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Functors on ∞- categories and the Yoneda embedding

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Abstract: Through the application of the Yoneda embedding in the context of the ∞- categories is obtained a classification of functors with their corresponding extended functors in the geometrical Langlands program. Also is obtained a functor formula that can be considered in the extending of functors to obtaining of generalized Verma modules. In this isomorphism formula are considered the Verma modules as classifying spaces of these functors.

Keywords: Deformed Category, Extended Functor, Full and faithfull Functor, ∞- Category, Moduli Problem, Yoneda Embedding

1. Introduction

This research begins with a Kontsevich-Soibelman work [1], and some ideas on extension of functors using tools of Yoneda embeddings and Grothendieck's "functor of points" considering the moduli space X, as equivalent to specifying of the functor

$$R \to X(R) = \mathrm{Hom}(\mathrm{Spec}R, X) \tag{1.1}$$

For other way, let k, be a field. The category Chain_k, of chain complexes over , admits a symmetric monoidal structure, given by the usual tensor product of chain complexes. A commutative algebra in the category Chain_k, is called a commutative differential graded algebra over k. The functor , is symmetric monoidal, and determines a functor

$$\varphi : \mathrm{CAlg}(\mathrm{Chain}_k) \to \mathrm{CAlg}(\mathrm{Mod}_k) \cong \mathrm{CAlg}_k, \tag{1.2}$$

We say that a morphism $f : A_\bullet \to B_\bullet$, in $\mathrm{CAlg}_k^{\mathrm{dg}}$, is a quasi-isomorphism if it induces a quasi-isomorphism between the underlying chain complexes of $A_\bullet$, and $B_\bullet$. The functor ϕ, carries every quasi-isomorphism of commutative differential graded algebras to an equivalence in CAlg_k. Here can appear an extended version of Penrose transform in the context of the moduli problems, that is to say, considering the schemes in the context Moduli_k, can be constructed a functor that comes from a sufficiently generalized Penrose transform such that the objects induced in an augmented algebra corresponds to geometrical objects of the functor Chain_k, (in this case we need that these objects are $\mathrm{CW}-$ complexes) applied in the context of the vector bundles that come from the commutative differential graded algebra over k. If k, is a field of characteristic zero, then ϕ, induces an equivalence

$$\mathrm{CAlg}(\mathrm{Chain}_k)[W^{-1}] \cong \mathrm{CAlg}_k, \tag{1.3}$$

where W, is the collection of quasi-isomorphism, that in the context of the $\infty-$categories of $E_\infty-$algebras over k, will be the obtained from the ordinary categories of commutative differential graded $k-$algebras by formally inverting the collection of quasi-isomorphisms. The medullar functors of these transformations are functores obtained through the Yoneda embedding [2]. An classification of these functors is obtained using the equivalences between objects in three levels: the formal moduli problems, the integral transforms and the Zuckerman functors,where this last results useful in the representation problem of Lie algebras in the geometrical context planted by the Langlands program with their geometrical correspondences.

2. Functores on $\infty-$Categories

Let C, be the category of all topological spaces and let , be the collection of weak homotopy equivalences. We will refer to $\mathrm{C}[W^{-1}]$, as the $\infty-$category of spaces and denote it by S. We describe the object of S, as the CW complexes [3], and whose property between CW complexes is:

(a) The objects of S, are CW complexes.
(b) For every pair of CW complexes X, and Y, we let $\mathrm{Hom}_{\mathrm{S}}(X,Y)$, denote the space of continuous maps from X, to Y, (endowed with the compact-open topology).

3. Yoneda Embedding on $\infty-$Categories

Let C, be a locally small category, and let locally small category $[\mathrm{C}^{\mathrm{op}},\mathrm{Set}]$, the category of pre-sheaves on C. Let $c\in\mathrm{C}$, be an object.

The Yoneda lemma asserts that the set of morphisms from the pre-sheaf represented by c, into any other pre-sheaf X, is in natural bijection with the set $X(c)$, that this pre-sheaf assigns to c.

Formally:

Proposition 3. 1. There is a canonical isomorphism

$$[\mathrm{C}^{\mathrm{op}},\mathrm{Set}](\mathrm{C}(-c),X)\cong X(c) \qquad (3.1),$$

natural in c.

