

# Analysis and Optimization of Addis Ababa Light Railway Ticketing Window

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**Abstract:** The main feature of Addis Ababa's light railway (AALR) ticketing window is either congestion or underutilized 106 – 137 and 12 - 18 percent respectively. That is why the study set the main objective to analyze and optimize AALR ticketing windows. So, the researcher first studied the problem for the specified ticketing windows. Secondly, establish and analyze the performance of a new model for the current and future design periods. And finally, it recommends the number of clerks based on the findings. The congestion and underutilization problem of each ticketing window is solved through a mathematical method called Queue Theory with a combination of special and statically analysis methods and train timetable optimization of urban railway by Arena. The study indicates that the congestion rate of the AALR at the congested station is between 106 & 137%. Similarly, the underutilization of the ticketing window is between 12 & 18%. Therefore, the result indicates that adding a single clerk could reduce the traffic intensity to 82% in congested windows. Similarly, reducing to 2 clerks can improve up to 35.5% the underutilization window. Finally, the optimum number of clerks required for the rest of the design period is determined and summarized using a combination of queuing theory, spatial and analytical method and Arena software timetable optimization.

**Keywords:** Optimization, Queue Theory, Waiting Time

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## 1. Introduction

A metro line is a railway infrastructure connecting the beginning and the end to a fixed number of stations. Analyzing the movement of trains on the metro includes the station, the number of trains, and the routing of the whole operation. In this case, the problem is limited to determine the number of ticket sellers at the metro station. This means its main purpose is to optimize the service capacity. There are two types of infrastructure features in AALR called the intermediate and end stations.

Several studies have analyzed the performance of railway stations in the transport and construction industry; and queue theory was applied at ticket windows of Lagos-[1], Salem [2], and Erode [3] railway ticket counters to analyze and optimize the railway ticketing service. It was also applied for the optimization of the subway station capacity of Beijing subway line 2 by combining queuing network modelling and Least-squares regression [4]. However, all of them are

conducted abroad. But the optimization of the AALR route [5] and the Performance of the Train Timetable [6] only were done. So, according to the best of the knowledge of the researchers, no comprehensive research has been done to optimize the ticket office of AALR. This gap has prompted the researchers to analyze the performance of windows at the selected station to conclude for the rest stations.

In addition, the scope of the study will focus on improving the service of AALR stations using queue theory at Torhailoch and Lideta stations. It would be based on direct data collected from the station. The study was limited to two months of data collection at Torhailoch and Lideta stations during peak operation hours to solve the whole stretch. Finally, this research significantly solves the waiting line problem in Torhailoch and Lideta ticketing windows specifically and recommends stations generally.

## 2. Literature Review

Concept of queuing: according to Taylor (1996), waiting in

lines is the most common occurrence in everyone's life every day. It would happen to anyone who has gone shopping or to a movie and that also happens to people, cars for service, or to buy fuel at a gas station, machines for repairs, files, etc. [7]. The main objective of the queue theory is the reduction of waiting time. It was started in 1905 by A. K. Erlang, a Danish telephone engineer, in telephonic calls [7]. It is the representative measure of performance, such as average queue length, average waiting time in the queue, and average facility utilization to use in the context of a cost and waiting for line optimization model.

### 2.1. Operating Characteristics of a Queuing System

A queuing system consists of the input source, calling population, arrival rate, arrival process, arrival distribution, service discipline, and service capacity. The arrivals are measured by the arrival rate; the service is measured by the service rate; and the length of the customer in the service and departure is the leave of the customer from the system [7]. In addition, the behavior of a queuing system is described by arrival rate, waiting time, service time, the probability that the system is empty, the average number of customers in the waiting line, average time spent in the queue line, and system, the average number of customers in the system, the probability that the time in the queue is 0, Utilization (traffic intensity), and so on [5]. These operating characteristics are emerging due to the interaction among the different variables of the queuing system and its numerical values can be calculated by using mathematical equations or by simulation [9, 10]. And, it is ruled by the first-come-first-served (FCFS), first-in, last-served (FIFS), priority, or random queue discipline. This property affects the operating characteristics of the queuing system [11, 12].

