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Hope for the Future: Overcoming the DEEP Ignorance on the CI (Confidence Intervals) and on the DOE (Design of Experiments)

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Abstract: The document shows the ideas to overcome the deep ignorance on the CI (Confidence Intervals) and on DOE (Design Of Experiments); the first part poses the problem that was originated in the RG (Research Gate): it analyses few of the answers, found in the forum, AND some wrong ideas one can find in Wikipedia; connection with the Test of Hypotheses is given; some figures are provided that make "intuitive" the concept of the Confidence Interval with the Theory (Classical Statistics). The second part considers some cases one can find in a very WWU (World Wide Used) Book: we show that high scores on documents do not prove the Quality of those documents. This paper is especially written to settle the matter for the researchers who use CI and DOE: Researchers must be alert in order to do a good job…. Many others cases should be shown: the paper should be 10 times longer; to make the paper shorter … I had to cancel pages providing the ideas on the "Scientificness", forgotten by many people and other providing ideas misleading the readers taken from Wikipedia.

Keywords: Confidence Intervals, Design of Experiments, Quality Methods, Scientific Approach, Intellectual Honesty

1. Introduction: "The Problem Outline"

The problem was originated, in the Research Gate, by Anvita Dharmarajan, B.Tech, M.Tech pursuing, Post Graduate Student, Manipal University, Department of Biomedical Engineering on October 2013. The question: In *most cases, the confidence level is taken as 95%? How do you get this value? What is the practical significance of this value?*

In the Research Gate database there were several answers, and various mistakes. [37] Analysis…

The complete set of answers is in the RG database; IF one reads them he finds that there is a problem of Statistical Knowledge, ACTUALLY of Statistical IGNORANCE! [5,6]

We do not present a literature review of the problem, because it will need at least hundreds of pages to be settled, both for Confidence Intervals and for Design of Experiments; we list here only few docs in the references [1-6,12-16,19-23,29,31-38].

To let the reader understand, I will analyze some answers:

- The first answers on October 2013
- And some on September 2014

ANALYSE the question: *What do you mean by confidence interval in statistical analysis?* NOTICE

1. There is difference between "What is the Confidence Interval?"
2. And "What *do you mean* by Confidence Interval?"
3. The Confidence Interval IS "definition" ……."
4. "I mean (it is only an Opinion!!!) THIS by Confidence Interval"

I will set WHAT IS Confidence Interval, NOT my opinion (what I mean…..).

We start our journey with the first answer [excerpt 1]; it was much appreciated: 31 people (out of 98 followers for 170 "answers") UPvoted it…. *I underlined and "italicized" the questions* [Q:] *to which Jochen provided an answer* [A:].

I will take one by one: the 1st answer was given by Jochen Wilhem, Justus-Liebig-University Grieβen (a researcher with very high scores and impact points: 158.36 and 334.21)

«« A: I suggest reading some books on statistics. It is a quite fundamental question. I will anyway give short answers to your questions, though...

Q: *What do you mean by confidence interval in statistical analysis?* A: It is an interval estimate for a parameter value. It is constructed in a way so that, in the long run, a given proportion of these intervals will include the unknown true parameter value. The proportion is given by the "level of confidence". For instance, you can expect that at least 90% of (a large series of) 90% confidence intervals will include the unknown true values of the parameters.

Q: *In most cases, the confidence level is taken as 95%?* A: Yes.

Q: *How do you get this value?* A: This depends on the parameter and the error model. Statistic software calculates such intervals, so a user actually doesn't need to know the technical details. A frequent problem is to give the CI for a mean value (xbar). This is calculated as xbar plusminus standarderror * t-quantile. The t-quantile is taken to get the desired confidence level.

Q: *What is the practical significance of this value?* A: It gives you an impression of the precision of the parameter estimate. Values spanned by this interval are seen as "not too unexpected to be true". CI's are actually a frequentist tool, but a further interpretation is Bayesian: given a flat prior, the CI is identical to the maximum a posteriori interval ("credible interval"). Here, the interpretation is inverse. Instead of saying that at least a given proportion of such intervals will include the true value, the Bayesian interpretation is that this particular interval includes the true value with a given probability. Looking at mean values, giving the CI is not in principle different to giving the standard errors (both are measures of precision), but the CI is much easier and clearer to interpret than the standard errors, since the directly give you a range of "not too unreasonable values" of the estimate. Further, the 95%-CIs include the information about the null hypothesis test on the 5% level (significance = 1-confidence). The null hypothesis can be rejected at the 5% level if the 95%CI does not include the null value. 31Oct, 2013»»

Excerpt 1. *(Jochen Wilhem answer, with many wrong points).*

Q: *What do you mean by confidence interval in statistical analysis?* A: It is an interval estimate for a parameter value. NOTICE the words: “parameter” and “VALUE”!!! What does that mean? That one cannot provide the “Confidence Interval” IF he does not have a value of the parameter? IF π is the symbol of a “generic parameter” in the formula f(x; π), can I not find the “Confidence Interval” for π, IF I do not know that the “generic parameter” π=3.14? The CAUSE of the wrong answer is in the following statements: HE says how to CONTRUCT (=CALCULATE) the Confidence Interval! *The DEFINITION and the Construction are DIFFERENT things!*

A: It is constructed in a way so that, in the long run, a given proportion of these intervals will include the unknown true parameter value. The proportion is given by the "level of confidence". For instance, you can expect that at least 90% of (a large series of) 90% confidence intervals will include the unknown true values of the parameters.

NOTICE: the PLURALS “parameterS” and “VALUES”!

A: Statistic softwares calculate such intervals, so a user actually doesn't need to know the technical details. A frequent problem is to give the CI for a mean value (xbar). This is calculated as xbar plusminus standarderror * t-quantile. The t-quantile is taken to get the desired confidence level

NOTICE: the example is based on the NORMAL Distribution” AND COMPLETE samples! The given RULE is NOT suitable for other DISTRIBUTIONS and samples!

We will see clearly in the next paragraphs….

Q: *How do you get this value?* A: This depends on the parameter and the error model. NOTICE: the answer is FALSE because there are involved the distribution (of the data) and the distribution of the ESTIMATOR of the “PARAMETER”! BUT what is the value that *you get?* IF the value is the CONFIDENCE LEVEL, it is NOT computed: it is FIXED, BEFORE the calculation of the CONFIDENCE INTERVAL!

Q: *What is the practical significance of this value?* A: It gives you an impression of the precision of the parameter estimate.

NOTICE: the answer is FALSE because the QUESTION is related to the CONFIDENCE LEVEL, while the answer is related to the CONFIDENCE INTERVAL! Regarding the “Credibility Interval (Bayesian)” see document of Fausto Galetto in the RG.

Let’s now analyze the last sentences of the answer:

A: Looking at mean values, giving the CI is not in principle different to giving the standard errors (both are measures of precision), but the CI is much easier and clearer to interpret than the standard errors, since the directly give you a range of "not too unreasonable values" of the estimate. Further, the 95%-CIs include the information about the null hypothesis test on the 5% level (significance=1-confidence). The null hypothesis can be rejected at the 5% level if the 95%CI does not include the null value.

We make the analysis by dividing them in two parts:

A: Looking at mean values, giving the CI is not in principle different to giving the standard errors (both are measures of precision), but the CI is much easier and clearer to interpret than the standard errors, since the directly give you a range of "not too unreasonable values" of the estimate.

NOTICE: «*looking at mean values*»; the answer is …. generally FALSE because one must prove that from standard errors he can compute CONFIDENCE INTERVAL: it is true ONLY for Normal distribution (&some related to it …)!

NOTICE: it is FALSE for any other parameter!

A: Further, the 95%-CIs include the information about the null hypothesis test on the 5% level (significance = 1-confidence). The null hypothesis can be rejected at the 5% level if the 95%CI does not include the null value.

NOTICE: the answer relates the CI with the “null hypothesis” of the <tests of Hypothesis> on any parameter…. To understand one MUST know the subject of the <tests of Hypothesis>. We try to provide the BASICS of <tests of Hypothesis> on any parameter….

Let π the parameter we want to “test”; previous to any collection of data we MUST state TWO Hypotheses and a probability α, named the “*significance level*”:

1. The “Null Hypothesis”, named H_0, where we assume, BEFORE any collection of data, a value for the parameter π; we indicate it with the symbol π_0; π_0 is a number, while π is the symbol of the parameter: we write H_0: [π=π_0]

2. The "Alternative Hypothesis", named H_1, where we assume, BEFORE any collection of data, another value for the parameter π; we indicate it with the symbol π_1; π_1 is a number different from π_0, while π is the symbol of the parameter: we write H_1: $[\pi=\pi_1]$
3. The probability α, the "*significance level*" that we assume, BEFORE any data collection and analysis of the data, is the <probability that we ACCEPT of being WRONG IF, AFTER the collection and the analysis of the data, we claim "*the Null Hypothesis* H_0*: [*$\pi=\pi_0$*] is REJECTED*", when ACTUALLY (and NOBODY knows it!) the "*the Null Hypothesis* H_0*: [*$\pi=\pi_0$*] SHOULD NOT be REJECTED*".

From the points 1, 2, 3, USING the Theory [5,6], we CAN, BEFORE any collection and analysis of the data, find TWO items:

- A "formula", named «Test Statistic», that will provide us with a number, AFTER the analysis of the data
- And an interval of the real line (real numbers) C, named «Critical Region» (or Rejection Region)
- Such that we REJECT «*the Null Hypothesis* H_0*: [*$\pi=\pi_0$*]*» IF $s \in C$.

Let's assume that we collect the data and analyze them, according to the Theory, and compute the number s; IF $s \in C$, THEN we, according to the Theory, MUST REJECT *the Null Hypothesis* H_0*: [*$\pi=\pi_0$*]*; IF $s \notin C$, we ACCEPT *the Null Hypothesis* H_0*: [*$\pi=\pi_0$*]*.

This idea is depicted in the figure 1

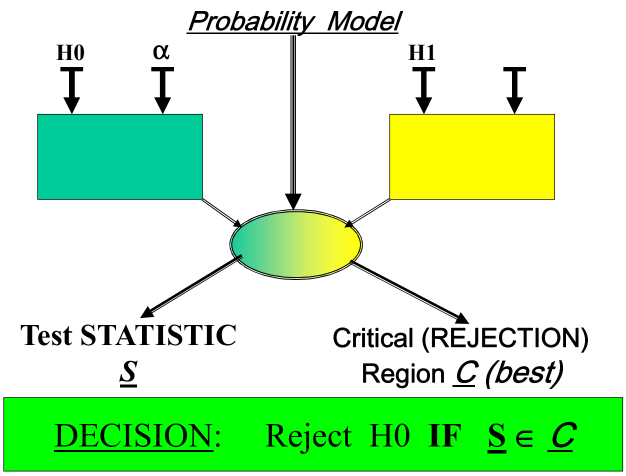


Figure 1. *Test of Hypothesis flow chart.*

In the figure 1 it is clearly shown that WE NEED the probability model SUITABLE to the analysis of our collected data for the *PARAMETER* we want to test!

In order to let the Researchers in the RG understand the BASICS, many and many times Fausto Galetto suggested considering problems like the following [5,6]:

«*You say "Statistical software such as SPSS, SAS etc. can calculate the CI. The CI shows the precision of the estimate, if it is narrower so the estimate is more precise. "Will those softwares provide the CI for the 2 cases?*

1. You have 10 atoms: 5 disintegrate and 5 do not disintegrate. Compute the CI (you can invent the data, as you like)
2. You have 100 atoms: 5 disintegrate (same time to disintegration as in 1.) and 95 do not disintegrate. Compute the CI (you can invent the data, as you like)

Which estimate is more precise?

The same is for "people dying"!

Notice: I did not FIX any parameter; I left the choice to the reader; the *question is valid for any parameter* the researchers want to analyze. NOTICE the answer (upvoted!) of Jochen Wilhem (158.36 and 334.21)

«« Fausto, I used R to calculate the CIs you requested:

5 of 10 atoms disintegrate. The estimated probability for disintegration for this data is p=0.5 with a 95%CI from 0.19 to 0.82.

5 of 100 atoms disintegrate. The estimated probability for disintegration for this data is p=0.05 with a 95%CI from 0.016 to 0.113.

However, your question "Which estimate is more precise?" cannot be answered for your example, because the variance is not constant and depends on the mean. From the presented data it seems that p=0.05 is a more precise estimate (the width of the CI is 0.094, whereas it is 0.63 for p=0.5. However, in simple terms, the relative precision (like the CV) is 1.9 for p=0.05 and 1.3 for p=0.5. Generally, proportions (binomial data) are analyzed on the logit scale, and there the width of the CIs are 2.0 for p=0.05 and 2.9 for p=0.5, indicating a higher precision in terms of the logits for p=0.05. This is only a rough estimate. A proper comparison is possible only for similar values of p, like comparing 5/10 with 50/100 (what has a width of the CI on the logit scale of 0.82).»»

Excerpt 2. *(Jochen Wilhem wrote to F. Galetto)*

NOTICE: the answer, in Excerpt 2, DOES NOT take into consideration the phenomenon "disintegration": the probabilities of disintegration depend from the interval considered (!), while those computed by Jochen are NOT time dependent e.g. they are related to DIFFERENT time intervals: the right way to compute the probability of disintegration is through the "disintegration rate" λ! For the same time t, the probability of disintegration of an atom is the same for the same interval 0____t! IF you go to paragraph 3, you see that the "precision" depends ONLY on g=5, NOT on n, in both cases! The very upvoted answer (31 upvotes) does not serve anything for this case [5,6]! Why people upvoted it?

They UPvoted the excerpt 2 due to their ignorance [5,6]. There is so vast ignorance in the RG that NOBODY accepted and considered that THERE IS a PROBLEM when the SAMPLES are INCOMPLETE and the distribution is NOT Normal!

Many and many researchers are BLIND AND DEAF. [5,6]

I am very sorry, BUT it is TRUE!!!!!!!!!!!!!!!

Notice: in the figure 1 we FIXED H_0, α, H_1, and we had to ASSUME the distribution of the "RANDOM VARIABLES" that will in future provide the data.

The width of the Rejection Region C, depends on the «number g of the RANDOM VARIABLES» providing the data we are going to collect; if g is "small" C is small (the acceptance region A, complementary set of C is large) and the probability $\beta(H_1)$ of rejecting H_1, in favor of H_0, will be "high": in this case, IF one computes the Confidence Interval, with Confidence Level CL=$\alpha+\beta$, he will find that H_0 and H_1 will be BOTH in the CI: one has NOT enough data (information) to distinguish between H_0 and H_1.

IF g increases C gets larger (the acceptance region A, complementary set of C gets smaller) and the probability $\beta(H_1)$ of rejecting H_1, in favor of H_0, will be "smaller": in this case, IF one computes the Confidence Interval, with Confidence Level CL=$\alpha+\beta$, he will find that H_0 and H_1 can be *either* BOTH in the CI *or* one alone $\in$ A: one has enough data (information) to distinguish between H_0 and H_1. BUT, at this point, the probability $\beta(H_1)$ [that is related to C], *can be* > β(WANTED):

$\beta(H_1) > \beta$(WANTED) ("small, as the researcher wants")

IF this is the case, we NEED [5,6] to INCREASE the "number g of the RANDOM VARIABLES" providing the data we are [NOT n] going to collect UNTIL we have

$\beta(H_1) < \beta$(WANTED) ("small, as the researcher wants").

The number g and the interval A are such that that we can distinguish H_0 and H_1 with the stated risks α and β, by using the RULE «ACCEPT *the Null Hypothesis* $[\pi=\pi_0]$ IF s $\in$ A».

For EXAMPLE…. Let's assume H_0: $[\pi(100)=\pi_0=0.90]$, versus H_1: $[\pi(100)=\pi_1=0.73]$, where $\pi(100)$ is the probability that an atom survive 100 years; we want to test our hypotheses with stated risks α=0.05 and β=0.10.

We MUST assume a distribution for the "time to disintegration" of the atoms: according to Physics we assume exponential distribution. Following what we said, we need that 8 atoms disintegrate; then we sum all the lives of the atoms we put on "test of disintegration"; this is the STATISTIC s; and we have to get s > 3781!

The formula for s is $s=t_1+t_2+t_3+t_4+t_5+t_6+t_7+(n-7)t_8$, where n [sample size] is the number of atoms we analyze for disintegration. The Acceptance Region is 3781 → ∞

NOTICE The sample size is n, while the number of random variable g is 8! The calendar time to get the decision depends on n; the POWER of the test depends on g! IF we put on test n=100000 atoms, we can decide about H_0 in 3871/100000 years that is 14 days…..

IF, after the test, we have the statistics s=4109, we find that the Confidence Interval, with Confidence Level CL=0.95, for the parameter $\pi(100)$ is 0.790____0.865; we see immediately that $\pi_1=0.73$ < 0.790____0.865 < $\pi_0=0.90$, that is the HYPOTISED values are at opposite sides of the Confidence Interval! (as it MUST be).

ALL the values in the Confidence Interval 0.790____0.865 are to be considered EQUIVALENT between them, and, since $\pi_0=0.90$ is accepted we can say 0.790____0.90 the set of numbers EQUIVALENT to the Hypothesis H_0. BUT the interval 0.790____0.90 is NOT the Confidence Interval, with Confidence Level CL=0.95!