Here $[\mathrm{C}^{\mathrm{op}},\mathrm{Set}]$, denotes the *functor category*, also denoted $\mathrm{Set}^{\mathrm{C}^{\mathrm{op}}}$, and $\mathrm{C}(-,c)$, the representable pre-sheaf. This is the standard notation used mostly in pure category theory and enriched category theory. In other parts of the literature it is customary to denote the pre-sheaf represented by c, as h_c. In that case the above is often written

$$\mathrm{Hom}(h_c,X)\cong X(c), \qquad (3.2)$$

or

$$\mathrm{Nat}(h_c,X)\cong X(c), \qquad (3.3)$$

to emphasize that the morphisms of pre-sheaves are natural transformations of the corresponding functors.

Proof. [3].

The Yoneda lemma given in the proposition 3. 1, embeds a category into the other category of their representable functors. Then implies the called Yoneda embedding between categories.

If we consider the CW complexes in the context of $\infty-$categories, we can establish that the homomorphism of $\mathrm{Set}^{\mathrm{C}^{\mathrm{op}}}$, is an isomorphism of type:

$$\mathrm{Hom}(\mathrm{C}^{\mathrm{op}}\times\mathrm{C},\mathrm{Set})\xrightarrow{\cong}\mathrm{Hom}(\mathrm{C}^{\mathrm{op}},[\mathrm{C},\ \mathrm{Set}]), \qquad (3.4)$$

where the space

$$\mathrm{Set}=\{\infty\text{-categories of spaces that are complexes}\},$$

If in particular these complexes are topological spaces with an associated homotopy theory, then Set, can be re-define as the space of objects of the section 2.

Then let S, be the set of objects that are CW complexes in the context of $\infty-$categories. Then, for any $\infty-$category C, one can define a *Yoneda embedding* [4] as the mapping

$$j:\mathrm{C}\to\mathrm{Fun}(\mathrm{C}^{\mathrm{top}},\mathrm{S}), \qquad (3.5)$$

given explicitly by

$$j(C)D=Hom_{\mathrm{C}}(D,C)\in\mathrm{S}, \qquad (3.6)$$

In this research we are interested in the $\infty-$category analogous more algebraic structures like commutative rings.

In this last point, we need the symmetric monoidal structure of $\infty-$category Sp, which endows to $\infty-$category CAlg, as "CRing" always when the objects of the commutative algebra are spectrum R, equipped with a multiplication $R\wedge R\to R$, which is unital, associative and commutative up to coherent homotopy. This objects are acquaintances as $E_\infty-$rings, and the space CAlg(Sp), as the $\infty-$category of $E_\infty-$rings. The commutative algebra of the spectrums R, equipped with a multiplication $R\wedge R\to R$, being this unital, associative and commutative up to coherent homotopy, conforms a Yoneda algebra when the $\infty-$categories are derived categories whose functor Ext, carries to the Yoneda $\infty-$algebra multiplication.

In the next section we consider the C, $\infty-$category of endo-functors $\mathrm{C}\to\mathrm{C}$.

4. Extension of Isomorphisms from the Hochschild co-Chain Complexes

An geometrical interpretation of the $\infty-$algebras (as the given by the Yoneda algebra) is the consideration of Hochschild chain complex of an $\infty-$algebras in terms of cyclic differential forms on the corresponding formal pointed dg$-$manifold.

Let k, be a field and let C, be a compactly generated k-linear $\infty-$category (*small differential graded category of* k).

Theorem 4. 1. Let k, be a field and let C, be a compactly generated k- linear $\infty-$ category. The Hochschild cohomology of $\infty-$ category C, is isomorphic to $\mathrm{Ext}^{\bullet}(\mathrm{Id}_{\mathrm{C}},\mathrm{Id}_{\mathrm{C}})$, taken in the $\infty-$category of endomorphisms $\mathrm{C}\to\mathrm{C}$.

Proof. Interpreting the Hochschild co-chains as vector field and functors as mappings, that is to say, the idea to treat $\mathrm{Ext}^{\bullet}(\mathrm{Id}_{\mathrm{C}},\mathrm{Id}_{\mathrm{C}})$, as the tangent space to deformations of the derived category S, through of the derived moduli problem $\mathrm{Fun}(\mathrm{CAlg}(\mathrm{Sp}),\mathrm{S})$ (enhanced triangulated category [5]). But by the theorem 6. 1, the functor $\mathrm{F}(\mathrm{C}):\mathrm{Alg}^{(n)}_{(\mathrm{sm})}\to\mathrm{S}$, is determined up to equivalence by the augmented E_n-algebra $A=\Phi\mathrm{F}(\mathrm{C})$ ($A\in\mathrm{Alg}^{(n)}_{\mathrm{aug}}$ and Φ acts on A-

