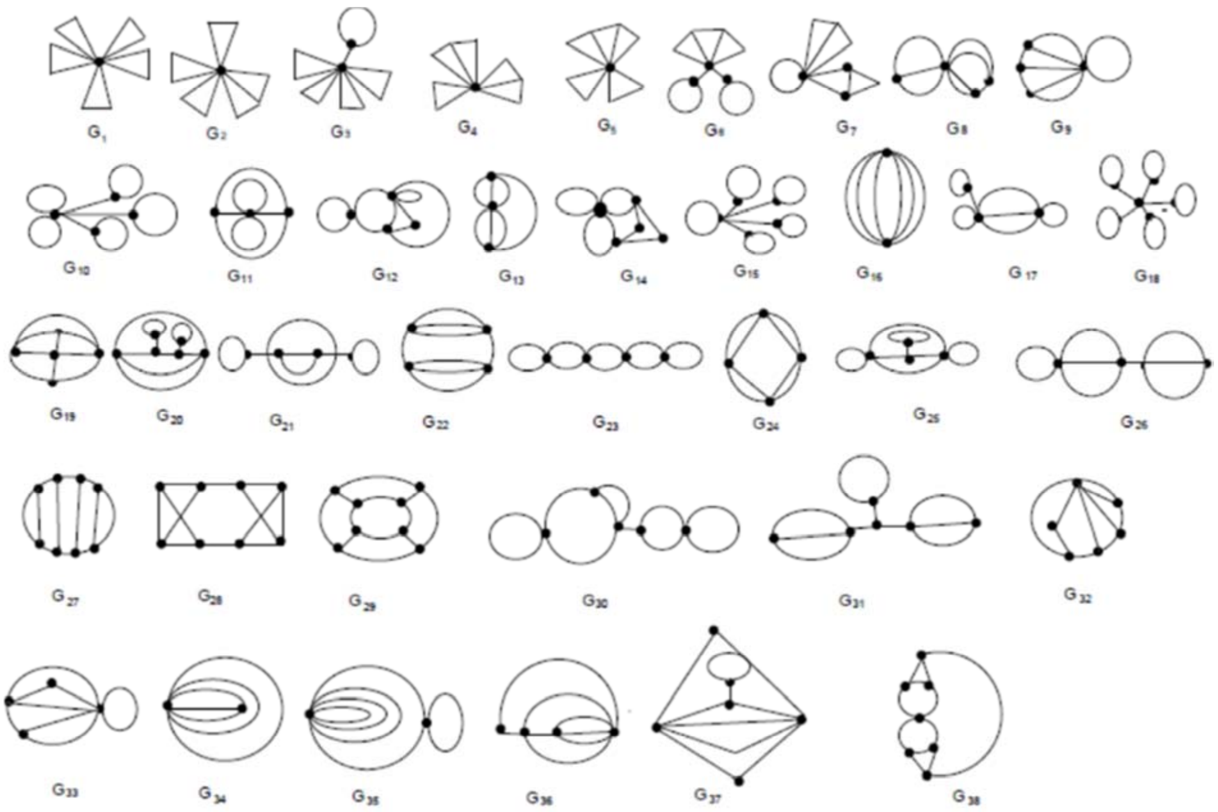


Figure 1. *Five Cyclic Graphs with n-Vertices and* $a_n = 1$.