NOTICE: The Wikipedia ideas are useless (IF NOT MISLEADING) for solving this problem!

Is the very upvoted answer (in excerpt 1) suitable to provide the right ideas? The previous ideas of Jochen Wihlem are useless also for the following case [September 2014]…. There are the usual MISconceptions! Another case where those ideas are useless ….. is related to the

Question «« How do you establish the minimal number of animals to test to get statistically significant data?

Could anyone suggest an established method, or formula, to calculate the minimal number of mice required to get statistical significance and adhere to the Replace, refine and reduce rule for animal use in experimental procedures. Best option will be to find an article to cite while writing grants or authorization to the ethical committee. Thank You »»

Excerpt 3. *(from Elena Adinolfi)*

For solving that problem, someone suggested to use the Software G_POWER, which is based on the NORMAL distribution! AGAIN NORMAL-drugged researchers…! HOW can anybody expect that Research and Decisions be good if people with high scores and high impact points are diffusing wrong ideas?

Another Upvoted (8 upvotes) answer was given by Viktor Witkovsky (23.28 and 37.02 Slovak Academy of Sciences):

««For a more comprehensive (and complicated) answer to your question look at the paper: "Confidence Distribution, the Frequentist Distribution Estimator of a Parameter: A Review" by Min-ge Xie and Kesar Singh. 8 / 0· Oct 9, 2013 http: //www.stat.rutgers.edu/home/mxie/RCPapers/insr.12000.» »

Excerpt 4a. *(suggestion of Viktor Witkovsky)*

The authors Min-ge and Singh are researchers NORMAL_DRUGGED! They write:

Suppose that a data set x is observed from a parametric family of densities $g_\mu(x)$, depending on an unknown parameter vector μ, and that inferences are desired for $\theta = t(\mu)$, a real-valued function of μ. Let $\theta_x(\alpha)$ be the upper endpoint of an exact or approximate one-sided level-α confidence interval for θ. The standard intervals for example have

$$\theta_x(\alpha) = \hat{\theta} + \hat{\sigma} z^{(\alpha)},$$

where $\hat{\theta}$ is the maximum likelihood estimate of θ, $\hat{\sigma}$ is the Fisher information estimate of standard error for $\hat{\theta}$, and $z^{(\alpha)}$ is the α-quantile of a standard normal distribution, $z^{(\alpha)} =$

Excerpt 4b. *(suggestion of Viktor Witkovsky)*

NOTICE: Normal distribution for the Confidence interval! The suggested paper is interesting BUT there is nothing that helps to solve the F. Galetto case (example) given before! *The same as WIKIPEDIA*…….! For EXAMPLE…. Let's assume H_0: $[\pi(100)=\pi_0=0.90]$, versus H_1: $[\pi(100)=\pi_1=0.73]$…

So again the Fausto Galetto question: Is the very upvoted answer suitable to provide the right ideas? All the answers (170! at November 2014) can be found in the RG. At the end of the 170 answers, one finds that Wikipedia is to be considered! SEE the sequence: D. I. Matthews, 20.08 and 32.15, Agri-Food and Bioscience Institute:

««When estimating something using a sample such as a mean, sampling theory allows the researcher to quantify the difference between the estimated value and true unknown value. The confidence interval is the value range in which the true population value lies (not to be mixed up with the sample estimate) given a level of certainty e.g. 5%. E.g. The true mean lies within the confidence interval.

Jochen Wilhem:

D.I. Matthews, I have to correct your answer. The CI does *NOT* give the range where the true population value lies. It is the range of values for the null hypotheses that would not be rejected. This has nothing to do with the true value of the population value.

What you were describing looks more like a credible interval. But this, too, is not about the true population value, but about the range of the most credible parameter value, given the current state of knowledge about the mode, including the data.

D. I. Matthews:

The entry on Wikipedia seems confirm my answer.

Fausto Galetto:

D.I. Matthews you should read various documents about Confidence Intervals....WIKIPEDIA several times is WRONG ....

Jochen Wilhem:

Possibly D.I. Matthews refers to this sentence (I looked up the English text in http://en.wikipedia.org/wiki/Confidence interval Meaning and interpretation):"There is a 90% probability that the calculated confidence interval encompasses the true value of the population parameter. “But this sentence is not standing alone in outer space there. There is an important explanation given right after this sentence: Note this is a probability statement about the confidence interval, not the population *parameter*.

The next point in this section of the Wiki article says: "The confidence interval represents values for the population parameter for which the difference between the parameter and the observed estimate is not statistically significant at the 10% level" - what is pretty much my explanation. I just avoid the term "population parameter" and call it "null hypothesis values" (or "hypothesized parameter values"), what to my opinion better hits the mark. D.I. Matthews was writing about a "level of certainty" what indicates that he is not referring to this sentence but rather to the previous one (but his "5%" do not fit then).

Thus, it would be nice, D.I. Matthews, if you could specify to what sentence of the Wiki article you are referring. We can then possibly reveal the source of the misunderstanding.

PS @Fausto: in my quick research I cannot see where the Wiki article is wrong (w.r.t to this statement). I think D.I. Matthews just interpreted something wrongly. Maybe others do a similar mistake, so it might be worth to find this out and discuss this.»

IF YOU GO to Wikipedia you find, inter alia…

««« For users of frequentist methods, various interpretations of a confidence interval can be given.

- The confidence interval can be expressed in terms of samples (or repeated samples): "*Were this procedure to be repeated on multiple samples, the calculated confidence interval (which would differ for each sample) would encompass the true population parameter 90% of the time."* Note that this does not refer to repeated measurement of the same sample, but repeated sampling.
- The confidence interval can be expressed in terms of a single sample: "*There is a 90% probability that the calculated confidence interval encompasses the true value of the population parameter.* Note this is a probability statement about the confidence interval, not the population parameter. This considers the probability associated with a confidence interval from a pre-experiment point of view, in the same context in which arguments for the random allocation of treatments to study items are made. Here the experimenter sets out the way in which they intend to calculate a confidence interval and know, before they do the actual experiment, that the interval they will end up calculating has a certain chance of covering the true but unknown value. This is very similar to the "repeated sample" interpretation above, except that it avoids relying on considering hypothetical repeats of a sampling procedure that may not be repeatable in any meaningful sense. See Neyman construction.
- The explanation of a confidence interval can amount to something like: "*The confidence interval represents values for the population parameter for which the difference between the parameter and the observed estimate is not statistically significant at the 10% level*". In fact, this relates to one particular way in which a confidence interval may be constructed.

In each of the above, the following applies: If the true value of the parameter lies outside the 90% confidence interval once it has been calculated, then an event has occurred which had a probability of 10% (or less) of happening by chance.»»

Excerpt 5. *(from Wikipedia)*

Jochen Wilhem states that there is nothing wrong with Wikipedia…Let’s see. I will analyze the Wikipedia….

- The confidence interval can be expressed in terms of samples (or repeated samples): "*Were this procedure to be repeated on multiple samples, the calculated confidence interval (which would differ for each sample) would encompass the true population parameter 90% of the time."* Note that this does not refer to repeated measurement of the same sample, but repeated sampling.

Wikipedia forgot to say that the statement refers to Confidence Intervals COMPUTED ASSUMING 90%=CL (Confidence Level)!

- The confidence interval can be expressed in terms of a single sample: "*There is a 90% probability that the calculated confidence interval encompasses the true value of the population parameter."* Note this is a probability statement about the confidence interval, not the population parameter. This considers the probability associated with a confidence interval from a pre-experiment point of view, in the same context in which arguments for the random allocation of treatments to study items are made. Here the experimenter sets out the way in which they intend to calculate a confidence interval and know, before they do the actual experiment, that the interval they will end up calculating has a certain chance of covering the true but unknown value. This is very similar to the "repeated sample" interpretation above, except that it avoids relying on considering hypothetical repeats of a sampling procedure that may not be repeatable in any meaningful sense. See Neyman construction.

Wikipedia forgot to say that the statement refers to Confidence Intervals COMPUTED ASSUMING 90%=CL (Confidence Level)! *AND* that the *CONFIDENCE LEVEL is NOT a probability*!

- The explanation of a confidence interval can amount to something like: "The confidence interval represents values for the population parameter for which the difference between the parameter and the observed estimate is not statistically significant at the 10% level". In fact, this relates to one particular way in which a confidence interval may be constructed. In each of the above, the following applies: If the true value of the parameter lies outside the 90% confidence interval once it has been calculated, then an event has occurred which had a probability of 10% (or less) of happening by chance.

Wikipedia forgot to say that the statement refers to Confidence Intervals COMPUTED ASSUMING 90% as Confidence Level AND that the *Calculated CONFIDENCE Interval* is a numeric interval which is *DIFFERENT with probability 100%* from the interval computed, BEFORE the Test by the ideas depicted in the figure 1. In the figure 1 it is clearly shown that WE NEED the probability model SUITABLE to the analysis of our data for the *PARAMETER* we want to test! We suggest reading what F. Galetto wrote about the Scientificness that it is needed in any Research.

2. Confidence Interval: Part 1

Any Manager needs data to take decisions, suitable to the case he has to solve. But it is not enough: he needs to analyze the data and transform them into VALID information. To get this he NEEDS methods: better it is if they are SCIENTIFIC. In my working life as Lecturer, Manager, Professor, … I have been seeing a huge number of Lecturers, Managers, Professors, … taking wrong decisions BECAUSE they used wrong methods, NOT APPLICABLE to the problems they wanted to solve! This is my long experience in the Quality field, as teacher, Manager, professor, papers writer, …When arguing on Scientific matters, everybody MUST act SCIENTIFICALLY.

We use here two scientific methods and others related to them: Maximum Likelihood Method (MLM) and Least Squares Method (LSM). We use the distribution of the Estimators (Probability Theory and Statistics Theory) to take the decisions.

To fix better the ideas I will use the following data on 10 items: the first 5 data are the TIME TO FAILURE [failures occur at 115, 149, 185, 251, 350 (unit of measurement are not given)] and the other 5 are data on items that did not fail [NON_Failures at 350, 350, 350, 350, 350 (they are also named "suspended items")]; such type of data are named "INCOMPLETE samples", because NOT ALL the data are "failures"; think to the data of survival of people to some drug cure: you do not wait until all die before taking decisions!!!! ASSUMING that the distribution function $F(t)=1-\exp[(t/\eta)^2]$ is the Weibull where the parameter is η and the Mean is μ, and fixing CL=90%, by making the right calculations we get that the Confidence Interval for the parameter μ is 270.7____582.8

GENERALLY the Statistics books do not consider the case of "INCOMPLETE samples"; they consider and provide only formulae for the "COMPLETE samples".

Many and many professors do not know the Reliability theory, EVEN THOUGH they teach Reliability.

To grasp the reality, LOOK at this exam exercise I used to give to my student: it is taken from a reliability book (3 incompetent authors!!!!) and refers to a reliability test where the time to failure distribution is assumed NORMAL!!!!Do not mind about the Italian language: I will translate for you. Macchina di prova=item on test, Tempo al guasto (ore)= Time to Failure (hours). 40 TTF are collected: the sample is complete (all the item failed). THREE incompetent professors say

vate al guasto dell'unità. Qualora alcune unità non arrivassero al guasto non è possibile considerare tale dato. Questo genera dei dati che non possono essere considerati ma che comunque generano dei costi di sperimentazione. |BMW csti !

[translation: If some of items do not fail it is not possible to use that datum. This generates data that cannot be considered but that in any case generate experimental costs]

The THREE SUPER_incompetent professors are highly rated in the so called «scientific community»!

««=====Esercizio n. 12 MOLTO ISTRUTTIVOrelativo ad un libro sull'Affidabilità di 3 BMWisti. Analyze the data of reliability tests …: THREE incompetent professors say, proving their whole IGNORANCE (they say that if some items do not fail by the end of the test the "suspended items" can NOT be considered in the computations)

YOU suppose that the test is truncated at 400 h: estimate the MTTF, WITHOUT neglecting the "suspended items". (the data are time to failure: data > 400 must be considered as non_failed at 400) BMWisti means ….

Misura e analisi dell'affidabilità **79**

Tabella 5.1 Esempio di dati rappresentabili con una distribuzione normale.

Macchina di prova	Tempo al guasto (ore)	Macchina di prova	Tempo al guasto (ore)
1	420	12	480
2	360	13	340
3	340	14	300
4	320	15	400
5	240	16	440
6	380	17	360
7	300	18	340
8	200	19	500
9	300	20	220
10	340	21	300
11	280	22	380

$$\mu = 343 \text{ ore}$$

$$\sigma = 77 \text{ ore}$$

Si noti come nelle prove sperimentali si sono considerate solo quelle che siano arrivate al guasto dell'unità. Qualora alcune unità non arrivassero al guasto non è possibile considerare tale dato. Questo genera dei dati che non possono essere considerati ma che comunque generano dei costi di sperimentazione. |BMW csti ! »»»

Excerpt 6. *(An exam exercise given by Galetto to his students)*

Poor students cheated and deceived by the professors they met and to be meted ….!YOU are guilty, because you do not use your brain! Can you be better than the great professorS?

Obviously my students could not be as stupid as those professors, to pass the test! Is so good one of the 3 authors Director of the Master on 6σ, met at the SIX SIGMA lessons?' HE knows and teaches wrong ideas. *Nevertheless* he is …. PhD, Visiting Prof. at MIT, author of 9 books, Master Black Belt, …., director of a Master on 6σ, …, Winner of the G. Taguchi Award on Robust Engineering, ….

LET'S HOPE that all those incompetent professors will consider their duty to teach scientifically, in order to satisfy the learning need of their students and of the whole society. See Deming, Gell-Mann, Galetto Fausto (figure 2), …

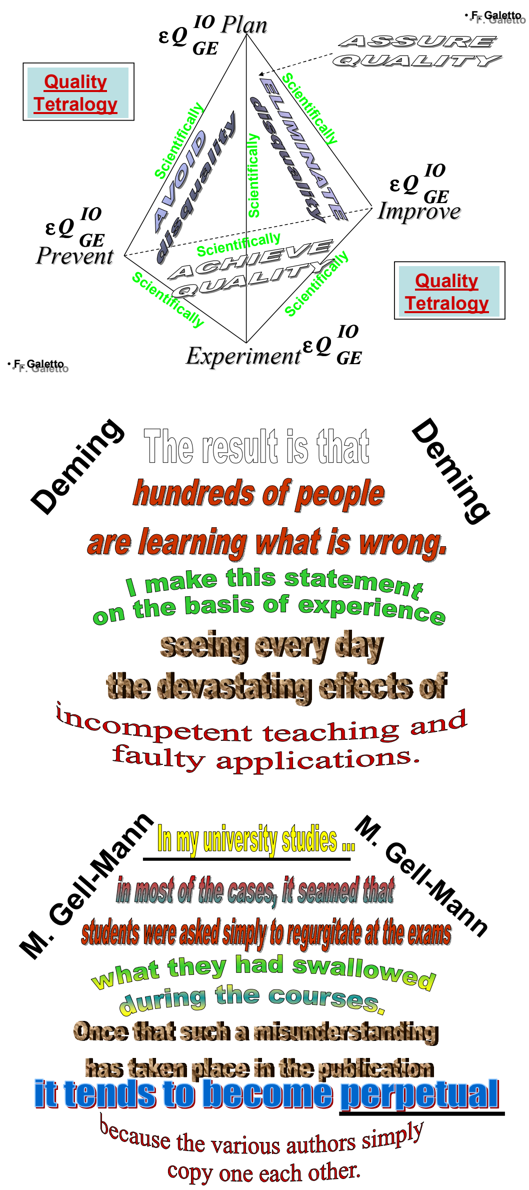


Figure 2. *Statements from Deming, Gell-Mann, Galetto ideas.*

Is there any Quality in wrong teaching? Teaching must be scientific for future managers, as Deming, Gell-Mann and Galetto say (figure 2).

To analyze the data we use a very powerful method: the Maximum Likelihood (ML) Method. [it is useful also for the Bayesian estimation]. To apply it we need to know the form of the distribution of the random variable T that generates the data D={t_1, t_2, …, t_n} [D is named "empirical sample, and n is the sample size]. This is a prerequisite; IF we do not know the form of the distribution of the random variable T generating the data, we need other methods. Since we are interested on defining the CONFIDENCE INTERVAL, we take advantage of the knowledge of the distribution F(t, θ) of the random variable T; θ is a vector that defines the parameters of the distribution; the density of the random variable T is indicated as f(t, θ). In the NORMAL case the distribution is the bell-shaped normal distribution N(t, μ, σ^2); μ in this case is the Mean (that is indicated with MU for the Greek letter μ), and σ^2 is the Variance (that is indicated with SIGMA_Squared, for the Greek letter σ^2).

Let's consider a sample D (of data) either incomplete or complete: the *Likelihood function L(θ, D)* is defined as

$$L(\theta, D)=\prod_i f(t_i;\theta) * \prod_j \left[1-F(t_j;\theta)\right]$$

where i refers to time to failures while j refers to survival times. GIVEN the data D, the function *L(θ, D) depends only on the vector of parameters θ*. The vector of the numbers $\tilde{\theta}$ maximizing the function *L(θ, D)* is called *Maximum Likelihood estimate*; it is the vector of values coming out from a RANDOM VARIABLE $\tilde{\Theta}$ called *Maximum Likelihood Estimator*[*MLE*].