modules). Equivalently, the functor $F(C)$, is determined by the non-unital E_n − algebra $\mathfrak{m}_A$. But $\mathfrak{m}_A$, can be identified with the endomorphism ring $\mathrm{Ext}_D(\mathrm{Id}_C, \mathrm{Id}_C)$, where D, is the stable ∞ − category of A - linear functors from C, to itself. Then the tangent complex of $F(C)$, is isomorphic to the corresponding mth -Hochschild group cohomology. The result stay proved.

Example 4. 1. Let the corresponding non-compact Lagrangian submanifolds L, as homotopy of L_0, where L_0, is the Lagrangian submanifold before of the action field, to know [6]:

$$HW^*(L_0, L_1) = H(CW^*(L_0, L_1)), \tag{4.1}$$

Using the *conjecture of Fukaya* considering $L = T_x^*$, a cotangent fiber, the A_∞ − structure on $CW^*(L_0, L_1)$, should be quasi-isomorphic to the algebra dg − structure on $C_{-*}(\Omega_x)$, where Ω_x, is the *based loop space1* of (Z, x), where Z, is differentiable manifold which involves a normalized geodesic flow accord to their Poincaré section.

To $L \subset M = T^*Z$, , one can associate the bundle E_L, of the Floer cohomologies:

$$E_x = HF^*(T_x^*, L), \tag{4.2}$$

One naturally wants to replace E_L, by an underlyings co-chains level objects E_L, which should be a "sheaf of complexes" in a suitable sense.

In these interpretations this will be a dg − module over a dg − algebra of Cěch co-chains (differential graded dg − algebras). We consider (unital right) dg − modules over A. We denote the dg − category of such module by $C = \mathrm{mod}(A)$. However, this category is not a dg model for the derived category, since there are acyclic modules which are non-trivial in $H(C)$, and as a consequence, quasi-isomorphism does not imply isomorphism in that category.

If Z, is simply connected and A, is the dg − algebra of Cech co-chains then B 2, is quasi-isomorphic to $C_{-*}(\Omega Z)$.

Hence reversing the functor above, we get a full and faithful embedding

$$H(\mathrm{mod}f(C_{-*}(\Omega Z))) \to H(C) \tag{4.3}$$

where $C = \mathrm{mod}(A)$, as before. Then by the conjecture of Fukaya mentioned before and the conjecture 2, given in the page foot3, it follows that W (M), itself is derived category equivalent to $H(\mathrm{mod}(C_{-*}(\Omega Z))$. The application of the *Yoneda embedding* is clear, and we can to give a full embedding of the wrapped Fukaya category into $H(C)$, avoiding the use of *Cěch complexes* altogether.

The before example give conditions to that a *quasi-isomorphism* can be established as an extension of a isomorphism and thus *imply an isomorphism in the category context.*

Considering Hochschild cohomology, the application of the Yoneda lemma is trivial and stay included to the conditions that this cohomology requires.

5. Classification Results

If we fix a field k, and an integer $n \geq 0$, the theorem 6. 1 in [4], asserts the existence of an equivalence of ∞ − category

$$\mathrm{Alg}^{(n)}_{\mathrm{aug}} \overset{\Phi^{-1}}{\cong} \mathrm{Moduli}_n \subseteq \mathrm{Fun}(\mathrm{A\,lg}^{(n)}_{\mathrm{sm}}, S), \tag{5.1}$$

Then by *Koszul duality*, the equivalence given in the first member of (5. 1) can be carried in the context of the contention of the second member of (5. 1) as a new equivalence of geometrical objects in S. These new objects class are the images of the extended functors via the formal moduli problems as *moduli stacks* and *new physics* [8] in field theory. Some classification of these new physics can be viewed in the work [9].

The production of extended functors in the *table 1*, obeys to a functoriality process over classifying stacks [10]. Of fact, these classifying stacks are module categories over D (BG).

Through to consider the structure of equivariant categories of D − modules as modules over D − modules over

1 In mathematics, the space of loops or (free) loop space of a topological space X, is the space of loops from the unit circle S^1, to X, together with the compact-open topology.

$\Omega X = C(S^1, X)$,

That is, a particular function space. In particular Ω_x, is the base loop space of ΩX.