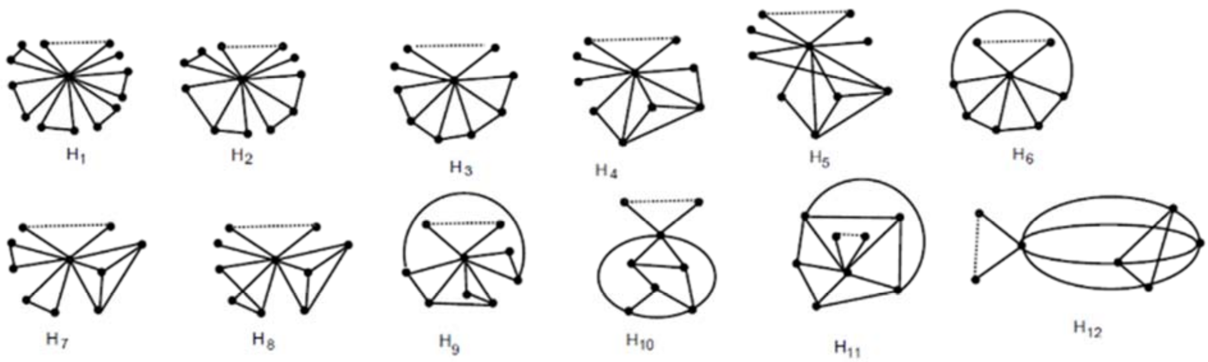


Figure 2. *Connected Five Cyclic Graphs with n-Vertices,* $a_1 = n-1$ *and* $a_n = 1$.

Subcase 2.1.5. If $a_1 = n-1, a_2 = \cdots = a_6 = 3,\ a_7 = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_6 in figure 2.

Subcase 2.1.6. If $a_1 = n-1, a_2 = a_3 = a_4 = 3,\ a_5 = a_6 = a_7 = a_8 = 2,\ a_9 = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_7 in figure 2.

Subcase 2.1.7. If $a_1 = n-1, a_2 = a_3 = a_4 = a_5 = 3, a_6 = a_7 = 2,\ a_8 = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_8 or H_9 in figure 2.

Subcase 2.1.8. If $a_1 = n-1, a_2 = a_3 = 4, a_4 = a_5 = 3,\ a_6 = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_5 or H_{11} in figure 2.

Subcase 2.1.9. If $a_1 = n-1, a_2 = 6, a_3 = \cdots = a_7 = 2,\ a_8 = \cdots = a_n = 1$, this degree sequence represents a unique graph isomorphic to H_{12} in figure 2.

Subcase 2.2. Suppose $a_1 \le n-2$ and $a_n = 1$. For each $1 \le k \le n-1$ if $a_k \le 2$ then $\sum_k a_k \le 2n-1$, which is a contradiction. There exists a maximum index m, $1 \le m \le n-1$, such that $a_m \ge 3$, and $a_{m+1} \le 2$ and $a_1 \ge a_2 \ge \cdots \ge a_{m-1} \ge a_m \ge \cdots \ge a_n \ge 1$. At minimum five members of the sequence $a_1, \cdots, a_{m-1}, a_m - 1, \cdots, a_{n-1}$ are greater than 2, for which $a_1 \le n-2$ and $a_1 + a_2 + \cdots + a_n = 2(n-1) + 8 = 2n+6$. For this degree sequence there exist $G \in G_n^5$, by induction hypothesis. A vertex of degree a_{m-1} is joined to a new vertex by an edge results a graph having four cycles with the degree sequence $a_1 \ge a_2 \ge \cdots \ge a_n = 1$. Hence proof of Lemma 1 is complete.

If for any vertex $v \in V(G), d(v) = t$, then $D(v) \ge 2n - t - 2$, and if for all $v \in V(G), d(v,y) \le 2$ then $D(v) = 2n - t - 2$. Consequently, $D'(G) = \sum_{v \in V(G)} d(v)D(v) \ge \frac{1}{2}\sum_{t=1}^{n-1} t d_t\,(2n - t - 2)$, where d_t denotes the number of vertices of degree $t, 1 \leq t \leq n-1$. By denoting as in [2], $F(x_1, x_2, \cdots, x_{n-1}) = \sum_{t=1}^{n-1} t d_t \,(2n - t - 2)$. We will find the minimum of $F(d_1, d_2, \cdots, d_{n-1})$ over all-natural numbers $d_1, d_2, \cdots, d_{n-1} \geq 0$ satisfying the conditions in above lemma. We have the following corollary:

Corollary: Let $n \geq 5$. The integers $d_1, d_2, \cdots, d_{n-1} \geq 0$ are the multiplicities of the degrees of a graph $G \in G_n^5$ iff