The Maximum Likelihood Estimators are always

- Asymptotically efficient (become efficient when n →∞)
- Consistent and
- functions of sufficient estimators
- and moreover often they are also efficient.

Another important property is that every functions of sufficient estimators coming out form the *Maximum Likelihood* is a sufficient estimator: that is *if G is a sufficient MLE then are sufficient estimators, e.g., G+3, exp(G), G /27, etc.* REMEMBER: the distribution F(t, θ) of the random variable T MUST be known to use *Likelihood function L(θ, D).*

Let θ={μ, σ^2} [2 parameters vector] and F(t, θ)=N(t, μ, σ^2). IF D is a complete sample, the *ML Estimate* of the unknown mean μ is $\bar{t}=\sum_1^n t_i/n$; I name it *empirical mean*. When the data are indicated D={x_1, x_2, …, x_n} the *empirical mean* is named x_bar (remember what said by Jochen Whilem. It is efficient if σ^2 is known, because it comes out from the efficient estimator $\bar{T} = \sum_1^n T_i / n$; this is the Random Variable MEAN!!!!IF both μ and σ^2 are unknown one can find a couple of sufficient estimators

$$\bar{T}=\sum_1^n T_i/n \quad \text{and} \quad \Sigma^2=\sum_1^n (T_i-\bar{T})^2/(n-1)$$

Both Estimators are correct

$$E[\bar{T}]=\mu \quad \text{and} \quad E[\Sigma^2]=\sigma^2$$

NOTICE that this property of CORRECTNESS does not depend on the Normal distribution; it is valid for any distribution, PROVIDED that the SAMPLE is COMPLETE. This explains why there is the denominator (n-1) [called "degrees of freedom, dof"]: many incompetent people say that 1 dof is lost! from n data!

In the case I will analyze where there are 10 data BUT 5 survival, how many are the dof? TRY to answer….

When the SAMPLE is INCOMPLETE the previous formulae are *NO LONGER VALID. That's why I gave to my students that exercise of the THREE SUPER_incompetent professors, highly rated in the so called «scientific community»!*

Let's go to the Confidence Interval and consider again a COMPLETE SAMPLE.

Again we assume $D=\{t_1, t_2, \ldots, t_n\}$, $F(t, \theta)=N(t, \mu, \sigma^2)$; $T\sim N(\mu, \sigma^2)$ and therefore the mean (r.v.) $\overline{T}\sim N(.., \mu, \sigma^2/n)$; when the variance σ^2 is NOT KNOWN we have to estimate it through the estimator of the variance σ^2.

It is $\Sigma^2=\sum_1^n (T_i-\overline{T})^2/(n-1)$ where $S=\sqrt{\Sigma^2}$ is the estimator of the standard deviation σ. We then write the probabilistic relationship $P(A<\overline{T}<B)=1-\alpha$ where the "constants" A and B are so chosen that the probability is $1-\alpha$. We can transform it into the following

$$P\left(\frac{A-\mu}{S/\sqrt{n}}<\frac{\overline{T}-\mu}{S/\sqrt{n}}<\frac{B-\mu}{S/\sqrt{n}}\right)=1-\alpha$$

Let's consider the quantity $L=(A-\mu)/(S/\sqrt{n})$ and $U=(B-\mu)/(S/\sqrt{n})$; it follows $A=\mu+LS/\sqrt{n}$; in the plane μ and Sample Mean $\overline{T}=\sum_1^n T_i/n$ the function $A=\mu+LS/\sqrt{n}$ provides a set of "Random" lines parallel to the bisector; analogously the function $B=\mu+US/\sqrt{n}$ provides a set of "Random" lines parallel to the bisector.

The random variable $(\overline{T}-\mu)/(S/\sqrt{n})$ is proved to follow the so called t distribution, with $\nu=(n-1)$ "degrees of freedom [dof]"; therefore, with $\alpha_1+\alpha_2=\alpha$

$$P\left(-t_{1-\alpha_1}=\frac{A-\mu}{S/\sqrt{n}}<\frac{\overline{T}-\mu}{S/\sqrt{n}}<\frac{B-\mu}{S/\sqrt{n}}=t_{1-\alpha_2}\right)=1-\alpha$$

equivalent to

$$P\left(\mu-t_{1-\alpha_1}\,S/\sqrt{n}<\overline{T}<\mu+t_{1-\alpha_2}\,S/\sqrt{n}\right)=1-\alpha$$

which is equivalent to

$$P\left(\overline{T}-t_{1-\alpha_2}\,S/\sqrt{n}<\mu<\overline{T}+t_{1-\alpha_1}\,S/\sqrt{n}\right)=1-\alpha$$

This is a PRECISE probability statement referring to RANDOM INTERVALS that "COVER" the unknown "true" value of the Mean μ [MU].

The functions

$$\mu-t_{1-\alpha_1}\,S/\sqrt{n} \quad\text{and}\quad \mu+t_{1-\alpha_2}\,S/\sqrt{n}$$

are an infinite number of PARALLEL random lines [*because S is a random variable*].

When we elaborate the data we get the empirical standard deviation s; so, having the value s, we have only TWO parallel lines, such that the probability is $1-\alpha$ of the random variable $\overline{T}$ being inside the lines, whatever μ [MU] is.

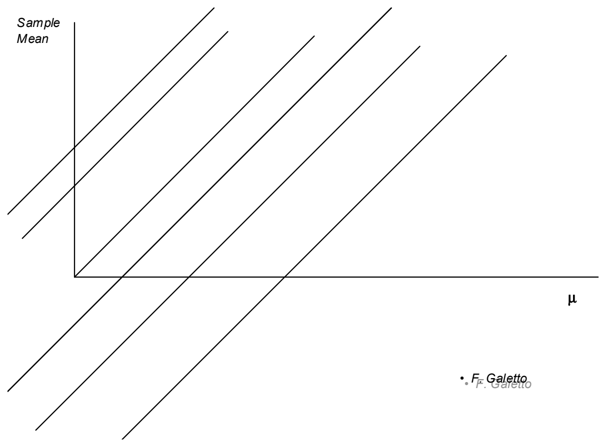


Figure 3. *Bisector and parallel lines*

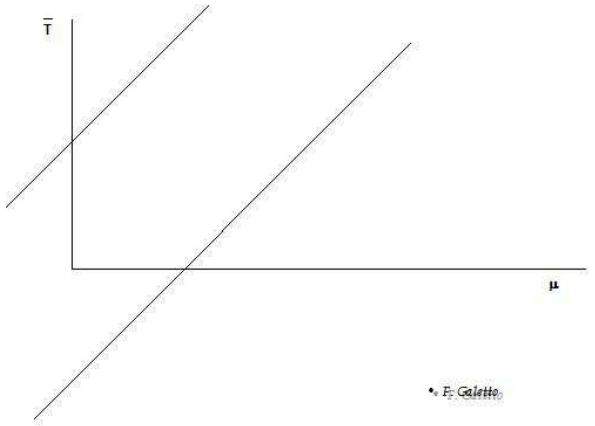


Figure 4. *Bisector and 2 parallel lines (after the computations)*

When we calculate the empirical mean $\bar{t}=\sum_1^n t_i/n$ we chose a point on the vertical axis.

By drawing the horizontal line we get TWO intersections, whose abscissas i and s are the lower and upper limits of the Confidence Interval.

SINCE our complete argument was done with the probability $1-\alpha$, chosen by US, we say that the Confidence Interval has the CONFIDENCE LEVEL $1-\alpha$! The numbers

$$\bar{t}-t_{1-\alpha_1}(\nu)\,s/\sqrt{n} \quad\text{and}\quad \bar{t}+t_{1-\alpha_2}(\nu)\,s/\sqrt{n}$$

are the lower and upper limits of the Confidence Interval [we show explicitly $\nu=(n-1)$ the "degrees of freedom (dof)"]

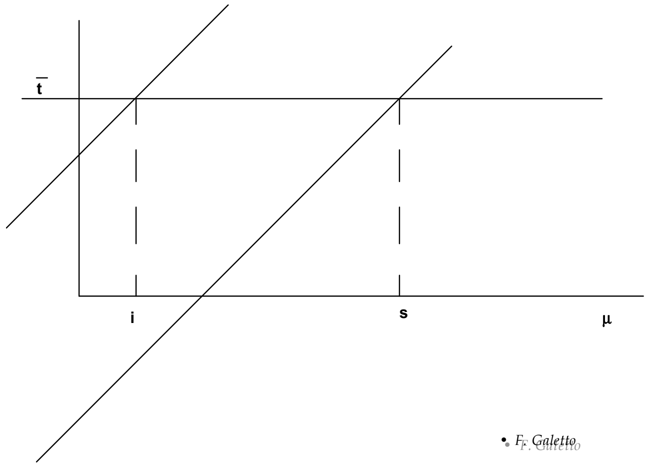


Figure 5. Intersections with the 2 parallel lines (after the computations)

The numeric interval is the CONFIDENCE Interval for the "true" Mean MU (μ) [NOT for the *empirical mean* x_bar!] with CONFIDENCE LEVEL 1-α.

$$\bar{t} - t_{1-\alpha_1}(\nu)\, {}^{s}\!/\!\sqrt{n} \qquad \text{———} \qquad \bar{t} + t_{1-\alpha_2}(\nu)\, {}^{s}\!/\!\sqrt{n}$$

The situation is depicted in the figure 6 (where there are few intervals; actually they are infinite!): some intervals comprise the TRUE mean μ [MU]: they are all with Confidence Level 1-α; (1-α)% of the intervals cover μ.

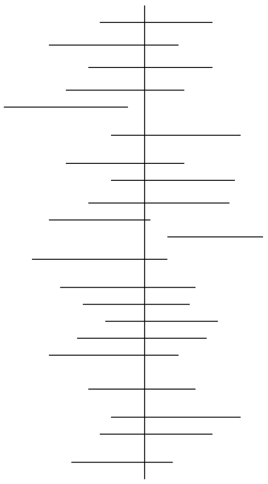


Figure 6. Confidence Intervals (each one after each test)

IF someone should say:

*Statistic softwares calculate such intervals, so a user actually doesn't need to know the technical details. A frequent problem is to give the CI for a mean value (xbar). This is calculated as xbar plusminus standarderror * t-quantile. The t-quantile is taken to get the desired confidence level* he is in error! Three mistakes are there:

- *the CI* (Confidence Interval) is NOT for xbar, BUT for μ, the unknown "true" Mean
- "*…calculated as xbar plusminus standarderror * t-quantile*", is MISLEADING: *standarderror* is s or s/√n ?
- "*Statistic softwares calculate such intervals, so a user actually doesn't need to know the technical details.*" The formula before is valid only for COMPLETE SAMPLES and for NORMAL DISTIBUTION, as done by MANY SOFTWARE!!!!! As a matter of fact, the reader must (as shown before) NOTICE that these formulae depend on the *Normal distribution*, PROVIDED that the SAMPLE is COMPLETE. ν=(n-1) is the "degrees of freedom [dof]" with many incompetents saying that 1 dof is lost!!!! from n data! In the case where there are 10 data BUT 5 survival, how many are the dof? TRY to answer….

When the SAMPLE is INCOMPLETE the previous formulae are NO LONGER VALID, also if the distribution is NORMAL. That's why I gave to my students that exercise of the THREE SUPER_incompetent professors, highly rated in the so called «scientific community» are in good company!

To let my student understand the meaning of the Confidence Interval, I used to tell them this story:

"Imagine you are in a room, completely dark, where there is a container full of Confidence Intervals, for the parameter you have chosen to estimate. The Statistics Goddess painted GREEN the infinite intervals that comprise the unknown "true" Mean μ and RED the infinite intervals that DO NOT comprises the unknown "true" Mean μ. The light is switched on and you see the room and the container. Computing the Confidence Interval is like drawing a CI from the container and looking at its color. When you have drown the CI you have just to look at the color and say if it comprise the unknown "true" Mean μ{or for the parameter you have chosen to estimate}. BUT the Statistics Goddess is a great joker and switch off the light when you look at the color: what is the probability that the color of the CI you have in your hand is GREEN? 1-α! How much can you be CONFIDENT that the color is GREEN? This is your CONFIDENCE LEVEL that you can be right: 1-α."

IF someone should say: *Looking at mean values, giving the CI is not in principle different to giving the standard errors (both are measures of precision), but the CI is much easier and clearer to interpret than the standard errors, since they directly give you a range of "not too unreasonable values" of the estimate. Surely, CIs are to be much preferred, since the actual meaning of SE depends on the sample size.* He is in error! Three mistakes are there:

- "giving the CI (Confidence Interval) is not in principle different to giving the standard errors (both are measures of precision)": standard error is s or s/√n ?
- It is hidden that the statement before is valid only for COMPLETE SAMPLES and for NORMAL DISTIBUTION.
- "Surely, CIs are to be much preferred, since the actual meaning of SE depends on the sample size." Is the amplitude of the interval independent [!!!!] on the sample size (for COMPLETE SAMPLES)?

IF someone should say: The frequentist properties are only assured for normal distributed data/errors. He is in error! One mistake is there:

- It is false that *"The frequentist properties* [of the Confidence Interval] *are only assured for normal distributed data/errors."*, because as we shall see the same type of interpretation is valid also for any other distribution. Here the NORMAL DISTIBUTION is mentioned, while it does NOT matter! Before, when the NORMAL DISTIBUTION did matter it was NOT mentioned.

IF someone should say: *Again, again, and again: I did not state that a CI is an interval for μ.* he is in error! Many mistakes are there, as many as the word "AGAIN" is repeated:

- It is false that *"I did not state that a CI is an interval for μ.* ACTUALLY the CONFIDENCE INTERVAL is for the parameter one WANTS to estimate.

The same type of reasoning, NOT the same FORMULAE, is applicable to any "chosen (by the Manager) parameter": e.g. for the percentiles B_x i.e the values such that $F(B_x)=x\%$

From the THEORY it follows the DEFINITION of the CI:

«The Confidence Interval, with a stated Confidence Level CL=($1-\alpha$), IS the set of all the numbers [*"equivalent"* numbers] about which we are confident [BUT nobody can know it] that the interval comprises the "true value" of the parameter we want to estimate.»

The parameter is not necessarily the mean!

This definition is valid for any parameter, any distribution, any sample (either incomplete or complete). See next …

3. Confidence Interval: Part 2

Now we use a distribution DIFFERENT from the Normal distribution. We shall see that «The same type of reasoning, NOT the same FORMULAE, is applicable [17,18,29]»

To fix better the ideas I will use the following data on 10 items: n=10; the first 5 data are the TIME TO FAILURE [failures occur at 115, 149, 185, 251, 350 (unit of measurement are not given)]:g=5, while the other 5 are data on items that did not fail [NON_Failures at 350, 350, 350, 350, 350 (they are also named "suspended items")]; n-g=5

Such type of data are named "INCOMPLETE samples", because NOT ALL the data are "failures"; think to the data of survival of people to some drug cure: you do not wait until all die before taking decisions!!!!

ASSUMING that the distribution function F(t)= 1-exp[$(t/\eta)^2$] is the Weibull where the parameter is η (the Greek letter ETA) and the Mean is μ [MU].

Fixing CL=90%, by making the right calculations we get that the Confidence Interval for MU is 272.850____588.123

NOTICE: all the formulae we will find are valid ONLY for the ASSUMED distribution (Weibull here with β=2).

IF it is TRUE, as we shall see it is TRUE, that, fixing CL=90%, by making the right calculations, we get that the Confidence Interval for μ is 272.850____588.123, we see that there are 4 different Confidence Intervals.

Which one, IF ANY, is the "right" interval? [17,18,29]

We use the index i for the failures: i=1, 2, …,5; we use the index j for the suspensions: j=6, 7, …,10; n is the total number of items tested.

$tttf(g)=\sum_1^5 t_i$ is the total time to failures, $ttts=\sum_6^{10} t_j$ is the total time to suspensions, $ttot=\sum_1^5 t_i+\sum_6^{10} t_j$ is the total time on test; this is the total of all the data (the same value as though we use the normal distribution).

From the 3 total times we can derive 3 mean values:

$\bar{t}(g)=\sum_1^5 t_i/g$ is the "observed" mean time to failure, $\bar{t}(s)=\sum_6^{10} t_j/(n-g)$ is the mean time to suspensions $\overline{tot}=(\sum_1^5 t_i+\sum_6^{10} t_j)/n$ is the mean time on test; this is the mean of all the data (the same value as though we use the normal distribution).

Table 1. *Table information*

Time to failure	Time to suspension		Total SAMPLE
115	350		
149	350		
185	350		
251	350		
350	350		
5	5	# of data	10
1050	1750	TOTAL	2800
210	350	Mean	280
93.075	0	Standard dev	96.409
USING the "normal" previous formulae and Confidence Level 90%			
170.316	350	Lower Limit of the CI	262.327
249.684	350	Upper Limit of the CI	297.673

IF we know that the distribution of the time to failures is the Weibull function F(t)= 1- exp[$(t/\eta)^2$] we know that the "shape parameter β" is 2 and the "scale parameter η (ETA)" is a value that we have to estimate from the data. The Mean is μ=ηΓ(1+1/β).

