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Anisotropic Stars with a Prescribed Form of Metric Potential Z

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Abstract: Assuming a linear equation of state and charged anisotropic matter, in this paper we obtain two new classes of exact solutions of the Einstein-Maxwell system with a particular form of the metric potential Z deduced for Malaver (2016). A physical analysis of electromagnetic field indicates that is regular in the origin and well behaved. The obtained models not admit singularities in the charge density and the matter at the centre.

Keywords: Linear Equation of State, Exact Solution, Charged Anisotropic Matter, Metric Potential, Charge Density

1. Introduction

From the development of Einstein´s theory of general relativity, the modeling of superdense matter configurations is an interesting research area [1, 2]. Some solutions found fundamental applications in astrophysics, cosmology and more recently in the developments inspired by string theory [2]. Different mathematical formulations that allow to solve Einstein´s field equations have been used to describe the behavior of objects submitted to strong gravitational fields known as neutron stars, quasars and white dwarfs [3, 4, 5].

In the construction of the first theoretical models of relativistic stars are important the works of Schwarzschild [6], Tolman [7], Oppenheimer and Volkoff [8]. Schwarzschild [6] found analytical solutions that allowed describing a star with uniform density, Tolman [7] developed a method to find solutions of static spheres of fluid and Oppenheimer and Volkoff [8] used Tolman's solutions to study the gravitational balance of neutron stars. It is important to mention Chandrasekhar's contributions [9] in the model production of white dwarfs in presence of relativistic effects and the works of Baade and Zwicky [10] who propose the concept of neutron stars and identify a astronomic dense objects known as supernovas.

The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT bag model [11]. In this model, the strange matter equation of state has a simple linear form given by $p=\frac{1}{3}(\rho-4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant.

In theoretical works of realistic stellar models, is important include the pressure anisotropy [12-14]. Bowers and Liang [12] extensively discuss the effect of pressure anisotropy in general relativity. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [15] or for the presence of an electrical field [16]. The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT-Bag model [11]. In this model, the strange matter equation of state has a simple linear form given by $p=\frac{1}{3}(\rho-4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model: Komathiraj and Maharaj [11] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure. Malaver [17, 18] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter. Thirukkanesh and Maharaj [19] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [20] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [21] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [22].

With the use of Einstein´ field equations, important progress have been made to model the interior of a star. Feroze and Siddiqui [23, 24] and Malaver [25, 26] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [27] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [28] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. More recently, Malaver [29, 30] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent. Raghoonundun and Hobill [31] found new analytical models for compact stars with the use of Tolman VII solution.

The main objective in this paper is to generate a new class for charged anisotropic matter with the barotropic equation of state that presents a linear relation between the energy density and the radial pressure in static spherically symmetric spacetime using a particular form for the metric potential $Z(x)$ deduced for Malaver [32] and two specific forms for the electrical field intensity. In this work has been obtained new classes of static spherically symmetrical models of charged matter without singularities in the charge distribution and the matter at the centre of the star. This article is organized as follows: Section 2 presents Einstein´s field equations. In Section 3, a gravitational potential $Z(x)$ was chosen in order to solve the field equations and obtain new models for charged anisotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

2. Einstein Field Equations of Anisotropic Fluid

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (1)$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by

$$\frac{1}{r^2}\left(1-e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho \quad (2)$$

$$-\frac{1}{r^2}\left(1-e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r \quad (3)$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p_t \quad (4)$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}(r^2E)' \quad (5)$$

where ρ is the energy density, p_r is the radial pressure, E is electric field intensity,

p_t is the tangential pressureand primes denote differentiations with respect to r. Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0, suggested by Durgapal and Bannerji [33], the metric (1) take the form

$$ds^2 = -A^2y^2(x)dt^2 + \frac{1}{4cxz}dx^2 + \frac{x}{c}(d\theta^2 + \sin^2\theta d\varphi^2) \quad (6)$$

and the Einstein field equations can be written as

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \quad (7)$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \quad (8)$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \quad (9)$$

$$p_t = p_r + \Delta \quad (10)$$

$$\frac{\Delta}{c} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x} - \frac{E^2}{c} \quad (11)$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \quad (12)$$