### 2.2. Model Specification

The researcher fixed a queuing model of M/M/c/∞/FCFS by assuming a Poisson probability distribution for arrival and an infinite calling population, exponentially distributed service time with first-come-first-served queue discipline to optimize the queue system. The performance measurement is given as follows [11, 12]:

Let  $\lambda$  = arrival rate,  $\mu$  = service rate,  $s$  = number of server and  $\rho$  = Traffic intensity ( $\lambda/s\mu$ )

The probability that the system is idle ( $P_0$ )

$$p_0 = \left[ a \sum_{s=0}^{s-1} \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s + \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \frac{1}{s!} \left( \frac{s\mu}{s\mu - \lambda} \right) \right]^{-1} \quad (1)$$

Probability of  $n$  passengers in the system ( $P_n$ )

$$P_n = \left( \frac{(\lambda/\mu)^n}{n!} \right) p_0 \text{ for } n \leq s \text{ but for } n > s, P_n = \frac{(\lambda/\mu)^n}{s! s^{n-s}} p_0 \quad (2)$$

The average waiting time for a passenger in the system ( $W$ ) and queue ( $W_q$ )

$$w = \left( \frac{(\lambda/\mu)^n \mu}{(s-1)!(s\mu - \lambda)^2} \right) p_0 + \frac{1}{\mu} \text{ and } w_q = \frac{(k\rho)^s}{s!(1-\rho)^2 s\mu} p_0 \quad (3)$$

The probability that an arriving customer must wait for service ( $P_w$ ).

$$p_w = \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \left( \frac{s\mu}{s\mu - \lambda} \right) p_0 \dots \text{where } \rho = \frac{\lambda}{s\mu} \quad (4)$$

The average number of passengers in the system ( $L$ ) and the queue ( $L_q$ )

$$L = \frac{\rho\mu(s\rho)^s}{s!(1-\rho)^2} p_0 + \frac{\lambda}{\mu} \dots \text{and } L_q = \frac{\rho\mu(s\rho)^s}{s!(1-\rho)^2} p_0 \quad (5)$$

Here, the necessary parameter for determining the performance of the model is the number of servers ( $s$ ), the mean arrival rate ( $\lambda$ ), the mean service rate ( $\mu$ ), and the traffic intensity ( $\rho$ ).

### 2.3. Spatial and Statistical Analysis Methods

The word analysis refers to data querying and data manipulation so spatial analysis refers to statistical analysis based on patterns and underlying processes. It can be done using various techniques by geographical and statistics information systems (GIS) [13]. This method can have divided in the following types Inverse Distance Weighting, Natural Neighbor Inverse Distance Weighted, Spline, Spline interpolation, Kriging etc. [9, 10].

It involves the GIS to build geographical data with be more informative than unorganized collected data. This association by using of map or statistical calculation, it would be the most effective way to realize and visualize direction changes in the characteristics of Earth's surface [14]. In spatial and statistical analysis methods were integrated to analyses the spatial accessibility, user population, and coverage of the Light Rail Transit of Addis Ababa used as the main source for determining the population coverage of the selected station of AALR [15].

### 2.4. Train Timetable of Urban Railways

Train timetable of urban railway is the most mandatory element for the good performance of a railway operation. It would have made faster, frequent, safe to more efficient for the optimal use of the available resources. It uses the headway time, dwell time, trip time, speed, braking distance, synchronization of trains etc. as a design parameter [9, 10].

## 3. Methodology

In the case study of AALR, transit is constructed in two directions called East-West (EW) and North-South (NS) corridors. There is a total of 39 stations which consist of 4 ends on the two routes of NS and EW corridors; 22 stations in the east-west (EW) line; 22 stations in the north-south (NS) line; and 5 stations in common. The platform of the AALR station has a length of 60m. It can be on the ground or cantilever.

The East-West corridor is congested relative to North-South corridors, and from the 39 stations Megenagna, Torhailoch, and Ayat are the most congested out of these

stations [5]. Other stations are grouped as less congested stations with similar operational characteristics according to an interview with the OCC worker of AALR. As a result, Torhailoch station from the most congested group and Lideta station from the less congested group were selected systematically for this case study.

This research design is designed for the case study research method. The data was obtained from the direct collection at the Torhailoch and Lideta ticket windows of AALR station. Recording sheets, stopwatches, and writing materials were the necessary tools used in data collection. It includes all passengers that purchase tickets. The sample for the study is conveniently drawn from all passengers and servers involved in the ticketing operation at the peak hours of the morning and afternoon of Lideta and Torhailoch stations for two months.

It is based on both primary and secondary data. The primary data is the number of passenger's arrivals and the capacity of

the servers that are directly collected from Torhailoch and Lideta stations at peak hours of the morning and afternoon. However, the secondary data was obtained from literature, the AALR file, and the website.