In homotopy theory *loop space* commonly refers to the same construction applied to pointed spaces, i.e. continuous maps respecting base points. In this setting there is a natural "concatenation operation" by which two elements of the loop space can be combined. With this operation, the loop space can be regarded as a magma or even as an A_∞ − space. Concatenation of loops is not strictly associative, but it is associative up to higher homotopies.

2 Conjecture 1 [6, 7]. Let $M = T^*Z$, be a cotangent fibre. Then the dg − module E_L (of Lagrangian submanifold L) is isomorphic to E_L in $H(\mathrm{M})$. Moreover, if , is simply connected G ,2 gives rise to a quasi-isomorphism $C_{-*}(\Omega_x) \cong CW^*(L, L) \to \hom_{\mathrm{M}}(\mathsf{E}_L, \mathsf{E}_L)$.

Here G is the A_∞ − functor:

G : W $(M) \to$ M $= \mathrm{mod}(C)$,

Resulted of associates to any exact Lagrangian submanifold , which is *Legendrian* at infinity, a dg − module E_L, over C .

Here a dg − algebra over our coefficient field K , with an augmentation $\varepsilon : C \to K$, whose kernel is denoted by I . Then is possible equip the free tensor co-algebra $T(\mathrm{I}[1])$, .with a differential and then dualizes it to a dg − algebra B $= T(\mathrm{I}[1])^\vee$.

3 Conjecture 2 [6, 7]. Any one fibre $L = T_x^*$, generates the derived category(taken, as usual, to be the homotopy category of twisted complexes) of W (M).

classifying stacks can be established the self-duality (*Koszul duality*, as the equivalence of type given in the first member of (5. 1)) of the $\infty-$category of $D-$modules. Then functors on $\infty-$ categories can be established as the extensions $\Phi+_{\text{geo. hypothesis}}$, understanding this geometrical hypothesis extension established by the formal moduli problem that require (5. 1).

Table 1. *Classification of functors with their corresponding extended versions.*

Functor	Scheme	Extended Functor
	Formal Moduli Problems	
$G\Psi_K$	k - Module	$G\Psi_R$
ΦF_K	CRing	ΦF_R
φ	Moduli_n	$\varphi(A)(B)$
ψ	Alg_k^{dg}	$\psi(A)(B)$
$\mathrm{Spec}(A)$	$\mathrm{Fun}(\mathrm{Alg}_{\mathrm{sm}}^{(n)}, \mathrm{S})$	$\mathrm{Spec}(A)(B)$
	Integral Transforms	
Φ	$\mathrm{M}_G(\mathfrak{g}, H)$	$\Phi+$geo. hypothesis
Ψ	$\mathrm{M}(D_{G/H}-$modules G -equivariants)	$\Psi+$geo. hypothesis
$R\Gamma_K^H$	$\mathrm{H}_{G^\wedge} \cong \mathrm{M}_\chi(\tilde{\mathfrak{g}}, \mathrm{Y})$ **	${}^L\Phi_\blacklozenge^\mu$
	Zuckerman Functors	
$\Gamma_{\mathfrak{g}, L\cap K}^{\mathfrak{g}, K}$	$R(\mathfrak{g}, K)$ *	$\Gamma_{G/H}^{\mathrm{equiv}G}$

* $R(\mathfrak{g}, K)$, is the *Hecke algebra* of G, of all distributions on G, with support in K, that are left and right K, finite.

**The Lie algebra $\tilde{\mathfrak{g}}$, is the loop extension of the loop algebra $\mathfrak{g}(t)$.

Remember that the geometrical hypothesis in the functor $\Phi+$geometrical hypothesis, comes established for the geometrical duality of Langlands which says that the derived category of coherent sheaves on a moduli space $\infty-$category of $D-$ modules is equivalent to the moduli space $D-$ modules on the moduli space $\infty-$category of $D-$modules the derived category of coherent sheaves on a moduli space of flatness.

The Zuckerman functor let be restricted to the sub-category of left $G-$equivariant $D_{G/H}-$ modules that are product of a triangulated sub-category as the given in [10], and for a factor category that comes from certain Zuckerman functor constructed from image of one derived category using the corresponding generalization of the integral transforms on $D-$modules.