How do we estimate μ?

With $\bar{t}(g)$? With $\bar{t}(s)$? With $\overline{tot}$?

Managers, professors, researchers MUST realize that, as W. E. Deming stated *"A figure without a theory tells nothing"*. This idea is not known in the WIKIPEDIA…WE NEED THEORY to estimate μ!

We need the *Likelihood function L(η, D)* defined as

$$L(\eta, D) = \prod_i f(t_i;\eta) * \prod_j \left[1 - F(t_j;\eta)\right]$$

and we MUST find the value that maximizes the *Likelihood function L(η, D)*.

Define $t_P tot = \sum_1^5 t_i^2 + \sum_6^{10} t_j^2$ as the total "POWERED" time on test. This is the total of all the data SQUARED (because β=2), one finds through MATHEMATICS that the *Maximum Likelihood estimate* of η is $\hat{\eta} = \sqrt{t_P tot / g}$

This is the SCIENTIFIC value estimating in the best way the parameter η.Since the Mean is μ=ηΓ(1+1/β), always from the THEORY we find that $\hat{\mu} = 369.175$ is the SCIENTIFIC value estimating in the best way the Mean μ

$$\hat{\mu} = \sqrt{t_P tot / g}\,\Gamma(1.5),$$

Compare this with the three values, wrongly found before:

Time to failure	Time to suspension		Total SAMPLE
210	350	Mean	280

ALL the three values, wrongly found before, UNDERestimate the "BEST" estimate 369.175

THEREFORE we have to expect that the Confidence Intervals, found before, are ALL WRONG….

Let's now see the way to find the Confidence Interval.

Since the total "POWERED" time on test $t_P tot = \sum_1^5 t_i^2 + \sum_6^{10} t_j^2$ is the FUNDAMENTAL quantity for estimating the scale parameter η, we use the Random Variable the total "POWERED" time on test $T_P tot$.

We write the probabilistic relationship $P(A < T_P tot < B) = 1 - \alpha$ where the "constants" A and B are so chosen that the probability is 1-α.

We can transform it into the following [17,18,29]

$$P\left(\frac{2A}{\eta^2} < \frac{2T_P tot}{\eta^2} < \frac{2B}{\eta^2}\right) = 1 - \alpha$$

It easily proved that the Random Variable $\frac{2T_P tot}{\eta^2} \approx \chi^2(2g)$ is distributed as a chi-square with 2g degrees of freedom.[17,18,29]

NOTICE: 2g degrees of freedom, NOT n-1! (as many people say!) The dof are 2 times the number of failures, NOT the numbers of the data minus 1!!!!!![17,18,29]

When $\alpha_1 = \alpha_2 = \alpha/2$, the previous probabilistic relationship is

$$P\left\{\chi^2_{\alpha/2}(2g) \le 2T_P tot / \eta^2 \le \chi^2_{1-\alpha/2}(2g)\right\} = 1 - \alpha$$ e.g.

$$P\left\{\eta^2 \chi^2_{\alpha/2}(2g)/2 \le T_P tot \le \eta^2 \chi^2_{1-\alpha/2}(2g)/2\right\} = 1 - \alpha$$ Putting

θ=η², the left hand of the equation $y = (\theta/2)\chi^2_{\alpha/2}(2g)$ and the right hand of the equation $y = (\theta/2)\chi^2_{1-\alpha/2}(2g)$ are two straight lines passing through the origin of the axes (θ, y):

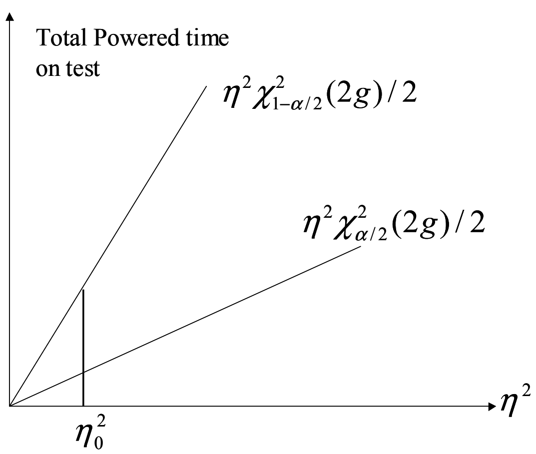


Figure 7. *Lines through the origin (axes θ and Random Variable "total POWERED time on test")*

Hence we find

$$P\left\{2T_P tot / \chi^2_{1-\alpha/2}(2g) \le \eta^2 \le 2T_P tot / \chi^2_{\alpha/2}(2g)\right\} = 1 - \alpha$$

where we see the random INTERVAL that includes the parameter θ=η²,

At the end of the test the Random Variable $T_P tot$, *Total Powered time on test*, assume its determination $t_P tot$ that we used for estimating the Mean μ!!!!!

The random INTERVAL then becomes a NUMERICAL interval

$$\left\{\sqrt{2t_P tot / \chi^2_{1-\alpha/2}(2g)}, \sqrt{2t_P tot / \chi^2_{\alpha/2}(2g)}\right\}$$

The Confidence Interval of the Mean μ [MU] is μ_L=272.850, μ_U=588.123; so we see that fixing CL=90%, by making the right calculations we get that the Confidence Interval for μ [MU] is 272.850____588.123

Compare this with the three couples of values, wrongly found before; the Confidence Interval for μ [MU] is 272.850____588.123 is got by drawing the horizontal line, at the ordinate $t_P tot$ that we used for estimating the Mean μ, and computing the abscissas θ_i and θ_s of the intersections, hence computing the square roots $\eta_i=\sqrt{\theta_i}$ and $\eta_s=\sqrt{\theta_s}$, and eventually μ_L and μ_U.

THEREFORE ⇒⇒⇒⇒⇒⇒⇒⇒

IF someone should say *The frequentist properties are only assured for normal distributed data/errors.* he is in error! One mistake is there:

- It is false that *"The frequentist properties* [of the Confidence Interval] *are only assured for normal distributed data/errors."*, because the same type of interpretation is valid also for any distribution. Here the NORMAL DISTIBUTION is mentioned, while it does NOT matter! Before, when the NORMAL

DISTIBUTION did matter it was NOT mentioned.

IF someone should say: Again, again, and again: I did not state that a CI is an interval for μ [MU]. he is in error!

Many mistakes are there, as many times as the word “AGAIN” is repeated:

- It is false that *“I did not state that a CI is an interval for μ [MU]*. ACTUALLY the CONFIDENCE INTERVAL is for the parameter one WANTS to estimate.

***Table 2.** Confidence Intervals (“wrong”)*

Time to failure	Time to suspension		Total SAMPLE
USING the “normal” previous formulae and Confidence Level 90%			
170.316	350	Lower Limit of the CI	262.327
249.684	350	Upper Limit of the CI	297.673

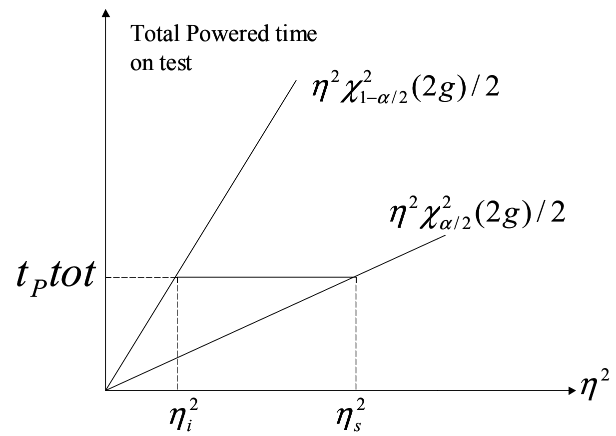


***Figure 8.** Lines through the origin (axes θ and “computed total POWERED time on test” intersecting the lines)*

The following figure is still applicable

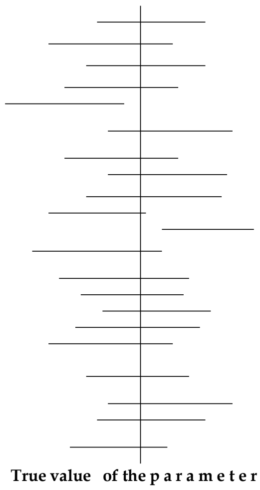


***Figure 9.** Confidence Intervals (each one after each test)*

NOTICE**:** All the arguments we developed DEPEND on the fact that we assume that the Distribution is known, apart from some parameters.

For the case of the Weibull distribution function F(t)= 1-exp[(t/η)^2], since F(B_{10})= 1- exp[(B_{10}/η)^2]=0.1 [10%] it is easy to find the estimate of B_{10} and its Confidence Interval; e.g. the same we can do for B_{50} [the median] and for B_{90} …

4. Confidence Interval: Part 3

The same type of reasoning can be done for any distribution; ONLY the formulae change, NOT the arguments and the interpretation.[17,18,29]

We know from Statistics that $Var(\overline{T}) = \sigma^2/n$ when σ^2 is known and n is the sample size of a COMPLETE SAMPLE and $Var(\overline{T}) = \sigma^2/(n-1)$ when σ^2 is Unknown n is the sample size of a COMPLETE SAMPLE. [17,18,29]

IF the sample is INCOMPLETE, we need some other formulae![17,18,29]

WHAT do you find in WIKIPEDIA? WRONG ideas!

««*After observing the sample we find values x for X and s for S, from which we compute the confidence interval*

$\left[\bar{x} - \frac{cs}{\sqrt{n}}, \bar{x} + \frac{cs}{\sqrt{n}}\right]$ »»» *WRONG ideas in WIKIPEDIA*

ASSUMING that the distribution function F(t)= 1-exp[(t/η)^2] is the Weibull with parameter η to be estimated we can prove that the estimator $\hat{H}^2 = \frac{T_P tot}{g}$ is Correct, Sufficient, Efficient, that is it is the best estimator we can find. Its variance is [17,18,29]

$$Var(\hat{H}^2) = \frac{\eta^4}{g}$$

THEREFORE we see clearly that the sample size n DOES NOT matter: ONLY “failures” are IMPORTANT and affect the Confidence Intervals. We get the usual formulae when g=n, that is the sample is complete![17,18,29]

IF you want to estimate a "parameter (any parameter)" of a DISTRIBUTION, you MUST find a random variable, the ESTIMATOR, which has its own DISTRIBUTION that depends on the distribution of the data (originated by the Random Variables). From that ESTIMATOR you can derive [making some LOGIC transformations] the *GI*=LL (Lower Limit) and the *GS*=UL (Upper Limit) of an INTERVAL, a PROBABILITY RANDOM INTERVAL, that has a fixed, by the manager, 1-α probability of comprising the parameter. WHEN you insert the collected data from a test INTO the LL and UL formulae, then you GET a NUMERICAL interval to which YOU attach a CONFIDENCE 1-alfa. IF one wants he/she can say that, IN THE LONG RUN, 1-α is the proportion of the infinite intervals, we can calculate with infinite tests that can be COVERING the ACTUAL “true” value of the parameter.

When θ is the parameter we want to estimate we can always write the *probabilistic* statement

$$P(GI \le \theta \le GS) = 1 - \alpha$$

where GI and GS are related to the Random variable we use to estimate the parameter θ, chosen by us.

4.1. One Application of Confidence Intervals Found in a Paper of the Magazine Total Quality Management

Fausto Galetto showed various wrong ideas contained in papers published in *Quality Magazines*. In this paper we will show only a case.

According to prof. F. Franceschini, papers published in *Quality Magazines* are, by definition, good papers: ACTUALLY many times that is not true.

The papers considered by Fausto Galetto were found by chance while looking for other papers for other ideas.

Let's, again, stand-back a bit and meditate, starting from a managerial point of view, using *published documents (found in magazines used by managers and professionals, and suggested to students)*, and analysing them from the point of view of the QUALITY PRINCIPLES, stated in ISO 9000:2000 standard.

Let's see the paper "Learning curves and p-charts for a preliminary estimation of asymptotic performances of a manufacturing process" [published in the magazine *Total Quality Management* Franceschini F. (2002)]. Franceschini suggests Montgomery book to his students and the data (non-conformity [nc]) he uses in the paper are from the Montgomery book; the 1st part of the table provides the data of 30 samples (with 50 sample size) while the 2nd part of the table provides the data of 24 samples (with 50 sample size); p is the non-conformance estimate for any sample:

From the data, a curve is interpolated whose equation is p=a/t + c; the coefficients [parameters] are estimated by the first row of formulae and with variances given by the second and third formulae

$$\hat{a} = (\overline{p_1} - \overline{p_2}) / (1/\overline{t_1} - 1/\overline{t_2}) \quad \hat{c} = \overline{p_1} - \hat{a}/\overline{t_1}$$

$$\sigma_a^2 = \left[\overline{t_1 t_2} / (\overline{t_2} - \overline{t_1})\right]^2 (\sigma_{p1}^2 + \sigma_{p2}^2)$$

$$\sigma_c^2 = \left[\overline{t_2} / (\overline{t_2} - \overline{t_1})\right]^2 \sigma_{p1}^2 + \left[\overline{t_1} / (\overline{t_2} - \overline{t_1})\right]^2 \sigma_{p2}^2)$$

Confidence Intervals (assuming normal distribution), for the parameters a and c, are calculated: F. Franceschini [*WELL rated in the ResearchGate database!*], estimates the parameters of the equation p=a/t + c and uses them to PREDICT the "asymptotic fraction of non-conformance p and its Confidence Interval"!

SINCE 0 belongs to the Confidence Intervals, computed by F. Franceschini, according to Franceschini formulae, the estimates are not significantly different from 0!; so $\hat{a} = 0$ and $\hat{c} = 0$; in spite of that the asymptotic fraction of nonconformity is predicted by Franceschini, BUT, in order to be coherent, a rational manager should not do that.

Franceschini did not realise that!

Where is the problem? Regression Theory provides different findings! A lot of errors are in the paper. The referee of the paper could not find what students can find.

If you look at the future data (given in Montgomery book) you find different results!

Table 3. *Data for a "wrong" Control Chart*

	nc	P		nc	p		Nc	p		nc	P		Nc	p		nc	P
1	12	0.24	2	15	0.30	3	8	0.16	4	10	0.20	5	4	0.08	6	7	0.14
7	16	0.32	8	9	0.18	9	14	0.28	10	10	0.20	11	5	0.10	12	6	0.12
13	17	0.34	14	12	0.24	15	22	0.44	16	8	0.16	17	10	0.20	18	5	0.10
19	13	0.26	20	11	0.22	21	20	0.40	22	18	0.36	23	24	0.48	24	15	0.30
25	9	0.18	26	12	0.24	27	7	0.14	28	13	0.26	29	9	0.18	30	6	0.12
1	9	0.18	2	6	0.12	3	12	0.24	4	5	0.10	5	6	0.12	6	4	0.08
7	6	0.12	8	3	0.06	9	7	0.14	10	6	0.12	11	2	0.04	12	4	0.08
13	3	0.06	14	6	0.12	15	5	0.10	16	4	0.08	17	8	0.16	18	5	0.10
19	6	0.12	20	7	0.14	21	5	0.10	22	6	0.12	23	3	0.06	24	5	0.10

4.2. One Application of Confidence Intervals [Wrong Ideas in Wikipedia]

To my question "*is in Control a Control Chart with trend and cycles?*" I had this answer from one researcher: "the following chart is in control".

To understand if the researcher in wrong we need to use the concept of Confidence Interval.

The reader is asked to found the basic of control charts.

Let's consider the problem of deciding if two means μ_1 and μ_2 are "significantly different".

Let's suppose that we have two samples, each of sample size n: we indicate as $\overline{x}_1$, and $\overline{x}_2$ the empirical means, and s_1 and s_2 the empirical standard deviations.

Extending the method we devised before (for the normal distribution), we can the confidence interval with CONFIDENCE LEVEL 1-α [s is the compounded standard deviation of s_1 and s_2]

for μ1: $\overline{x}_1 - t_{1-\alpha_1}(\nu)\, s/\sqrt{n}$, $\overline{x}_1 + t_{1-\alpha_2}(\nu)\, s/\sqrt{n}$

for μ2: $\overline{x}_2 - t_{1-\alpha_1}(\nu)\, s/\sqrt{n}$, $\overline{x}_2 + t_{1-\alpha_2}(\nu)\, s/\sqrt{n}$

IF it happens that $\overline{x}_1$ is in the SECOND interval, and at the same time, $\overline{x}_2$ is in the FIRST interval THEN the two means μ_1 and μ_2 are "NOT significantly different".

We can apply these ideas to any of the points of the control chart below.

It is easily proved that the points 3rd and 9th are such that μ_3 and μ_9[17,18,29]are "*significantly different*"; moreover there is trend; THEREFORE the Control chart in *Wikipedia IS OUT OF CONTROL!*

We think that the YOUNG Researchers MUST be ALERT if they want to LEARN: THEY MUST know the THEORY!