(i) $\sum_{t=1}^{n-1} d_t = n$

(ii) $\sum_{t=1}^{n-1} t d_t = 2n + 8$

(iii) $d_1 \leq n - 5$

Let the set of vectors of non-negative integers $d_1, d_2, \cdots, d_{n-1}$ be denoted by Δ satisfying the conditions (i)-(iii) of corollary. Let us define transformations T_1 and T_2 $l \geq 2,\ m > 0, l + m \leq n - 2, x_l \geq 1, x_m \geq 1$, by

$T_1(d_1, \cdots, d_{n-1}) = (d_1', \cdots, d_{n-1}')$= $(d_1, \cdots, d_{l-1} + 1, d_l - 1, \cdots, d_{l+m} - 1, d_{l+m+1} + 1, \cdots, d_{n-1})$ and

$T_2(d_1, \cdots, d_{n-1}) = (d_1', \cdots, d_{n-1}')$=$(d_1, \cdots, d_{l-1} + 1, d_l - 2,\ d_{l+1} + 1, \cdots, d_{n-1})$ we have $d_i = d_i'$ for $i \neq \{l-1, l, l+m, l+m+1\}$.

Lemma 2. Let $(d_1, \cdots, d_{n-1}) \in \Delta$ then

(a) $T_1(d_1, \cdots, d_{n-1}) \in \Delta$ if $l \neq 2$ and $x_1 \neq n - 5$, moreover $F(\,T_1(d_1, \cdots, d_{n-1})) < F(d_1, \cdots, d_{n-1})$

(b) $T_2(d_1, \cdots, d_{n-1}) \in \Delta$ if $l \neq 2$ and $x_1 \neq n - 5$, moreover

$F(\,T_2(d_1, \cdots, d_{n-1})) < F(d_1, \cdots, d_{n-1})$

Proof: (a). As $\sum_{t=1}^{n-1} d_t = \sum_{t=1}^{n-1} d_t'$ and $\sum_{t=1}^{n-1} t d_t = \sum_{t=1}^{n-1} t d_t' = 2n + 6$. If $(d_1, \cdots, d_{n-1}) \in \Delta$, $m = 2$ and $n - 5$ then $d_1' > n - 5$ a contradiction. Also $F(d_1, \cdots, d_{n-1}) - F(\,T_1(d_1, \cdots, d_{n-1})) = 2p + 2 > 0$.

Similarly, (b) also hold.

3. Main Result

Theorem: Let $G \in G_n^5$,

(a) If $n = 5$ then $minD'(G) = 78$, where G is a graph obtained from $W_5 +$ an edge joining two vertices shown in figure 3.

(b) If $n = 6$ then $minD'(G) = 124$ and all the extremal graphs are $A_6$and A_8 in figure 4.

(b) If $n \geq 7$ then $minD'(G) = 3n^2 + 13n - 62$, then all the extremal graphs are isomorphic to the graphs E_1 and E_2 in figure 5.

Proof. In order to find $\min F(d_1, \cdots, d_{n-1})$ where $(x_1, \cdots, x_{n-1}) \in \Delta$.

Firstly, let $n = 5$ then unique graph is given in Figure 3, and $D'(G) = 78$.

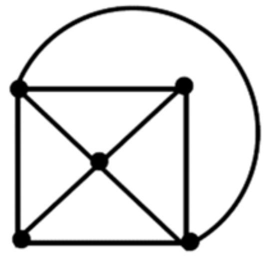


Figure 3. *Five Cyclic Graph with five Vertices.*

Secondly, let us consider $n = 6$. Then all graphs $G \in G_6^5$ are A_i for $1 \leq i \leq 8$ are shown in Figure 4. Here $D'(A_1) = D'(A_3) = 130, D'(A_2) = 132, D'(A_4) = D'(A_7) = 128, D'(A_5) = 126$ and $D'(A_6) = D'(A_8) = 124$, respectively.