Example 1. [10]. If we consider as derived category of coherent sheaves on a moduli space $\infty-$category of $D-$ modules the Hecke category $\mathrm{H}_G = \mathrm{M}(\mathbb{D}_\lambda^{G/H})$, $\forall\lambda\in\mathfrak{h}_{\mathbb{Z}}^*$, then $\mathrm{H}_G = \mathrm{M}(\mathrm{B}\backslash G/\mathrm{B})$, from the group $G = G^{\mathbb{C}}$, B-equivariant $D-$ module on the flag manifold $X = G/\mathrm{B}$. Then the isomorphism the Hecke categories is:

$$\mathrm{H}_{G^\wedge} \cong \mathrm{M}(\tilde{\mathfrak{g}}, \mathrm{Y}), \tag{5.2}$$

under duality of the cycles on $G(t)$ (B-equivariant $D-$ modules on the flag variety G/B) and using their images of kernels corresponding to the integral transforms. The Lie algebra $\tilde{\mathfrak{g}}$, is the *loop extension* of the loop algebra $\mathfrak{g}(t)$. The functor in the equivalence (5. 2) is ${}^L\Phi_\blacklozenge^\mu$, which is the extended functor of $R\Gamma_K^H$.

The functor $R\Gamma_K^H \in \mathrm{Fun}(\mathrm{Alg}_{\mathrm{sm}}^{(n)}, \mathrm{S})$. Then the isomorphism are given in the context of equivalences $\mathrm{Alg}_{\mathrm{aug}}^{(n)} \overset{\Phi^{-1}}{\cong} \mathrm{Moduli}_n$ to moduli problems. These are represented by (5. 2). But we want that the reverse functor has an full and faithful embedding in the space $\mathrm{Fun}(\mathrm{Alg}_{\mathrm{sm}}^{(n)}, \mathrm{S})$, that is to say, that all quasi-isomorphism that extends the isomorphism $\mathrm{Alg}_{\mathrm{aug}}^{(n)} \cong \mathrm{Moduli}_n$, let an isomorphism from Moduli_n, to $\mathrm{Fun}(\mathrm{Alg}_{\mathrm{sm}}^{(n)}, \mathrm{S})$, such that S, has the character of derived category. Then to an adequate election of co-chains defined by the co-chain complex $\mathrm{Hom}_A(C_\bullet^{bar}, k)$, 4 the category $\mathrm{Alg}(\mathrm{Chain}_k)$, can be identified with the category of differential graded algebras over k. Then is induced an equivalence as (1. 3) through of a quasi-isomorphism.

Let M, be a non-commutative formal pointed $dg-$ manifold and let $C^\bullet(A, A) := C^\bullet(M, M) := Vect(M)[-1]$, the Hochschild co-chain complex of A (*as the determined in the theorem 4. 1*). As a $\mathbb{Z}-$graded vector space we have:

$$C^\bullet(A, A) = \prod \underline{Hom}_C(A[1]^{\otimes n}, A), \tag{5.3}$$

Algebraically $C^\bullet(A, A)[1]$, is a dg- Lie algebra of derivations5 of the co-algebra $T(A[1])$ [5, 7].

Theorem (M. Kontsevich, Y Soilbelman) 5. 1. Let M, and $C^\bullet(A, A)$, the $dg-$ manifold and Hochschild co-chain complex defined before. Then one has the following quasi-isomorphism of complexes:

$$C^\bullet(M, M) \cong T_{\mathrm{Id}_M}(Maps(M, M)), \tag{5.4}$$

where T_{Id_M}, denotes the tangent complex at the identity map.

Proof. [7].

The tangent complex T_{Id_M} is such that $T_{M,x}(0) = M_x(k[\varepsilon]/(\varepsilon^2))$, which can be identified with a classifying space for the grupoid of projective $k[\varepsilon]/(\varepsilon^2)-$ modules V, 6which deform V_0 7. This grupoid has only object up to isomorphism given by the tensor product $k[\varepsilon]/(\varepsilon^2)\otimes V_0$. Then $T_{M,x}(0)$, can be identified with the classifying space BG, for the group , of automorphisms of $k[\varepsilon]/(\varepsilon^2)\otimes V_0$, which reduces to the identity moduli ε ($\varepsilon\to 0$ and the equivalence is risked in this level). One of these automorphisms can be written as $1+\varepsilon\tau$, where