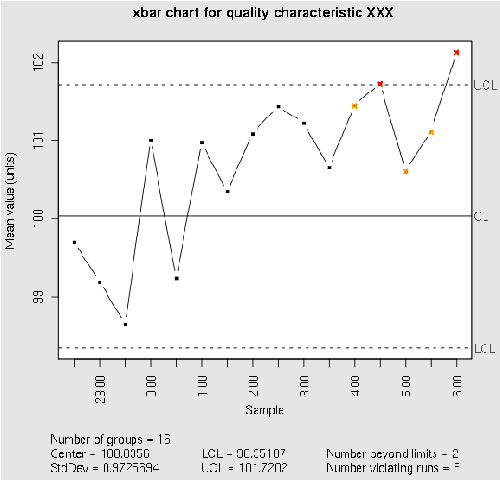


Figure 10. *Control Chart from Wikipedia)*

5. Maximum Likelihood and Least Squares Methods

In the RG Questions&Answers forum there were two debate points: the Least Squares Estimates [LSE] and the Maximum Likelihood Method [MLM] to find the Maximum Likelihood Estimates [MLE]; it is important to have the correct ideas on the areas of applications.

This is especially important for YOUNG Researchers.

On POURPOSE some data have been modified in order not to let the authors to be identifiable: I wanted to help them to CORRECT their errors. Doing that the SIGNIFICANCE of the results was NOT modified…..This short paragraph shows very clearly that professors dealing with DOE theory (as those dealing with Probability and Statistics theory) sometimes show "theories" (are they THEORIES???) that provide wrong teaching to other researchers [17,18,29]: YOU, researcher, MUST be ALERT, in order NOT to be cheated!

Let's use the following MODIFIED DATA (from a paper).

On POURPOSE the data have been modified in order not to let the authors to be identifiable: I wanted to help them to CORRECT their errors. Doing that the SIGNIFICANCE of the results was NOT modified…..

Table 4. *Data "modified" from a paper on RG*

	Factors			Response
State	A	B	C	S/N ratio
1	-1	-1	-1	44.94
2	-1	0	0	45.29
3	-1	1	1	43.80
4	0	-1	0	44.80
5	0	0	1	44.17
6	0	1	-1	45.30
7	1	-1	1	44.63
8	1	0	-1	45.06
9	1	1	0	45.00

The Experiment was a 3^{3-1} FRACTIONAL Factorial design: 3 factors, that I name A, B, C, at 3 levels. The values of y [the RESPONSE variable] are the S/N ratios (as it always done by TAGUCHIANS!)[17,18,29]

The FULL factorial design has 27 test states, while the FRACTIONAL Factorial design has only 9 test states.

It is obvious that the FRACTIONAL Factorial design CANNOT provide the same information provided by the FULL design… A problem arises: any FRACTIONAL Factorial design has ALIASES![17,18,29]

Let's consider the Least Squares Method [LSM] to find the Least Squares Estimates [LSE] [which includes the ANOVA Estimates from DOE (Design Of Experiments)].

We assume that any datum y_{ijkr} is defined as made by the following MODEL, [FULL MODEL] (for A, B, C factors)

$$y_{ijkr} = \mu + \alpha_i + \beta_j + \gamma_k + \alpha\beta_{ij} + \alpha\gamma_{ik} + \beta\gamma_{jk} + \alpha\beta\gamma_{ijk} + e_{ijkr}$$

μ provides the influence of the MEAN of the random variable Y_{ijkr}, α_i provides the influence of the factor A, β_j provides the influence of the factor B, γ_k provides the influence of the factor C, $\alpha\beta_{ij}$ provides the influence of the interaction AB (due to the factor A and the factor B), $\alpha\gamma_{ik}$ provides the influence of the interaction AC (due to the factor A and the factor C), $\beta\gamma_{jk}$ provides the influence of the interaction BC (due to the factor B and the factor C), $\alpha\beta\gamma_{ijk}$ provides the influence of the interaction ABC (due to ALL the 3 factors).

ALL the above quantities are the PARAMETERS of the model, while e_{ijkr} provides the influence of the "random errors" due to the random variables E_{ijkr}. The suffix r stands for "replication". We can write the model in matrix form (using the Random Vectors Y and E) as

$$Y = X\beta + E$$

The vector Y has dimensions n x 1 [we will collect n data]. The matrix X, the Design Matrix, is a known n x p matrix that contains only 0's and 1's (related to the presence of the parameter: be CAREFUL, the matrix X has rank m, where $m<p\le n$. β is a vector of the unknown parameters; WE WANT to estimate them!!!!!E is a vector of the unknown random variables: WE CANNOT observe them!!

ALL WE CAN OBSERVE is any datum y_{ijkr} from the random variable Y_{ijkr} .We write the vector product (inner product, where the (apex) symbol «′» means the operation transpose of the vector or the matrix) that provides the Sum of Squares of the "errors"

$$SS = E'E = (X\beta - Y)'(X\beta - Y)$$

NOW we have to assume that the random variables E_{ijkr} are UNCORRELATED with Mean 0 AND Variance σ^2.

We then derive SS (Sum of Squares of the "errors") with respect to the elements of the vector β of the unknown parameters; then we set the derivatives equal to 0.

We get the NORMAL EQUATIONS (nothing to do with the "normal distribution"!!!!!!)[17,18,29]

$$X'X\beta = X'Y$$

TWO cases can arise:

1. EITHER the matrix X'X is of FULL rank, that is m=p and therefore it has an inverse
2. OR the matrix X′X HAS NOT FULL rank (it is SINGULAR) and therefore there is NOT an INVERSE

In the 1st case the Normal Equations have a unique solution vector $\hat{\beta}$ whose entries are the POINT estimators for the elements of the vector β of the unknown parameters.

In the 2nd case the Normal Equations there may be TWO situations [17,18,29]:

(1) There is no vector $\hat{\beta}$ which satisfies the Normal Equations

(2) OR there are an infinite number of vectors $\hat{\beta}$ which satisfies the Normal Equations

NOTICE that the case (2) is not very satisfactory: TWO experimenters with the same model and the same data get the same Normal Equations, BUT each of them gets a DIFFERENT estimate of the vector β of the unknown parameters.IN THIS case there is NO unbiased estimator of the vector β of the unknown parameters.

THIS is the case of the Design Of Experiments where we apply the ANOVA (Analysis Of Variance)

NOW we assume that The random variables E_{ijkr} are NORMAL variables UNCORRELATED with Mean 0 AND Variance σ^2.In this case we have the Likelihood function L(e; β, σ^2) given by the formula

$$L(e;\beta,\sigma^2) \propto \frac{1}{\sigma^n} e^{-e'e/(2\sigma^2)} = (1/\sigma^n)e^{-(X\beta-Y)'(X\beta-Y)/(2\sigma^2)}$$

From the calculus we have the MAXIMUM of the L(e; β, σ^2) when the exponent of the number e is MINIMUM; in any case one gets the Normal equation we got BEFORE

$$X'X\beta = X'Y$$

Therefore we conclude that (NOTICE)

For COMPLETE SAMPLES *and Normal distributed data* the MLE and the LSE are identical.[17,18,29]

Let's see what I got from a German guy (in Deutch language!). It is easily seen that he did not consider the case that the matrix X′X is singular and therefore has no inverse: this is generally the case in the ANOVA!!!!

4.6.1 Regressionsmodelle: Vergleich mit der LSE-Methode

Als erstes Beispiel wenden wir die ML-Methode auf das vollständig nach Gauß-Markow spezifizierte lineare multivariate Regressionsmodell (2.2) an.[23] Das Modell lautet also

$$Y(\boldsymbol{x};\beta) = \sum_{m=0}^{M} \beta_m x_m + \epsilon = \beta\boldsymbol{x} + \epsilon, \quad \epsilon \sim N(0,\sigma_\epsilon^2)$$

und die dazugehörigen Systemgleichungen

$$\boldsymbol{Y} = \mathbf{X}\beta + \epsilon, \quad \text{mit} \quad \boldsymbol{\epsilon} = (\epsilon_1, ..., \epsilon_i, ..., \epsilon_n)^{\mathrm{T}}, \quad \epsilon_i \sim \text{i.i.d } N(0,\sigma_\epsilon^2).$$

Die Wahrscheinlichkeit, dass das Modell die Messung i beschreibt, oder genauer, die Wahrscheinlichkeitsdichte des Modells mit den exogenen Variablenvektor $\boldsymbol{x}_i$ an der Stelle y_i lautet

$$f_i(y_i) = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp\left[-\frac{(y_i - \beta\boldsymbol{x}_i)^2}{2\sigma_\epsilon^2}\right].$$

Das Rechnen und Ableiten mit Produkten ist umständlich. Deshalb berücksichtigt man, dass sich die Stelle $\hat{\beta}$, an der die Likelihood-Dichte ihr Maximum aufweist, durch Anwenden einer streng monoton steigenden Funktion auf L nicht ändert. Insbesondere ist der Logarithmus eine solche Funktion und er erweist sich als günstig, da er Produkte in Summen umwandelt. Dies ergibt hier die **Log-Likelihood**(dichte)

$$\begin{aligned} \tilde{L}(\beta) &= \ln L(\beta) \\ &= \sum_{i=1}^{n} \left\{ -\frac{1}{2}(\ln 2\pi + \ln \sigma_\epsilon^2) - \left[\frac{(y_i - \beta\boldsymbol{x}_i)^2}{2\sigma_\epsilon^2}\right] \right\} \\ &= -\frac{n}{2}(\ln 2\pi + \ln \sigma_\epsilon^2) - \frac{1}{2\sigma_\epsilon^2}(\boldsymbol{y} - \mathbf{X}\beta)^{\mathrm{T}}(\boldsymbol{y} - \mathbf{X}\beta). \end{aligned} \quad (4.48)$$

Wie der Name schon sagt, maximiert man bei der ML-Methode die *Likelihood*. Als notwendige (und hier hinreichende) Bedingungen muss der Gradient, d.h. alle partiellen Ableitungen, verschwinden. Mit den Ableitungsregeln in Kap. 2.10.3 ergibt dies

$$\frac{\partial \tilde{L}}{\partial \beta} = \frac{1}{2\sigma_\epsilon^2}(2\mathbf{X}^{\mathrm{T}}\boldsymbol{y} - 2\mathbf{X}^{\mathrm{T}}\mathbf{X}\beta) \stackrel{!}{=} 0 \quad \Rightarrow \quad \hat{\beta} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\boldsymbol{y} \quad (4.49)$$

Excerpt 7. *(doc got from a German guy (in Deutch language!)*

The FULL factorial design has 27 test states, while the FRACTIONAL Factorial design has only 9 test states (table 4)…. Therefore we consider only the factors two A and B. The model, named FULL model for A and B [Fm(AB) for short], is

$$y_{ij} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + e_{ij}$$

because we DO NOT have replications: 9 states, 9 data!!!!!

One finds the NORMAL EQUATIONS for Fm(AB): The NORMAL EQUATIONS are 16 equations with 16 unknown quantities; X′X is a 16 x 16 matrix and the unknown vector β is 16 x 1; the vector X′Y is 16 x 1.

X′X is a SINGULAR matrix and therefore there is an infinite number of solutions of the Normal Equations!!!!

Let's indicate with $\hat{\beta}_{Fm(AB)}$ any vector solution of the NORMAL Equations for the FULL model Fm(AB)

$$(X'X)_{Fm(AB)}\hat{\beta}_{Fm(AB)} = (X'Y)_{Fm(AB)}$$

$\hat{\beta}_{Fm(AB)}$ provides the estimates of the parameters of the model.[17,18,29]

The "scalar" (or the "dot") product of the solution $\hat{\beta}_{Fm(AB)}$ with the known term $(X'Y)_{Fm(AB)}$ provides the Sum of Squares "explained" by the Full model: we named it Sum of Squares of the Regression and we write

$$SS\,\mathrm{Re}\,g[Fm(AB)] = \hat{\beta}_{Fm(AB)}(X'Y)_{Fm(AB)}$$

We can use $y_{ijr} = \mu + \alpha_i + \beta_j + e_{ijr}$ the ADDITIVE model to analyse the data; we use the symbol Am(AB) for it.

Let's indicate with $\hat{\beta}_{Am(AB)}$ any vector solution of the NORMAL Equations for the ADDITIVE model Am(AB)

$$(X'X)_{Am(AB)}\hat{\beta}_{Am(AB)} = (X'Y)_{Am(AB)}$$

$\hat{\beta}_{Am(AB)}$ provides the estimates of the parameters of the model. The product of the solution $\hat{\beta}_{Am(AB)}$ with the known term $(X'Y)_{Am(AB)}$ provides the Sum of Squares "explained" by the ADDITIVE model: we named it Sum of Squares of the

Regression and we write

$$SS\,\mathrm{Re}\,g[Am(AB)] = \hat{\beta}_{Am(AB)}(X'Y)_{Am(AB)}$$

The difference of SSReg[Fm(AB)] with the SSReg[Am(AB)] provides the Sum of Squares due to the INTERACTION A*B, indicated as SS(A*B), that is

$$SS(A*B) = SS\,\mathrm{Re}\,g[Fm(AB)] - SS\,\mathrm{Re}\,g[Am(AB)]$$

WHEN we have the same number of data in any CELL of the matrix A\B as in the case we are analysing [1 datum in any cell]we can get the SS(B), the Sum of Squares due to the factor B by writing $y_{ijr} = \mu + \alpha_i + e_{ijr}$ the REDUCED model

Let's indicate with $\hat{\beta}_{\mu+\alpha}$ any vector solution of the NORMAL Equations for the REDUCED model

$$(X'X)_{\mu+\alpha}\hat{\beta}_{\mu+\alpha} = (X'Y)_{\mu+\alpha}$$

$\hat{\beta}_{\mu+\alpha}$ provides the estimates of the parameters of the REDUCED model.

The product of the solution $\hat{\beta}_{\mu+\alpha}$ with the known term $(X'Y)_{\mu+\alpha}$ provides the Sum of Squares "explained" by the REDUCED model: we named it Sum of Squares of the Regression and we write

$$SS\,\mathrm{Re}\,g[\mu+\alpha] = \hat{\beta}_{\mu+\alpha}(X'Y)_{\mu+\alpha}$$

The influence of the Factor B is given by the difference

$$SS(B) = SS\,\mathrm{Re}\,g[Am(AB)] - SS\,\mathrm{Re}\,g[\mu+\alpha]$$

IN THE SAME MANNER we can get the SS(A), the Sum of Squares due to the factor A, by writing $y_{ijr} = \mu + \beta_j + e_{ijr}$ (REDUCED model); let's indicate with $\hat{\beta}_{\mu+\beta}$ any vector solution of the NORMAL Equations for the REDUCED model

$$(X'X)_{\mu+\beta}\hat{\beta}_{\mu+\beta} = (X'Y)_{\mu+\beta}$$

$\hat{\beta}_{\mu+\beta}$ provides the estimates of the parameters of the REDUCED model. The product of the solution $\hat{\beta}_{\mu+\beta}$ with the known term $(X'Y)_{\mu+\beta}$ provides the Sum of Squares "explained" by the REDUCED model: we name it Sum of Squares of the Regression and we write

$$SS\,\mathrm{Re}\,g[\mu+\beta] = \hat{\beta}_{\mu+\beta}(X'Y)_{\mu+\beta}$$

The influence of the Factor A is given by the difference

$$SS(A) = SS\,\mathrm{Re}\,g[Am(AB)] - SS\,\mathrm{Re}\,g[\mu+\beta]$$

NOTICE: When we estimate the MEAN of any distribution we use the MOST REDUCED Model

$$y_{ijkr} = \mu + e_{ijkr}$$

In this case there is ONLY ONE NORMAL EQUATION; the vector β has ONLY ONE element the unknown parameter μ!

NOTICE:

The use of the NORMAL EQUATIONS is APPLICABLE to ANY distribution.

The Normal Distribution is NOT important for the ESTIMATION of the parameters.

NOTICE:

We MUST know the distribution involved when we have to TEST the significance on the estimates of the parameters.

We can consider various models with 2 factors; putting all together we have, BE CAREFULL…….

Table 5a. *Elaboration of the data "modified" from a paper on RG*

Element	Symbol SS	SS	dof	MS
Factor A	SS(A)	0.0744	2	0.0372
Factor B	SS(B)	0.0302	2	0.0151
Factor C	SS(C)	1.5038	2	0.7519
Inter. A*B	SS(A*B)	1.9336	4	0.4834
Inter. A*C	SS(A*C)	0.4600	4	0.1150
Inter. B*C	SS(B*C)	0.5042	4	0.1260

NOTICE: We have estimated 6 elements for 16 degrees of freedom dof….. while we have ONLY 9 data!!!

There MUST BE something we did not say up to now.

ACTUALLY any element is ENTANGLED with several other elements; we write this in the following way [& symbol of ENTANGLEMENT]

A & B*C & …. & …., B & A*C & …. & ….,

C & A*B & …. & ….

We see that the factor C seems "more important" that the interactions A*B, B*C and A*C.

In any case we see that the factor A and B seem "LESS important" that the interactions A*B, B*C and A*C.

THEREFORE IF one wants to find the OPTIMUM SETTING of the LEVELS of the FACTORS he MUST consider the INTERACTIONS.

That's why the INCOMPETENTS, which follow the STUPID ideas of Taguchi, MANY and MANY times find WRONG OPTIMUM SETTING!

SEE Fausto Galetto papers on this…………..