σ is the charge density, $\Delta = p_t - p_r$ is the anisotropic factor and dots denote differentiation with respect to x. With the transformations of [33], the mass within a radius r of the sphere take the form

$$M(x) = \frac{1}{4c^{3/2}}\int_0^x \sqrt{x}\rho(x)dx \quad (13)$$

In this paper, it has been assumed the following lineal equation of state within the framework of MIT-Bag model

$$p_r = \frac{1}{3}\rho \quad (14)$$

3. The New Physical Models

Following Malaver [32], we take the form of the gravitational potential $Z(x)$ given by

$$Z(x) = \frac{(1-ax)^6}{(1+ax)^6} \tag{15}$$

where a is a real constant. This potential is regular at the origin and well behaved in the interior of the sphere. Two particular forms for the electrical field have been considered.

3.1. Model I

In this model the form of the electrical field is proposed for Bibi et al. [34]

$$E^2 = \frac{Kax(5+ax)}{(1+ax)^3} \tag{16}$$

Using $Z(x)$ and eq. (16) in eq. (7) it is obtained

$$\rho = \frac{\left[-Ka^6x^6 - \left(24a^6c + 9Ka^5\right)x^5 + \left(264a^5c - 26Ka^4\right)x^4 - \left(400a^4c + 34Ka^3\right)x^3 + \left(560a^3c - 21Ka^2\right)x^2 + \left(216a^2c + 5Ka\right)x + 72ac\right]}{2(1+ax)^7} \tag{17}$$

Substituting (17) in eq. (14), the radial pressure can be written in the form

$$P_r = \frac{\left[-Ka^6x^6 - \left(24a^6c + 9Ka^5\right)x^5 + \left(264a^5c - 26Ka^4\right)x^4 - \left(400a^4c + 34Ka^3\right)x^3 + \left(560a^3c - 21Ka^2\right)x^2 + \left(216a^2c + 5Ka\right)x + 72ac\right]}{6(1+ax)^7} \tag{18}$$

Using (17) in (13), the expression of the mass function is

$$M(x) = \frac{x^{1/2}}{c^{1/2}}\left[-\frac{K}{4ac} - \frac{32}{(1+ax)^6} + \frac{96}{(1+ax)^5} - \frac{120}{(1+ax)^4} + \frac{80}{(1+ax)^3} - \frac{30}{(1+ax)^2} + \frac{6}{(1+ax)} - \frac{K}{4ac(1+ax)^2} + \frac{K}{2ac(1+ax)}\right] \tag{19}$$

With (16) and $Z(x)$ in (12), the charge density is

$$\sigma^2 = \frac{Kca(1-ax)^6\left(a^2x^2 + 4ax + 15\right)^2}{(1+ax)^{11}(5+ax)} \tag{20}$$

Substituting (16) and (18) in (8)

$$\frac{\dot{y}}{y} = \frac{\left[-Ka^6x^6 - \left(24a^6c + 9Ka^5\right)x^5 + \left(264a^5c - 26Ka^4\right)x^4 - \left(400a^4c + 34Ka^3\right)x^3 + \left(560a^3c - 21Ka^2\right)x^2 + \left(216a^2c + 5Ka\right)x + 72ac\right]}{2(1+ax)^7} + \frac{12a + 40a^3x^2 + 12a^5x^4}{4(1-ax)^6} - \frac{Kax(5+ax)(1+ax)^3}{8c(1+ax)^6} \tag{21}$$