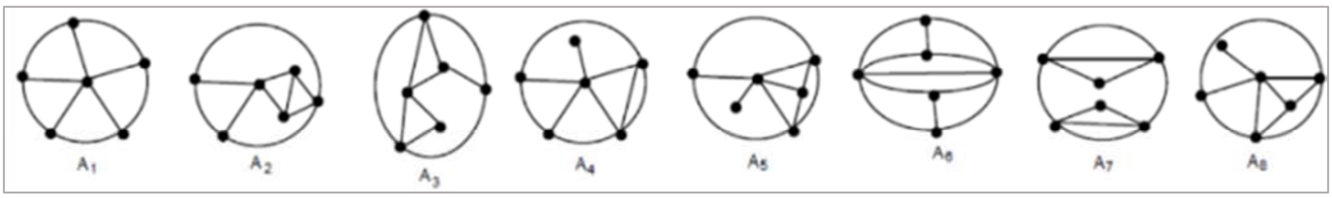


Figure 4. *All Connected Six Cyclic Graphs with Six Vertices.*

Finally, $n \geq 7$. If $d_{n-1} \geq 2$, take two distinct vertices $y_1, y_2 \in V(G)$ such that $d(y_1) = d(y_2) = n - 1$. As $n \geq 7$, by choosing at least seven different vertices which are distinct and adjacent to y_1, y_2 there exist at least six cycles, which is a contradiction. Therefore $d_{n-1} \ngtr 1$.

Now we determine the possible values of $d_1, d_2, \cdots, d_{n-2}$. If there exist $7 < l, m < n - 2$ such that $d_l \geq 1$ and $d_m \geq 1$ then a new vector $(d_1', \cdots, d_{n-1}') \in \Delta$ is obtained for which $\mathrm{F}(d_1', \cdots, d_{n-1}') < F(d_1, \cdots, d_{n-1})$ by the application of the transformation T_1 for the position l and m. we have for Similarly, if there exist $6 < l, m < n - 2$ such that $d_l \geq 2$ then a new degree sequence in Δ is obtained such that $\mathrm{F}\,(d_1', \cdots, d_{n-1}') < F(d_1, \cdots, d_{n-1})$ by T_2. Now we consider two cases:

Case 1. Suppose that there exist distinct indices i, k with $7 < l,\ k < n - 2$ such that $d_l = 1$ and $d_k = 0$. In this case, if $d_6 \geq 1$ then a smaller value of F is obtained by applying the T_1 for position 6 and l. Suppose that $d_6 = 0$. Since $d_{n-1} \in \{0, 1\}$, two cases will be considered separately.

(a) In this case $d_{n-1} = d_l = 1$, where $7 < l$ and $d_5 = 0$. Pick different vertices $\mathrm{u, v, w, x, y, p, q} \in \mathrm{V(G)}$ such that $\mathrm{d(u) = n - 1, d(v)} = l \geq 7$ then w, x, y, p, q are all adjacent to u and v respectively. Also u and v are adjacent too, this leads to existence of six cycles which contradicts our hypothesis.

(b) If $d_{n-1} = 0$ then $d_5 = 0$ and $d_l = 1$, $(7 < l, n - 2)$ and Δ is characterized by the equations

$d_1 + d_2 + d_3 + d_4 = n - 1$ and $d_1 + 2d_2 + 3d_3 + 4d_4 = 2n + 8 - l$ which implies that $d_2 + 2d_3 = n + 8 - l$ by solving for d_2 and d_3 and then by applying the transformation for position 2 and l or 3 and l or 4 and l, we obtain smaller value of F.

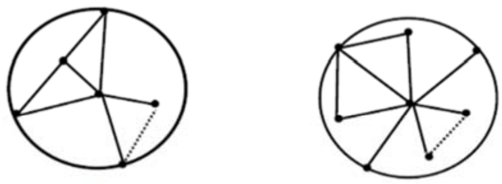
Case 2. Suppose that $d_7, \cdots, d_{n-1} = 0$, hold and the degree sequence is

$(d_1,\ d_2, d_3,\ d_4, d_5, d_6, 0,\ \cdots, 0,\ d_n)$. As $d_{n-1} \in \{0, 1\}$, so we have to analyze two cases:

(a'). If $dx_{n-1} = 0$, then $d_2 + 2d_3 + 3d_4 + 4d_5 + 5d_6 = n + 8$. This equation does not hold. If all $d_2, d_3,\ d_4, d_5$ and d_6 are not greater than 2, then $d_2 + 2d_3 + 3d_4 + 4d_5 + 5d_6 \leq 30$ which contradicts the hypothesis $n \geq 7$. If one of them is greater than two, then by using T_2 for the corresponding position, we obtain a smaller value of F.