The authors of the documents I used (with the modified data) say «The optimum conditions according to ANOVA is the level 1 for the factor A, the level 1 for the factor B and the level -1 for the factor C.» NOTICING that they do not consider the interaction! Their statement is FALSE!

Actually using the interaction and the MS one finds that "The optimum conditions according to the MS, are the level -1 for the factor A, 0 for the factor B and 0 for the factor C.

Therefore we see how much one can be in error IF we DO NOT consider the INTERECTIONS! [17,18,29]

I ASKED to the German_GUY <<<<PLEASE, consider the exercise 6 that I gave to my students in the

document ""'BMWvsPROF_ExamTests_ SET5-forRG Researchers"" that you can find in the RG database….. SOLVE that case I used to give to my STUDENTS!>>>>>

The ANOVA of those data DOES NOT use the NORMAL distribution AND DOES NOT REQUIRE the NORMAL distribution to TEST the difference within the products!

I told him that HIS <> IS NOT VALID for ANOVA, UNLESS … HE NEVER sent the solution!

I ASKED to the German_GUY <<<<to USE the data EXPONENTIALLY distributed AND ESTIMATE the "failure rate"… with LS….>>>>>>

***Table 5b.** Elaboration of the data "modified" from a paper on RG*

0.409211	0.222132
0.668679	0.078471
0.221406	0.740956
0.069011	0.192883
0.328767	0.185932

HE NEVER sent the solution!

See the REFERENCES and documents of F. Galetto in the RG for other case of NONSENSE in the "Quality field".

We have shown that the YOUNG Researchers MUST be ALERT if they want to LEARN: THEY MUST know the THEORY!

The same attitude was show by a professor, referee of a thesis at Bologna University, Engineering Faculty Prof. Peretto) [35]: he refused to consider my analysis of data.

The case is absolutely identical to the previous one: a Taguchi design, fractional. The data were copied from J. Z. Zhang et al. [36], "*Surface roughness optimization in an end-milling operation using the Taguchi design Method*", Journal of Material Processing Technology (ELSEVIER), 184(2007) 233-239. The Taguchi design [36] is a 3^{3-1} fractional design as the one in table 4.

***Table 6.** Data from the Taguchi design of the paper [36] "Surface …" (A, B, C:inner factors; X, Y:outer factors)*

		X	1	1	2	2	S/N
		Y	**1**	**2**	**1**	**2**	
A	B	C	N1	N2	N3	N4	η
1	1	1	35,5	47,0	71,5	58,5	-34.77
1	2	2	59,5	58,5	51,0	69,0	-35.54
1	3	3	68,5	56,5	96,5	133,0	-39.41
2	1	2	26,0	23,5	82,5	53,6	-34.36
2	2	3	31,0	40,0	56,0	26,0	-32.02
2	3	1	45,0	41,0	58,5	49,0	-33.77
3	1	3	23,5	26,5	76,5	30,5	-33.03
3	2	1	24,5	22,5	51,0	56,5	-32.37
3	3	2	31,5	38,0	82,0	48,0	-34.57

The last column of table 6 provides the S/N ratio η, on which one can make the same analysis of table 4: with the same conclusions! [we MUST provide it to show….]

The authors, Italian and American, found the effect of factors A, B, C, as did wrongly the researcher on RG, we saw before: *stupid ideas last very long*!

Doing that they found [36] the "optimum cutting conditions as A3B2C1 (they did not consider the interactions!)".

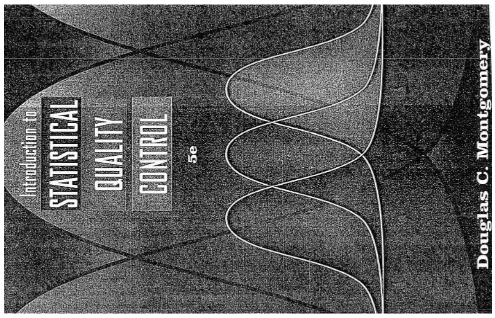
Using all the factors, controlled factors and noise factors, and all the data, the Galetto's "optimum cutting conditions (considering the interactions)" are as A3B2C3 (and noise factors X, Y both at low level): 1/3 was wrong for the 4 professors, the Italian [35] (L. Peretto, Bologna University) and American [36] (J. Z. Zhang et al., University of Northern Iowa, and Iowa State University).

6. Design of Experiments. Applications in a Book

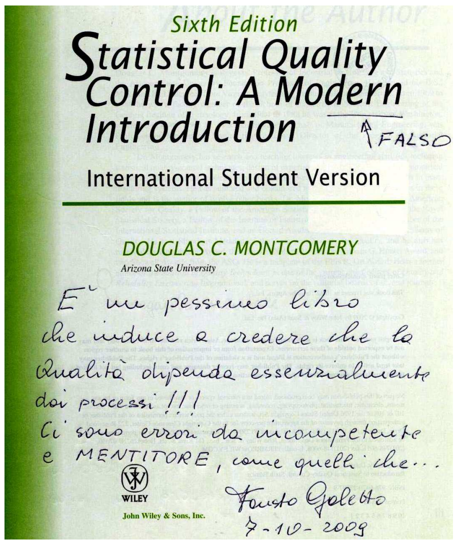
To show how big is the problem of the incompetence in the field of DOE (Design Of Experiments) we will use some cases found in a WWU (World Wide Used) book (D. C. Montgomery), used in many and many universities……..

The Normal Distribution is assumed in the book and the same we do here for comparison.

Notice, that in his book [all editions!! The front page of two are given here], prof. Montgomery does not know how to analyze data exponentially distributed!!! It is important to give them for my argument!



***Excerpt 8a.** (front page from the 5th edition,)*



***Excerpt 8b.** (front page from the 6th edition, found in the Politecnico Library and commented by F. Galetto)*

We ask the reader to look at the books (the 5th and 6th editions) to find all the errors in there.

We consider here the FALSENESSES of prof. Montgomery, (in the 5th and 6th editions) related to an application of the Design Of Experiments, the DOE. The prof. Montgomery is a liar!!!!!!!!!!!!!!!

He writes (excerpt 8c):

EXAMPLE 12-8

An article in *Solid State Technology* ("Orthogonal Design for Process Optimization and Its Application in Plasma Etching," May 1987, pp. 127–132) describes the application of factorial designs in developing a nitride etch process on a single-wafer plasma etcher. The process uses C_2F_6 as the reactant gas. It is possible to vary the gas flow, the power applied to the cathode, the pressure in the reactor chamber, and the spacing between the anode and the cathode (gap). Several response variables would usually be of interest in this process, but in this example we will concentrate on etch rate for silicon nitride.

We will use a single replicate of a 2^4 design to investigate this process. Since it is unlikely that the three-factor and four-factor interactions are significant, we will tentatively plan to combine them as an estimate of error. The factor levels used in the design

Excerpt 8c. *(from the 5th edition; also present in previous editions)*

NOTICE IMMEDIATELY: (in the example 12-8, 5th ed.) the statement «SINCE it is UNLIKELY that the three-factor and four-factor interactions are SIGNIFICANT, we will plan to combine them as an estimate of error» a very stupid statements, as we shall SEE! NOTICE IMMEDIATELY: (in the example 13.8, 6th ed.) the statements «Three-factor and higher interactions are USUALLY NEGLIGIBLE. … a common practice is to combine the higher interactions as an estimate of error» a very stupid statements, as we shall SEE!

The case is taken from a paper [33] on «Plasma Etching Process» [excerpt 8c and 8d]; it was a 3^{4-2} Fractional Factorial Design. Prof. Montgomery presented it (lying) as a single replicate 2^4 design…

Many times Fausto Galetto wrote statements like the following ones:

<<<<<<<Very few people take care of "Quality of Quality Methods" and that is very dangerous for Quality achievement. [6]

In order to provide the proof of the deep ignorance [16, 20, 21, 25, 29] (the contrary of Deming "profound knowledge" [5, 6]) of Quality matters and Quality Methods, we will use some published papers on DOE (Design Of Experiments).

Often these papers show conclusions based on data collected through experiments; the data are not in the papers. If you ask the data, in order to understand the conclusions, the authors refuse to provide the data: "the data are secret". Sometimes you find the data in the papers. Using the Scientific Approach [17, 18, 29], you can then analyse and understand the conclusions.

Generally the conclusions provided by the (professors) authors of those papers are based on methods that they found in books suggested to the students attending "Quality Courses" (given in Universities).

If one looks at universities web-sites (in Internet) he finds often mentioned the Montgomery book [30]: many professors suggest its use. Would that mean it is a good book? Really it means something completely different! [29]>>>>>>

VERY; VERY FEW give them some consideration…….

NOTICE IMMEDIATELY: my comment (in Italian) says «The INTERACTION ABCD, here not considered, is SIGNIFICANT!». It was the same for the 5th edition…

13.5.3 A Single Replicate of the 2^k Design

As the number of factors in a factorial experiment grows, the number of effects that can be estimated also grows. For example, a 2^4 experiment has 4 main effects, 6 two-factor interactions, 4 three-factor interactions, and 1 four-factor interaction, whereas a 2^6 experiment has 6 main effects, 15 two-factor interactions, 20 three-factor interactions, 15 four-factor interactions, 6 five-factor interactions, and 1 six-factor interaction. In most situations the **sparsity of effects principle** applies; that is, the system is usually dominated by the main effects and low-order interactions. Three-factor and higher interactions are usually negligible. Therefore, when the number of factors is moderately large—say, $k \geq 4$ or 5—a common practice is to run only a single replicate of the 2^k design and then pool or combine the higher-order interactions as an estimate of error.

KAZZATA

vedi l'es. seguente !!! Elaborare bene i dati !! !!

EXAMPLE 13.8 Characterizing a Plasma Etching Process

An article in *Solid State Technology* ("Orthogonal Design for Process Optimization and Its Application in Plasma Etching," May 1987, pp. 127–132) describes the application of factorial designs in developing a nitride etch process on a single-wafer plasma etcher. The process uses C_2F_6 as the reactant gas. It is possible to vary the gas flow, the power applied to the cathode, the pressure in the reactor chamber, and the spacing between the anode and the cathode (gap). Several response variables would usually be of interest in this process, but in this example we will concentrate on etch rate for silicon nitride. Perform an appropriate experiment to characterize the performance of this etching process with respect to the four process variables.

Design Factor Level	Gap A (cm)	Pressure B (m Torr)	C_2F_6 Flow C (SCCM)	Power D (W)
Low (−)	0.80	450	125	275
High (+)	1.20	550	200	325

SOLUTION

The authors used a single replicate of a 2^4 design to investigate this process. Since it is unlikely that the three-factor and four-factor interactions are significant, we will tentatively plan to combine them as an estimate of error. The factor levels used in the design are shown here:

Table 13.15 presents the data from the 16 runs of the 2^4 design. The design is shown geometrically in Fig. 13.30. Table 13.16 is the table of plus and minus signs for the 2^4 design. The signs in the columns of this table can be used to estimate the

(continued)

FALSO; hanno fatto un piano 3^{4-2}

KAZZATA !!!

sono più importanti ...

SIGNIFICATIVA

Excerpt 8d. *(from the 6th edition, found in the Politecnico Library and commented by F. Galetto)*

DATI TUTTI FALSI

580 Chapter 13 ■ Basic Experimental Design for Process Improvement

■ **TABLE 13.15**

The 2^4 Design for the Plasma Etch Experiment

Run	A (Gap)	B (Pressure)	C (C_2F_6 flow)	D (Power)	Etch Rate (Å/min)
1	−1	−1	−1	−1	550
2	1	−1	−1	−1	669
3	−1	1	−1	−1	604
4	1	1	−1	−1	650
5	−1	−1	1	−1	633
6	1	−1	1	−1	642
7	−1	1	1	−1	601
8	1	1	1	−1	635
9	−1	−1	−1	1	1037
10	1	−1	−1	1	749
11	−1	1	−1	1	1052
12	1	1	−1	1	868
13	−1	−1	1	1	1075
14	1	−1	1	1	860
15	−1	1	1	1	1063
16	1	1	1	1	729

factor effects. To illustrate, the estimate of the effect of gap on factor *A* is

$$A = \frac{1}{8}[a + ab + ac + abc + ad + abd + acd + abcd - (1) - b - c - d - bc - bd - cd - bcd]$$

$$= \frac{1}{8}[669 + 650 + 642 + 635 + 749 + 868 + 860 + 729 - 550 - 604 - 633 - 601 - 1037 - 1052 - 1075 - 1063]$$

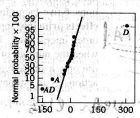
$$= -101.625$$

Thus, the effect of increasing the gap between the anode and the cathode from 0.80 cm to 1.20 cm is to decrease the etch rate by 101.625 angstroms per minute. It is easy to verify that the complete set of effect estimates is

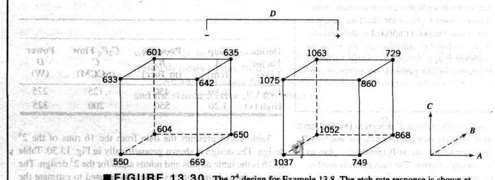
$A =$	−101.625	$AD =$	−153.625
$B =$	−1.625	$BD =$	−0.625
$AB =$	−7.875	$ABD =$	4.125
$C =$	7.375	$CD =$	−2.125
$AC =$	−24.875	$ACD =$	5.625
$BC =$	−43.875	$BCD =$	−25.375
$ABC =$	−15.625	$ABCD =$	−40.125
$D =$	306.125		

A very helpful method in judging the significance of factors in a 2^k experiment is to construct a **normal probability plot** of the effect estimates. If none of the effects is significant, then the estimates will behave like a random sample drawn from a normal distribution with zero mean, and the plotted effects will lie approximately along a straight line. Those effects that do not plot on the line are significant factors.

The normal probability plot of effect estimates from the plasma etch experiment is shown in Fig. 13.31. Clearly, the main effects of *A* and *D* and the *AD* interaction are significant, as they fall far from the line passing through the other points. The analysis of variance summarized in Table 13.17 confirms these



■ **FIGURE 13.31** Normal probability plot of effects, Example 13.8.



■ **FIGURE 13.30** The 2^4 design for Example 13.8. The etch rate response is shown at the corners of the cubes.

Excerpt 9. *(from the 6th edition, found in the Politecnico Library and commented by F. Galetto)*

We said that Montgomery is a liar: we see why. Consider the two matrices in table 7: data of the article [33] in *Solid State Technology [REAL DATA, on the right]*. NOTICE that in the same rows, of the two matrices, there are the SAME DATA! As you see the data on the left are the ones of the process optimisation dealt in Montgomery's book [the book suggested to students, in several Universities] (see the excerpts!) where he wrote (in the example 11-4, page 545 of [30]):*"An article in Solid State Technology* [33] *("Orthogonal Design for Process Optimisation and its Application in Plasma Etching")* *describes the application of factorial design in developing a nitride etch process on a single-wafer plasma etcher. ... we will concentrate on etch rate for silicon nitride. We will use a single replicate of a 2^4 design to investigate this process. Since it is unlikely that the three-factor and four-factor interactions are significant, we combine them as an estimate of error."*

Table 12-15 The 2^4 Design for the Plasma Etch Experiment

Run	A (Gap)	B (Pressure)	C (C_2F_6 flow)	D (Power)	Etch Rate (Å/min)
1	−1	−1	−1	−1	550
2	1	−1	−1	−1	669
3	−1	1	−1	−1	604
4	1	1	−1	−1	650
5	−1	−1	1	−1	633
6	1	−1	1	−1	642
7	−1	1	1	−1	601
8	1	1	1	−1	635
9	−1	−1	−1	1	1037
10	1	−1	−1	1	749
11	−1	1	−1	1	1052
12	1	1	−1	1	868
13	−1	−1	1	1	1075
14	1	−1	1	1	860
15	−1	1	1	1	1063
16	1	1	1	1	729

Excerpt 10. *(FALSE DATA from the 5th edition; also present in previous editions)*

■ TABLE 13.17
Analysis of Variance for the Plasma Etch Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	41,310.563	1	41,310.563	20.28
B	10.563	1	10.563	< 1
C	217.563	1	217.563	< 1
D	374,850.063	1	374,850.063	183.99
AB	248.063	1	248.063	< 1
AC	2,475.063	1	2,475.063	1.21
AD	94,402.563	1	99,402.563	46.34
BC	7,700.063	1	7,700.063	3.78
BD	1.563	1	1.563	< 1
CD	18.063	1	18.063	< 1
Error	10,186.815	5	2,037.363	
Total	531,420.938	15		

l'interaz. ABCD, qui trascurata, è SIGNIFICATIVA

Excerpt 11. *(from the 6th edition, found in the Politecnico Library and commented by F. Galetto)*

Those effects that do not plot on the ...
The normal probability plot of effect estimates from the plasma etch experiment is shown in Fig. 12-31. Clearly, the main effects of A and D and the AD interaction are significant, as they fall far from the line passing through the other points. The analysis of variance summarized in Table 12-17 confirms these findings. Note that in the analysis of variance we have pooled the three- and four-factor interactions to form the error mean square. If the normal probability plot had indicated that any of these interactions were important, they would not be included in the error term.
... the gap between the cathode and anode

Excerpt 12a. *(from the 5th edition,)*

Table 7. *Data for an ANOVA ("false/wrong" data on the left)*

Montgomery's false data						Data of the actual experiment [33]					
State	Gap	Pressure	Flow-Rate	Power	RESPONSE	State	Gap	Pressure	Flow-Rate	Power	RESPONSE
	A	B	C	D			C'	B'	D'	A'	
1	-1	-1	-1	-1	550						
2	1	-1	-1	-1	669	7	1	-1	0	1	669
3	-1	1	-1	-1	604						
4	1	1	-1	-1	650						
5	-1	-1	1	-1	633	2	0	0	0	-1	633
6	1	-1	1	-1	642						
7	-1	1	1	-1	601						
8	1	1	1	-1	635						
9	-1	-1	-1	1	1037	1"	-1	-1	-1	-1	1037
10	1	-1	-1	1	749	9	0	1	-1	1	749
11	-1	1	-1	1	1052	1'	-1	-1	-1	-1	1052
12	1	1	-1	1	868	6	-1	1	0	0	868
13	-1	-1	1	1	1075	1	-1	-1	-1	-1	1075
14	1	-1	1	1	860	4	0	-1	1	0	860
15	-1	1	1	1	1063						
16	1	1	1	1	729						
FALSE Montgomery's Design						3	1	1	1	-1	406
						5	1	0	-1	0	561
						8	-1	0	1	1	1138

Various attendants to my presentation at 4th AITEM Conference (Brescia 1999) [26] shared this Montgomery's same idea: … *it is unlikely that the three-factor and four-factor interactions are significant*! INCOMPETENTS…

The paper [33] *"Orthogonal Design for Process Optimisation and its Application in Plasma Etching", May 1987*, provides the data (in the right matrix) of a FRACTIONAL FACTORIAL 3^{4-2} design! MONTGOMERY used the data for a 2^4 FULL Factorial Design, without saying that he was doing that for didactical reasons!!!!!!!!!!???????................