Integrating (21), it is obtained for $y(x)$

$$y(x) = c_1(1+ax)(1-ax)^A \exp[B(x)] \tag{22}$$

where

$A = -\dfrac{K + 6ac}{6ac}$ and

$$B(x) = -\frac{\left[\left(360a^5c - 195Ka^4\right)x^4 + \left(330Ka^3 - 720a^4c\right)x^3 + \left(1120a^3c - 460Ka^2\right)x^2 + \left(230Ka - 560a^2c\right)x + 184ac - 49K\right]}{90ac(1-ax)^5} \tag{23}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda} = \frac{(1+ax)^6}{(1-ax)^6} \tag{24}$$

$$e^{2\nu} = A^2 c_1{}^2 (1+ax)^2 (1-ax)^{2A} \exp[2B(x)] \tag{25}$$

With $Z(x)$, the tangential pressure is given by for

$$P_t = 4x\frac{(1-ax)^6}{(1+ax)^6}\frac{\ddot{y}}{y} + \frac{4(1-ax)^5(1-a^2x^2-6ax)}{(1+ax)^7}\frac{\dot{y}}{y} - \frac{12a(1-ax)^5}{(1+ax)^7} - \frac{Kax(5+ax)}{2c(1+ax)^3} \tag{26}$$

The metric for this model is

$$ds^2 = -A^2c_1^2(1+ax)^2(1-ax)^{2A}\exp[2B(x)]dt^2 + \frac{(1+ax)^6}{4cx(1-ax)^6}dx^2 + \frac{x}{c}(d\theta^2 + \sin^2\theta d\varphi^2) \tag{27}$$

3.2. Model II

Following Mafa Takisa and Maharaj [27], we choose the form of electrical field given by

$$E^2 = \frac{Kx}{(1+ax)^2} \tag{28}$$

Substituting $Z(x)$ and E^2 in eq. (7)

$$\rho = \frac{\left[-Ka^5x^6 - (24a^6c + 5Ka^4)x^5 + (264a^5c - 10Ka^3)x^4 - (400a^4c + 10Ka^2)x^3 + (560a^3c - 5Ka)x^2 - (216a^2c + K)x + 72ac\right]}{2(1+ax)^7} \tag{29}$$

With (29) in eq. (14)

$$P_r = \frac{\left[-Ka^5x^6 - (24a^6c + 5Ka^4)x^5 + (264a^5c - 10Ka^3)x^4 - (400a^4c + 10Ka^2)x^3 + (560a^3c - 5Ka)x^2 - (216a^2c + K)x + 72ac\right]}{6(1+ax)^7} \tag{30}$$

Using (29) in (13), the expression of the mass function is

$$\begin{aligned} M(x) = &-\frac{K\sqrt{x}}{4a^2c\sqrt{c}} - \frac{K\sqrt{x}}{8a^2c\sqrt{c}(1+ax)} + \frac{3K\operatorname{arctag}(\sqrt{ax})}{8a^2c\sqrt{ac}} - \frac{32\sqrt{x}}{\sqrt{c}(1+ax)^6} + \frac{96\sqrt{x}}{\sqrt{c}(1+ax)^5} \\ &- \frac{120\sqrt{x}}{\sqrt{c}(1+ax)^4} + \frac{80\sqrt{x}}{\sqrt{c}(1+ax)^3} - \frac{30\sqrt{x}}{\sqrt{c}(1+ax)^2} + \frac{6\sqrt{x}}{\sqrt{c}(1+ax)} \end{aligned} \tag{31}$$

and for charge density

$$\sigma^2 = \frac{Kc(1-ax)^6(a^2x^2 + 6ax + 9)^2}{(1+ax)^{10}} \tag{32}$$

Replacing (30), (28) and (15) in (8) it is obtained

$$\frac{\dot{y}}{y}=\frac{\left[-Ka^5x^6-\left(24a^6c+5Ka^4\right)x^5+\left(264a^5c-10Ka^3\right)x^4-\left(400a^4c+10Ka^2\right)x^3+\left(560a^3c-5Ka\right)x^2-\left(216a^2c+K\right)+72ac\right]}{24c(1+ax)(1-ax)^6}$$
$$+\frac{12a+40a^3x^2+12a^5x^4}{4(1-ax)^6}-\frac{Kx(1+ax)^4}{8c(1+ax)^6} \tag{33}$$