The data collected in the study were used for the computation of performance of the current queuing system (congestion condition), development of a new queue model, and estimation and forecasting of population coverage of selected stations.

Different statistical tools are used in this study to achieve the objective of this study.

M/M/c/∞/FCFS model to study the congestion condition for existing system performance analysis.

Special and statically analysis methods for population coverage analysis.

Arena software – for investigation of the Performance of Train Timetable of AALR.

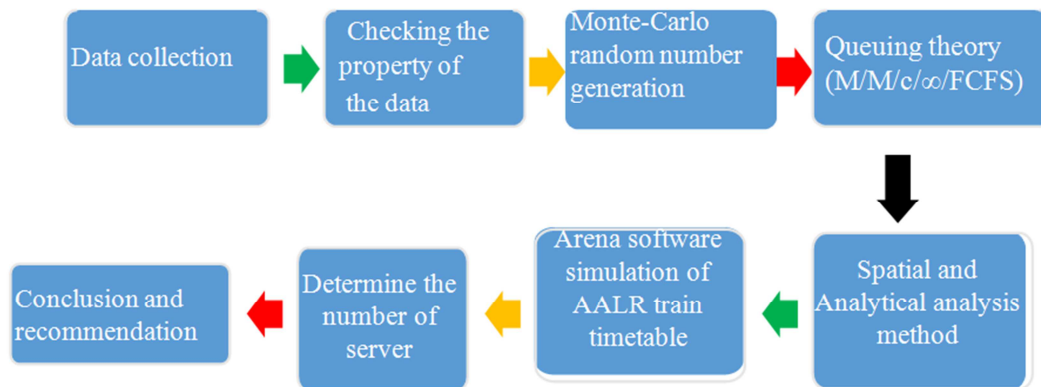


Figure 1. Flow chart of step and method of research.

The above Figure 1 indicates the step and the method of the research by collecting of the passenger arrivals and analysis at peak hours in the morning and afternoon to check the property of the data and 100 random generate using Monte Carlo simulation between the maximum and the minimum. This Monte Carlo random number simulation is used to have the mean arrival and service rate and to calculate the performance of the current queuing system using the M/M/c/∞/FCFS model. By this model, the researcher developed a new queue model for passenger arrival at peak hours and analysis of the performance of the new model to fix the exact queue model.

After a new model was developed, the researchers used special and statically analysis methods to determine the population coverage. It is used to categorize the AALR station as well as to forecast the population coverage and then to have a new model for each design year of the selected station. Finally, the new model would be fixed based on investigation of the performance of Train Timetable of AALR using Arena software.

## 4. Analysis and Discussion

The step-by-step data analysis was undertaken for the evaluation of the existing as well as development and evaluation of a new model. It is achieved by answering all

specific objectives. The probability distribution of all stations was assumed to be Poisson distribution for passenger arrival and exponential for service rate. Using mathematical queue theory, the congestion of each station was checked. After the existing condition was analyzed to know the current performance, a new model would be developed using queue theory. The new model would be used to recommend a better performing server for Torhailoch and Lideta stations, for 39 stations in general.

All stations operate in two shifts (morning and afternoon) for 16 hours. The morning shift starts at 6:00 AM and ends up at 2:00 PM. Similarly, the afternoon shift starts at 2:00 PM and ends up at 10:00 PM. The study indicated that the peak hours are from 7 - 9 AM and 4 - 6 PM [5]. The data were collected for the two-month peak hours from January 14, 2019, up to March 3, 2019. All in all, this research tries to represent the actual condition by dividing the days and consisting the period and peak hours to increase accuracy.

### 4.1. Data Collection and Summary

The two-month data summary of Lideta and Torhailoch stations in the morning and afternoon peak hours is summarized in Table 1. The data was recorded at the morning peak hours from 7 AM up to 3 AM; and the afternoon peak

hours from 4 PM up to 6 PM. The statistical distribution of passengers' arrival for the morning and afternoon peak hours of the two stations are also summarized in Table 1. The data of Lideta was collected after Torhailoch was completed.

#### 4.2. Checking the Property of the Data

The data is checked for the probability distribution using Excel add-in @Risk software and the result of goodness of fitting is uniform, binomial, and negative binomial distribution [3]. It indicates that the result is not solved using

queue theory, so the researcher is randomly generated 100 times. So, this increasing of the data can improve the accuracy of assuming Poisson and exponential probability distribution for arrival and service rate in queuing theory.