(b'). If $d_{n-1} = 1$, then $d_2 + 2d_3 + 3d_4 + 4d_5 + 5d_6 = 10$. If $d_6 \geq 3$, then $d_2 + 2d_3 + 3d_4 + 4d_5 + 5d_6 \geq 15$. So $d_6 \leq 2$, if $d_6 = 2$ then $d_2 + 2d_3 + 3d_4 + 4d_5 = 0$, which implies that $d_2 = d_3 = d_4 = d_5 = 0$ and $d_1 = n - 3$ which is a contradiction as $d_1 \leq n - 5$. So $d_6 \neq 2$. Thus either $d_6 = 0$ or $d_6 = 1$. If $d_6 = 1$, then $d_2 + 2d_3 + 3d_4 + 4d_5 = 5$, all the possible solutions are $d_2 = 1,\ d_3 = 2,\ d_4 = 0, d_5 = 0$ or $d_2 = 3,\ d_3 = 1,\ d_4 = 0, d_5 = 0$ or $d_2 = 0,\ d_3 = 1,\ d_4 = 1,\ d_5 = 0$ or $d_2 = 1,\ d_3 = 0,\ d_4 = 0, d_5 = 1$. The third and fourth case should be removed as they contradicted corollary. From first and second case we obtain a degree sequence $(\mathrm{n} - 6, 3, 1, 0, 0, 1, 0, \cdots, 0, 1)$ and $(\mathrm{n} - 5, 2, 0, 1, 0, \cdots, 0, 1)$ which is not graphical.

Next consider if $d_6 = 0$, then then $d_2 + 2d_3 + 3d_4 + 4d_5 = 10$, all possible solution of $d_2,\ d_3,\ d_4$ and d_5 which follows the above corollary and construct the nine degree sequences which are graphical. The sequences are $(\mathrm{n} - 5,0,2,2,0,\cdots,0,1)$, $(\mathrm{n} - 6,0,5,0,\cdots,0,1)$, $(\mathrm{n} - 6,2,2,0,1,0,\cdots,0,1)$, $(\mathrm{n} - 6,2,1,2,0,\cdots,0,1)$, $(\mathrm{n} - 7,4,1,0,1,0,\cdots,0,1)$, $(\mathrm{n} - 7,4,0,2,0,\cdots,0,1)$, $(\mathrm{n} - 7,3,2,1,0,\cdots,0,1)$, $(\mathrm{n} - 8,5,1,1,0,\cdots,0,1)$ and $(\mathrm{n} - 8,4,3,0,\cdots,0,1)$. By applying transformations T_1 and T_2 these degree sequences are transformed to either the sequence $(\mathrm{n} - 5,0,2,2,0,\cdots,0,1)$ or $(\mathrm{n} - 6,2,2,0,1,0,\cdots,0,1)$ represented by graphs E_1 and E_2, respectively. And Hence it is proved that $F(\mathrm{n} - 5,0,2,2,0,\cdots,0,1) = F(\mathrm{n} - 6,2,20,1,0,\cdots,0,1) = 3n^2 + 13n - 62$.



E_1 E_2

Figure 5. *Connected Six Cyclic Graphs with n-Vertices.*

4. Conclusion

The minimum degree distance of connected five cyclic graphs G_n^5 is investigated. The graph contains five cycles if the sum of degrees of n- vertices is equal to n and sum of $\sum_{t=1}^{n} td_t$=2n+8. A graph $G \in G_n^5$ if G contains at least five vertices. The degree distance of $G \in G_5^5$ is $D'(G) = 78$ and $G \in G_6^5$ is $D'(G) = 124$. For $n \geq 7$, the minimum degree distance of $G \in G_n^5$ is $D'(G) = 3n^2 + 13n - 62$.

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