Integrating (33)

$$y(x)=c_2(1+ax)(1-ax)^C\exp[D(x)] \tag{34}$$

where

$C=-\dfrac{K+6a^2c}{6a^2c}$ and

$$D(x)=-\frac{\left[\left(360a^6c-135Ka^4\right)x^4+\left(330Ka^3-720a^5c\right)x^3+\left(1120a^4c-370Ka^2\right)x^2+\left(200Ka-560a^3c\right)x+184a^2c-43K\right]}{90a^2c(1-ax)^5} \tag{35}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda}=\frac{(1+ax)^6}{(1-ax)^6} \tag{36}$$

$$e^{2\nu}=A^2c_2^2(1+ax)^2(1-ax)^{2C}\exp[2D(x)] \tag{37}$$

the tangential pressure is given by for

$$P_t=4x\frac{(1-ax)^6}{(1+ax)^6}\frac{\ddot{y}}{y}+\frac{4(1-ax)^5\left(1-a^2x^2-6ax\right)}{(1+ax)^7}\frac{\dot{y}}{y}-\frac{12a(1-ax)^5}{(1+ax)^7}-\frac{Kx}{2c(1+ax)^2} \tag{38}$$

and the metric for this model is

$$ds^2=-A^2c_1^2(1+ax)^2(1-ax)^{2C}\exp[2D(x)]dt^2+\frac{(1+ax)^6}{4cx(1-ax)^6}dx^2+\frac{x}{c}(d\theta^2+\sin^2\theta d\varphi^2) \tag{39}$$

4. Physical Characteristics of the New Models

The new generate models allow solve to the Einstein-Maxwell system (7) - (12) and constitute another new family of solutions for a charged matter with anisotropy. These models must satisfy the following physical properties [28, 34]:

(i) The energy density is positive and a decreasing function of the radial coordinate;
(ii) The radial pressure should be positive, finite and a decreasing function of the radial coordinate;
(iii) Regularity of the gravitational potentials in the origin;
(iv) Radial pressure must be finite at the centre;
(v) Electric field intensity E must be regular and well defined inside the solution.

In the new obtained solutions, the metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written in terms of elementary functions and the variables energy density, radial pressure, charge density and tangential pressure also are represented analytical. In the model I, $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_1^2\exp\left[\dfrac{49K-184ac}{45ac}\right]$ in the origin and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$ at the centrer=0. This analysis demonstrates that the gravitational potentials are regular in the origin. The energy density is positive throughout the interior of the star, regular at the centre with value $\rho=36ac$. The radial pressure p_r is regular at the centre with value $p_r=12ac$. In the model II, $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_2^2\exp\left[\dfrac{43K-184a^2c}{45a^2c}\right]$ at the centre and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$. Again, as in the model I, the gravitational potentials are regular at the centre. The energy density andradial pressure take the values of $\rho = 36ac$ and $p_r = 12ac$ in r=0, respectively. In both classes of models, the mass function is continuous and behaves well inside the star and the charge density σ not present singularity at the centre.

5. Conclusions

In this paper, we obtain two new solutions of the Einstein-Maxwell field equations for charged anisotropic matter with a barotropic equation of state and a particular form of the metric potential $Z(x)$. The new class of obtained solutions is physically acceptable and may be used to model relativistic stars in different astrophysical scenes. In the new obtained models the gravitational potentials are regular at the origin r=0 and well behaved. The radial pressure and energy density are regular and positive throughout the stellar interior. The charge distribution not admits singularities at the centre and the mass function is an increasing function, continuous and finite. The new obtained solutions show the usefulness of the Einstein-Maxwell system of equations in many astrophysical applications.

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