#### 4.3. Performance and Optimization of the Existing Ticketing Window Model

The parameter of this research is calculated as follows to optimize by M/M/c/ $\infty$ /FCFS model in Tables 2 and 3 of Torhailoch and Lideta station respectively.

**Table 1.** Summary of passenger arrival at peak hours of Torhailoch and Lideta station.

Statically description	Passengers' arrival Peak hours							
	Torhailoch				Lideta			
	Morning		Afternoon		Morning		Afternoon	
	7 - 8	8 - 9	4 - 5	5 - 6	7 - 8	8 - 9	4 - 5	5 - 6
Minimum	980	842	711	660	83	94	95	101
Maximum	1,348	1,266	1,409	1,284	195	179	217	222
Mean	1164	1054	913.43	912.21	139	133	156	147
Mode	1,123	1,196	711	660	160	140	193	201
Median	1,201	1,148	905	916	160	140	168	171
Std. Deviation	104.87	116.48	152.98	139.2	36.16	25.57	39.52	39.17

Arrival rate ( $\lambda$ ) = 1184.14, 1118.11, 913.43, 912.21 at 7-8 AM, 8-9 AM, 4-5 PM, and 5- 6 PM respectively.

$\mu$ , the service rate of the server = 287.79 in the morning and 228.20 in the afternoon.

The number of servers (s) is 4.

The performance of the existing window is stated in Table 2 and at Torhailoch station the server would not be idle at peak hours; the utilization of the server is between 99-137%. While, at Lideta station, the server became idle from 49- 63% at peak hours. The performance or ticket-selling service of the existing model at Torhailoch is congested, and the traffic intensity is more than 1. This indicates the requirement for a new ticket-selling service model. Similarly, the performance or ticket-selling service at Lideta is underutilized; the traffic intensity is less than 20%. This also requires new ticket-selling service model optimization.

Similarly, the optimization of existing windows was conducted as stated in Table 3. In Torhailoch, with the addition of one clerk to the system, the server would be idle about 1%. The average number of customers in the queue is between 2.09 and 3.69, whereas in the system is between 6.05 and 7.94. Even if there is no time spent in the queue, the system has the waiting time about 0.4 minutes. The traffic intensity is also computed as less than one. Therefore, the service performance is improved due to one clerk increment in the system. However, the current model at Lideta station is underperformed and for the new model, 2 servers would increase the performance of ticketing windows. From this, the probability that the system is empty is between 48% and 53%;

the average number of customers in the queue is 1% whereas in the system there are 2 persons; and there is waiting time spent in the queue reduced to 0.6 minutes. The traffic intensity estimated is less than one. Therefore, the service performance improved due to the reduction of clerks in the system.

#### 4.4. The Server Required for the Whole Design Period

The AALR is designed for 100 years. To determine the future server required, the researcher has taken the platform length of 60m and the tramcar length of 28.7m from the previous study [6]. Through observation at the time of data collection, the current ticketing window accommodates a maximum of 8 ticket sellers, the reliability factor is 98% [16], and the maximum reliable maintenance of the railway is 27 days or 96% which is greater than 91% per 10,000 km per train. The labor productivity of Ethiopia is 94% [17] and the accuracy of population coverage is 97.03% [15].

The design period is from 2015 up to 2115. The population of Addis Ababa was 2,738,248 and the population growth is 3.8% [5]. The population coverage of East-West line was conducted by Andualem and Takele in 2016; and they used Spatial and analytical methods by combining The GIS and Descriptive Analysis method.

**Table 2.** Steady-State Operating Characteristics and performance of the existing system at Torhailoch and Lideta station.

Station		Torhailoch (congested station)				Lideta (underutilized station)			
Shift		Morning		Afternoon		Morning		Afternoon	
Pick hours		7 - 8	8 - 9	4 - 5	5 - 6	7 - 8	8 - 9	4 - 5	5 - 6
Inputs: -	Arrival rate ( $\lambda$ )	1164	1054	913.43	912.21	139	133	156	147
	Service rate per server ( $\mu$ )	287.79	287.79	228.2	228.2	287.79	287.79	228.2	228.2
	Number of servers (s)	4	4	4	4	4	4	4	4
The probability that the system is empty, $p_0$		-0.01	0	-0.06	-0.06	0.59	0.63	0.5	0.49