4 Definition. $C_\bullet^{bar}$ is the normalized bar resolution of $k\in A-$mod.

5 Also is known as differential graded Lie algebra (DGLA).

6 Remember that V, is a $k-$ vector space.

7 V_0, is the $k-$ vector space before the deforming.

$\tau \in \mathrm{End}(V_0)$. Then at least $T_{\mathrm{Id}_M} Maps(k,k)$, to any element in $Maps(k,k)$, represents an homomorphism of $\infty-$ algebras $A^{\mathrm{op}} \to \mathrm{Hom}_{A-\mathrm{mod}}(k[\varepsilon]/(\varepsilon^2)\otimes V, k)$, for all A, an $\infty-$ algebra 8 . But $Maps(\mathrm{Spec}(k[\varepsilon]/(\varepsilon^2))\otimes V_0, k)$, is a non-commutative $dg-\mathrm{ind}-$ manifold of vector fields of M .

Proposition 5. 1. Let V , a $k-$ vector space that is an Harish-Chandra module to the pair $(\tilde{g}, G(t))$. The extended functors in the *table 1*, corresponding to this extended algebra $\tilde{g}$, involve a quasi-isomorphism of complexes

$$\mathrm{Ext}^{\bullet}_{HC}(V,V) \cong T_{\mathrm{Id}_{\tilde{g}}} \mathrm{End}(V,V), \tag{5.5}$$

Here V , is the classifying space or Verma module to $G(t)$.

Proof. We can interpreted newly the Hochschild co-chains as vector field and functors as mappings, and use the theorem 5. 1, that is to say, the idea is to treat $\mathrm{Ext}^{\bullet}_{HC}(V,V)$, as the tangent space to deformations of the derived category $\mathrm{D}(BG)$ [11, 12]. Analogously to the argument given before to the tangent complex T_{Id_M} , we can to give an argument considering an $A_\infty-$ algebra with the Yoneda structure having that to tangent complex $T_{\mathrm{Id}_{\tilde{g}}}$ we have of consider to $A^{\mathrm{op}} \to \mathrm{Hom}_{\tilde{g}}(V,V)$, due to that $T_{\mathrm{Id}_{\tilde{g}}}$, can be identified as the enveloping algebra $U_{c+\varepsilon k}\tilde{g}$, reducing to the identity moduli when $\varepsilon \to 0$, (that is to say to the central element of the group) One of the automorphisms of the group of $U_{c+\varepsilon k}\tilde{g}$, can be written as $\mathrm{End}(V,V)$. Then at least $T_{\mathrm{Id}_{\tilde{g}}} Maps(V,V)$, to any element in $Maps(V,V)$, represents an homomorphism of $\infty-$ algebras. Then by the theorem 4. 1, and the theorem 5. 1 $\mathrm{Ext}^{\bullet}_{HC}(V,V)$, is quasi-isomorphic to $T_{\mathrm{Id}_{\tilde{g}}} Maps(V,V)$.

Functors are viewed as maps then the Ext, groups are computed in a suitable defined category of Harish-Chandra modules for the pair $(\tilde{g}, G[z])$, of critical level (that is to say, to the corresponding generalized Verma modules and their self-extensions). This will be equivalent to extend the action of central elements of $U_{c+\varepsilon k}\tilde{g}$, to the entire Yoneda Ext- algebra of Verma modules of critical level.

6. Conclusions

The functor from projective $\tilde{g}-$ modules to $\mathrm{D}(BG)-$ modules is exact on the full, exact subcategory consisting of finite successive extensions of Verma modules of critical level [13, 14]. Inducing on the length can be showed that the localization functor is concentrated in degree zero (and exact) , but this functor follows being an A_∞ morphism of Ext , algebras (as the Yoneda Ext - algebra of Verma modules of critical level). The Hochschild complex $\mathrm{Ext}^{\bullet}_{HC}(V,V)$, in the Harish-Chandra category is generated by Ext^1, and the commutative relations are enforced by formal deformations which (to all orders) belong to the subcategory of finite successive extensions of critical Verma modules, having the succession

$$V \to \mathrm{E}_1 \to \mathrm{E}_2 \to \ldots \to \mathrm{E}_k \to V, \tag{6.1}$$

where we can associate functorially a self-extensions of categories in $\mathrm{D}(BG)$, using the generalized Penrose transform that defines the functors class $\Phi + {}_{\mathrm{geo.hypothesis}}$, as was signed in the *theorem 4. 1*. Finally we can say, from the point of view of the Yoneda structure, that this is a functor in the Yoneda extension classes in exact categories.

8 We have a structure in the $k-$ vector space as an $A-$ module.

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