Neglecting the interactions, as it is usually done by "Taguchians" and "Robust Design practitioners", from HIS DATA (false data, left of table 7) Montgomery finds and writes

[30]: "*clearly, the main effects of A and D and the AD interaction are significant ...*".

Table 12-17 Analysis of Variance for the Plasma Etch Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	41,310.563	1	41,310.563	20.28
B	10.563	1	10.563	< 1
C	217.563	1	217.563	< 1
D	374,850.063	1	374,850.063	183.99
AB	248.063	1	248.063	< 1
AC	2,475.063	1	2,475.063	1.21
AD	94,402.563	1	99,402.563	46.34
BC	7,700.063	1	7,700.063	3.78
BD	1.563	1	1.563	< 1
CD	18.063	1	18.063	< 1
Error	10,186.815	5	2,037.363	
Total	531,420.938	15		

Excerpt 12b. *(from the 5th edition,)*

Besides lying on data, Montgomery makes a "strange" data analysis. In table 7 it is easily seen he uses the same data for very different experiments (notice the data in the same rows): the actual experiment was a 3^{4-2} *design (fractional, with 3 replicates of state 1)*; the actual experiment has a very complicate ALIAS Structure, forgotten by Yin and Jillie, and by Montgomery who invented (falsely, without saying) the experiment as a 2^4 *design and made wrong assessment of significance*. Using the G-Method [the method that uses the Normal Equations and the Gauss-Markov Theorem, F. Galetto, 7, 13, 14, 15, 19, 22, 23, 27, 28, 29] one can find the ANOVA table 8 [using 2 interactions and factor B (not significant) as the Residual Error estimate].

Table 8. *Data analysis of Montgomery's false data on the left of table 7*

Source	Df	SS	MS	Fcalc	Sign.
Total	16	10167789			α=10%
Mean	1	9636368			
Tot Cor.	15	531420.9			
A	1	41310.56	41310.56	4105.40	*
B	1	10.56	10.56	1.05	
AB	1	248.06	248.06	24.65	*
C	1	217.56	217.56	21.62	*
AC	1	2475.06	2475.06	245.97	*
BC	1	7700.06	7700.06	765.22	*
ABC	1	976.56	976.56	97.05	*
D	1	374850.1	374850.1	37252.18	*
AD	1	94402.56	94402.56	9381.62	*
BD	1	1.56	1.56	0.15	
ABD	1	68.06	68.06	6.76	*
CD	1	18.06	18.06	1.79	
ACD	1	126.56	126.56	12.58	*
BCD	1	2575.56	2575.56	255.96	*
ABCD	1	6440.06	6440.06	640.00	*
Error	3	30.19	10.06		

Montgomery makes decisions based on data analysis and applies correctly the (ISO 9000:2000 and 9004:2000) seventh principle *"Factual approach to decision making"* which states: *"Effective decisions are based on the analysis of data and information"*. BUT his decisions *are NOT effective*: they are wrong!

It is interesting noting that Prof. Montgomery missed many interactions that are more important than factors.!(table 8).

Why professors suggest his book [30] to students [17,18,29]??? [17,18,29]???[17,18,29]?

The scientific analysis [7, 12, 13, 19, 23, 29] of the actual data provides a very different picture about the significance of factors and interactions for the etching process (see table 9): all the factors and all the interactions (1st order) are significant! *But there is an important hoax always hidden by Taguchi and his lovers.* When you carry out a part of the entire test you should do (this is called "fractional replication design") you can NOT obtain the same information of the complete design: you cannot separate factors effect and interactions effects: they are inevitably entangled (symbol & for "entanglement relation" in table 10) [19, 20, 21, 23, 27, 29].

Table 9. *Data analysis of Solid State Technologydata on the right of table 7*

Source	Df	SS	MS	Signif (α=10%)
A' & ...	2	33030.89	16515.44	*
B' & ...	2	56336.22	28168.11	*
C' & ...	2	351020.22	175510.11	*
D' & ...	2	11260.22	5630.11	*
A'*B' & ...	4	362280.44	90570.11	*
A'*C' & ...	4	67596.44	16899.11	*
A'*D' & ...	4	407356.44	101839.11	*
B'*C' & ...	4	44291.11	11072.78	*
B'*D' & ...	4	384051.11	96012.78	*
C'*D' & ...	4	89367.11	22341.78	*
Residual Error	2	732.67	366.33	

The actual experimental design is a "fractional factorial 3^{4-2} design" in the controlled factors A', B', C', D'. There are several ways to get it; a very interesting method is mentioned in [LEVI R., LOMBARDO A. (1997) Nuove frontiere nella programmazione degli esperimenti, *Convegno SIS*, 210-215] and shown in [PISTONE G., WYNN H. P. (1996) Generalised confounding with Groebner bases, *Biometrica 83 (1)*]: find the solution to the following system of equations (factors are x_i)

$$x_2^3 - x_2 = 0, \quad 4x_4 + 9x_1^2x_2^2 + 3x_1x_2^2 - 6x_2^2 - 3x_1^2x_2 + 3x_1x_2 + 2x_2 + 4 - 2x_1 - 6x_1^2 = 0$$

$$x_1^3 - x_1 = 0, \quad 4x_3 - 9x_1^2x_2^2 - 3x_1x_2^2 + 6x_2^2 - 3x_1^2x_2 + 3x_1x_2 + 2x_2 - 4 + 2x_1 + 6x_1^2 = 0$$

The solution provides you with the treatment combinations, not the confounding pattern as, on the contrary, is written in [LEVI R., LOMBARDO A. (1997)]: it is said there that "the left-side two equations confound the 1st and the 3rd order interaction of each factor, as it is taken for granted for 3 level factors design. The right-side two equations confound the 4th (3rd) factor with a complex combination of interactions (of the 1st two factors)". The authors do not provide the "alias structure", as always do the "Taguchi lovers". If they had used the G-method [29] they would had found that every factor is "entangled" with various interactions (we use the symbols "&" for the "entanglement relation" and "..." for the not shown higher order interactions): A'&B'*C'&B'*D'&C'*D'&A'B'C'&...; B'&A'*C'&A'*D'&C'*D'&...; C'&A'*B'&A'*D'&B'*D'&...; D'&A'*B'&A'*C'&B'*C'&...

"Entanglement" is an "equivalence relation", in a logical sense. More precisely, there is also the ALIAS structure (the symbol @ stands for "equivalent to"), neglected by all professors...:

Table 10. *Entanglement for the Solid State Technology design, right of table 7*

(A'+B') @ C'*D'@.	(A'+C') @ B'*D'@...	(A'+D') @ B'*C'@...	(B'+C') @ A'*D'@...	(B'+D') @ A'*C'@...	(C'+D') @ A'*B'@...

This means that changing "additively" any two factors is exactly the same as changing "interactively" the other two factors and .... As a consequence you cannot choose the best levels of factors as though they were independent, which is, on the contrary, "a magic feature of Taguchi orthogonal arrays". Yin and Jillie missed that point. [33]

You can show all that using the G-Method [13, 15, 19, 21, 22, 23, 26, 27, 29]; in the Galetto book [29] it is mentioned a method that allows you to find the bias of the estimate of the parameters of a "reduced model"; the same idea can be used for finding the alias structure. From that it is easily seen that

⇨ when a full design is carried out and a reduced model is used the estimator of β_1 is biased

⇨ when a *fractional design* is carried out only a reduced model β_1, ALIASED, can be estimated.

It is not scientific and not managerial say the contrary. The right tools can be used if managers do use correctly the "Knowledge Matrix" [22, 29]. IF skilful people make such kind of pitfalls, what can we expect form incompetent ones? These last use "Robust Design" and Taguchi Methods and claim: "TM work", BUT they did not read Taguchi books: it very amazing asking them "Did you read Taguchi books?". I always had the reply NO!!! Why people act that way? I have been looking for the answer for at least 15 years: I found it during 1998 holidays in [32]: the truth does not influence them: only their conviction is reality!!! If skilful people slip into such pitfalls what can you expect from unskilled managers who act like "tamed monkeys monkeying their incompetent consultants and teachers"?

For many years, since 1988, Levi and Galetto have been suggesting to be cautious in using blindly some Taguchi ideas. Then the name "G-method" was invented; many applications of it were made before (one of the firsts was [7]): actually the G-method is, in few words, the correct use of the Normal Equations [29]. Many times interactions are important; then it is *quite unmanagerial pretending, before any test, to say (Taguchi) "... when there is interaction, it is because insufficient research has been done on the characteristic values.", or to say, after a test (Phadke), "... if we observe that for a particular objective function the interactions among the control factors are strong, we should look for the possibility that the objective function may have been selected incorrectly*".

Again, Montgomery, Yin and Jillie make decisions based on data analysis and apply correctly the (ISO 9000:2000 and 9004:2000) seventh principle *"Factual approach to decision making"* which states: *"Effective decisions are based on the analysis of data and information"*. BUT their decisions *are NOT effective*: they are wrong!

It is evident that the author D.C. MONTGOMERY made a COMPLETELY WRONG analysis!!!!!!!!!

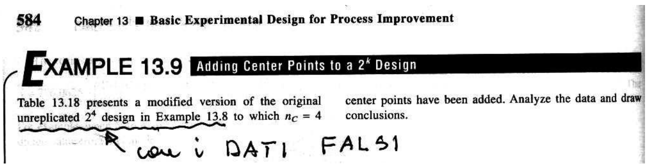
I asked my students to be BETTER than Montgomery

See the documents of F. Galetto in the RG for other case of NONSENSE in the "Quality field"

Can anybody be happy and pleased that authors do not know the matter they publish?

Can anybody be happy and pleased that authors have very high Impact Points and Scores (of any type) and publish wrong ideas?

Now we are going to see other mistakes of Montgomery


584 Chapter 13 ■ Basic Experimental Design for Process Improvement

EXAMPLE 13.9 Adding Center Points to a 2^k Design

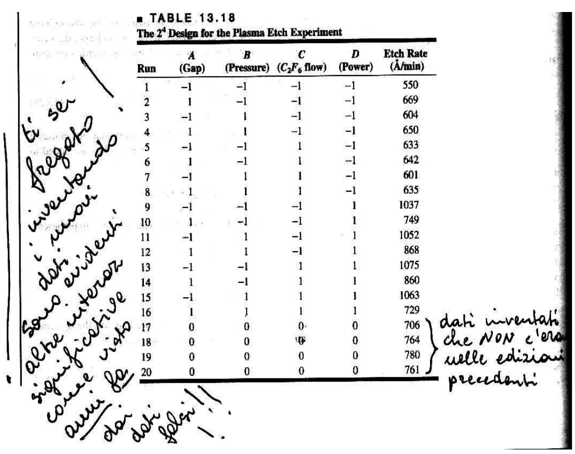
Table 13.18 presents a modified version of the original unreplicated 2^4 design in Example 13.8 to which $n_C = 4$ center points have been added. Analyze the data and draw conclusions.

con i DATI FALSI

Excerpt 13. *(from the 6th edition, found in the Politecnico Library and commented by F. Galetto)*

The data of EXAMPLE 13.9 are the ones "false" of table 7, with 4 new rows added, as you can see in the "TABLE 13.18", that you find next, as Excerpt 14 (from the 6th edition that I found in the library and I commented!!!!!)

The "false" findings of Montgomery drawn from the "false" data (in the excerpt 15).


■ TABLE 13.18
The 2^4 Design for the Plasma Etch Experiment

Run	A (Gap)	B (Pressure)	C (C_2F_6 flow)	D (Power)	Etch Rate (Å/min)
1	−1	−1	−1	−1	550
2	1	−1	−1	−1	669
3	−1	1	−1	−1	604
4	1	1	−1	−1	650
5	−1	−1	1	−1	633
6	1	−1	1	−1	642
7	−1	1	1	−1	601
8	1	1	1	−1	635
9	−1	−1	−1	1	1037
10	1	−1	−1	1	749
11	−1	1	−1	1	1052
12	1	1	−1	1	868
13	−1	−1	1	1	1075
14	1	−1	1	1	860
15	−1	1	1	1	1063
16	1	1	1	1	729
17	0	0	0	0	706
18	0	0	0	0	764
19	0	0	0	0	780
20	0	0	0	0	761

Excerpt 14. *(from the 6th edition, found in the Politecnico Library and commented by F. Galetto)*

Let's see the findings of Montgomery.(in the excerpt 15).

ONE CAN expect that the analysis of the "NEW and FALSE" data by Montgomery could have some drawbacks. Let's see them.

13.5 The 2^k Factorial Design 585

■ TABLE 13.19
Analysis of Variance Output from Minitab for Example 13.9

Estimated Effects and Coefficients for Etch Rate (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		776.06	10.20	76.11	0.000
A	-101.62	-50.81	10.20	-4.98	0.001
B	-1.63	-0.81	10.20	-0.08	0.938
C	7.37	3.69	10.20	0.36	0.727
D	306.12	153.06	10.20	15.01	0.000
A*B	-7.88	-3.94	10.20	-0.39	0.709
A*C	-24.88	-12.44	10.20	-1.22	0.257
A*D	-153.63	-76.81	10.20	-7.53	0.000
B*C	-43.87	-21.94	10.20	-2.15	0.064
B*D	-0.63	-0.31	10.20	-0.03	0.976
C*D	-2.13	-1.06	10.20	-0.10	0.920
Ct Pt		-23.31	22.80	-1.02	0.337

Analysis of Variance for Etch (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	416389	416389	104097	62.57	0.000
2-Way Interactions	6	104845	104845	17474	10.50	0.002
Curvature	1	1739	1739	1739	1.05	0.337
Residual Error	8	13310	13310	1664		
Lack of Fit	5	10187	10187	2037	1.96	0.308
Pure Error	3	3123	3123	1041		
Total	19	536283				

Excerpt 15. *(from the 6th edition, found in the Politecnico Library and commented by F. Galetto)*

LOOK at the ANOVA in the Excerpt 15 [that Montgomery got with MINITAB] (compare with table 11):

1. The main effects, with 4 dof, are significant (p=0.000 in the P column, at the right of the F column).
2. ACTUALLY only the LINEAR Effects Al and Dl of the factors A and D are significant!
3. the LINEAR Effects Bl and Cl of the factors B and C and ALL the quadratic effects of A, B, C, D [all equal to the "Curvature" in the excerpt 15: see Aq, Bq, Cq, Dq in the table 8 (where they are significant due to the error term 381.32 with 10 dof)] are NOT significant!
4. ACTUALLY only the LINEAR Effects Al * Dl of the interaction A * D is significant, in the excerpt 15!
5. And NOT the "2-way interactions" that you see in …!
6. the *Lack of Fit IS ACTUALLY* due to the stupid idea of Montgomery of pooling all the interactions as he did in the Example 13.8 (and in the previous editions!)
7. the Pure Error is due to the 4 Central Points… and SHOULD be used to test the significance of ALL the effects!
8. IF Montgomery DID that, HE could have found that the interactions Al * Dl and Bl * Cl are significant !
9. The "Curvature" is the same for all the factors, [all equal to the "Curvature" in the excerpt 15: see Aq, Bq, Cq, Dq in the table 8 (where they are significant because of the error term 381.32 with 10 dof)] due to the plan that was made; Montgomery found that it is NOT significant!
10. ACTUALLY, for Fausto Galetto analysis the "Curvature" (that is the same for all the factors: Aq=Bq=Cq=Dq!!!!, due to the plan that was made) IS significant [see table 11]!