Station	Torhailoch (congested station)				Lideta (underutilized station)			
Shift	Morning		Afternoon		Morning		Afternoon	
Pick hours	7 – 8	8 – 9	4 – 5	5 – 6	7 – 8	8 – 9	4 – 5	5 – 6
The average number of customers in the waiting line, $L_q$	-19.57	123.45	-6.51	-6.56	0	0	0	0
Average time spent in the queue line, $W_q$ (min)	-0.02	0.11	-0.01	-0.01	0	0	0	0
Average time spent in the system, $W$ (min)	-0.02	0.11	-0.01	-0.01	0	0	0	0
The average number of customers in the system, $L$	-23.82	127.41	-10.64	-10.65	0.53	0.47	0.7	0.71
The probability that the time in the queue is 0, $W_q(0)$	-0.14	0.02	-0.78	-0.75	1	1	0.99	0.99
Utilization (traffic intensity), $\rho$	1.06	0.99	1.37	1.37	0.13	0.12	0.18	0.18

Table 3. Optimization of AALR ticketing service at Torhailoch and Lideta station and its performance.

Station	Torhailoch (congested station)				Lideta (underutilized station)			
Shift	Morning		Afternoon		Morning		Afternoon	
Pick hours	7 – 8	8 – 9	4 – 5	5 – 6	7 – 8	8 – 9	4 – 5	5 – 6
Inputs: -								
Arrival rate ( $\lambda$ )	1164	1054	913.43	912.21	139	133	156	147
Service rate per server ( $\mu$ )	287.79	287.79	228.2	228.2	287.79	287.79	228.2	228.2
Number of servers ( $s$ )	5	5	5	5	1	1	2	2
The probability that the system is empty, $p_0$	0.01	0.01	0.01	0.01	0.48	0.53	0.48	0.48
The average number of customers in the queue line, $L_q$	3.69	2.09	2.84	2.68	0.58	0.41	0.1	0.1
Average time spent in the queue line, $W_q$	0	0	0	0	0	0	0	0
Average time spent in the system, $W$	0.01	0.01	0.01	0.01	0.01	0.01	0	0.01
The average number of customers in the system, $L$	7.94	6.05	6.97	6.77	1.1	0.88	0.8	0.81
The probability that the time in the queue is 0, $W_q(0)$	0.35	0.46	0.4	0.41	0.48	0.53	0.82	0.81
Utilization (traffic intensity), $\rho$	0.85	0.79	0.82	0.82	0.52	0.47	0.35	0.36

This population coverage was developed by using Network Analyst extension tool in ArcMap for Road Network Dataset Building and Service Area Coverage Analysis, Population Accessibility Analysis, Duplicate Service Area Coverage Analysis and Uncovered Area (Gap) Analysis. So, the population covered by Torhailoch (congested) and Lideta (underutilized) station is 79,911 and 65,224, respectively, by using Spatial and analytical method [15].

The number of tramcars is fixed by the length of the platform and the length of the tramcar. A maximum of a double tramcar can be accommodated on the existing platform. The optimal headway time of 7.5 minutes' is obtained for the available trains at a speed of 30 km/h and 5.55 minutes of headway by using Arena software in the

timetable is optimized by AALR [6]. So, the maximum platform capacity is 4576 passengers and it became 4090.21 when it considered the reliability factor of 98%, the labor productivity of Ethiopia at 94%, and the accuracy of population coverage of the East-West line of AALR would be 97.03%.

Then, the population coverage of Torhailoch and Lideta is forecasted to find the average number of clerks for each expected design year. From the research, AALR's platform can serve up to a maximum of 4090.21 passengers. So, the maximum number of clerks required is 15 in 2017 and 2054 at Lideta and Torhailoch station, respectively. The overall server requirement for each year is summarized in Table 4.

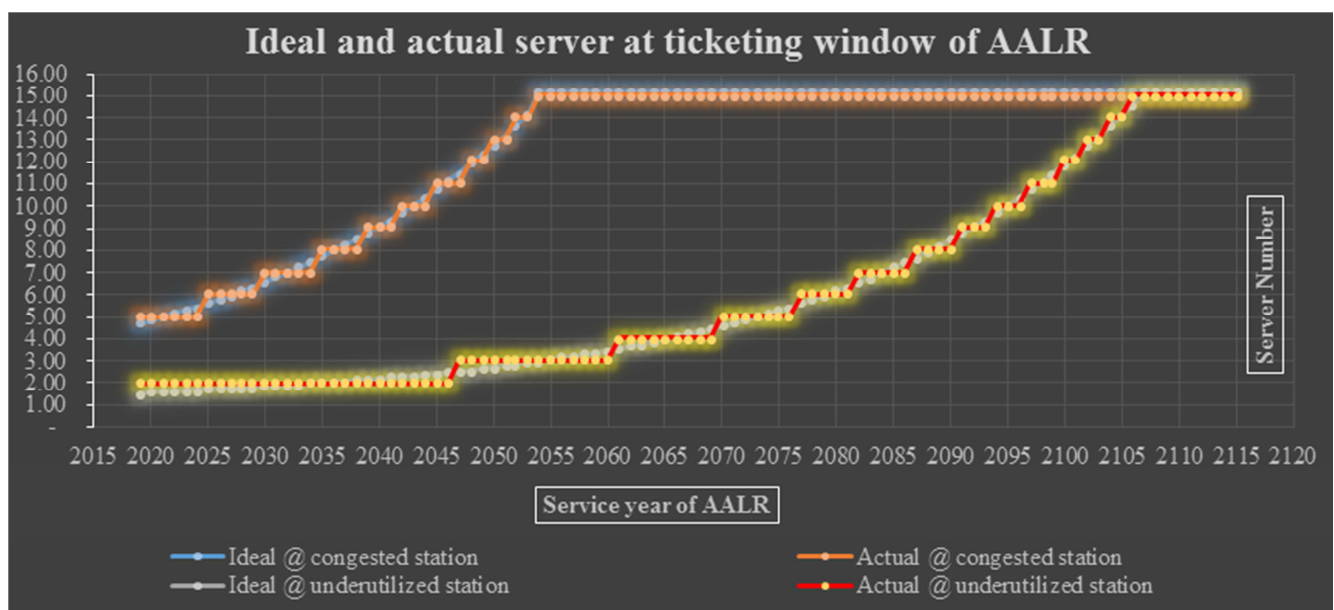


Figure 2. Ideal and actual server determined the ticketing window of AALR.

**Table 4.** The optimum server required at the AALR ticketing window of Torhailoch and Lideta stations.

No of Server	Torhailoch/congested station period/Year	Lideta/underutilized station period/Year
2		2019 - 2047
3		2047 - 2061
4		2062 - 2070
5	2019 – 2025	2071 - 2077
6	2026 – 2031	2078 - 2082
7	2032 – 2035	2083 - 2087
8	2036 – 2039	2087 - 2090
9	2040 – 2042	2091 - 2094
10	2043 – 2045	2095 - 2097
11	2046 – 2048	2097 - 2100
12	2049 – 2050	2101 - 2102
13	2051 – 2053	2103 - 2104
14		2105 - 2106
15	2054 – 2115	2107 - 2115

The researcher can conclude that the Torhailoch station result is used to represent Megenagn, Ayat, stadium, Hayahult 1&2, and Tegbareed, and the rest station is represented by Lideta station ticketing window optimization.

The AALR platform cannot accommodate the additional population from 2054, which restricts the additional tramcars that cannot be provided by the current structure.

An additional 2m by 2m ticketing office shall be added in 2040 and 2091 to the congested and underutilized stations, respectively. The ideal and actual are almost identical, however the variation occurred because of positive integer count for clerks.

Overall, the approach of this research implements by combination of queuing theory, Spatial and analytical method, Monte Carlo simulation with Arena builds on previously described approaches of queuing theory [1-3, 7], Monte Carlo simulation [8], combination of queuing theory with Monte Carlo simulation [5], Arena Simulation [6], combined simulation-optimization model [18], and queuing theory with lest-squares regression [4] and the analyses indicate that it improves on them in terms of model fit to the data of two sampling station from 39 stations.

## 5. Conclusion

AALR station is not properly optimized and it is either congested or underutilized. Based on the new optimization model developed, the optimum clerk would be 5 (Five) at congested stations and 2 (Two) at underutilized stations in 2019. This model would improve the probability that the system is empty, the average number of customers in the waiting line and the system, and the average time spent in the queue line as well as in the system by either increasing or decreasing the congested and underutilized stations respectively.

All in all, the AALR transit station ticketing window is congested at Torhailoch, Megenagn, Ayat, stadium, Hayahult 1&2, and Tegbareed but the rest station is underutilized and a 2m by 2m office station shall be added. The modification increases the satisfaction of customers and the profit of the

company.

## Data Availability

The data used to support the findings of this study can be obtained from the author upon request.

## Conflict of Interest

On behalf of all authors, the author states that there is no conflict of interest.

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