The findings of Fausto Galetto are in the table 11 (significant effects are indicate by the "asterisk *", and by bold character).

NOTICE that the SS are not completely independent; the "squared effects of factors provide only the "common" curvature of the response surface.

Is that enough to understand how many students and researchers were, are and will be learning wrong ideas from D. C. Montgomery? [17,18,29]

Table 11. *Galetto's Analysis of data of the excerpt 14*

Source	Df	SS	MS	Fcalc	Sign.
Total	20	12437442			α=10%
Mean	1	11901159.2			
Tot Cor.	19	536282.8			
Al	1	41310.56	41310.56	108.34	*
Aq	1	1739.11	1739.11	4.56	*
Bl	1	10.56	10.56	0.03	
Bq	1	1739.11	1739.11	4.56	*
AlBl	1	248.06	248.06	0.65	
AlBq	1	41310.56	41310.56	108.34	*
AqBl	1	10.56	10.56	0.03	
Cl	1	217.56	217.56	0.57	
Cq	1	1739.11	1739.11	4.56	*
AlCl	1	2475.06	2475.06	6.49	*
AqCl	1	217.56	217.56	0.57	
AlCq	1	41310.56	41310.56	108.34	*
BlCl	1	7700.06	7700.06	20.19	*
BlCq	1	10.56	10.56	0.03	
AlBlCl	1	976.56	976.56	2.56	
Dl	1	374850.1	374850.1	983.04	*
Dq	1	1739.11	1739.11	4.56	*
AlDl	1	94402.56	94402.56	247.57	*
AlDq	1	41310.56	41310.56	108.34	*
AqDl	1	374850.1	374850.1	983.04	*
BlDl	1	1.56	1.56	0.004	
BlDq	1	10.56	10.56	0.03	
BqDl	1	374850.1	374850.1	983.04	*
AlBlDl	1	68.06	68.06	0.18	
ClDl	1	18.06	18.06	0.05	
ClDq	1	217.56	217.56	0.57	
AlClDl	1	126.56	126.56	0.33	
BlClDl	1	2575.56	2575.56	6.75	*
AlBlClDl	1	6440.06	6440.06	16.89	*
Error	10	3813.19	381.32		

(error with df=10 by pooling the variance of the Central Points and NON_significant effects)

FROM table 11 it is easily seen that various effects are ENTANGLED, because we have NOT enough df: Montgomery missed this point![17,18,29]

7. Conclusion

We analyzed some very few cases, taken from the Questions & Answers of the Research Gate link and from some paper and books, to show the deep ignorance existing on two fundamental methods used in research and management for making decisions: Confidence Intervals and Design of Experiments; the problem of ignorance is so huge [1-6,12-16,19-23,29,31-38] that a profound change of mind (metanoia, Deming) [5,6] is NEEDED.

See all the figures (mostly figures 12, 13, 14, 15).

The following statements of great scientists and managers are important for any person that wants to make QUALITY

Decisions on QUALITY matters.

We think that the YOUNG Researchers MUST be ALERT if they want to LEARN: THEY MUST know the THEORY!

The author Galetto always invited people to be intellectually honest in teaching and taking decisions: THEORY is fundamental in both cases.

From above we see that Fausto Galetto taking into account the following statements by great people, as always did, could provide a sensible advice for any Researcher, in any university.

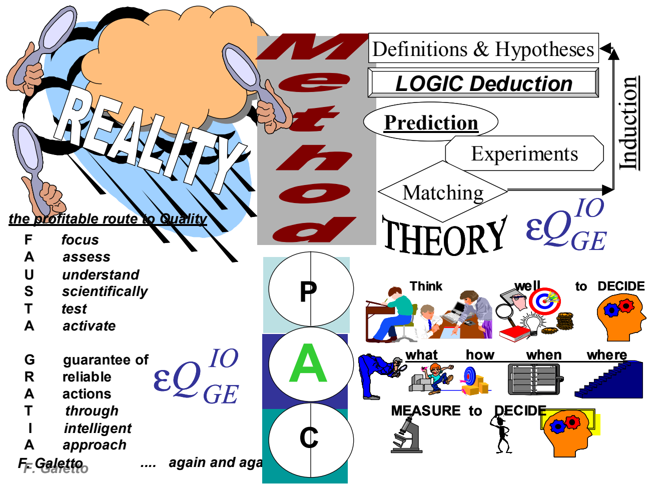


Figure 12. *FAUSTA GRATIA for Quality in order to avoid the Disquality*

===

W. E. DEMING *"It is a hazard to copy". "It is necessary to understand the theory of what one wishes to do or to make." "Without theory, experience has no meaning." "A figure without a theory tells nothing". <<<The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications.>>>>*

M. GELL-MANN *"In my university studies ..., in most of the cases, it seemed that students were asked simply to regurgitate at the exams what they had swallowed during the courses."*. Some of those students later could have become researchers and then professors, writing “A_scientific” papers and books … For these last, another statement of the Nobel Prize M. Gell-Mann is relevant: *<<<<<"Once that such a misunderstanding has taken place in the publication, it tends to become perpetual, because the various authors simply copy one each other."....>>>>>*, similar to *"Imitatores, servum pecus"* [Horatius, 18 B.C.] and *"Gravior et validior est decem virorum bonorum sententia quam totius multitudinis imperitiae"* [Cicero].

P. B. CROSBY Paraphrasing P. B. CROSBY one could say *"Professors may or may not realize what has to be done to achieve quality. Or worse, they may feel, mistakenly, that they do understand what has to be done. Those types can cause the most harm."*

What do have in common Crosby, Deming and Gell-Mann statements? The fact that professors and students betray an important characteristic of human beings: rationality [the “Adult state” of E. Berne]

A. EINSTEIN "*Only two things are infinite: the Universe and the Stupidity of people; and I'm not sure about the former*".

GALILEO GALILEI Before EINSTEIN, GALILEO GALILEI had said [in the Saggiatore] something similar "*Infinite is the mob of fools*".

=====================================

The scientific community as a whole must judge [κρινω] the work of its members by the objectivity and the rigour with which that work has been conducted; in this way the scientific method should prevail. Any professor and any Statistical Consultant should know Probability Theory and Statistics!

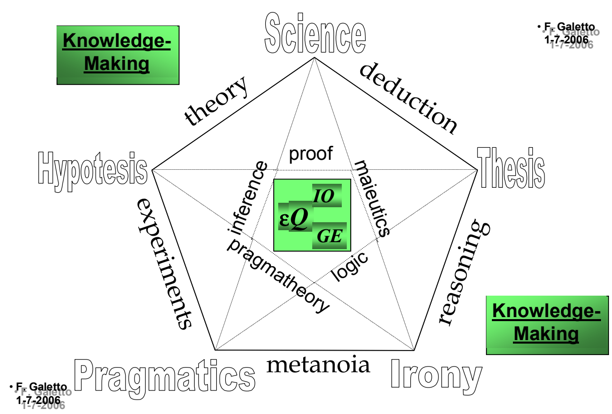


Figure 13. *Quality Tools and Quality Methods: avoid the Disquality*

I always said to my students: «««*IF a guy suggests book and papers written by incompetents he is TWICE incompetent, because he does not recognize wrong ideas and suggests to read wrong ideas*»»»».

Please see well the figures 12 and 13 and see IF …

Researchers shall use their intelligence in order to make knowledge for the improvement of people and their life.

Researchers MUST not cheat people and act according figures 12 and 13.

Any Intellectually hOnest person that loves QUALITY and hates DISquality will Focus on the problems [potential and/or actual], Assess their importance (money, impact, consequences, risks, …), Understand all the previous items SCIENTIFICALLY and SCIENTIFICALLY Test for finding the causes; when a solution is found anybody will Activate to implement the solution, in order to Guaranty that Reliable Actions (preventive and corrective) are taken Through an Intelligent Approach (approach that uses intelligence, ingenuity and science, avoiding misdeeds).

Eric Berne devised the Transactional Analysis “Theory” [that actually is not a theory in the scientific sense] with the 3 EGO_States: Parent, Adult, Child.

The Parent ego_state is a set of thoughts, feelings, and behaviours that are learned or “borrowed” from our parents or other caretakers. Two parts are comprised: the Nurturing Parent ego_state soft, loving, and permission giving, and Prejudiced Parent, the part of our personality that contains the prejudged thoughts, feelings, and beliefs that we learned from our parents.

The Adult ego_state is our data processing centre. It is the part of our personality that formulate hypotheses to be verified by experiments, uses LOGIC and SCIENCE, invents METHODS to test ideas and to process data accurately, that sees, hears, thinks, and can come up with solutions to problems [potential and/or actual] based on the facts and not solely on our pre-judged thoughts or childlike emotions: it denounces misdeeds. You can see its capacities on the right hand of the figure 12. Qualitatis FAUSTA GRATIA is related to the Adult ego_state.

The Child ego_state is the part of our personality that is the seat of emotions, thoughts, and feelings and all of the feeling state "memories" that we have of ourselves from childhood. The Child ego_state can also be divided into two parts: the Free Child ego_substate is the seat of spontaneous feeling and behaviour. It is the side of us that experiences the world in a direct and immediate way. Our Free Child ego_substate can be playful, authentic, expressive, and emotional, and the Adapted Child ego_substate that is the part of our personality that has learned to comply with the parental messages (from everywhere and everybody) we received growing up; if we are faced with parental messages (from everywhere and everybody) that are restricting, instead of complying with them, we rebel against them...

The Adult ego_state [17,18,29]is embodied in the $\varepsilon\, Q^{IO}_{GE}$ *symbol*(the epsilon-Quality, see also figure 14)

Figure 14. *The epsilon-Quality to avoid the Disquality*

Intellectually hOnest people use as much as possible their rationality and Logic, in order not to deceive other people.

Deming, Einstein, Gell-Mann are beacons for the Quality Journey.

If we want to achieve QUALITY, MANAGERS (now students) NEED TO BE EDUCATED ON QUALITY $\varepsilon\, Q_{IO_G}$ by Quality Professors, EDUCATED on Quality.

I could, at last, paraphrase ST John "And there are also many other things, the which, if they should be written everyone, I suppose that even the world itself could not contain the books that should be written."[1-37]

Will someone want to see the truth? Only God knows that …

The personal conclusion is left to the Intellectually Honest reader to whom is offered the Quality Tetralogy: Prevent, Experiment, Improve, Plan, SCIENTIFICALLY to avoid disquality, to eliminate disquality, to achieve Quality, to assure Quality, using Intellectual Honesty: we wish them to use correctly the Decision-Making Tetrahedron (fig. 15).

Quality Tetralogy and Decision-Making are much better than ISO 9004:2008 because Quality Tetralogy and Decision-Making Tetrahedron take into account explicitly the need for scientific behavior either of people or of organizations that really want to make Quality. Moreover they show clearly that prevention is very important for Quality and Good Management is strongly related to Good Knowledge for Business Excellence.

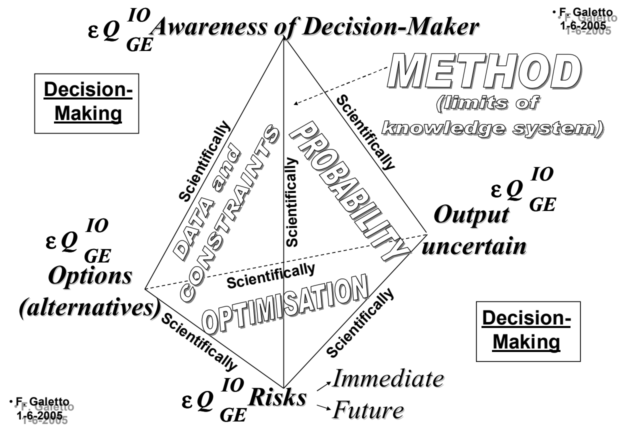


Figure 15. *The Decision-Making Tetrahedron.*

Brain is the most important asset: let's not forget it, *IF we want that our students be better that their professors.*

We repeat

YOUNG Researchers MUST be ALERT
if they want to LEARN:
THEY MUST know the THEORY!

«*«The truth sets you free»*»

Professors and researchers WHO DO NOT ARE Intellectually hOnest will not grow students and researchers fond of Quality (see figures 2, 12, 13, 14, 15) and [32].

References

[1] BARONE S, ERTO P, LANZOTTI A (2001), "Beyond robust design methodologies: an elementary example of synergy between statistics and advanced engineering design", SIS Sperimentare per la Qualità, Turin February 2001.

[2] BORSELLINO C., LO CASTO S., MARAGIOGLIO G., RUISI V. (1999), "Thermovision temperature mapsevaluation in turningoperation", 3rd AITEM 99, Brescia.

[3] CAPELLO E., SEMERARO Q. (1999), "Residual stresses in turning", 3rd AITEM 99, Brescia.

[4] CARRINO L., GIULIANO G., POLINI W. (2000), "New method to characterise superplastic materials in comparison with alternative methods", Int. Manufacturing Conference in China, Hong Kong.

[5] DEMING W.E. (1986), Out of the crisis, Cambridge Press.

[6] DEMING W.E. (1997), The new economics for industry, government, education, Cambridge Press.

[7] GALETTO F. (1978), “An application of experimental design in the Automotive field”, SIA Congress.

[8] GALETTO F. (1984) “Assessment of Product Reliability”, World Quality Congress '84, Brighton.

[9] GALETTO F. (1986)“Quality/Reliability: How to get results”, EOQC (Automotive Section), Madrid.

[10] GALETTO F. (1987) “Quality and Reliability, the Iveco way”, Mgt Dev. Review by MCE, Brussells.

[11] GALETTO F. (1988) “Quality and reliability. A must for industry”, ISATA, Montecarlo.

[12] GALETTO F. (1989) “Quality of methods for quality is important”, EOQC Conference, Vienna.

[13] GALETTO F. (1990) “Basic and managerial concerns on Taguchi Methods”, ISATA, Florence.

[14] GALETTO F., LEVI R. (93) “Planned Experiments: key factors for product Quality”, 3rd AMST 93, Udine.

[15] GALETTO F. (1993) “DOE. Importanti idee sulla Qualità per i manager”, DEINDE, Torino.

[16] GALETTO F. (1993) “Which kind of Quality? Of products, of processes, of Management?”, 1st AITEM, Ancona.

[17] GALETTO F. (1995) Affidabilità, Volume 1: Teoria e Metodi di Calcolo, CLEUP, Padova.

[18] GALETTO F. (1995) Affidabilità, Volume 2: Prove di affidabilità, CLEUP, Padova.

[19] GALETTO F. (1996) “Managerial Issues for Design of Experiments”, 4th AMST 96, Udine.

[20] GALETTO F. (1997) “We need Quality of Managers”, Quality 97 6th Int. Conf., Ostrava, Czeh Republic.

[21] GALETTO F. (1998) “Quality Education on Quality for Future Managers”, 1st Conference on TQM for Higher Education Institutions, Toulon.

[22] GALETTO F. (1999) “GIQA, the Golden Integral Quality Approach: from Management of Quality to Quality of Management”, Total Quality Management, Vol. 10, No. 1.

[23] GALETTO F. (1999) “Quality Methods for Design of Experiments”, 5th AMST 99, Udine.

[24] GALETTO F. (1999) “Quality Function Deployment, Some Managerial Concerns”, AITEM99, Brescia.

[25] GALETTO F., GENTILI E. (1999), “The need of Quality Methods used for Quality”, CAPE '99, Durham, UK.

[26] GALETTO F., GENTILI E. (1999), “Quality of the Quality Methods”, AITEM 99 Conference, Brescia.

[27] GALETTO F. (99)“Quality Education on Total Quality Management” 2nd Conf. on TQM for HEI, Verona.

[28] GALETTO F., GENTILI E. (2000), “In search of Quality in QFD and Taguchi methods”, CAPE.

[29] GALETTO F. (2000) Qualità. Alcuni metodi statistici da Manager, CLUT, Torino.

[30] MONTGOMERY D.C. (1996), Introduction to Statistical Quality Control, Wiley & Sons (many drawbacks).

[31] PARK S. (1996), Robust Design and Analysis for Quality Engineering, Chapmann& Hall, London.

[32] WATZLAWITCK P. (1976) La realtà della realtà, Astrolabio, Roma.

[33] YIN G.Z., JILLIE D.W. (1987), “Orthogonal Design for Process Optimisation and its Application in Plasma Etching”, Solid State Technology, (may 87).

[34] W. Eureka, N. E. Ryan The Customer driven Company: Managerial Perspectives on QFD, ASI Press, 1989.

[35] C. Casciano (L. Peretto referee), Thesis on the Taguchi Method, Bologna University 2014.

[36] J. Z. Zhang et al., “Surface roughness optimization … using the Taguchi design Method”, Journal of Material Processing Technology 184(2007) 233-239.

[37] Research Gate, Papers and Questions & Answers.

[38] GALETTO F. (2014) “Bibiliometrics: Help or Hoax for Quality?” UJER 2(4), DOI: 10.13189/ujer.2014.020404.

[39] GALETTO F. (2014) Quality Pedia Project. Volume 1, Chapter 1, Research Gate.

[40] GALETTO F. (2014) “Riemann Hypothesis Proved”, Academia Arena 2014;6(12):19-22]. (ISSN 1553-992X).